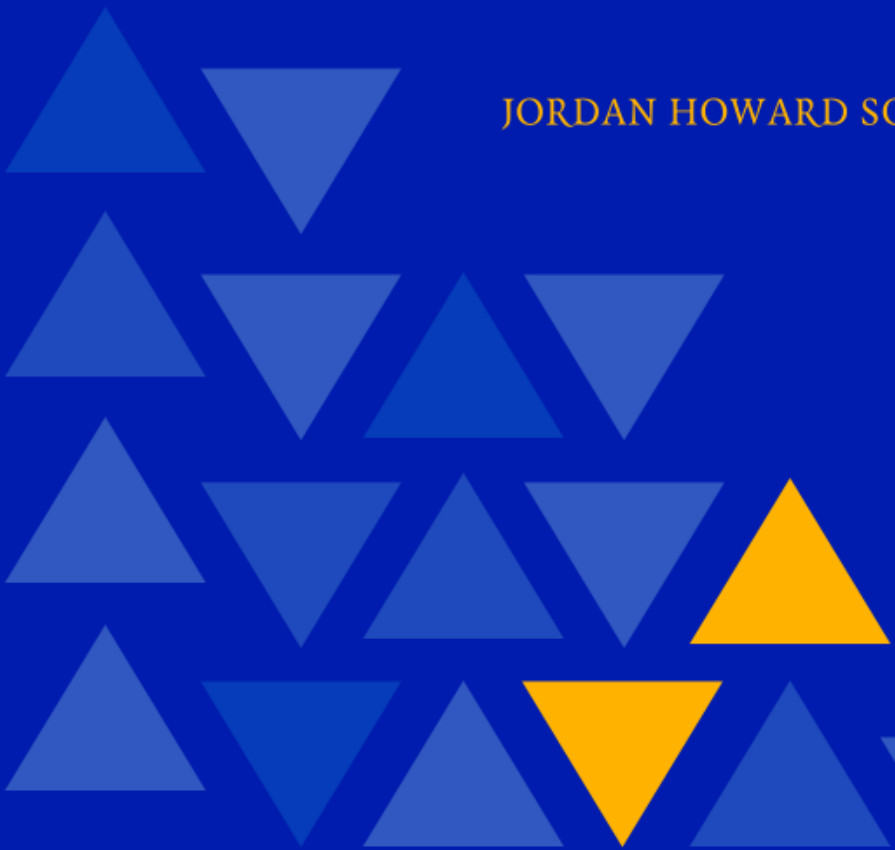
Logic and Theism

Arguments For and Against Beliefs in God

JORDAN HOWARD SOBEL



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This is a wide-ranging book about arguments for and against beliefs in God. Arguments for the existence of God analyzed in the first six chapters include ontological arguments from Anselm through Gödel, the cosmological arguments of Aquinas and Leibniz, and arguments from evidence for design and miracles. Following these chapters are two chapters considering arguments against that existence. The last chapter examines Pascalian arguments for and against belief regardless of existence. There are discussions of Cantorian problems for omniscience, of challenges to divine omnipotence, and of the compatibility of everlasting complete knowledge of the world with free will. For readers with a technical background in logic there are appendices that present formal proofs in a system for quantified modal logic, a theory of possible worlds, notes on Cantorian set theory, and remarks concerning nonstandard hyperreal numbers.

This book will be a valuable resource for philosophers of religion and theologians and will interest logicians and mathematicians as well.

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University of Toronto

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There is No God

‘There is no God,’ the wicked saith,

‘And truly it’s a blessing,
For what he might have done with us
It’s better only guessing.’

‘There is no God,’a youngster thinks,
‘Or really, if there may be,
He surely didn’t mean a man
Always to be a baby.’

‘There is no God, or if there is,’
The tradesman thinks, ‘twere funny
If he should take it ill in me
To make a little money.’

‘Whether there be,’ the rich man says,
‘It matters very little,
For I and mine, thank somebody,
Are not in want of victual.’

Some others, also to themselves,
Who scarce so much as doubt it,
Think there is none, when they are well,
And do not think about it.

But country folks who live beneath
The shadow of the steeple;
The parson and the parson’s wife,
And mostly married people;

Youths green and happy in first love,
So thankful for illusion;
And men caught out in what the world
Calls guilt, in first confusion;

And almost everyone when age
Disease, or sorrows strike him,
Inclines to think there is a God,
Or something very like him.

Anthur Hugh Clough

Dover Beach

The sea is calm to-night
The tide is full, the moon lies fair
Upon the straits; – on the French coast the light
Gleams and is gone; the cliffs of England stand,
Glimmering and vast, out in the tranquil bay.
Come to the window, sweet is the night-air!
Only, from the long line of spray
Where the sea meets the moon-blanched land,
Listen! you hear the grating roar
Of pebbles which the waves draw back, and fling,
At their return, up the high strand,
Begin, and cease, and then again begin
With tremulous cadence slow, and bring
The eternal note of sadness in.

Sophocles long ago
Heard it on the Aegean, and it brought
Into his mind the turbid ebb and flow
Of human misery; we
Find also in the sound a thought,
Hearing it by this distant northern sea.

The Sea of Faith
Was once, too, at the full, and round earth's shore
Lay like the folds of a bright girdle furled,
But now I only hear
Its melancholy, long, withdrawing roar,
Retreating, to the breath
Of the night-wind, down the vast edges drear
And naked shingles of the world.

Ah, love, let us be true
To one another! for the world, which seems
To lie before us like a land of dreams,
So various, so beautiful, so new,
Hath really neither joy, nor love, nor light,
Nor certitude, nor peace, nor help for pain;
And we are here as on a darkling plain
Swept with confused alarms of struggle and flight,
Where ignorant armies clash by night.

Matthew Arnold

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Preface

This book is about beliefs in the existence of God in two senses. It is about beliefs in God in the sense of their objects, the propositions believed. And it is about beliefs in God in the sense of states of minds. Classical arguments and evidence for and against propositions affirming God's existence are studied, as well as Pascalian practical arguments for and against cultivating states of belief in God.

Questions of truth, and of belief, concerning God's existence come after questions regarding what would be God's nature. Discussions of arguments in this book are predicated on several conceptions often combined, and sometimes moderated, of what God would be like. Chapter I, " 'GOD' and 'god', and God," goes into these conceptions. Its business is to establish the broad perspective from which issues concerning God's existence, and what would be This One's nature, are taken.

Then come chapters about theoretical arguments for beliefs in God. Chapters II through IV are about demonstrative arguments that would establish God's existence without the aid of contingent assumptions or premises – the classical ontological arguments of René Descartes, St. Anselm, and Baruch Spinoza (Chapter II); the modal ontological argument of Charles Hartshorne and Alvin Plantinga (Chapter III); and Kurt Gödel's ontological proof (Chapter IV). Chapters V and VI examine connected ordinary deductive arguments that make use of contingent premises: Thomas Aquinas' argument for a first cause, an argument of David Hume's character Demea, Samuel Clarke, and especially Gottfried Leibniz for ultimate reasons. Chapter VII reviews and extends David Hume's masterful study of arguments from design, which arguments purport not to deduce God's existence, but only to make a case for it, that is, to marshal evidence that, while not strictly entailing the hypothesis of a designing God's existence, make it probable. Chapter VIII interprets and discusses Hume's critique of testimonial evidence for miracles, and through them for particular gods.

Next come chapters on what would be salient parts of God's nature. Chapter IX attends to challenges to omnipotence considered alone and in combination with other possible divine attributes. Chapter X takes up challenges to omniscience considered alone. Going with these chapters is a substantial appendix to Chapter XII that studies the issue of divine omniscience and freedom of choice.

There are then chapters on theoretical arguments against the existence of God. Chapter XI attends briefly to demonstrative arguments that would, if successful, establish that neither God nor 'anything very like him' *could* exist. It attends at greater length to 'the empirical problem of evil' and to arguments that would establish that 'on the evidence of evil' there is probably no god. Chapter XII is about the problem of evil in its classical form: Examined are deductive arguments in a series that starts with 'Epicurus' old problem,' arguments that would if successful establish that the existence of a perfect god is logically incompatible with the existence of evil, or, if not with that, then with this world's not being a best possible world, or, if not with that, then with this world's not being a best possible world that a perfect god could have actualized for sure, or, if not that . . .

Chapter XIII is about beliefs in God as states of mind. It studies practical arguments for cultivating beliefs in God, arguments that work with values for and probabilities of possible consequences of theistic beliefs, and of steps taken to acquire them. (Not conjured for comment are 'anti-Pascalian' practical arguments for avoiding theistic beliefs and eliminating them, because of consequences of having, and of acquiring, them.)

The logic and mathematics used are explained as required. Notations of elementary predicate logic come into discussions of René Descartes's ontological argument, as well as of St. Anselm's, and of Thomas Aquinas's first cause argument (Chapters II and V). It helps to bring out certain amphibolies or structural ambiguities of English sentences that feature words of quantity such as 'a' and 'something.' Russell's theory of descriptions is used to sort out more elusive amphibolies of relevance to Spinoza's ontological argument (Chapter II). Modal logic figures in discussions of Hartshorne's and Plantinga's and Gödel's ontological arguments (Chapters III and IV), of cosmological arguments (Chapter VI), of omnipotence (Chapter IX), of arguments for evil and objections to them (Chapter XII), and of omniscience and freedom (Chapter XII, Appendix). A natural deduction system for first-order quantified modal logic is explained in an appendix for Chapter III and used to confirm several informal arguments of that chapter. That system is extended in an appendix for Chapter IV to accommodate articulations of Gödel's reasoning in his ontological proof, and to confirm other informal arguments of the chapter. Bayesian confirmation theory is explained for discussions of the evidence of design for a designer, of the evidence for miracles of testimony to miracles, and of the evidence of evil against a perfect god (Chapters VII, VIII, and XI). Chapter XIII includes a series of exercises in Bayesian rational choice

theory. Particular attention is paid to the play in these Pascalian arguments of infinities, both standard Cantorian cardinal infinities and nonstandard Robinsonian hyperreal 'infinimals.' Hyperreal infinitesimals enter explications floated of Hume's ideas of 'degrees of proof' and of 'highest certainties' (Chapter VIII).

With four exceptions chapters of this book do not include previously published material. The exceptions are "Mr. Spinoza, Meet Mr. Russell" which is Part Three of "Classical Ontological Arguments" (Chapter II), Gödel's *Ontologischer Beweis*" (Chapter IV), "Clouds of Witnesses" (Chapter VIII) and "Pascalian Wagers" (Chapter XIII). The first of these revises and substantially expands a paper that appeared in *Logica* (1999). The second substantially revises a contribution to *On Being and Saying: Essays for Richard Cartwright* (1987). The third merges and substantially expands revisions of two papers that appeared in *The Philosophical Quarterly* (1987, 1991). The fourth revises and expands somewhat a paper that appeared in *Synthesé* (1996).

Most chapters began either as material for the philosophy of religion section of introductions to philosophy taught at the University of California at Los Angeles in the 1960s and subsequently here at the University of Toronto, or as material for advanced undergraduate courses and graduate seminars conducted there and at Uppsala University. Exceptions are "Gödel's Ontological Proof" and the chapter, "Clouds of Witnesses" which began in comments I made on a paper by David Owen presented at the meeting of the Canadian Philosophical Association in 1984. Chapters V and VI were re-written for Burman Lectures delivered at the University of Umeå in May of 1998. That work took place during my tenure as Research Fellow at the Swedish Collegium for Advanced Study in the Social Sciences in Uppsala, for the supportive and stimulating conditions of which I am very grateful.

Debts are noted in these chapters to several colleagues, students, readers, and auditors for valuable criticisms and suggestions. I owe most to Willa Fowler Freeman Sobel, for her help through the years with this work and for much else.

REGARDING TECHNICAL MATERIAL

All formal derivations and models are relegated to appendices. Some technical material within chapters can be skipped over without loss of continuity.

DIVINITY

I

‘God’, ‘god’, and God

> If we have no idea *what* God [would be], then what sense is there in asking whether God *exists* or not?
>
> (Peterson et al. 1991, p. 49)

1. EXISTENCE AND ESSENCE QUESTIONS

This book would be about God, that is, it is about God, supposing This One exists. But since this book would be about God, it is appropriate to begin by saying something about what this book would be about. For, to say it would be about God is not so much to say as to advert or point. ‘God’, after all, is a proper name. And it is for proper names to name, not to describe. Generally we have descriptions in mind when we use names, but people can, when they converse, have in mind different descriptions. And although this need not make trouble for the identifying functions of names, since different descriptions can pick out the same things, it does mean that communication is uncertain when the existence of things that people would speak of with a name is at issue.

People, when they use names, generally intend to name things, but they do not always succeed, and when it is a question whether a name does name, it becomes important to discussion that there be a shared understanding of the character of what it would name. That I know Anne Marie personally, or not knowing her personally, know her only as the librarian while you, not knowing her personally, know her only as the short-stop, makes no difference when we are talking about where she was born and whether she wears glasses. But when I say that this book would be about God, mainly whether God exists, and whether in any case we should believe that God exists – you can wonder exactly how I would identify This One, and what sort of being I have in mind. It would be much worse if I said that this book would be about the existence of Medlo, for then you would have no idea of the sort of entity I had in mind, not even whether it would be a ‘being’ (for example, a tenth-century

English magician) or a thing but not a being (for example, a park in eastern Connecticut). When there is a question of the existence of a name's referent, one should be as explicit as possible concerning what would be its referent's character.

2. NAMES IN QUESTIONS OF EXISTENCE AND BELIEF

2.1. Suppose the question, Did Atlantis really exist? One would want to know what Atlantis is supposed to have been. Learning that it would have been a city located west of Gibraltar that sank into the ocean, one could begin to look. One would know what one was looking for – a city – and where to look – west of Gibraltar, under the sea. Learning, instead, that Atlantis is the name of "a **mythical** island in the Atlantic Ocean, first mentioned by Plato, **supposedly** west of Gibraltar, **said** to have finally sunk into the sea" (*American College Dictionary*, New York: Random House, 1953; bold emphasis added), one would know not to look. *This* answer to the description question would entail without further investigation a negative answer to the existence-question. The question, Did Atlantis exist?, presupposes some answer to the question what sort of thing would Atlantis have been. Similarly for the questions, Does Santa Claus exist? and Are there unicorns? These presuppose answers to the questions, What sort of thing would Santa Claus be, that is, would 'Santa Claus' name?, and, What sort of thing would a unicorn be, that is, to what sort of thing would the common name 'unicorn' apply?

2.2. Similarly for, Does God exist? and, Are there good reasons for believing that God exists? These questions presuppose at least a sketchy answer to the nature question, What sort of thing would God be if This One did exist? I do not say that our main questions about God take us back to the question, What does 'God' mean? For 'God' is a 'proper' name, and these, in contrast with 'common' names, generally do not have meanings. Perhaps, however, 'God' is, near enough, an exception to this rule. For even if best semantical theories say that, since it is a name, it does not have a meaning; it does plainly have in use an 'expressive function' or 'expressive force' fixed by convention in the way in which meanings are fixed. **'God' (uppercase)** does, by a natural and compelling convention of language – explicable in terms of its etymology – purport to name what would be ***the one and only true god (lowercase)***. This is its intended referent when used by believers, and the common name 'god' has a meaning. Saying what that meaning is all but settles what 'the one and only true god' means. In other times, it may have been fixed by convention for those who spoke of God, that He would be not the god, but only the main god, 'the god of gods' (*Deuteronomy* 10:17). To explain the main thing at those times was still to say what the common name 'god', or whatever name stood in its place in the language and culture, meant.

2.3. Many names come from words that have meanings. Avrum Stroll informs that "dictionaries tell us that 'Sarah' in Hebrew means 'princess'" (Stroll 1998, p. 528). Of course 'Sarah' does not mean, or nowadays 'connote,' that royal position, or even the condition of being metaphorically 'a princess' (being special and precious, young, and unmarried). It has no such suggestions in English or in Hebrew. The truth Stroll finds in dictionaries is that 'Sarah' comes from 'sarah,' which in Hebrew meant, and means, what 'princess' means in English: 'Sarah' only 'Pickwickian' means princess in Hebrew. Perhaps most names are without indefeasible connotations or suggestions, even of sex. 'Mary' was, I understand, the private name of J. Edgar Hoover.

'God' is not like most names, if, as I think, that it would name in common use One who would be the true god is settled by linguistic convention and is not defeasible. Suppose believers became convinced that what they had been meaning to refer to with the name 'God' – the god of Abraham, Isaac, and Jacob – was in fact no god at all, but an ancient all-too-human practical joker named David who burnt bushes and the rest. These believers would not say, "That joker David, what a come-down for God." They would say that they had been mistaken in thinking that they, and those before them, Abraham, Isaac, and Jacob, had been referring to *God* (cf., Gellman 1995, p. 541), for David was no god, and That One would need to have been one.

2.4. Notes on another divine name. Hasker says that, "'Yahweh', which was used by the ancient Hebrews to refer to their God," was used by them "simply as a non-connotative proper name referring to that individual who in fact was, and is, the God of Abraham, Isaac, and Jacob" (Hasker 1989, pp. 170–1). 'Yahweh', Hasker reports, did not express or suggest any concept. It was, in this, like the general run of proper names, and different from the English name 'God'. I wonder. 'George' is a name that can without linguistic impropriety be given to, or taken by, female as well as male persons, weak as well as strong persons, and so on. Was 'Yahweh' similarly unconstrained linguistically? Did 'Yahweh' come from a common name or description with which it was still semantically connected? Did it come from a common name or description with which it was no longer semantically connected (as 'George', though it comes from the Greek for worker of the soil, no longer has that connotation). Was it made up out of whole cloth? According to *The Columbia Encyclopaedia*, (i) the "tretragrammaton . . . is of unknown origin," and "the reconstruction . . . *Jahweh* is not now regarded as reliable." Perhaps, however, as some have speculated, it, the tretragrammaton, came from the Hebrew verb *hawah'* (which means to become), whose consonants are those of the tretragrammaton minus the first one (*yodh*). If so, it is plausible that, for the ancient Hebrews, the tetragrammaton, however they pronounced it, was in the beginning and perhaps always was a 'connotative proper name' that expressed a status somehow related to becoming and/or being. Might 'Jahweh' have recommended itself as the name of one who *is*, from whom everything else *comes*?[1]

2.5. St. Thomas Aquinas held that 'God', since it is an 'appellative name' that signifies 'the divine nature' in the thing that possesses it, is not a proper name. (*Summa Theologica* I,q13,a9). The opposition is, I think, unnecessary. A name can be a proper name, and signifying or expressive. 'God' is such a name. Hasker agrees: "'God'," he writes, while a proper name, "expresses our concept of God" (Hasker 1989, p. 170). I come back, in Section 6.2.3, to differences between Hasker and me concerning the character of the concept expressed.

3. ETYMOLOGY AND SEMANTICS

3.1. To speculate amateurishly regarding ancient counterparts of 'God,' first there were gods, then 'our' god, and finally 'the' one god, the one true god of all:[2] First came the common name 'god', or, more accurately, first came counterparts of this common name in ancient languages including Hebrew; then came to the Hebrews the idea that their descriptive word applied to a being to whom they stood in a special relation; close on this conceit came the name 'God' (or, more accurately again, its counterpart in ancient Hebrew) for that being for whom they had other names and titles; finally came the idea that the descriptive word 'god' applied properly to only one being to whom no one is special and for whom the already in play name 'God' was naturally adopted. Relevant to the second stage of this story are these lines from *Exodus* 19:25–20:6, King James Version unrevised:

> So Moses went down unto the
> people, and spake unto them.
> And God, spake all these words, saying.
> I am the Lord thy God, which
> have brought thee out of the land of
> Egypt, out of the house of bondage.
> Thou shalt have no other gods
> before me.
> Thou shalt not make unto thee any
> graven image, or any likeness of any
> thing that is in heaven above, or that
> is in the earth beneath, or that is in
> the water under the earth:
> Thou shalt not bow down thyself
> to them: for I the Lord thy God am
> a jealous God. . . .

Moses used a proper name for God ('*Elohim*') to tell Hebrews assembled that God had identified Himself in the phrase 'the Lord thy God'. The phrase 'the Lord thy God' in Hebrew consists of the personal name of God ('Yahweh') and a title-term, '*eloheyka*', a cognate of '*Elohim*', meaning god-of-you, standing,

I assume, in apposition. A better translation would be 'the Lord, thy god' or, better still, 'Yahweh, thy god'.[3]

Moses did not report that God identified Himself as simply 'the god' or as 'everyone's god'.[4] Moses has God identifying Himself to Moses (and through him, to them) as either specifically, or at least especially, his and their god. He is represented as having spoken in a manner that allows, that perhaps implies, that there are other gods whom they might be tempted to take before Him and of whom they might be tempted to carve images to which to bow. He is reported to have described Himself as a jealous god ('*el*', of which '*Elohim*' is a cognate), suggesting that perhaps other gods ('*elohim*') are not jealous. I consider the name '*Elohim*', not the personal name 'Yahweh', to be a 'counterpart' in Hebrew of the English name 'God'.[5]

I have speculated with *The NIV Interlinear Hebrew-English Old Testament* (ed. J. R. Kohlenberger III, Grand Rapids, MI: Zondervan Publishing House, 1987) open before me, and the following in mind: "In the Old Testament various names for God are used. YHWH [first occurrence at *Genesis* 2:4, report of first use by humans at *Genesis* 4:26] is the most celebrated of these; the Hebrews considered the name ineffable and, in reading, substituted the name *Adonai*.[6] The ineffable name, or tetragrammaton [Gr.,=four-letter form], is of unknown origin; the reconstruction *Jehovah* was based on a mistake, the form *Jahweh* is not now regarded as reliable. . . . The most common name for God in the Old Testament is *Elohim*, a plural form, but used as a singular when speaking of God. . . . The name *Shaddai* . . . appears rarely. Of these names only *Adonai* has a satisfactory etymology. It is generally not possible to tell from English translations of the Bible what was the exact form of the name of God in the original" (*The Columbia Encyclopaedia*, Fifth Edition, New York: Columbia University Press, 1993.) I have had in mind also the incantation (here inexpertly transliterated), *Smie Yisra-ale, Adonoi eloheynu, Adonoi echod*. In some English translations: Hear Oh Israel, the Lord our God, the Lord is one.[7] (Amen.)

3.2. My semantic proposal is that the name 'God' today expresses our concept of a unique god. It expresses our concept of what would be the one and only true god, even if this concept is not strictly speaking, the sense or meaning of this name. And so, even if, according to the best semantic theories, 'God', since a proper name is necessarily meaningless, there is a pertinent meaning near-to-hand into which we can inquire, specifically, the meaning of the common name 'god' and connectedly that of 'what would be the one and only true god'.

Philosopher-linguists sometimes say that 'God' is not a proper name but a title-term analogous to 'The President', a kind of compressed-in-one-word definite description. At least they sometimes say that they shall use 'God' in this way in their 'scientific' discussions of its would-be referent's nature and existence. Witness: "Following Pike's presentation . . . , I assume that the term 'God' is a descriptive expression used to mark a certain *role*, rather than

a proper name" (Fischer 1989, p. 87). I regard as hardly controversial, and as not calling for argument, that 'God' in religious discourse and literature is a proper name, not a title-term. So does *The Oxford English Dictionary*. So does Jerome Gellman.: "In the day to day religious life of the West, in prayer and 'religious language,' typically the word 'God' functions as a proper name . . . the typical believer, when speaking . . . of God, does not intend to be speaking . . . of 'whatever it is' that satisfies a certain description, or cluster of them" (Gellman 1995, p. 536). In my view this is true as well of the play of 'God' in philosophy and theology that is, regarding the semantics of 'God', naive. It seems preferable, therefore, to continue this practice in semantically sophisticated discussions, and to use for related title and role-functions elevated (uppercase) definite descriptions such as 'The One God' and 'The True God' or, perhaps best, the already established title 'The Lord'. In this way 'God' can be retained for the subject-position in answers to such questions as, Who is The Lord? While the name 'God' and these definite descriptions and titles would be necessarily co-referential, they would 'reach out' to their objects in different ways, the name by way of a "referential chain leading back to initial baptisms" (op. cit., p. 542) and the descriptions and titles by their meanings.

My primary object in this chapter is, however, to establish a perspective for discussions of subsequent chapters. For this purpose, little if anything depends on whether I am right and title-theorists are wrong on the semantic points and/or practices that divide us. To this observation may be added that it is certain that nothing turns on my reluctance, since I consider 'God' to be a name, to follow Gellman, and refer to it also as a word.

3.3. 'This book would be about God.' "'*Would be* about God'?" Yes, 'would be.' Talk of God in this book in which the issue is often whether God exists does not, of course, take for granted that God exists. Subjunctive forms are often used as explicit reminders that existence is not presupposed. "But do not best semantic-theories say that proper names necessarily denote, and that denotations of subject terms of sentences are presupposed by statements one would make with them, so that, when these terms fail to refer, no statements, true or false, are made?" No. *Best* semantic-theories say neither of these things. The curiously popular view that proper names in correct use necessarily denote, refer, tag, or what have you, is plainly false of 'names' as ordinarily understood, as is the thesis that failures of reference of names always result in failures to make statements either true or false. Against the first view, we have that 'Santa Claus', 'Sherlock Holmes', and 'God' are names in frequent unfaultable use, though most people do not believe in Santa Claus, though everyone knows that Sherlock Holmes is a purely fictional character, and though there is, for many, a question of whether or not God exists, and for others, who deny that God exists, no question. Against the thesis, we have that it may be true that

God does not exist: The sentence 'God does not exist' may express a true statement, even though, if it does, its grammatical subject, 'God', fails to refer (cf., Stirton 1995, and Stroll 1997).[8,9]

"What makes 'God' a name is the *intention*" (Gellman 1995, p. 543n2) with which it is used by believers to refer *not* by way of a concept or description that would pick out its referent, but by tying "into a referential chain that culminates . . . [in an] initial act or acts of naming" (Gellman 1995, p. 536) its referent. No matter to the semantic status of 'God' if the initial acts of naming went astray and nothing was actually named in them. No matter, if, for example, Moses was mistaken when he thought someone was speaking to him and naming Himself, 'YHWH' (*Exodus* 3:14) – no matter, if he was then dreaming. No matter, indeed, if that bit of *Exodus* is pure fiction, with the line in which God is quoted as naming itself having the status of the first speech in *Moby Dick*. The status of 'God' as a name is settled by the *intention* of believers when using it to refer by tying into a referential chain that goes back to a named being, whether or not they succeed in their intention. The use of this name by nonbelievers is parasitic on its use by believers.

4. THE CORE ATTITUDINAL CONCEPTION OF GOD

Whatever else [would] be true of God, it must at least be said that God [would be] a *worthy object of worship*. (Peterson et al. 1991)

4.1. There is not complete agreement regarding what would be the attributes of God, or about the meaning – the *proper* meaning – of the words 'the *true* god'. Indeed, this use of 'true' suggests disagreement. But there is an important *point* of agreement, and it is possible to see central parts of *the common conception of God* – that is, the conception of the god of the old and new Bible and of the Koran, as elaborated by "traditional theologians" (Rowe 1993, p. 5) – as a response to this point of agreement that, I am saying, is settled by, (i) *the meaning of 'god'* and, (ii) *the conventional tie of 'God' to 'the god'*.

John Findlay is right to "pin God down . . . as the 'adequate object of religious attitudes'" (Findlay 1955[1948], p. 48[176]). Findlay speaks in this regard of reverence, adoration, abasement, awe, wonder, extreme gratitude, and, above and before all others not included in it, of ***worship***.[10] God, it seems agreed by all, would need to be an appropriate object for at least some attitudes or emotions and behavior, including this one. That much is fixed by the meaning of 'the true god'. It would be very odd to say that God exists or that some being is the one and only true god, and to say this with indifference or while countenancing indifference. It is a plausible linguistic conjecture that at least part of the meaning of 'god' is that, even though psychologically possible, such indifference would be *inappropriate* on the part of those who believingly speak God's name and in a way *impossible* for those who speak with understanding of

The One of whom they would speak. God, it seems, just *must*, at least in the end and on full reflection and appreciation, matter to, and be worshipped at least in their hearts by, those who believe that God exists. Those who ***believe that* God exists** just *must*, at least in the end, ***believe in* God**, where this includes, in addition to the belief that, a worshipful attitude. In Plato's opinion The Good, an impersonal Idea or Form, must matter and be loved by all who know it. He considered its indifferent apprehension to be quite impossible. God would, according to ordinary religious thought and talk, be like that. *God would be in an objectively normative manner a proper object for religious attitudes*. This is a fixed point of agreement in our use of the name 'God' in religious discourse.

4.2. My semantic hypothesis is consonant with the following note on the etymology and semantics, which, though it is offered for the name 'God', is, as far as it goes, adequate only for the word 'god'. "***God*** . . . The ulterior etymology is disputed. Apart from the unlikely hypothesis of adoption from some foreign tongue, the OTeut. *gudo*^{m} implies as its pre-Teut. type either *ghudho-m* or *ghuto-m*. The former does not appear to admit of explanation; but the latter would represent the neut. of the [passive] pple. of the root *gheu-*. There are two Aryan roots of the required form. . . . ; one meaning 'to invoke' . . . , the other 'to pour, to offer sacrifice'. . . . Hence *ghuto-m* has been variously interpreted as 'what is invoked' . . . and as 'what is worshipped by sacrifice'. . . . Either of these conjectures is fairly plausible, as they both yield a sense practically coincident with **the most obvious definition deducible from the actual use of the word, 'an object of worship'**" (*The Compact Edition of the Oxford English Dictionary*, Volume A–0, Oxford University Press, 1971, p. 1168, being p. 267 in the volume for 'G' of *The Oxford English Dictionary* of 1933; bold emphasis added). My position is that the strongest semantic demand on the correct use of 'God' in evidence in its actual religious use is that God would be *the* one and only *proper* object of worship.

There is confirmation of this hypothesis even in the name's wider uses. Consider: "'Ladies and gentleman, your attention please. God is in the house!'," said Fats Waller, "when he was playing in a nightclub, and told . . . that Art Tatum had just walked in" (Gelly 1986, p. 61). This Wallerian hyperbole was for the assembled being in the presence of a musician of singular magnificence, a higher being, as it were, who was, as a pianist, simply awesome. Other speakers in other circumstances could use those words of a poet or a wide-receiver. Constant, as the idea expressed by these declarations, is the awesomeness of some person in some dimension, and this person's being *as if* worthy of worship *as a whatever*. Variable, and only suggested by a given context, would be the manner or dimension of this awesomeness, and also the nature and/or accomplishments of this person that made him awesome in that manner, or on which his awesomeness was consequent, and somewhat similarly, I think, for the primary use of 'God'. For differences, this name expresses the idea of awesomeness in every dimension and of being really worthy of worship without

qualification for all that it is. For a similarity, the primary use also leaves unsaid and at most only suggests the nature and history of its purported bearer that would make God totally awesome and worthy of unqualified worship, or on which This One's appropriateness as an object of religious emotions and behavior would be consequent.

4.3. Confirmation of another sort for my semantic proposal that, bottom line, God would be the proper object of worship can be found in ***the common conception of God*** elaborated from the Bible by traditional theologians. It is as if, at least in its major parts, this conception were tailored for the attitudinal office that I say is assigned by linguistic convention to a being properly named 'God'. God, according to this common conception, would be a proper object of religious attitudes, par excellence. He would be "worthy of worship . . . in virtue of his [nature and his possession of certain] properties" (Swinburne 1993, p. 292).[11] These would be properties that made appropriate other more specific religious attitudes, as well as this major one. The common conception details a wide consensus concerning these properties. This consensus is arguably summed up in what I term ***the philosophers's conception of God***, which would, in a single formula, make God the proper object par excellence, a *proper* object if any object could be, of religious attitudes and behavior, including centrally again those of worship. To establish a perspective, and to make some distinctions, for chapters to come, I start with the philosopher's conception and go from it to the common one.[12]

5. THE PHILOSOPHERS' CONCEPTION OF GOD – GOD AS A PERFECT BEING

If God would be a proper object of worship – if it would make sense that we should bend to God – it is required that This One should be *superior* to us, that This One should be great. "[R]eligious attitudes presume *superiority* in their objects, and such superiority, moreover, as reduces us, who feel the attitudes, to comparative nothingness" (Findlay 1955[1948], p. 51 [179]). How much superior to us and how much greater than us do we want God to be? The more superior the better. "[H]aving described a worshipful attitude as one in which we feel disposed to bend the knee before some object, to defer to it wholly, and the like, we find it natural to say that such an attitude can only be fitting where the reverenced *exceeds* us very vastly. . . . To feel religiously is . . . to presume surpassing greatness in some object. . . ." (Ibid). To feel religiously, Findlay maintains, is to presume for the object of one's feelings *unsurpassable* greatness. "[N]ow we advance further – in company with a large number of theologians and philosophers, who have added . . . touches to . . . portraits of deity . . . to make their object worthier of our worship . . . we are led . . . to [the] demand that our religious object should have an *unsurpassable* supremacy . . . that it should tower *infinitely* above all other objects" (Ibid).

We are led, Findlay is saying, to the god of the philosophers, to René Descartes's ***perfect being***, to St. Anselm's ***that than which nothing greater can be conceived***, to a being superior not only to us, but to everything that is and that *might be*. We are led to *this* idea of *the Supreme Being*. Having formed the idea of such greatness, it can seem that it would be proper to bend before, and to worship, nothing less – that it is either this God of the philosophers or no god at all. "This conception of God, as the absolutely perfect being, is one that . . . is . . . plausibly thought to be implied by the very idea of worship . . ." (Peterson et. al. 1991, p. 51). I doubt this, but what is certain is that this conception of God, that God as a perfect being, fuels most contemporary and much historic, philosophic discussion of God, as well as much popular discussion.

6. THE COMMON CONCEPTION OF TRADITIONAL THEOLOGY

According to the philosophers' conception, God would be unsurpassably great. We can recover and in a manner explain 'the common conception' by asking, Great in what ways, and in what manner? I am not suggesting that the common conception was reached by deduction from the philosophers' conception. I think that historically most parts of common conception were articulated before the synoptic philosophers' conception. My claim is only that the philosophers' conception can come first in order of explanation, and that all parts of the common conception can be understood as elaborations of it.

6.1 ***Ways*** *in which God would be great*

6.1.1 In general. God would be great in all ways that would contribute to being a proper object for religious attitudes. That, I think, is a better summary than that God would be great in all "valued" ways (Findlay 1955[1948], p. 51 [179]) or great in all ways that it is "intrinsically better" to be than not be (Morris 1987, p. 12). I explain what would be God's greateness in terms of the core ideas of the worshipful, awesome, venerable, and so on. This being's attributes would contribute to its being *unsurpassably worthy* of religious attitudes and emotions. God's attributes would 'make' God worshipful and the rest, and thus great, as attributes are thought to make persons good and lovable and their actions praiseworthy and right.

6.1.2 In particular

6.1.2.1. God would be unsurpassably powerful, *omnipotent*, and capable of doing absolutely everything that it is conceivable that anyone should do. That greatness would contribute to God's being *maximally awesome*. The deity would be unsurpassably knowledgeable, *omniscient*, and knowledgeable of absolutely everything that is knowable. That greatness would make sense of aspects of *unreserved respect*, for example, respect for God's authority regarding

what is for the best, and right and wrong. God would be unsurpassably good, *perfectly good*. "For if a being were to fall short of perfect goodness, it would not be worthy of unreserved praise and worship. So, God [would be] not just a good being, his goodness [would be] unsurpassable" (Rowe 1993, p. 8). One might dread an omnipotent and omniscient devil and bend and scrape before it, but, I take Rowe to imply, it would not make sense for anyone to praise and adore it in his heart or sincerely to worship it.

6.1.2.2. However, to quibble this last point, what about 'devil worship'? One possibility is to say that devil-worshippers see the devil not as seriously evil, but only as 'seriously evil,' that is, only as what other people say is seriously evil. Elaborating, devil-worshippers may see the devil's so-called evil as what makes that one so wonderful, so very good. Another possibility, however, is that the coherence of devil worship for a few people shows that goodness, in contrast perhaps with great power and knowledge, is, contrary to Rowe's suggestion, not in everyone's fully reflective and considered view essential to something's being a proper object of praise and worship. In any case, however, goodness is, in almost everyone's considered view, required if a being is to be worthy of praise and worship, which explains the perfection of it in the conception of traditional theologians.

Rowe suggests that God's perfect goodness would include, in addition to being and doing good, 'having it good' (Rowe 1993, p. 9). But that strikes an odd note and is at best an uncertain element of the common conception. Connectedly, for the thesis I am developing, it is doubtful that having everything good would suit God for any religious attitude or emotion. Having everything good could suit God for unreserved envy, but that is not a religious attitude. Left open are questions concerning unreserved admiration, whether precisely it would be a proper response to God's having it good. As for the unreserved praise of which Rowe speaks, we have it on no less an authority than Aristotle that that would not be a proper response to God's having it good in unsurpassable measure. "[N]o one praises happiness in the same sense in which he praises justice, but he exalts its bliss . . ." (*Nicomachean Ethics* 1101b26, tr. M. Ostwald). Perhaps, then, the appropriate response to God's having it good would be rejoicing with This One. That does have a clear religious ring to it.

6.1.2.3. Returning to firmer ground, certainly God, according to the common conception, would be *responsible* at least ultimately for the existence of all that exists. Absolutely every existent would be *due to* God at least ultimately and would, for its existence, *depend on* this being, which greatness would make sense of attitudes of *unreserved gratitude* ('cosmic gratitude,' I once heard George Nahknikian suggest), since in view of this unsurpassable existential responsibility, but for God we would be nothing. But for This One, nothing that need not be, that is, nothing that exists only contingently, would be. According to this idea, God would be ***The Creator*** of the world and of everything in it and also ***The Sustainer*** and every-moment-ratifier of creation, 'the ground of

all being' (dark saying), 'a personal ground of being' (Swinburne 1993, p. 291). "God," Thomas Morris implies, would be "the ultimate reality responsible for the existence and activity of everything else" (Morris 1987, p. 11). Anything less "would not be God" nor that which we would worship (p. 20).

What would be God's 'existential,' creative, and sustaining responsibility needs, however, to be distinguished from what would be God's 'moral' responsibility. While it is part of the common conception that God would be ultimately responsible for the existence of everything that exists, it is not part of that conception that This One would be responsible for everything that existents do and that happens to them. It is part of the common conception that creatures of God would include free beings that were themselves ultimately responsible for at least some of their actions, especially bad ones that would be entirely their faults and not at all God's.

6.1.3. The central ways of greatness of the common conception are omnipotence, omniscience, perfect goodness, and being the Creator and Sustainer of the universe. There is more to the common conception, and to versions of it, some with and some without easy relations to the task of making God worthy of worship. There is, for example, the agreed incorporeality of God, though this does not seem important to worshipfulness. And there are questions concerning what would be God's relations to space and time. Some traditional theologians would make This One atemporal (never) and aspatial (nowhere), somewhat as numbers are, while others would make God always everywhere. There is a *prima facie* advantage to the less rarefied, with us and with all of us everywhere at once, conception in terms of worshipfulness, making a kind of sense as it can of intimate and direct prayer and dedication. And yet, if I am not mistaken, the majority opinion amongst advanced philosopher-theologians has been for some time that God would be outside of time and space.

6.2 The **Manner** *in which God would be great*

6.2.1. Many philosopher-theologians say that God would not be simply unsurpassably great in many ways, but that This One would be in all of these ways ***essentially*** unsurpassably great. They say that God would not merely happen to be omnipotent and the rest, but that ***God*** could not be other than omnipotent and the rest. Thus Findlay writes: "[I]t is . . . contrary to [demands inherent in religious attitudes] that [their object] should *possess its various excellences* in some merely . . . **contingent** matter . . . an adequate object of our worship must possess its various qualities *in some* ***necessary*** *manner*" (Findlay 1955[1948], pp. 52–3 [180–1]; bold emphasis added). It would, he says, be *idolatrous* to worship something that was merely unsurpassably great, to worship it, one is tempted to elaborate, for its good fortune. The common amendment of essentialism does not add to the usual attributes. Rather, advocates suppose, it enhances what would be God's possession of them. As "Buckingham Palace

[not only will not but] could not cease to be a house and become a road" (Swinburne 1993, p. 250). So, according to the present elaboration of the common conception, God could not cease to be omnipotent. God could not *be* without being omnipotent, and similarly for omniscience and perfect goodness. No one who was not all of these things *could* be God.

'Compleat' perfect-being theology is *essential* perfect-being theology. According to it, God would be twice removed from us in power, knowledge, and the rest. For we are limited in these ways in which the deity would be perfect and unlimited. And it is certainly not essential to our beings and existence that we have even the meager power, knowledge, and so on that we do have. For instance, things could have worked out so that the very person we know as Descartes should have been uneducated and dim. This is not so in God's case, theistic essentialists say. God, they say, could not be without omniscience, omnipotence, and the rest. To put this modal point in now standard philosophical pictorial terms, God would be omniscient and the rest in every possible world in which God existed.[13]

6.2.2. On being essentially, merely, and simply P

6.2.2.1. Suppose there are just three *possible worlds*, just three *total ways in which things might have been*: the actual world, a second possible world, and a third possible world. This is presumably not true. There are presumably *many* distinct total ways in which things might have been. But it is true, as regards a single entity X and property P things can have only if they exist, that there can be at most three *kinds* of possible worlds, or total ways in which things might have been. At most, there can be worlds in which X exists and has P, worlds in which X exists but does not have P, and worlds in which X does not exist. The pretense of exactly three possible worlds makes possible pictorial explanations of certain distinctions In the following diagrams, each world is represented by a box.[14] The letter 'X' is in a box if and only if entity X exists in the world of this box. The formula 'P(X)' is in a box if and only if X has property P in this world. P is to be a property that only existent things can have, a property such as *being visible*, and not such as *being imaginable* – Santa Claus is imaginable but not visible.

X **is *essentially P*** if, for example,

the actual world	second possible world	third possible world	
P(X)	P(X)		(1)

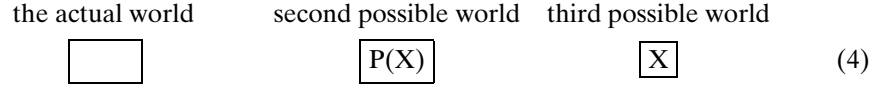
X exists and is P in every world in which it exists. X only **would be *essentially P*** if, for example,

the actual world	second possible world	third possible world	
	P(X)	P(X)	(2)

Although X does not exist, X is P in every world in which it does exist. If, for contrast,

(3)

then X ***is merely P***, which is the same as being actually but not essentially P. To be merely P entails being P and, thus because of the kind of property P is, existing. In the next example, X ***is not*** merely P but ***could be*** merely P, that is, X is possibly merely P.



(4)

This case (4) features only a thing that is possibly merely P.

I say that X ***is simply P*** *(that is, P simply)* if either X is essentially P or X is merely P and, similarly, that X ***would be simply P*** or could be simply P. Ordinary unqualified 'is' or 'would be' predications of P will be short for 'is simply' and 'would be simply' predications. The modal elaborations of 'essentially' and 'merely', and the disjunctions of these for which I have 'simply', are not in the vocabularies of most people. When I have occasion to use them in coming chapters, I will say again what they mean.

Incidentally, the four cases diagrammed all feature a contingent X that does not exist in every world. For practice with the ideas, one might sketch boxes for necessary existents that are essentially P, merely P, and only possibly P (that is, merely but not actually P). Perfect-being theologies not infrequently not only 'go essential' regarding God's attributes, but 'go necessary' regarding God's existence.

6.2.2.2. A thing's essential properties are sometimes said to be those that it has in a "necessary manner" (Findlay 1955[1948], p. 53 [181]) or those that "it has necessarily" (Cartwright 1987, p. 150). Cf.: "An *essential* attribute of a being is an attribute that the being *necessarily* possesses" (Rowe 1999, p. 112n21). That agrees with my usage of 'X is *essentially* P', provided that 'X is necessarily P' is *not* taken in the sense of 'it is necessary that X is P', which says not only that it is true at every world in which X exists that X is P, but that this is true at absolutely every world. P can be such that it is true at a world that X is P only if X exists in this world. That, as said, can be the kind of property P is. For some X and P, therefore, it can be that X is essentially P, in the sense I have explained, although it is not necessary that X is P: In notation explained in Chapter III, that X is essentially P, $\Box$ [E!(X) $\supset$ P(X)], is not logically equivalent to its being necessary that X is p, $\Box$ P(X). These formal conditions, although distinct and not logically equivalent, are, however, equivalent for necessary existents: That is, for any X, that X is a necessary existent, $\Box$E!x, strictly entails that $\Box$[E!(X) $\supset$ P(X)] $\equiv$ $\Box$ P(X). So, for 'all in' perfect-being theists who say that God is a necessary existent who is omniscient and the rest essentially, these

two conditions, taking P to be the property of perfection, come to the same thing.

6.2.2.3. I take logical or metaphysical necessity to be 'truth in every possible world' and logical or metaphysical possibility to be 'truth in some possible world.' Exact formal theories of modalities are generally framed in terms of an 'accessibility relation' of one world from another, with necessity and possibility of one or another kind in some world being truth in every or in some world 'accessible' from that world. Using that terminology, I am assuming that, for the strictly logical or metaphysical notions of necessity and possibility, the accessibility relation is 'universal,' with every world being accessible from every world. That assumption leads to the same formal theories and valid principles for these modalities – they are called S5 formal theories and valid principles – as does the assumption that the accessibility relation proper is an equivalence relation, which is to say that it is a symmetric, reflexive, and transitive relation, as the universal accessibility relation is.[15]

6.2.3 'God', and 'god', and essential attributes. According to Hasker, various essential attributes – for example, "essential infallibility" – are included in our *ordinary* concept of god and are thus expressed by ordinary uses of the name 'God' (Hasker 1989, p. 170). I disagree. For just as Hasker says that 'Jahweh' was, so is 'God' now "used [by many people] with no thought or connotation of such metaphysical attributes as essential omniscience, essential everlastingness, and the like" (Hasker 1989, p. 170) or, for that matter, 'mere' omniscience and the like. Most people have no ideas of such attributes in contrast with simple omniscience, everlastingness, and the rest, of which those essential attributes are recondite philosophical elaborations.

My position, implicit in Section 4, is that none of these attributes, even unenhanced, is a part of the shared ordinary concept of God in the modern community of global discourse in English, although these ideas – since widely possessed by members of this linguistic community – are at least candidates for inclusion. My semantic hypothesis is that the ordinary concept of God that is expressed by 'God' is *confined* to the idea of a being worthy of worship. For evidence that this exhausts the ordinary concept of God fixed by semantic convention, there is the apparent 'openness' of questions such as – Would God be omniscient? – in which they contrast, I think, with the question, Would God be worthy of worship? (Would God be a god?!!). Compare this with the Moorean questions – Are all pleasant things good? – and, Are all valuable things good? (Are all good things good?!!). (Cf., Moore 1993, Sections 11, 13, and 24.)

6.3 And then there is what would be the mystery of God. There is, additional to the ways and manner in which God would be great, a 'metapart' to the common conception of God of traditional theologians, according to which the 'first-order part' of this conception is profoundly inadequate to its

subject. Although it is not part of the common conception that the nature of God would be beyond our comprehension, it is thought, even if not frequently said, that the nature of God would be unsurpassably beyond it, so that nothing could harbor deeper and more wonderful mysteries. What finitely little we could know of God would be as nothing to the infinite reality of the divine and would be inadequate even to aspects of the divine to which it was addressed.

> [I]t is part of almost all religious forms of life to say that God . . . is not properly conceptualizable by us . . . that whatever one says about God falls hopelessly, ridiculously, short of describing God, is almost universal. (Putnam 1997, p. 410)[16]

And this too may be said to be essential to the being of God. The mystery of the divine nature could contribute to wonder and humility and be found "adorable" (Hume 1991, Part 2, p. 112).

But would not God's 'hiddenness' make a problem for belief? That depends on the kind of hiddenness proposed. Hiddenness in the just stated sense, the hiddenness of God's nature, the condition of being for the most part incomprehensible, would not pose a problem for belief, which would have as a possible 'purchase' the little part that by hypothesis is comprehensible. In contrast, God's hiddenness in the sense of God's existence not being knowable for sure could make a problem for belief. It is argued in Schellenberg (1993) that epistemic hiddenness does make a problem for belief: "The weakness of evidence for theism, I maintain, is itself evidence against it" (p. 2). I mention this only incidentally, and without suggesting that this second sort of hiddenness – hiddenness of God's existence – is a part of the common conception of God, or that it is a condition that would contribute to a being's worshipfulness and so on. I think it is not, and that it would not.

. . . . . .

Briefly, in Sections 4 through 6, according to *the philosophers's conception*, God would be a *perfect* being, a being than which none greater could be *conceived*. This philosophers's conception was cast as a synoptic response to the demands of *the core attitudinal conception*, which is that God would be the proper object of religious attitudes and behavior, especially worship. Then elements and aspects of *the common conception drawn by traditional theologians* were presented as articulations of the conception of the philosophers as details of the ways of what would be God's greatness according to traditional theologians that would make This One the proper object of worship and other religious attitudes and behavior. According to the common conception as 'refined' by 'compleat' perfect-being theologicans, God would be not merely perfect and great in ways that make appropriate religious attitudes, This One would be not merely omnipotent, omniscient, perfectly good, and the rest, but essentially so.

QUESTIONS ABOUT GOD AND GODS
7. MIGHT THERE **BE** A GOD, EVEN IF THERE **IS NOT** A PERFECT BEING?

Suppose that though there exists a greatest being who is great indeed, "it is entirely possible that there should have been a being *superior*" (Peterson et al. 1991, p. 50) – suppose that that *great* being is not a being than which no greater can be conceived, that though the greatest, that though not even remotely *approached*, it is not the greatest conceivable. Would that mean that there is no God, properly so-called – "**would** [that] . . . inevitably detract from the unreserved devotion required for worship" (p. 50; bold emphasis added), and so make worship, properly so-called, inappropriate (consider p. 64n5)? Or (contrary to Peterson et al.) might one reasonably, under those circumstances, settle for this lesser being and worship it in the manner due to God? Might, as William James suggested, God be to us as we are to our cats and dogs?

7.1 Could God be merely perfect?

7.1.1. What if arguments establish only that, in Swinburne's terms, a 'contingent god' who is *merely* absolutely perfect and unsurpassably powerful and the rest, and not essentially so, exists? Might it be reasonable to settle for *this* 'lesser god'?[17] It is argued in Chapter 15 of Swinburne (1993) that a 'necessary god' – that is, an essentially eternal, omnipresent, perfectly free, omnipotent, omniscient, perfectly good, source of moral obligation, free creator of all spirit (Swinburne 1989, p. 291) – *would* be worthy of worship. Swinburne does not say that a contingent god – for which, replace 'essentially' by 'merely' – *would not* be worthy of worship, although some of his arguments imply that a contingent god would be at any rate *less* worthy of worship (see, for example, p. 300).

7.1.2. I think that Swinburne's contingent god would be, to most minds, not a *bit* less worthy of worship than his necessary god. Its perceived deficiencies, if any, would be, to most minds, not at all *spiritual* or *religious*, and as an object of worship, but entirely *philosophical*. For example, a contingent god could not serve the *metaphysical* function of a final reason that would explain everything, the kind of reason that it has been thought that a necessary being could serve (see Chapter VI). For a second example, some philosophers claim not metaphysical, but *argumentative*, advantages for necessary existence combined in a conception with essential greatness and perfection. Charles Hartshorne and Alvin Plantinga, for example, claim that it is demonstrable that the combination of necessary existence and essential greatness is instantiated in this world. Plantinga stresses that the same cannot be said of the combination of necessary existence and mere greatness (see Chapter III).

Reviewing "sources of views on the nature of God," Edward Wierenga writes that

> No doubt the primary source is . . . the Bible. . . . Another source is **human experience**. . . . John Calvin, for example, held that 'there is within an awareness of divinity . . . [which] typically issues in worship. . . . Reflection on **what is involved in being worthy of worship** can lead to views about God's nature, for no being is worthy of worship unless that being is supremely excellent. . . . Another source of ideas about God's nature is **philosophical**. Some philosophers have given arguments for the existence of a being they take to be God, and these arguments demand that the being in question possess certain features. . . . [Wierenga mentions Aquinas, Leibniz, and Anselm]. (Wierenga 1989, pp. 2–5; bold emphasis added)

This is true. But there is a lesson that Wierenga does not draw, namely, that reflection on what is involved in being worthy of worship need not agree with the requirements of philosophy and its arguments and systems that would concern God. Not only may these sources not agree in their contributions to what would be God's nature, but there is the possibility, given their very different characters, of their disagreeing. According to Peter Geach, Richard Price would argue that Swinburne's necessary god is not only no *more* worthy for worship than his contingent god, but that it is *less* worthy.

7.1.3. Swinburne's necessary god – *Swinburne's* 'necessary god' – though essentially perfect, would, just as his contingent god, *not* be a necessary existent: His necessary god exists in only some possible worlds. Swinburne's paragon of godliness would be "truly ultimate, itself without explanation of any kind, that which explains all other things" (Swinburne 1993, p. 301). His necessary god would simply be, without necessity and without explanation of any kind. Swinburne does not think that that detracts in the least from his necessary god's worthiness for worship. The point Geach attributes to Price, a point with which I am sympathetic, is in part that similarly Swinburne's contingent god's being merely, that is, nonessentially, perfect does not detract from its worthiness for worship. The rest of that point, to which I will return, is that this aspect of Swinburne's contingent god is actually important to its being worthy of worship.

There is, incidentally, if not a paradox, then at least a curiosity, to do with Swinburne's gods, both his contingent god and his necessary god, and the proposition that either would be worthy of worship. For God, according to Swinburne, would be among other things *the source of moral obligation*, and Swinburne suggests the theist's claim that a being is *worthy of worship* wants to be "the stronger claim that men **ought morally** to worship [this being]" (p. 292; bold emphasis added): "Most theists would wish to make the stronger claim, and for that reason I shall examine it, and I shall understand the claim that God is worthy of worship in this sense" (Ibid). Swinburne must say that God would be the source of any moral obligation to worship. Depending on

what it *means* for God to be the source of moral obligation, and the extent to which Swinburne would have moral obligations be *up to* God, Swinburne may have a problem if he embraces 'the stronger claim' that there would be a moral obligation to worship God. I recall that Swinburne wants to say that God would be "**worthy of worship** . . . [in part] in virtue of his [nature and his having certain] essential properties" (Swinburne 1993, p. 292; bold emphasis added) and thus *presumably*, although Swinburne does not say this, not in virtue of His will or by His fiat. Indeed, issues of consistency aside, I doubt that it makes sense – that it is possible – that any being should be morally worthy of worship, respect, love, or any emotion, because it decrees that its nature (including that it is the source of moral obligation!) makes it morally worthy of worship.

7.1.4. There are problems with the idea of a *god* who would be an *essentially* omnipotent, omniscient, and a perfectly good creator. For one, it seems that such a being would exist only in the best worlds it could create, and that it would *need* to create a world than which there was no better world that it could create. It seems that an *essentially* perfect creator *would have no choice* but to create a best world from those that it is possible for it to create. The limited capacity of this creator, its *incapacity* to choose less than a best world that it could create, must detract from admiration and gratitude that its power and perfect goodness, and the goodness of its world, might initially inspire. "If there is no possible world in which a person makes a certain choice, it cannot be that the person, nevertheless, has it within his power to make that choice" (Rowe 1999, p. 112n24). If "God chooses to do what is best of necessity . . . we are left with no reason to thank God or be grateful to him for choosing and acting in accordance with his knowledge of what is best" (op. cit., p. 110b; cf., p. 108b) The question we are laboring is, Might God *not* be *essentially* perfect? The answer *seems* to be, Yes, indeed! Contrast your regard for a person you believe to be constitutionally incapable of telling lies (it might be something complicated to do with his brain) with that for a person you think can lie with the rest of us but, as a matter of principle, and determination to do the right thing, never does. Consider the 'chemistry' of the comic relief of a largely tragic old circus movie delivered with the lines: "I can't short-change the customers It has something to do with the lengths of my fingers."

7.1.5. Peter Geach writes: "Richard Price . . . argues . . .: if God is a free moral being, even as we are, it must **not** be absolutely impossible for God to do something wicked . . . or [else] God isn't free and isn't therefore **laudable** for his goodness" (Geach 1973a, p. 16; bold emphasis added). Geach is scandalized by this line of argument: "The way this reverend gentleman commends his Maker's morals is so startling that you may suspect me of misrepresentation; I can only ask any sceptic to check Daiches Raphael's edition of Price's work! Further comment on my part is I hope needless" (Ibid). For my part, I am

puzzled by Geach's reaction, since what he reports that Price says has for me the ring of truth. Certainly there was nothing laudable about that circus-hand's straight dealings with ticket buyers.

Geach implies that it makes no sense to say of Almighty God that "He has the bodily and mental power to do so-and-so, but he certainly will not, it would be pointlessly silly and wicked" (Ibid). That, he suggests, makes "the Scholastic distinction between God"s *potentia absoluta* [His power considered in abstraction from His wisdom and goodness] and *potentia ordinata* [His power considered as controlled in its exercise by His wisdom and goodness] . . . wholly frivolous" (Ibid.). Cf.: "There is no sense in distinguishing what God simply could do from what he could do wisely and well, since he cannot act except wisely and well" (Geach 1973b, p. 328). Again, however, what Geach implies makes no sense, and it seems at least to all the unphilosophical world not only intelligible, but quite sensible. Nor does that audience find frivolous the scholastic distinction between God's power extending to the silly and wicked, and God's conduct, which, pursuant to his wisdom and goodness, although it could be, never is silly or wicked. It is not an argument against this common sense – that it is not consistent with God's being essentially wise and good – as some philosophical theologians would have the deity be. On the contrary, this sense argues in Pricean fashion against the would-be God of worship of ordinary discourse (which has no philosophical axes to grind) and of religious people's lives, being *essentially* wise and good. It argues that, if '*praise*-be-to-God' for good works, God had better be capable of lesser work, for which capacity it *seems* necessary that the essentiality of at least one of omniscience, omnipotence, goodness be 'lost' (and – see Morris (1987, p. 35) – that the *immutability* of God's nature not be insisted upon).

Geach, when sending sceptics to Daiches Raphael's edition of Price's work, neglects to provide its title and year, or to mention relevant pages. On consulting Price (1787), I have found clear evidence that he thought that God is capable of acting wrongly, since He is in all His acts free, but no clear evidence that Price thought that if God were not capable of acting otherwise than rightly, He would not be 'laudable for his goodness'. To the first point are the lines:

> whenever I represent *necessity* as the account of the *rectitude* of the Deity, or speak of *goodness* as *essential* to him, it is the *principle* of rectitude I mean; and not the *actual exercise* of this principle. . . . All voluntary action is, *by the terms*, free, and implies *physical possibility* of forbearing it. What is meant by this *possibility* is not the least inconsistent with the utmost *certainty of event*. (p. 422)

According to Price, we can be sure that God never will exercise His "power of deviating from rectitude" (p. 426). But, Price is saying, He can deviate from righteousness, *he* has the power, and He is not *essentially right-acting*. This is 'nice'. It is a way of saying that, although God is essentially omnipotent, omniscient, and a perfectly good creator, there are worlds in which He exists

in which he acts out of character and does *not* do the best creating of which he is capable.

Price considered the condition of being essentially *exclusively right-acting* to be inconsistent with divine freedom. He may, as Geach reports, have considered it to be also inconsistent with divine laudableness. Others have found in the condition of essential omniscience a challenge to human freedom. This challenge is discussed in the Appendix to Chapter XII. There are, I argue in Chapter IX, not only problems between mere omnipotence and essential goodness, and mere omnipotence and omniscience, but problems intrinsic to the would-be condition of essential omnipotence alone. Theistic essentialisms not only, as said in Section 7.1.2, promise certain metaphysical and argumentative advantages, but, depending on their details, harbor interesting problems. The popularity of theistic essentialisms amongst philosophers has these two sources. Some philosophers like theistic essentialisms for what it seems they can do with them, while other philosophers are attracted to them for the discussions they start and for the nice points they think they can score against them.

7.1.6. It is a problem, Rowe says, "that God's perfect goodness results in the necessity of his always doing what he judges to be best, thus depriving him of the freedom not to do what is best and depriving us of any grounds to praise him or thank him for so acting. [This] casts serious doubts on the plausibility of the theistic picture of God" (Rowe 1999, p. 111b). I say that it is only God's being *essentially* perfectly good that can have these consequences; that it has this consequence only if it includes not only essentially having a good character, but of being essentially one who never acts out of character; and that what is made implausible is not *the* theistic picture of God, but only *a* theistic picture of God. It is the picture of a being not merely perfect in *every* way, but of a being *essentially* perfect in *every* way, that philosophers have conjured not for the spiritual purposes of those who would be faithful, but for their own (dubious) philosophic purposes. The picture, this Anselmian picture, especially when necessary existence is added to in every way essential perfection, *ill* serves the former purposes.

7.2 Could God be Simply Imperfect *(that is, neither essentially nor merely perfect)?* Could God be a lesser god? Why yes. 'The God of the Bible and the rabbis', according to Howard Wettstein, *would be* a lesser god at least in power, "a God whose awesome power [was] nevertheless limited" (Wettstein 1997, p. 426). This God would, Wettstein suggests, perhaps also fall short of moral perfection: "It is at least an interesting question whether the God of the Bible and the Rabbis exhibits . . . ethical impeccability. Abraham, after all, argues with God that God's impending destruction of Sodom would be unjust" (Ibid.). What is necessary, I have been saying, if 'the God of the Bible'

would be *God*, is that He would be worshipful. Arguing that He would be worshipful are the demonstrations of worship of the rabbis and the legion of their students who, having thought well about the matter, 'find' their God worshipful and an object of awe, adoration, gratitude, and so on. See how they pray.

8. MIGHT THERE **NOT** BE A GOD, EVEN IF THERE **IS** A PERFECT BEING?

There are two ways in which one may maintain that, even if there is a perfect being, there is no god. There is the way of 'the objective humanist', and there is the way of the 'normative sceptic'. Each is a way to the radical negation that there is no *god*, no matter *what* there is, an essentially perfect being, a merely perfect being, the imperfect God of the Bible and the rabbis, or whatever!

8.1. An 'objective humanist' might say that there can be nothing to which it would not be *beneath the proper dignity of a human being* to bend and to worship. He might say that, far from ever being appropriate, *worship* done by a human being, regardless of its object, would be disrespectful of his humanity, *and wrong*. Let a being be perfect, he says with a nod to philosophers and theologians. Let it be whatever 'turns on' religious spirits, he adds with a nod to Wettstein's rabbis. This being is even so not a proper object of worship, because, as a matter of fact, *worship* is always for every human being and every possible object an *improper* attitude. Given that a god would be a proper object of worship *for a human being* (which, until now, has been left implicit), it follows, our objective humanist concludes, that *there is no* ***god***, even if *there* ***is*** *a being that is perfect and everything for which a religious spirit might ask*.

8.2. Suppose, however, that our objective humanist is mistaken when he says that it is as a matter fact improper and wrong for a human being to worship. He could still be right when he says that not even a perfect being would be as a matter of fact a proper object of worship. He could still be right about that, *if* there are *no facts of matters of* ***propriety***. Suppose this is so. Suppose there are in the vicinity only *psychological facts* concerning what if any religious attitudes this or that person would, or would not, on reflection entertain toward various beings, including perfect beings. Suppose there are not in addition *normative facts* concerning what attitudes are proper and prescribed, and what attitudes are improper and proscribed, whether or not they are forthcoming for persons. Then, while our objective humanist would be mistaken in his reason for bending and scraping's being 'beneath us', he would be correct in his conclusion that not even a perfect being would be a proper object of worship. The correct reason for this, according to the present line, is that 'proper', that is, 'objectively proper', never correctly applies to attitudes. This way to say

'you are no god' to an essentially perfect being, and to *every* being, is the way of a 'normative sceptic'. It is a 'Mackiean way.'

8.3. John Mackie says that *there are no objective values* (Mackie 1977, Chapter 1). He says that there are no objective *goods* or values of universal validity that everyone ought to cherish, whether or not they would be so moved in the end, on fully informed reflection. He holds that there are only subjective values, this or that person's values, where a particular person's goods are the things he would in the end be moved to value. Even so, he maintains, ordinary value thought and talk involves commitments to objective values. Unhedged use of the language of values to ascribe objective values to things is, therefore, Mackie says, in error and undermined. Affirmations of objective values are neither true nor false, since there are no such values to be correctly or incorrectly ascribed.[18] The suggestion of the previous section is that perhaps the case is similar for *gods*. Perhaps, although there are no possible 'objective gods', ordinary 'God-talk' of both believers and their opponents expresses, in both affirmations and denials, the idea of a being who would be an 'objective god.' It could be that, (a) ordinary God-talk, especially impassioned ordinary God-talk, presupposes the *possibility* of a being who would be *objectively worthy* of worship, notwithstanding that, (b) as Mackie might say, this idea of *objective worthiness* for worship is without instantitiation in any possible world, which is to say that a being that would be objectively worthy of worship is not so much as a *possibility*. That (a)-and-(b) condition would be a plague on the houses of both theists and many atheists.

"Questions, conjectures, enough! How about some answers, please? What do you think?" For what my opinion on recent difficult matters is worth, I think that the ordinary God-talk of both believers and disbelievers does presuppose the possibility of a being objectively worthy of worship and the rest an objective god. And I think, for Mackiean reasons, that there cannot be an objective god, a being such that there would be a prescription, valid and authoritative for all, that those who believe in its existence must worship this being. I do not believe in the possibility of such prescriptions. So I think that the ordinary God-talk and God-thought of believers and disbelievers alike involves an undermining error, which is that there could be an objective god. But it is the business of this chapter to establish a perspective for discussions to come of arguments for and against the existence of God, and this is served by a canvass that opens, without exhaustively treating, questions relevant to literature and issues to be discussed. Having shown my hand, and where I tentatively stand on these deep matters, it is not necessary for present purposes to say more, had I much more to say to them.

ARGUMENTS FOR THE EXISTENCE OF GOD

II

Classical Ontological Arguments

There's no getting blood out of a turnip.
Captain Marryat

1. INTRODUCTION

1.1 On 'the very idea of an ontological proof'

1.1.1. Ontological arguments would be demonstrations of God's existence, deductions from scratch without aid of contingent premises. But cannot such existence proofs be rejected out of hand and without detailed consideration on the general ground that it is not possible to demonstrate the existence of anything? This has been said.

Cleanthes: [T]here is an evident absurdity in pretending to demonstrate a matter of fact, or to prove it by any arguments *a priori*. Nothing is demonstrable unless the contrary implies a contradiction. Whatever we conceive as existent, we can also conceive as non-existent. There is no being, therefore, whose non-existence implies a contradiction. Consequently there is no being whose existence is demonstrable. I propose this as entirely decisive. (Hume 1991, Part 10, p. 149)[1]

However, while certainly nothing is demonstrable that is not itself necessary, it does not follow from that that there is nothing whose existence is demonstrable, for there are things that exist necessarily. For example, the number 23: It is necessary that there exists a prime number greater than 20 and less than 25, and it is 23. "But surely," Cleanthes might complain, "you quibble. For no one supposes that God is a number, or anything like one. Numbers can of course be great, but not in ways that call for reverence, adoration, praise, or worship. Numbers are not beings, let alone worshipful ones! And even if not absolutely everything we can we conceive as existent, can be also conceived as non-existent, this is so of every *being* we can conceive as existent."

1.1.2 Aspecial privilege. Ontological arguers agree that God is in many ways unlike a number, but they say that, even so, He is like a number in His existence, which is necessary much as is the existence of numbers. He is in this, they say, unique amongst beings, indeed amongst all 'real entities', where 'real entities' are the things with which you and I and the Taj Mahal stand in 'causal relations' broadly construed. This necessity, Gottfried Leibniz might have said, "is in fact an excellent privilege of the divine nature" (Leibniz 1951, p. 324).[2] St. Anselm does say this: "[W]hatever else there is, except thee [O Lord, my God] alone, can be conceived not to exist. To thee alone, therefore, it belongs to exist more truly than all other beings, and hence in a higher degree than all others" (*Proslogion* III, 1903 translation by Sidney Norton Deane). One might suspect that Hume had Anselm in mind, incidentally to contradict him, when he gave Cleanthes the line, "***Whatever*** we conceive as existent, we can also conceive as non-existent" (bold emphasis added).[3]

Ontological proofs would show by doing that God's existence is demonstrable. Sometimes they would explain how the supposition that there is no god, suitably spelled out, implies a contradiction. What we want, in the case of articulated ontological proofs, are explanations of their defects, if such there be. We want that, even if there are plausible reasons such as might be worked up from Cleanthes' speech for thinking that each must be in some way or other defective. Cf.: "I do not deny that [there is intuitive plausibility] in favor of the claim that no real existence is logically necessary – so long as no conclusive proof of the logically necessary existence of some real being has been discovered. . . . What I do deny is that such intuitive plausibility can rightly serve as a refutation, in advance, of any argument for logically necessary existence" (Adams 1987, p. 200). Even if, as is unlikely, general reasons against ontological proofs were not merely plausible, but obviously decisive, I would want to understand the defects of the proofs, for likely instruction given the prowess of their authors.

1.2 Five proofs will be examined. The present chapter takes up classical proofs, starting with René Descartes's in the *Fifth Meditation* because it is the simplest, going forward to Baruch's Spinoza's,which is better, and having saved the best for last, going back to the argument that started the fun, St. Anselm's. The implicit logic of these arguments is nonmodal quantified; their strategies of proof are at points indirect. The next chapter is about the modal ontological proof of Charles Hartshorne and Alvin Plantinga. Its logic is modal sentential, and its strategy throughout is direct. It is an update of the first ontological argument of St. Anselm. The principal objection to it updates the principal objection of Anselm's opponent, the Monk Gaunilon, to Anselm's reasoning. (It is not his most famous objection.) Chapter IV is about Kurt Gödel's ontological proof. Its logic is third-order quantified modal with identity (and more), and its strategy is direct.

2. PART ONE: RENÉ DESCARTES'S ONTOLOGICAL PROOF

But if, from the mere fact that I can bring forth from my thought the idea of something, it follows that all that I clearly and distinctly perceive to pertain to something really does pertain to it, then is this not an argument by which to prove the existence of God? Certainly I discover within me an idea of God, that is, of a supremely perfect being. . . . And I understand clearly and distinctly that it pertains to his nature that he always exists. . . .

[I]t . . . becomes obvious to a very diligent attentive person that the existence of God can no more be separated from his essence than the essence of a triangle can be separated from the fact that its three internal angles equal two right angles. . . . So it is no less repugnant to think of a God (that is, a supremely perfect being) lacking existence (that is, lacking some perfection), than it is to think of a mountain lacking a valley. . . .[4]

[A]s often as I think of a being first and supreme . . . I must of necessity ascribe all perfection to it. . . . This necessity plainly suffices so that afterwards, when I consider that existence is a perfection, I rightly conclude that a first and supreme being exists. (Descartes 1979, Meditation Five, pp. 42–3)

2.1. Descartes's first sentences say that the idea of a supremely perfect being's nature or essence includes existence. The second sentences suggest an 'indirect' proof of God's existence, given the involvement of existence in the nature of this supremely perfect being: 'Try to think of a supremely perfect being's *not* existing. You will see that such thinking "fights with itself" into a contradiction.' The third sentences recapitulate without allusion to an indirect thought-experiment: A being first and supreme has every perfection, and existence is a perfection, from which it follows that *a first and supreme being exists*. 'It is as simple as that.'

2.2 On toward God. Given that there is at least one perfect being, it can be an easy inference to the conclusion that exactly one exists, for it can be ruled implicit in the idea of supreme perfection that at most one supremely perfect being exists. It can be maintained that being unparalleled is itself a perfection, and that a supremely perfect being would have this perfection, as it would have every perfection.[5] And, given the existence of exactly one perfect being, it can seem that nothing else could be a proper object for total deference and unqualified reverence and worship. In fact, however, a person might hold out for more than perfection or, indeed, settle for less, even given a choice between a perfect being and an imperfect one who had other things going for it that recommended it for exclusive worship. What other things? One answer could be, "Contingent personal connections. It would be nice if we were the freely chosen of a towering being, or if such a being had offered up some personal sacrifice for our sins." Cf.:

The theist normally claims that God is worthy of worship ***both*** in virtue of his having [certain] . . . essential properties [such as omnipotence, omniscience, and perfect

goodness] . . . ***and also*** in virtue of his having done of his own free will various actions (e.g., rescued the Jews from Egypt and brought them to the Promised Land). (Swinburne 1993, p. 292; bold emphasis added.)

As has been said (Section 7 of the previous chapter), a proof that there is a perfect being would be of debatable significance for the existence of God. "And for the identity of God," we may add.

2.3 An articulation of the argument. Here, for the light discussion of it can cast on Descartes's reasoning, is a somewhat plainer deduction. Let it be ***our Cartesian argument***. It is a 'demonstration' of the existence of a supremely perfect being, that is, a deduction without aid of contingent premises. To be proved 'from scratch':

(i) A supremely perfect being exists.

Assume, for purposes of argument, that this is *not* so. That is, assume for argument that

(ii) A supremely perfect being *does not exist.*

Observe that the following are available as premises, since they are, for Descartes, necessary truths:

(iii) A supremely perfect being has every perfection.
(iv) Existence is a perfection.

Premise (iii) is for anyone a necessary truth. Premise (iv) is for Descartes necessary in the manner in which the propositions that yellow is a color and that 2 is a number are necessary. From (iii) and (iv) it follows that,

(v) A supremely perfect being *does* exist

This contradicts (ii). Given (iii) and (iv), (ii) is 'no less repugnant to thought than is a mountain without a valley'. Premises (iii) and (iv) are not consistent with (ii). It is not possible that they should be true unless not (ii), but what it negates, (i), is also true. They entail it. Since they are necessary truths, deducing it from them 'demonstrates' it, which is what was to be done.

2.4 A friendly criticism. Leibniz writes of Descartes's argument:

It is not a paralogism, but it is an imperfect demonstration . . . , which assumes something that must still be proved . . . ; it is tacitly assumed that this idea of the all-great or all-perfect being . . . implies no contradiction. [But] it is already something that . . . it is proved that, *assuming that God is possible, he exists*, which is the privilege of divinity alone. [Furthermore] we have the right to presume the possibility of every being, and especially that of God, until some one proves the contrary." (Leibnitz, 1949, p. 504)

Leibniz was right about one thing: Descartes's argument of the *Fifth Meditation* does "assume something that must still be proved."[6] But he was wrong about two other things: Descartes does not prove even the conditional that *if God is possible, then God exists* – I will get back to this!![7] There is also not a general right to presume possibilities, and certainly not when there are considerations that positively raise questions. We know that there is not a greatest number, but only bigger and bigger ones, and we may well suspect that the case is similar regarding things in general. Perhaps there is not a greatest possible being, or a perfect possible being, but only an endless series of greater and progressively improved possible beings. I will not presently go further with this difficulty, since, although it is real, the main trouble with the Descartes's argument lies elsewhere.[8]

2.5 Two objections – 'the usual suspects'

2.5.1 'It would open the floodgates.' The first objection that may come to mind is that similar arguments would establish the existence of things we think do not exist, for example, supremely perfect islands; things we know do not exist, for example, supremely perfect colleagues; and things we hope do not exist, for example, supremely corrupt and bad beings. Famously, the monk Gaunilon made an objection of this kind to the first ontological argument.[9] But while such reflections raise presumptions against ontological arguments and suggest that something is wrong with them, they do not say what is wrong with them. They leave open the possibility that the cases of supreme perfection and maximal greatness are different in ways that make an argument from essence or nature to actual. Critics cannot be content with putting an ontological argument in bad company. They must explain what is wrong with the arguments, including it, of this company.

Descartes, in his reply to objections of Pierre Gassendi, distinguishes perfect bodies such as islands from perfect beings to explain why only the latter necessarily exist. The difference, he says, is that

> existence does not arise out of the other bodily perfections because it can equally well be affirmed or denied of them. . . . But instead of a body, let us now take a thing – whatever this thing turns out to be – which possesses all perfections which can exist together. . . . [W]hen we attend to the immense power of this being, we shall . . . recogniz[e] that it can exist by its own power; and we shall infer from this that this being does really exist and has existed from eternity, since it is quite evident by the natural light that what can exist by its own power always exists. . . . [W]e shall come to understand that . . . existence is contained in the idea of a supremely powerful being. (Descartes 1986, pp. 101–2).

There are two problems with this reply. First, what Descartes says is not *relevant* to his *Fifth Meditation* proof of the existence of a supremely perfect being. That proof does not require that the perfection of existence should be inseparable from the other perfections of a perfect being. It requires only that

it be among them. Second, there is a problem of *substance*. Descartes contends that, whereas the perfection of a particular divine shape such as that of Michelangelo's *David* that belongs to a perfect body is separable from existence, the perfection of omnipotence that belongs to a perfect being is not separable from existence. That, he says, "is quite evident" (Ibid.). But it is not. The idea of a mythical omnipotent is *prima facie* coherent. (I do not say that the idea of a mythical being that could exist by its own power is *prima facie* coherent. But this is because I do not, as Descartes thought he did, find the idea of a being that could exist by its own power *prima facie* coherent. If this idea is coherent, then there is no evident difficulty with joining it to the idea of mythicality.) The first difficulty with Descartes's reply is plainer and more important to our study than the second. He does show that his argument is relevantly different from arguments that must be bad because their conclusions are false. However, as said, even if the charge that it fails with them is correct and conceded, left would be the question of what is wrong with it, and them. A 'blessed isle' refutation may persuade, but it cannot completely satisfy.

2.5.2 Existence is not a predicate, 'don'tcha know'

The most celebrated criticism of this form of argument [is that] existence is not a real predicate [that can serve] as part of [a] definition. . . . [W]hether or not there exists something that corresponds to a concept cannot be settled by definition. The existence of God cannot be deduced from the concept of a perfect being because existence is not contained in the concept or definition of a perfect being. (Quinn 1995, p. 608.)

2.5.2.1. The fault, we are taught, lies in the premise that existence is a perfection. Perfections must be properties, things to be mentioned in full descriptions of objects that have them, and, they say, existence is not a property, but a precondition for having properties. Further, only properties can be included in definitions of kinds. So, existence cannot be so included. Typically, this last point is buttressed. Existence cannot be included in definitions of kinds *because*, it is frequently explained, if it were, we could, simply by including it, define into existence all kinds of things, whereas it is plain that we cannot do that. Leibniz, who sees nothing wrong with including existence in the concept of being, writes that:

Descartes's reasoning . . . assumed that such a being can be conceived or is possible. If it is granted that there is such a concept [that such a being is possible], it follows at once that this being exists, **because we set up this very concept in such a way that it at once contains existence**. (Leibniz 1976, p. 168; bold emphasis added.)

But, our present objectors say, that is precisely what we cannot do; we cannot define sundry things into existence. So we cannot include existence in our

concepts of things, and, since we could if it were a property, the conclusion is that it is not a property of things.

> To say that something does nor does not exist is to make a claim of quite a different order from a claim about what sort of thing it is . . . **[i] we can never, merely by examining a concept deduce that anyone using it is referring to an actually existing thing**. [ii] **We could only deduce this if existence were contained in the concept itself**. . . . From the examination of the concept of deity **we could never discover that God exists; we could do this only if his existence were part of the concept of deity, and indeed Anselm and Descartes talk as if it were – but it cannot be part of any concept**. (Penelhum 1974, pp. 16–17; bold emphasis added.)

That existence cannot be part of any concept of a kind of thing follows from [i] and [ii]. Since any possible property of things can be included in concepts of them, it follows from that that existence is not a property, and therefore not a perfection.[10]

2.5.2.2. This hard line is at best contentious. Is it not part of the idea of a historical person that such a person once existed? Also, one supposes that a person can mean what he wants to by words, as long as he makes his intentions clear and sticks to them. And it seems that a philosopher should be free to *stipulate* that existence is a 'perfection' and thus, by implication, stipulate that it is part of the essence or nature or idea of supreme perfection. Cf., "the ontological arguer is . . . entitled to whatever standards of greatness he wants . . . [as long as he] stick[s] to [them]" (Lewis 1983, p. 13). Leaving aside the idea of perfection, it seems that we should be free to include existence in definitions of all sorts of things, for example, Gassendi's example of 'existing lions' (Descartes 1986, p. 99). Why not? Contrary to Penelhum and others, it is not as if making existence part of the concept of deity were tantamount to defining deities into existence, in the way in which making marriage part of the concept of a husband settles that husbands are one and all married. Only confusions make it appear so.[11] The hard line, that would, *to block Descartes's argument*, ban existence from definitions of kinds of things, whatever the merits of the ban per se, is inspired by an error about his argument. Hard-liners say that one *could*, in Descartes's way, demonstrate the existence of all sorts of things, *if* one could include existence in sundry definitions. That is not true. There is not *this* harm in allowing existence entry into definitions of kinds.[12]

2.6 The flaw of the argument – it is a matter of an article. The trouble with our Cartesian argument is not that it assumes without argument the possibility of a perfect being, and so is incomplete. Nor is the problem that it assumes that existence is a perfection. Its trouble is not metaphysical or theological, nor is it deep. Its trouble is logical and elementary. To bring out the trouble we should not resist definitions that make true by definition that a supremely perfect being exists! We should allow this, so that we may see clearly that there is no

harm in it. Definitions may settle some existential issues, for example, issues in mathematics, but the definitions proposed for supremely perfect beings do not settle the existential issue for them. That they can seem to is due to play that is possible on an ambiguity of 'a' and an amphiboly of the sentence 'a supremely perfect being exists'.[13]

Sentence (i) of our Cartesian argument,

P: ***A*** supremely perfect being exists

has the following interpretations:

P_1: ***Any*** supremely perfect being exists
$(x)(Sx \supset Ex)$

and

P_2: ***At least one*** supremely perfect being exists.
$(\exists x)(Sx \,\&\, Ex)$

Logical symbols are explained with illustrations in Appendix A. The initial article '*A*' in P is ambiguous between '*any*' and '*at least one*'. Here, '(x)' is a universal quantifier, '(∃x)' is an existential quantifier, 'Sx' abbreviates 'x is supremely perfect', and 'Ex' abbreviates 'x exists', so that our symbolic interpretations of the sentence P spelled out in 'official' translations are

P'_1: For each x, if x is supremely perfect, then x exists

and

P'_2: There is at least one x such that, x is supremely perfect and x exists.

P_1 is general-hypothetical. It does *not* say that there is a supremely perfect being that exists, but only that, for anything, *if* it is a supremely perfect being, then it exists. P_2, on the other hand, is existential and says *precisely* that there is a supremely perfect being that exists. As said, P is amphibolous between P_1 and P_2. The sentences

F: A friend of mine has a car

and

H: A husband has a wife

are similarly amphibolous.[14]

P_1 and P'_1 are a little odd, since, given the stipulation that existence is included in the idea of supreme perfection, their predicates do not add to their subjects. It could go without saying that any supremely perfect being *exists*, since the words 'supremely perfect', by present stipulations, already *say* that; compare it with 'any husband has a wife'. P2 is, given that stipulation, similarly odd. P'_2 is a bit odder, given its proximity to the explicitly redundant sentence,

'There *exists* at least one x such that x is supremely perfect and x *exists*'. This remark of redundancy assumes that quantifiers range over things that 'exist' in the sense in which Descartes wants his supremely perfect being to exist. Descartes of course wants God to 'exist in reality' or to 'actually exist'. This oddity can thus be removed by assuming that their generalizations range over things that 'exist in the mind' or that 'possibly exist', of which things that actually exist is a subset. It is best for our purposes to enforce this assumption, so that henceforth the sentences 'every supremely perfect being exists' and 'at least one supremely perfect being exists' are short for '**for every thing that possibly exists, if it is supremely perfect, then it actually exists**' and '**there is at least one thing that possibly exists such that it is supremely perfect and it actually exists**', and similarly for other sentences involved in interpretations of our Cartesian argument.[15]

P_1, which is not existential, is necessarily true, given that by a natural definition a supremely perfect being would have every perfection, and by either natural necessity, as Descartes would claim, or a stipulative definition, as it would be his right to make, existence is a perfection. The status of P_2, which is existential, is so far at best problematic. One *assumes* that, if true, it is not necessarily true. One assumes that it is in this way like the proposition that there is at least one husband who has a wife, which is equivalent to the plainly contingent proposition that there is at least one husband.

2.7 Negative complications. Sentence (ii) of our argument,

P': A supremely perfect being *does not exist*

can be read in either of the following ways:

P'_a: Every supremely perfect being does not exist
P'_b: At least one supremely perfect being does not exist

Furthermore, each of these sentences is, I think, open to two readings, depending on the scope assigned to its negation. Thus, for P'_a, which may be compared with 'everything that glistens is not gold', there are readings

P'_a1: It is not the case that (just) any (that is, that every) supremely perfect being exists

$$\sim(x)(Sx \supset Ex)$$

and

P'_a2: Every supremely perfect being does not exist.

$$\sim(x)(Sx \supset \sim Ex)$$

And P'_b, which may be compared with 'at least one guest was not surprised', has the out-of-context readings

P′$_b$1: There is a supremely being such that it does not exist
(∃x)(Sx & ∼Ex)

and

P′$_b$2: It is not the case that there is a supremely perfect being that exists.
∼(∃x)(Sx & Ex)

It is, however, useful to observe that, of the four readings, P′$_a$1 and P′$_b$1 are equivalent [consider that ***′∼(x)(Sx ⊃ Ex)′*** is equivalent to ′(∃x)∼(Sx ⊃ Ex)′, this to ′(∃x)∼(∼Sx ∨ Ex)′, this to ′(∃x)(∼∼Sx & ∼Ex)′, and this finally to ***′(∃x)(Sx & ∼Ex)′***] and P′$_b$2 and P′$_b$2 are equivalent [consider that '***(x)(Sx ⊃ ∼Ex)***' is equivalent to ′∼∼(x)(Sx ⊃ ∼Ex), this to ′∼(∃x)∼(Sx ⊃ ∼Ex)′, this to ′∼(∃x)∼(∼Sx ∨ Ex)′, this to ′∼(∃x)(∼∼Sx & ∼∼Ex)′, and this finally to ***′∼(∃x)(Sx & Ex)′***]. So we have just two *significantly* different readings for P′, which we can name, dropping alphabetic, and resuming numeric subscripts, P′$_1$ and P′$_2$. Of these, given the stipulation that existence is a perfection, P′$_1$ is necessarily false, for it says that there is a being that, although it has every perfection, lacks the perfection of existence. Regarding P′$_2$, however, one assumes, pending clear reasons to the contrary, that, even if false, it is not necessarily false. One assumes, pending clear reasons to the contrary, that there might *not* be – that there *might* not be – a supremely perfect being.

2.8 *Deflating our Cartesian argument.* The argument *claims* that the sentence P: (1) expresses an existential proposition, a proposition that says that there *exists* a supremely perfect being, and (2) expresses a necessary truth. Claim (1) is implied by the *point* of the argument, which is to demonstrate the existence of a supremely perfect being. Claim (2) is implied by the argument's pretence to be a *demonstration* that proceeds without aid of contingent premises. Successful demonstrations establish that their conclusions are necessarily true. We have seen that P does, when interpreted in one way, P$_2$, express an existential proposition, and that it does, when interpreted in another way, P$_1$, express a proposition that, given premises that we may accept themselves as necessary truths, is a necessary truth. But we have not seen that P expresses a proposition that is *both* existential and necessarily true. Certainly the general hypothetical, that for each x, *if* x is supremely perfect, then x exists, *is not existential*. And we assume that the existential, that there is at least one x such that x is supremely perfect and x exists, *is not necessarily true*. More to the point, however, is that our argument has not demonstrated that it is, for it has not shown that *this* proposition is entailed by its premises.

To make plain this last negative point, we may go through the putative demonstration of the existence of a supremely perfect being, avoiding all troublesome ambiguities. Here we go. To be proved 'from scratch':

(i*) At least one supremely perfect being exists.
(∃x)(Sx & Ex)

Assume, for purposes of argument, that this is *not* so. That is, assume for argument that,

(ii*) No supremely perfect being exists.
$\sim(\exists x)(Sx \,\&\, Ex)$

The following premises are available for a 'demonstration', since they are, for Descartes, necessary truths:

(iii*) Every supremely perfect being has every perfection.
(iv*) Existence is a perfection.

From (iii) and (iv) it follows that

(v*) Every supremely perfect being exists
$(x)(Sx \supset Ex)$

or, equivalently,

(v**) No supremely perfect being *fails* to exist.
$\sim(\exists x)(Sx \,\&\, \sim Ex)$

However, the 'proof' stops here, for (v**) does not contradict (ii*). They are both true, if there is not a supremely perfect being.[16] They can be made to *look* like contradictories by using, for their expressions, the amphibolous sentences '*a supremely perfect being does not exist*' and '*a supremely perfect being exists*,' but this is no longer even an excuse for saying that they are contradictories.

Our argument does not work for the conclusion it is after. Indeed, no argument *can* work for it, since, assuming the correctness of the following symbolizations, it is *not valid*. In these symbolizations 'Py' abbreviates 'y is a perfection', 'H(x,y)' abbreviates 'x has y', 'E' abbreviates 'existence', and 'P(E)' and 'H(x,E)' abbreviate, respectively, 'existence is a perfection' and 'x has existence'.

(iii*) Every supremely perfect being has every perfection.
$(x)[Sx \supset (y)[Py \supset H(x,y)]$
(iv) Existence is a perfection.
$P(E)$
$\therefore$ At least one supremely perfect being exists.
$(\exists x)[Sx \,\&\, H(x,E)]$

The symbolic argument is not valid: This is established by a model in Section B1 of Appendix B, in which 'P(E)' is made true and nothing x is such that Sx, so that (iii*) is true and the conclusion is false. On the other hand, our Cartesian argument *works* when interpreted to demonstrate the general hypothetical that, for each x, if x is supremely perfect, then x exists, which may now be symbolized by '$(x)[Sx \supset H(x,E)]$'. A derivation in Section B1 of Appendix B, using the just-displayed symbolizations, confirms this last point, which is small

comfort for a philosopher who aspires to a demonstration of the existence of a supremely perfect being.[17]

2.9 *Our minds are open.* We are ready to entertain reasons for thinking that

P_2. At least one supremely perfect being exists

like

P_1. Any supremely perfect being exists

is necessarily true, though no reasons are in view. It is not a reason, or even an excuse, for thinking that the sentence 'a supremely perfect being exists' is amphibolous between existential P_2 and necessary P_1.[18]

3. PART TWO: "MR. SPINOZA, MEET MR. RUSSELL"

I will pay great attention in future to the article in question.
Dorothy Sayers[19]

3.1. Ian Hacking writes that "[T]here are of course a great many ontological arguments. Some of them start with what purports to be a name, 'God'; some let that word stand for a definite description, 'The supreme being'" (Hacking 1978, p. 629). Funny that he should have said so, for English translations of Descartes's and Anselm's arguments, 'work' neither the name 'God' nor related definite descriptions. They feature not the definite article 'the', but the indefinite article 'a' and terms such as 'something' with the same force, and 'indefinite' rather than 'definite' descriptions, as Bertrand Russell might say (Russell 1919, p. 167). I think that no ontological argument of significance features in an available translation a definite description for God or exploits possible conditions for the use of the name 'God'. There is, however, an argument – Baruch Spinoza's somewhat neglected entry in the classical line-up – that would have the former character translations better than available ones.

3.2 Spinoza's 'Ontological Argument.' It is his primary, short and sweet proof of Proposition 11.[20]

PROPOSITIO XI.

Deus, sive substantia constans infinitis attributis, quorum unumquodque æternam, & infinitam essentiam exprimit, necesessariò existit.

DEMONSTRATIO.

Si negas, concipe, si fieri potest, Deum non existere. Ergo (*per Axiom. 7.*) ejus essentia non involvit existentiam. Atqui hoc (*per Proposit. 7.*) est absurdum: Ergo Deus necessariò existit. *Q.E.D.* (*Spinoza Opera* II, Heidelberg 1925, p. 8.)

Now comes Samuel Shirley's translation, which is article-free as far as '*substantia*' goes.

Proposition 11. *God, or substance consisting of infinite attributes, each of which expresses an infinite essence, necessarily exists*. Proof. If you deny this, conceive, if you can, that God does not exist. Therefore (Ax.7) his essence does not involve existence. But this is absurd (Pr.7). Therefore God necessarily exists.

Citations are to:

[Axiom] 7. *If a thing can be conceived as not existing, its essence does not involve existence.*

Proposition 7. *Existence belongs to the nature of substance*.[21]

Also important to the argument is

Definition 6. By God I mean an absolutely infinite being; that is, substance consisting of infinite attributes, each of which expresses eternal and infinite essence.

Spinoza's primary demonstration of Proposition 11, his four-sentence proof of it, is similar in several ways to arguments of Anselm and Descartes. Like theirs, it purports to deduce the existence of God from what would be God's nature; it proceeds at a point indirectly, and it purports to be a demonstration, that is, a deduction without aid of contingent premises. These are reasons enough to say that it is an 'ontological argument'. It is in the sense of the one who I believe coined this label.[22]

3.3 What was Spinoza's argument? The words 'or substance consisting of infinite attributes' present a problem.

3.3.1 On 'substantia' in Spinoza's Ethics. Samuel Shirley writes:

Spinoza's *Ethics*, composed in . . . Latin . . . presents to the reader a . . . challenging task. My purpose in this Foreword is . . . to help the reader to acclimatize himself to a terminology that is at first . . . forbidding. . . . Substance (*substantia*). . . . For Aristotle the word had several connotations, of which one signified independent primary existence: a substance was the basic metaphysical individual that could exist by itself.

Shirley indicates that Spinoza uses the word in this sense (Spinoza 1982, pp. 21, 22–3). Edwin Curley writes:

there are two . . . strains in [traditional uses of the concept of substance] . . . [i] the concept of an unknowable subject of predicates, and [ii] the concept of an independent being. . . . [O]nly the second is of much importance for understanding of Spinoza. (Curley in Spinoza 1985, p. 404.)

'Substantia' is for Spinoza a count-noun for independent beings. There is, Shirley and Curley agree, no question about that. And Spinoza himself says

so: "By substance I mean that which is in itself and is conceived through itself; that is, that the conception of which does not require the conception of another thing from which it has to be formed" (Spinoza 1982, p. 31.) A consequence is that, when translating the *Ethics* into English, occurrences of 'substance' want to be preceded by occurrences of articles, by 'a' or 'the' or other words to the same effects. It is therefore remarkable that occurrences of 'substance' are rarely if ever embellished in these ways in Shirley's translation. The case is very different in Curley's translation. There they are almost always embellished, usually with an indefinite article. Curley does however caution that "[t]here are no articles . . . in classical Latin. So whenever the translation . . . has either a definite or indefinite article, the reader should be aware that this involves an element of interpretation on the part of the translator" (Curley in Spinoza 1985, p. xv). Presumably the explanation of Shirley's practice is that he wants to leave certain matters of interpretation to readers.

3.3.2. How then are we to *complete* Shirley's translation? Which shall it be for 'substance' in Proposition 11, 'the' or 'a'? Boyle says that 'a' is right: "Prop. XI. God or a substance consisting of . . ." (Spinoza 1910, p. 7). Curley agrees: "*God*, or *a substance consisting of* . . ." (Spinoza 1985, p. 417.) I think that, for four reasons, 'the' is best. First, Spinoza has demonstrated to his satisfaction in Proposition 5 (which is in the path taken to Proposition 11) that "there cannot be two or more substances of the same nature or attribute" (op. cit., p. 411). Why therefore say that he means only *a* in Proposition 11, when he can as well, in view of Proposition 5, mean *the*? (I assume that Proposition 11 specifies a substance-nature, namely, *infinite attributes, each of which expresses eternal and infinite essence*.) Second, the construction 'God, ***or a*** substance' can point two ways, neither of which could have been Spinoza's intention: It can suggest a hedge, as 'Donald Kalish, or at any rate *an* author of *Logic* . . . ' does, and it can suggest a disjunction as 'the speaker, or a member of the audience . . . '. 'God, or a substance' contrasts with 'God, ***a*** substance', which would leave untranslated Spinoza's 'sive', and 'God, ***or the*** substance', which does not raise the problems of these indefinite article constructions. Third, 'sive' is in this text Latin presumably for 'or' in the sense of 'or in other words'.[23] While 'the substance' can begin what could be other words for 'God', 'a substance' cannot: 'God' purports to make a singular reference; 'the substance' begins words for such a reference while 'a substance' does not.[24] A fourth reason implicit in what is to come is that the definite description interpretation makes for better reasoning, which I study in Sections 4 through 8. Not good reasoning, but better. It is simpler and more seductive than the best that I can make of indefinite description interpretation, which I spell out and discuss briefly in Section 9, and less is wrong with it. Clear reasons are needed for making something worse of a text than its words allow.

3.4 Spinoza's reasoning[25] *construed as for the necessary existence of* ***the*** *"substance consisting of infinite attributes, each of which expresses eternal and infinite essence" (i.e., for short, 'the infinite substance').*

3.4.1 The idea of his reasoning so construed. God would be the infinite substance. If you require proof that God necessarily exists (cf., "if you deny this"), suppose that the infinite substance can be conceived not to exist (cf., "conceive, if you can, that God does not exist"). That, given Axiom 7 and Proposition 7, leads quickly to a contradiction. Therefore, given them, we have that the infinite substance cannot be conceived not to exist, which is to say that it necessarily exists. But then, by definition, *God* necessarily exists.

3.4.2 An articulation of this idea.

TO BE PROVED: God, the infinite substance, necessarily exists.

TWO PREMISES

(1) "*If a thing can be conceived not to exist, its essence [or nature] does not involve existence.*" (Axiom 7)

This is dubious. Might not our conception of a thing leave out things essential to it? The 'real essence' of water, some say, is and always has been H_2O, even when no one's conception of water included anything like that. Why should existence be different? But the merits of his premises are beside the issues of my critical discussion of his ontological reasoning, which is not of its *soundness*, but of its *cogency*.

(2) "*Existence belongs to the nature [or essence] of [a] substance.*" (Proposition 7)[26]

This premise may have more going for it. Hume can seem to lend it support: He claimed that "**[w]hatever** we conceive, we conceive to be existent"[27] (Hume 1888, p. 67; bold emphasis added) and that we have no "idea of existence . . . separable from the idea of particular objects" (Hume 1888, p. 623). Spinoza stipulated "that there pertains to the essence of a thing . . . that without which the thing can neither be nor be conceived" (Spinoza 1982, p. 63, Part II, Definition 2). Certainly *nothing* can be that does not exist. Proposition 7 implies that we cannot so much as *conceive* of a *substance* without existence. But no more. As said, the merits of Spinoza's premises are beside my main purpose.

FIRST INFERENCE

(3) *The infinite substance* ***cannot*** *be conceived not to exist.*

Assume, for purposes of an indirect subsidiary demonstration of (3) from (1) and (2) that, to the contrary, (-3) *the infinite substance*

***can** be conceived not to exist*.* [Cf.: "conceive, if you can, that God does not exist."] Then, from (1), *the essence or nature of the infinite substance **does not** involve existence*. But, since the infinite substance is a substance, from (2) it follows that *the essence or nature of the infinite substance **does** involve existence*. Contradiction! [*Assumption (-3) is not the negation of (3). Assumption (-3) is (3) 'unnegated. I read it 'dash 3' not 'not 3'.)]

TWO MORE PREMISES

(4) *If an infinite substance exists, then God is the infinite substance.*

This spells out in one way my 'material mode' take on Definition 6, which I expressed above in the words 'God would by definition be the infinite substance'. Spinoza's words are again, in Shirley's translation: "By God I mean an absolutely infinite being: that is, substance consisting of infinite attributes, each of which expresses eternal and infinite essence" (Spinoza 1982, p. 31). I wish Shirley had not taken the liberty of adding 'an' before 'absolutely infinite being'. Encouraged by the invitation to read in an article, instead of (4) I would have, in its place, if *the* infinite substance exists, that is, if *there is exactly one* infinite substance, then God is the infinite substance. The argument would sound better, and would be served as well, by this weaker conditional premise. Spinoza's conclusion is that God *necessarily* exists, and so its question would not be begged by a categorical premise that simply identified God with the infinite substance. Even if such a premise would 'presuppose' that God, the infinite substance, *exists*, it would not presuppose that this one under two 'names' *necessarily* exists. Still we should not suppose that Spinoza considered as settled by Definition 6 that an absolutely infinite substance exists and that God is the one such substance, so that in Spinoza's mind Proposition 11 merely added that God, this infinite substance, *necessarily* exists. Premise (4) avoids that suggestion. It identifies God with the infinite substance, not categorically, but only conditionally on there being an infinite substance, and, let me add, it intends 'narrow scope' for the definite description 'the infinite substance' in the consequent of (4). Premise (4) leaves to be proved that there exists an infinite substance, as well as that the infinite substance God necessarily exists, and allows the argument's conclusion, (6), silently to include *and so certainly exists*. According, as premise (4) is meant to do, 'narrow scope' to the name 'God' leaves to be proved also that God exists.[28] A 'formal mode' rendering of Definition 6 could be

DfGod 'God' shall be short for 'the infinite substance'.

That would have the advertised virtues of (4). If used instead, it would leave in place the essential points of criticisms to come of this ontological reasoning and simplify some developments at the cost of notational complications and anomalies. Several notes will explain.

(5) *What cannot be conceived not to exist, exists necessarily.*

This is the one point at which I read into his argument something that Spinoza does not make absolutely explicit. The connection that this premise makes between conceivability and necessity is, however, plainly implicit in the compact text of his proof. It is something that Spinoza could have considered so obvious as not to need explicit statement. "Why does what cannot be conceived not to exist, exist necessarily? Because, *what cannot be conceived is impossible*. And if it is impossible that something should not exist, then it is necessary that this thing exists." The essential point, Hume might say, is that "whatever *appears* impossible and contradictory upon comparison of . . . ideas, must be *really* impossible" (Hume 1888, p. 29). He means, I think, that whatever appears impossible because it *is* contradictory upon comparison of ideas is really impossible. Similarly, for Spinoza, 'cannot be conceived' in (5) needs to be short for 'cannot be conceived *without contradiction*', where these are other words for *is* 'contradictory upon comparison of ideas.' And so that they should connect with 'conceived' in (5), also in (1) and (3) 'conceived' needs to be short for 'conceived without contradiction'.

SECOND INFERENCE

(6) *God, the infinite substance, necessarily exists!!*

This follows directly by a sequence of inferences. From (3) we have that the infinite substance exists, for otherwise it could not be true of it, as (3) says that it *is* true of it, that *it* cannot be conceived not to exist. So certainly *an* infinite substance exists. With that we can 'detach' from (4) that *God is the infinite substance*, from which, with (3) again, it follows that *God* cannot be conceived not to exist, for what is true of a thing is true of anything identical with it. So, by (5), *God necessarily exists*. And there, folding the two into one, you have it: *God, the infinite substance, necessarily exists*.

I have answered one question about my articulation of Spinoza's demonstration of Proposition 11. I have said, under (4), why though there is nothing *explicitly* conditional about it, I have placed a conditional in the middle of it. Another question concerns the prominence in my articulation of the description

'the infinite substance', given that, though I argue for an occurrence of a definite description in good translations of Proposition 11, there are none in good translations of Spinoza's demonstration. This, in Shirley's translation (and others that I have consulted), runs in terms of the name 'God' and the pronoun 'his'. I explain Spinoza's choice of words in this way. His words for Proposition 11 recall Definition 6 of God, the terms of which definition are important to his reasoning. It is in particular important that God would be a *substance*. Spinoza does not repeat the description 'the substance consisting of infinite attributes, each of which expresses eternal and infinite essence' in the text of his reasoning but uses 'God' as short for it, because that description is so *long*. He had it in mind, however, and, in a way to be explained, it worked for him. For that explanation, it is best that the description 'the infinite substance' should be more prominent in my articulation of his reasoning than the name 'God'.

3.5 What are we to make of Spinoza's demonstration? I will impugn its logic. I will not, however, simply show that his argument is invalid but will go into his reasoning for it and say how it managed to make the argument seem valid to him. I will not, in addition to this curiously unusual line of attack,[29] also go into possible adaptations of common criticisms of ontological arguments to Spinoza's argument, though for their possible interest I insert here brief remarks about three of them. (1) Spinoza does not explicitly demonstrate in his text the *possibility* of an absolutely infinite substance, as Leibniz would have had him do. Perhaps he would have begun an explicit demonstration by recalling his point that, "if something is absolutely infinite, whatever expresses essence **and involves no negation** pertains to its essence" (Spinoza 1985, p. 409; bold emphasis added). Leibniz begins his argument for the compossibility of all perfections, which he communicated to Spinoza during his meetings with him in November of 1676, thus: "By a *perfection* I mean every simple quality which is positive and absolute or which expresses whatever it expresses without any limits . . ." (Leibniz 1976, p. 167). He might have persuaded Spinoza that only simple and unanalyzable qualities pertain to the essence of what is absolutely infinite, thus enabling Spinoza to use Leibniz's argument to settle the possibility of an absolutely infinite substance. Consider: "I showed this reasoning to Mr. Spinoza . . . He thought it sound, for when he contradicted it at first, I put it in writing" (p. 168).[30] (2) To a Gaunilon, Spinoza could say that his proof of the existence of an absolutely infinite substance cannot be adapted to prove that there are perfect islands. His proof, he could say, is specific to *substances*, of which it can be demonstrated that there is exactly one! There is, he could say, no queue at the gates. (I doubt that this response to "the familiar 'floodgates' objection" (Bennett 1982, p. 75) would be in the end, after sympathetic reflection on Spinoza's principles and deductions, satisfactory.) (3) 'The most celebrated criticism,' according to which existence cannot be part of a definition of a kind of being, does not apply. Existence

comes into Spinoza's argument not as part of a definition of the object of his proof, but as part of the nature of any substance, and thus of what would be the nature of God.[31] Jonathan Bennett writes that Spinoza's ontological argument "is unlike any other in *how* it gets 'existent' into the definiens [of 'God']" (Bennett 1984, p. 72). Spinoza's argument does not do that. A problem, Bennett implies, "is that if we allow existence as a property at all, then it belongs to the essence of everything" (op. cit., p. 72). That, however, is not a problem for Spinoza. It is, after all, what Proposition 7 *says*, for this proposition is about substances, and they, for Spinoza, are the only 'things' there are. Furthermore, there is the problem of what is supposed to be the *problem* with including existence in the essence of a kind of thing. It is not (I cannot resist repeating) as if one would thereby ensure that there exists a thing of this kind!

3.6. To study and assess Spinoza's not-so-simple ontological reasoning, we should, since we can, *symbolize* it. My principal interpretive thesis concerning this reasoning is that symbolizations adequate to it in a monadic quantifier calculus with identity and Russellian descriptions can be based on the following economical *scheme of abbreviation* (in the sense of Kalish et al., 1980, pp. 127–8).

K: *a* can be conceived not to exist;	I: *a* consists of infinite attributes;
V: the essence of *a* involves existence;	S: *a* is a substance;
X: *a* necessarily exists;	G: God.

Predicate letters 'K' for 'c' in 'conceived'(!), 'V' for 'v' in 'involves', 'X' for 'x' in 'necessarily exists', and 'I', 'S', and 'G' for obvious reasons. We may let the domain of quantification include *everything that is*, without begging questions to which Spinoza's argument is addressed. It is not, as has been observed, addressed to the question of whether God is or exists, but to the question of whether God, the infinite substance, necessarily exists. Now come, using this scheme, symbolizations in a monadic quantifier calculus with identity and Russellian descriptions of six sentences featured in my articulation of his reasoning. (Appendix A explains symbols and symbolizations including such as these.)

(1) If a thing can be conceived not to exist, its essence or nature does not involve existence.

$$(1') \ (x)(Kx \supset \sim Vx)$$

(2) Existence belongs to the nature of a substance.

$$(2') \ (x)(Sx \supset Vx)$$

(-3) The infinite substance can be conceived not to exist.

$$(-3') \ \{\neg x(Ix \ \& \ Sx)\}K\neg x(Ix \ \& \ Sx)^{32}$$

or equivalently

$$(\exists y)((x)[(Ix \ \& \ Sx) \equiv x = y] \ \& \ Ky)$$

(4) If an infinite substance exists, then God is the infinite substance.[33]

$$(4') \ (\exists x)(Ix \ \& \ Sx) \supset \{\urcorner x(Ix \ \& \ Sx)\}[G = \urcorner x(Ix \ \& \ Sx)]^{34}$$

or equivalently

$$(\exists x)(Ix \ \& \ Sx) \supset (\exists y)((x)[(Ix \ \& \ Sx) \equiv x = y] \ \& \ G = y)$$

(5) What cannot be conceived not to exist, exists necessarily.

$$(5') \ (x)[\sim Kx \supset X(x)]$$

(6) God, the infinite substance, necessarily exists!!

$$(6') \ \{\urcorner x(Ix \ \& \ Sx)\}[G = \urcorner x(Ix \ \& \ Sx)] \ \& \ X(G)$$

or equivalently

$$(\exists y)((x)[(Ix \ \& \ Sx) \equiv x = y] \ \& \ G = y) \ \& \ X(G)^{35}$$

Left for symbolization is the *problem* of this interpretation of Spinoza's reasoning,

(3) The infinite substance cannot be conceived not to exist.

More elaborate symbolizations are of course possible, and could facilitate study of Spinoza's premises, additional to our study of his reasoning. For example, using the additional abbreviations

N^2: the nature of *a* involves *b*; H^2: *a* has *b*; and E^0: existence

one might symbolize in a full quantifier calculus, (2), that existence belongs to the nature of substance,

$$(x)[Sx \supset N(x, E)],$$

or better, in a full modal quantified calculus with identity, by

$$(x)(Sx \supset \Box[(\exists y)(y = x) \supset (\exists y)(y = x)]),$$

or equivalently in a calculus with an existence-operator *E!* (for which see in *On Originality*, page 57)

$$(x)[Sx \supset \Box(E!x \supset E!x)].$$

These symbolizations suppose that a condition is 'of the essence' of x if and only if it pertains to x in every world in which x exists. And one might symbolize premise (5) – that what cannot be conceived not to exist, exists necessarily – by

$$(x)[\sim K(x) \supset \Box H(x,E)],$$

or, in an identity calculus with an existence operator and the operator '⊗' for 'it is conceivable that,' by

$$(x)(\sim\otimes\sim E!x \supset \Box E!x).$$

Premise (1) might be symbolized to match that by,

$$(x)[\otimes\sim E!x \supset \sim\Box(E!x \supset E!x)].$$

But these and other more elaborate symbolizations would, I think, make only more complicated symbolic arguments, not better or, in what might be supposed to make them work, significantly different ones.[36] If this is right (a note coming says that it is right for the complications just floated), then though his argument does not work, we can see with my simple symbolizations at hand everything of how Spinoza, reasoning of necessity informally, could have thought that it worked.

In the next section I show that symbolic premises (1′), (2′), (4′), and (5′) do not entail conclusion (6′). In Section 3.8 I explain how informal premises (1), (2), (4), and (5) can *seem* to entail (6) *by way of* (3). That is the core of my critique of Spinoza's primary reasoning under Proposition 11, construed as for the necessary existence of God, *the* infinite substance. These logical claims are for conditions in *standard quantifier logic*. In any proper application of this logic, quantifiers range over a *nonempty domain* and *all terms abbreviated denote in this domain*. In the current application, the range is *things that are or exist*, and so it must be supposed for this application that 'God', which term is abbreviated by 'G', denotes something that is or exists. As has been observed, no questions at issue in Spinoza's argument are begged by this supposition that would be, in other contexts, contentious. Standard logic recommends itself for present purposes, because it is familiar and simpler than *free logic,* in which domains of quantification can be empty and terms abbreviated need not denote in an application's domain, *and* because logical claims made below concerning Spinoza's reasoning construed as for the necessary existence of God, *the* infinite substance, all hold as well in *free quantifier logic*: This is explicitly confirmed for major claims in Appendix B. Looking beyond to Section 9 and his reasoning construed as for the necessary existence of God, *an* infinite substance, it will be noted that one inessential claim made there is correct only for standard logic.

3.7 The argument is not valid. Symbolic premises sentences, (1′) (x)(Kx ⊃ ∼Vx), (2′) (x)(Sx ⊃ Vx), (4′) ∃x)(Ix & Sx) ⊃ (∃y)((x)[(Ix & Sx) ≡ x = y] & G = y), and (5′) (x)[∼Kx ⊃ X(x)], do not entail (6′) (∃y)((x)[(Ix & Sx) ≡ x = y] & G = y) & X(G).

For it is possible that there is exactly one thing G (Spinoza would have no quarrel with that!) and that, while K is true of this thing, S, X, V, and I are true of nothing. It can be seen that, in that case, (1′) is true, (2′) and (5′) are 'vacuously true,' (4′) has a false antecedent and is thus true, and (6′) is 'doubly false.' This 'refutation' of the argument – (1′), (2′), (4′), (5′)} ∴ (6′) – is spelled out in Section B3 of Appendix B in a model of which these premises express truths, though this conclusion expresses a falsehood.[37] The symbolic argument, {(1′), (2′), (4′), (5′) ∴ (6′), is not valid, which means that, if – to recall my principal interpretive claim – my symbolization of an articulation of Spinoza's argument is *adequate* and brings out everything that he would

suppose made his argument work, that articulation, {(1), (2), (4), (5)} ∴ (6), and his argument are not valid.[38]

3.8 The heart of the matter – the reasoning of this faulty demonstration trades on an amphiboly of scope. The argument {(1), (2), (4), (5)} ∴ (6) is not valid, but reasoning can *seem* to validate it. For premises (1) and (2) as symbolized really do entail the intermediate sentence

(3) ***The infinite substance cannot be conceived not to exist.***

when it is understood in *one* way; and premises (4) and (5) as symbolized, when conjoined with intermediate sentence (3) understood in *another* way, really do entail (6). Furthermore, while this 'bridging sentence' (3) is in neither way up to *both* of its roles, it is easy and natural in a process of informal reasoning 'to have it both ways' as one labors first from premises {(1), (2)} to (3) and then from {(3), (4), (5)} to (6) – it is easy and natural to have it 'now' up to its entailee role, and 'now' up to its entailor role.

3.8.1. Premise (3) is amphibolous in the way the slightly less idiomatic sentence,

it is not the case that the infinite substance can be conceived not to exist.

is more plainly amphibolous. Both this sentence and (3) itself are open to interpretations that assign their definite description 'narrow scope,'

(3′a) $\sim\{\neg\, x(Ix \,\&\, Sx)\}K[\neg x\,(Ix \,\&\, Sx)]$

and interpretations that assign it 'wide scope,'

(3′b) $\{\neg x(x \,\&\, Ix)\}\sim K[\neg x(Ix \,\&\, Sx)]$

or equivalently

$(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, \sim Ky).$

Indeed, *every* sentence in which 'the' relates to 'not' as it does in (3) has in some contexts a 'narrow-scope' sense and in other contexts a 'wide-scope' sense. Suppose, for example, you were to say, while pointing to a frog in my pond, "The frog in your pond is green." I might to correct say: "No. *The frog in my pond is* ***not*** *green*. It is only the yellow light that makes it seem so."

$$(\exists y)[(x)(Fx \equiv x = y)\& \sim Gy].$$

Or, *using the same words*, but with different emphasis, I might correct differently, saying: "No. ***The*** *frog in my pond is not green.* For that frog, whatever its real color, has company. Look over there!"

$$\sim(\exists y)[(x)(Fx \equiv x = y) \,\&\, Gy].$$

Better words here than '*the* frog in my pond is not green' would be 'it is not the case that *the* frog in my is green'.[39]

3.8.2. The sentence

(3′a) $\sim\{\neg x(Ix \,\&\, Sx)\}\, K[\neg x\, (Ix \,\&\, Sx)]$

or equivalently

$$\sim(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, Ky)$$

is up to the entailee-role: It is entailed by the sentences

(1′) $(x)(Kx \supset \sim Vx)$

and

(2′) $(x)(Sx \supset Vx)$

For entailment, consider that the *negation* of (3′a) says that there is something such that it and only it is both an I and an S, and that it is also a K. From that, and (1′) and (2′), it follows that it is both not a V, and a V, which is impossible. This means that the negation of (3′a) is not consistent with (1′) and (2′), which is to say that *it is impossible that they should be true and it false*. And *that* is to say that they *entail* it. Section B4 of Appendix B contains formal derivations for this entailment that spell out this informal deduction. It contains formal derivations both in standard logic for denoting terms and in free logic for possibly nondenoting terms.

So (3′a) is entailed by (1′) and (2′). *However*, (3′a) is not up to the entailor-role. It, together with (4′) and (5′), does *not* entail (6′). The possibility used in Section 3.7 makes this point as well (as does the model in Section B3 of Appendix B). For, as seen, (4′) and (5′) are true in the case of that possibility, though (6′) is false in it. And, as can be seen, (3'a) is true in this case. For this it is sufficient that it is the negation of

(-3′) $(\exists y)\,((x)\,[(Ix \,\&\, Sx) \equiv x = y] \,\&\, Ky)$,

which says 'for openers' that there is something of which both I and S are true. That is false in this case. Since (-3′) is false in this case, its negation (3′a) is true in it.[40]

3.8.3. The sentence

(3′b) $\{\neg x(Ix \,\&\, Sx)\} \sim K[\neg x\, (Ix \,\&\, Sx)]$

or equivalently

$(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, \sim Ky)$

is up to the entailor-role. It makes with sentences

(4′) $(\exists x)\,(Ix \,\&\, Sx) \supset \{\neg x(Ix \,\&\, Sx)\}[G = \neg x(Ix \,\&\, Sx)]$

or equivalently

(4′) $(\exists x)\,(Ix \,\&\, Sx) \supset (\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, G = y)$

and

(5′) $(x)\,[\sim Kx \supset X(x)]$,

sentences that entail

(6′) $\{\neg x(Ix \,\&\, Sx\}[G = \neg x(Ix \,\&\, Sx)] \,\&\, X(G)$

or equivalently

(6′) $(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, G = y) \,\&\, X(G)$.

For *this* entailment, consider that (3′b) says *inter alia* that there is something of which both I and S are true. Given that, one can detach the consequent of (4′), which says that *the thing of which both I and S are true is identical with G.* Going back to (3'b), we have that it says that ∼K is true of this thing. From that and (5′) it follows that X is true of it, and thus, recalling the consequent of (4′), that *X is true of G*. Putting these two emphasized consequences together makes (6′). Section B5 of Appendix B contains formal derivations that spell out this somewhat involuted informal deduction.

However, while (3'b) is up to the entailor-role, it is not up to the entailee-role. Sentences (1′) and (2′) do not entail (3′b). The possibility used in Section 3.7 is sufficient to this point as well (as is the model of Section B3 of Appendix B). As seen, (1′) and (2′) are true in the case of that possibility. And, as can be seen, (3′b) is false in it. For this it is sufficient (i) that (3′b) says 'for openers' that there is something of which both I and S are true and (ii) that in the case there is nothing of which either is true.

3.8.4 In short. The argument –

> (1) If a thing can be conceived not to exist, its essence or nature does not involve existence: $(x)(Kx \supset \sim Vx)$.
> (2) Existence belongs to the nature of a substance: $(x)(Sx \supset Vx)$.
> (4) If an infinite substance exists, then God is the infinite substance: $(\exists x)(Ix \,\&\, Sx) \supset (\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, G = y)$.
> (5) What cannot be conceived not to exist, exists necessarily: $(x)[\sim Kx \supset X(x)]$.
> ∴ (6) God, the infinite substance, necessarily exists: $(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, G = y) \,\&\, X(G)$ –

is not valid.

Sentence (3)

> 'the infinite substance cannot be conceived not to exist'

is amphibolous between

(3a) It is not the case that there is a thing such that, (i) it and only it is an infinite substance, and (ii) it can be conceived not to exist: $\sim(\exists y)((x)[(Ix \ \& \ Sx) \equiv x = y] \ \& \ Ky)$.

and

(3b) There is a thing such that (i) it and only it is an infinite substance, and (ii) it is not the case that it can be conceived not to exist: $(\exists y)((x)[(Ix \ \& \ Sx) \equiv x = y] \ \& \sim Ky)$.

*The **proof** falters at one point or another, depending on how (3) is made definite*. Its first inference, if from (1) and (2) to **(3a)**, is valid. But *the second inference, if from **(3a)**, (4), and (5) to (6), is not valid*. And though the second inference in the proof, if from **(3b)**, (4), and (5) *to* (6), is valid, *the first inference, if from (1) and (2) to **(3b)**, is not valid*.

Thanks to our charitable natures – by which we are disposed to make the best of English sentence (3) when it is derived, and again, some time, some *real time,* later, when it is used – this sentence can seem to do the impossible. For though (1), (2), (4), and (5) do not entail (6), (1) and (2) do entail *something* that sentence (3) can express; and (4) and (5), taken with *something else* that sentence (3) can express, do entail (6). It is easy not to notice the switch, and that it is only something *else* that carries on to the conclusion (6).

*3.9. Spinoza's ontological reasoning construed as for the necessary existence of **an** infinite substance.* Spinoza's succinct primary reasoning for Proposition 11 loses much of its charm when one attends to the amphiboly of (3). To make his argument 'quite go away,' we shall, now that we are done with a definite description interpretation, study in a similar manner its less likely, albeit more common, indefinite description translations and readings. I will be brief here, notwithstanding that there are more problems with indefinite description than with definite description interpretations of Spinoza's argument.

3.9.1 An articulation of the reasoning so construed.

TO BE PROVED: God, an infinite substance, exists.

$$(\exists x)([I(x) \ \& \ S(x)] \ \& \ G = x) \ \& \ X(G)$$

or equivalently

$$[I(G) \ \& \ S(G)] \ \& \ X(G)$$

TWO PREMISES

(1) "*If a thing can be conceived not to exist, its essence or nature does not involve existence*." (Axiom 7)

$$(1') \ (x)(Kx \supset \sim Vx)$$

(2) "*Existence belongs to the nature of [a] substance*." (Proposition 7)
(2′) (x)(Sx ⊃ Vx)

FIRST INFERENCE

(3*) *An infinite substance cannot be conceived not to exist*. (From 1 and 2)

Assume, for purposes of an indirect argument to the contrary, that **(-3*)** an infinite substance can be conceived not to exist. Then, from (1), *the essence or nature of an infinite substance **does not** involve existence*. But, from (2), since an infinite substance is a substance, it follows that *the essence or nature of an infinite substance **does** involve existence*. Contradiction!

TWO MORE PREMISES

(4*) If there is an infinite substance, God is an infinite substance.
(∃x)(Ix & Sx) ⊃ [(I(G) & S(G)]

This is another take, still conditional, on Spinoza's Definition 6 for 'God'. Now, it does not conditionally identify God with a certain thing but merely conditionally characterizes God as a certain kind of thing.

(5) What cannot be conceived not to exist, exists necessarily.
(5′) (x)[∼Kx ⊃ X(x)]

SECOND INFERENCE

(6) *God, an infinite substance, necessarily exists!!* (From 4, 5, and 3.)
(6′) [I(G) & S(G)] & X(G)

Given (3*), which speaks of an infinite substance, we may gather that there is one, and with that settled go to (4*) and 'detach' that God is such a substance. But then, according to (3*) again, it follows that God cannot be conceived not to exist. So, given (5), God is an infinite substance that necessarily exists.

3.9.2. New sentence (3*) is amphibolous between

(3*a) (x)[(Ix & Sx) ⊃ ∼Kx]

and

(3*b) (∃x)[(Ix&Sx) & ∼Kx].

New sentence (-3*) is amphibolous between,

(-3*)a (x)[(Ix & Sx) ⊃ Kx]

or equivalently

∼(3*b) ∼(∃x)[(Ix & Sx) & ∼Kx],

and

(-3*)b $\quad (\exists x)[(Ix \,\&\, Sx) \,\&\, Kx)]$

or equivalently

$\sim$(3*a) $\quad \sim (x)[(Ix \,\&\, Sx) \supset \sim Kx]$.

3.9.3. Now for inferences in this reasoning, and the argument of it. The *first inference* 'works' for (3*a) but not for (3*b). Premises (1′) and (2′) entail (3*a) but not (3*b). Connectedly, (-3*)b is equivalent to $\sim$(3*a), and (-3*)b together with (1′) and (2′) does entail a contradiction. However, only (-3*)a is equivalent to $\sim$(3*b), and (-3*)a is consistent with (1′) and (2′). The subsidiary argument for the first inference works exactly for (3*a).

The *second inference* breaks down into two. The first of these goes to [I(G) & S(G)], using a version of (3*) and (4*), and the second goes to X(G) by way of $\sim$K(G), which is to come from that first result by (5) and a version of (3*); see the informal deduction under (6) in the preceding section. However, while (3*b) and (4*)' entail [I(G) & S(G)], to get from that to $\sim$K(G), and then on to X(G) by (5), one needs (3*a). No single interpretation of (3*) serves the second inference, let alone serving not only it, but the first inference.

Regarding the argument of this reasoning – Spinoza's argument construed as for the necessary existence of God, *an* infinite substance – it can be confirmed that (1′), (2′), (4*), and (5′) do not entail (6′).

Lastly, for good measure we may observe that an argument for (6′) that is valid in standard logic does result, if, instead of the conditional (4*), we suppose that Spinoza's Definition 6, "By God I understand a being absolutely infinite" delivers to go with (1′), (2′), and (5′), the categorical consequent of (4*), God is an infinite substance,

(4**) $\quad I(G) \,\&\, S(G)$

or equivalently in standard logic

(4**′) $(\exists x)([I(x) \,\&\, S(x)] \,\&\, x = G)$.

It is, however, not plausible that Spinoza understood his Definition 6 in a way that made it in this manner 'creative.'[41]

3.10. With these comments on an indefinite description interpretation of Spinoza's reasoning added to those on a more likely definite description interpretation, we may be done with it. His argument is not valid in the manner he supposed that it was. It is not valid, as he would have had it be, merely by principles for quantifiers with identity, and descriptions, and of course without begging its compound question of God's necessary existence and relation to infinite substance. Appearances of validity derive from 'logical illusions'

that have to do with descriptions, especially, I think, definite descriptions, and ways in which descriptions can relate to negation. There is no reason to think that there lurks in his text a demonstration that uses additional logical principles. I trust, however, that we can see how his argument could have seemed to him demonstrably valid, this so that we may appreciate his bother, as well perhaps as that of another of our betters, Leibniz,[42] and be ourselves free of it.

It is a great resource of modern philosophy, of which it does not always take full advantage, that it has as its disposal modern symbolic logic, including the formal development of "that paradigm of philosophy, Russell's theory of descriptions" (Frank Ramsey, quoted in Moore 1944, p. 177). Great philosophers of ontological-argument-fame, have sometimes been seriously disadvantaged by being without this logic. Had Descartes, Spinoza, and Leibniz had it, they might have seen through amphibolies of definite and indefinite descriptions and not have perpetrated their ontological arguments.[43]

ON ORIGINALITY

The relation I have made of Russell's theory of descriptions to ontological reasoning may be original. William Alston gives a Russellian analysis of 'the P does not exist' to bring out its "self-defeating character" (Alston 1965, p. 90), without noticing that that sentence has another Russellian analysis that is not self-defeating. He does not use Russell's scope indicator or suggest that definite description ontological arguments might thrive on not noticing the scope indeterminacy of 'the P does not exist'. The problem of ontological arguments is, in his view, that they treat existence as a predicate, whereas it is not (p. 28). John Hicks implies that Bertrand Russell used his theory of descriptions against ontological arguments, but Hicks explains, and by implication attributes to Russell, a critique that merely updates the 'existence is not a predicate' line (Hick 1967, pp. 539–40). Jan Berg considers 'reconstructions' of Anselm's argument that would prove that *the* x such that nothing greater can be conceived exists (for Anselm it was 'a'). His reconstructions do not go to *how* this existence is supposed to follow, directly or indirectly. So, while, without using classical scope-notation, he does distinguish wide- and narrow-scope negations (Berg 1961, pp. 101, 106);[44] he does not use the distinction to explain how Anselm's actual argument, though not valid, can seem to be valid. His object is only a 'reasonable reproduction' (p. 106). Jaako Hintikka tries "to reconstruct some of the most important aspects of the ontological argument in terms having . . . to do . . . nothing whatsoever with definite descriptions" (Hintikka 1969, p. 85).

Russell and Whitehead, when commenting on negative existential sentences whose subject terms are definite descriptions, do not use scope notation. They write as if these sentences had only narrow-scope interpretations – "Suppose we say: 'The round square does not exist.' It seems plain that this is

a true proposition. . . . Generally, when 'the so-and-so' is said not to exist, we have a proposition of the form

$$\text{`}\sim E!(\urcorner x)(\phi x),\text{'}$$

i.e.,

$$\sim\{(\exists c)\colon \phi_x \,.\, \equiv_x \,.\, x = c\}\text{''}$$

(Whitehead and Russell 1962, p. 66). The latter is in our notation $\ulcorner\sim(\exists c)(x)(\phi \equiv x = c)\urcorner$, where x is free in ϕ and c is not free in ϕ, which 'says' that it is not the case that there is exactly one ϕ. This analysis of propositions of the form $\ulcorner$the ϕ does not exist$\urcorner$ is short for the redundant form $\ulcorner$it is not the case that there exists exactly one ϕ that exists$\urcorner$, which is equivalent to the narrow-scope interpretation of $\ulcorner$it is not the case that the ϕ exists$\urcorner$, or equivalently, $\ulcorner$it is not the case that there exists something to which the ϕ is identical$\urcorner$; in our notation,

$$\sim\{\urcorner\alpha\phi\}(\exists\beta)\urcorner\alpha\phi = \beta.$$

As said, the wide-scope interpretation of sentences of the form $\ulcorner$the ϕ does not exist$\urcorner$

$$\{\urcorner\alpha\phi\} \sim (\exists\beta)\urcorner\alpha\phi = \beta$$

and the sense of 'the round square does not exist,' in which it expresses a *false* proposition, indeed a *necessarily* false proposition, are ignored by the authors of *Principia Mathematica*. They miss in the quoted passage the scope-amphibolies of sentences of the form 'the so-and-so does not exist'.

Noteworthy is the studied absence of their scope indicator '$[(\urcorner x)(\phi x)]$' from '$E!\urcorner\alpha\phi$'. They did not extend their apparatus of description scope to embedded occurrences of $E!(\urcorner x\phi)$-formulas. These *sans* scope description $E!$-formulas are not addressed by

$$\text{“14.01}\qquad [(\urcorner x)(\phi x)] \cdot \psi(\urcorner x)\phi x) \,.=\colon (\exists b) : \phi x \,.\, \equiv_x \,.\, x = b : \psi b \;\; \text{Df”}$$

They are covered and explained only by

$$\text{“14.02}\qquad E!(\urcorner x)(\phi x) \,.=\colon (\exists b) : \phi x \,.\; \equiv_x \;.\, x = b \;\; \text{Df”}$$

(Whitehead and Russell 1962, p. 175). Whitehead and Russell would have said that $\{\urcorner\alpha\phi\}\ E!\urcorner\alpha\phi$-formulas, if countenanced, would not be covered and explained by 14.01. Suppose an application of 14.01 to '$\{(\urcorner x)(Fx)\}\ E!(\urcorner x)(Fx)$'; that could lead to '$(\exists x)[(y)(Fx \equiv y = x)\ \&\ E!x]$'. They say, however, that their "definition give[s] no meaning to '$E!x$'" (p. 175). They thought that no meaning could be found for such applications of an existence operator and that existence operators only 'make sense' when applied to descriptions: "[T]here is no reason, in philosophy, to suppose that a meaning of existence could be found which would be applicable to immediately given

[not given by description] subjects" (p. 175). That is their 'update' of 'existence is not a predicate': It makes no sense, they say, as a 'term-operator' either.

They were wrong about that. Although there is not much point to it in standard denoting-term logic, one can in this logic have an operator '*E!*' such that, for any term τ, $\ulcorner E!\tau \urcorner$ is a formula and every formula

$$E!\tau \equiv (\exists\alpha)\ \tau = \alpha$$

(α a variable other than τ if τ is a variable, is an axiom), so that $\ulcorner E!\tau \urcorner$ can be read $\ulcorner \tau$ exists$\urcorner$. That makes sense, as Whitehead and Russell would agree, since they are committed to $[(\exists\alpha)\ \tau = \alpha]$-formulas making sense (see 13.195, op. cit., p. 168). Had they established this existence operator, they would have had every formula

$$\{\urcorner\alpha\phi\}E!\urcorner\alpha\phi \leftrightarrow (\exists\beta)(\alpha)(\phi \equiv \alpha = \beta),$$

(α free, β not free, in ϕ), as a *theorem*, for they would have had every formula

$$\{\urcorner\alpha\phi\}E!\urcorner\alpha\phi \leftrightarrow \{\urcorner\alpha\phi\}(\exists\beta)\urcorner\alpha\phi = \beta,$$

and they already have every formula

$$\{\urcorner\alpha\phi\}(\exists\beta)\urcorner\alpha\phi = \beta \equiv (\exists\beta)(\alpha)(\phi \equiv \alpha = \beta)$$

(α free, β not free, in ϕ) as a theorem. They could have deleted from their manuscript definition 14.02 and left everything in other numbered principles unchanged, given their practice of suppressing indicators for descriptions with narrowest scope. The establishment of diverse possible scopes for descriptions in embedded $E!\urcorner\alpha\phi$-expressions[45] would have made this difference; however, it could have rung useful bells when these turned up in philosophic asides of possible relevance to, for example, ontological arguments.

I believe that it never occurred to Russell that with his scope apparatus he could make hay of some ontological arguments. It is a shame. He would have enjoyed doing that. Incidentally, even if Russell, for a reason to do with the "meaning of existence" (op. cit., p. 175), refused to use his scoped $\urcorner$-notation when symbolizing 'the infinite substance does not exist', he could have done 'my number' on an argument such as Spinoza's that runs in terms of *necessary existence*. He could have used his theory on an indirect argument for the proposition that the infinite substance necessarily exists, which used 'the infinite substance does not necessarily exist' to float its assumption, and worked by playing the scope-amphiboly of that, or on an indirect argument for a purported equivalent to the proposition that the infinite substance necessarily exists expressed by 'the infinite substance cannot be conceived not to exist'. A possible answer to, Why didn't he do anything like that?, is that he had not thought of such permutations on the theme of ontological reasoning, as he might well not have done, if he was satisfied with available *indefinite* description translations of Spinoza's Proposition 11.

4. PART THREE. ST. ANSELM'S ARGUMENT OF *PROSLOGION* II

[1] Well then, Lord, You who give understanding to faith, grant me that I may understand, as much as You see fit, that You exist as we believe You to exist, and that You are what we believe You to be. [2] Now we believe that You are something than which nothing greater can be thought. [3] Or can it be that a thing of such a nature does not exist, since "the Fool has said in his heart, there is no God?" [*Psalms* 14, l. 1, and 53, l. 1.] [4] But surely, when this same Fool hears what I am talking about, namely, "something-than-which-nothing-greater-can-be-thought", he understands what he hears, and what he understands is in his mind [intellect, understanding], even if he does not understand that it actually exists. [5] For it is one thing for an object to exist in the mind, and another thing to understand that an object actually exists. [6] Thus, when a painter plans beforehand what he is going to execute, he has [it] in his mind, but does not yet think that it actually exists because he has not yet executed it. [7] However, when he has actually painted it, then he both has it in his mind and understands that it exists because he has now made it. [8] Even the Fool, then, is forced to agree that something-than-which-nothing-greater-can-be-thought exists in the mind, since he understands this when he hears it, and whatever is understood is in the mind. [9] And surely that-than-which-a-greater-cannot-be-thought cannot exist in the mind alone. [10] For if it exists solely in the mind even, it can be thought to exist in reality also, which is greater. [11] If then that-than-which-a-greater-cannot-be-thought exists in the mind alone, this same that-than-which-a-greater-*cannot*-be-thought is that-than-which-a-greater-*can*-be-thought. [12] But this is obviously impossible. [13] Therefore there is absolutely no doubt that something-than-which-a-greater-cannot-be-thought exists both in the mind and in reality. (Anselm 1965, translation by M. J. Charlesworth.)

4.1. Anselm's grammatically definite terms 'something-that-than-which-nothing-greater-can-be-thought' and 'that-than-which-a-greater-cannot-be-thought' can remind one of the common practice in natural deduction proofs of existential instantiation, whereby, when it is given that there is something of a kind, for purposes of logical calculations one 'gets down to a particular of this kind.' Anselm offers a proof that *something than which nothing greater can be thought exists* (see his first three sentences), by which, looking ahead, he means 'exists in reality.' But he does not actually draw this conclusion in the last sentence of *Proslogion* II, which is not an existential generalization, but a statement about that particular something-than-which-nothing-can-be-thought' introduced in sentence (4). This allows Anselm to continue, in *Proslogion* III, speaking of this individual. To study the argument of *Proslogion* II, I supply for it the existential general conclusion that something than which nothing greater can be thought exists both in the mind and in reality.

The argument of *Proslogion* II consists, I shall say of a *preliminary argument*, sentences (4) through (8) and the *major argument*, sentences (8) through (13). The job of the preliminary argument is to deliver a premise to the major argument. Now, in broad strokes, the preliminary argument: There is something than which nothing greater can be conceived *in the Fool's mind*, for he

understands these words when he hears Anselm say what he is about to prove. From this Anselm gathers that a particular something-than-which-nothing-greater-can-be-thought is in the Fool's mind. And now the major argument: But even the Fool who says there is no God can see that it is not possible that this thing should exist only in a mind, and not also in reality. For if it did, it would *not* exist both in a mind and in reality, and it *would* exist both in the mind and in reality, which is obviously impossible. Therefore, it exists both in the mind and in reality and so, as was to be proved, *something than which nothing greater can be thought exists both in the mind and in reality*.

4.2 'Something' or, in another word, 'a'. This argument of Anselm's purports to establish that,

> ***something*** *than which a greater cannot be thought exists both in the mind and in reality*

or, in other words that,

> ***a being*** *than which nothing greater can be conceived exists both in the mind and in reality.*

How shall its intended conclusion be understood? Let 'Gx', 'Mx', and 'Rx' abbreviate respectively 'x is such that nothing greater can be thought', 'x exists in the mind', and 'x exists in reality'. We know that possible symbolizations of Anselm's conclusion are

$$(x)[Gx \supset (Mx \,\&\, Rx)]$$

and

$$(\exists x)[Gx \,\&\, (Mx \,\&\, Rx)],$$

with quantifiers ranging over things that exist either in the mind or in reality.[46] Peter van Inwagen observes that, though "Latin . . . has no word corresponding to 'a' or 'an' . . . there is [an] ambiguity in Latin," there is in Latin an ambiguity corresponding to the one "rooted in two different functions performed by [these English] indefinite article[s]" (van Inwagen 1994, p. 80). M. J. Charlesworth seems at pains in his translation of 1965 to maintain this ambiguity.

I conjecture in ignorance of the Latin of Descartes's and Anselm's days that indeterminancies in their writings that correspond to that of the indefinite article's ambiguity were more elusive and seductive than this one of modern English. I also conjecture that they either lacked resources perspicuously to sort out these indeterminancies or had resources that were much inferior to those of modern logic. The problem, I suspect, was not so much with their logical theories as with their language for logical definiteness and calculation. They did their logic in a somewhat stylized form for the purpose of Latin, and not, as we can, in thoroughly regimented symbolic languages

that are by careful design free of ambiguities and amphibolies. Certainly Anselm intends in *Proslogion* II to establish the existential that there is at least one thing that than which nothing greater can be thought that exists not only in the mind but in reality. Still, the universal/existential ambiguity could exercise illicit persuasion. For a proof of the universal generalization could be mistaken for a proof of its lookalike existential generalization. And a proof of this universal generalization from material in *Proslogion* II is possible. This proof – to which Anselm would have no objection, though it is not the proof he intended – comes next. It is followed by a proof – this time essentially the proof Anselm did intend – for that existential generalization itself.

4.2.1 That any *thing than which nothing greater can be thought exists both in the mind and in reality. To be proved*: (1) Any thing than which nothing greater can be thought exists both in the mind and in reality:

$$(x)[Gx \supset (Mx \,\&\, Rx)]$$

or equivalently

$$\sim(\exists x)[Gx \,\&\, \sim(Mx \,\&\, Rx)].$$

For purposes of an indirect proof of (1) we suppose: (2) It is not the case that every thing than which nothing greater can be thought exists both in the mind and in reality:

$$\sim(x)[Gx \supset (Mx \,\&\, Rx)]$$

or equivalently, (3) There is a thing than which nothing greater can be thought that does not exist both in the mind and in reality:

$$(\exists x)[Gx \,\&\, \sim(Mx \,\&\, Rx)]$$

> Let j be a thing such as (3) says there is at least one, that is, let 'j' abbreviate the indefinite descriptive term 'something-than-which-nothing-greater-can-be-thought-that-does-not-exist-both-in-the-mind-and-in-reality'.

(4) j is a thing than which nothing greater can be thought, and j does not exist both in the mind and in reality:

$$Gj \,\&\, \sim(Mj \,\&\, Rj).$$

(5) j exists in the mind:

$$M_j.$$

> *Informal subsidiary argument*. We understand 'j', that is, 'something-than-which-nothing-greater-can-be-thought-that-

does-not-exist-both-in-the-mind-and-in-reality', and what we understand is in the mind. There is no contradiction in this description, for, if there were, then, contrary to (3), there would not be a thing of this description. (Cf. previous note.)

(6) j is a thing than which nothing greater can be thought: from (4)
G_j.

(7) j does not exist both in the mind and in reality: from (4)
$\sim(M_j \& R_j)$.

(8) j does not exist in reality: from (5) and (7)
$\sim R_j$.

(9) j is not a thing than which nothing great can be thought,
$\sim G_j$.

Subsidiary argument for (9) *from* (5) *and* (8). We can *think* of something j′ that is exactly like j, *except* that, though j by (5) and (8) exists in the mind but not in reality, j′ exists in reality as well as in the mind. Even if it existed only in the mind, it could be *thought* to exist in reality as well.[47] This thing j′ of which we can think is greater than j, for existing both in the mind and reality is greater than existing only in the mind. Therefore, (9), j is not a thing than which nothing greater can be thought. *End of subsidiary argument*.

From the contradiction of (6) and (9) we may gather that the supposition (2) made for purposes of this argument is untenable, and that, contrary to it, as was to be proved, *a thing than which nothing greater can be thought exists both in the mind and in reality*: $(x)[Gx \supset (Mx \& Rx)]$. That *could be* mistaken for the similar-sounding existential generalization $(\exists x)[Gx \& (Mx \& Rx)]$. The additional possibility of an 'indefinite description reading' in which 'something' functions as a term could help one to the conclusion that there exists *exactly one* being than which nothing greater can be thought, as he seems to take for granted in the second stage of his argument when he writes of "this being" (Charlesworth translation) and argues that "it cannot even be thought not to exist," and in the third stage when he again writes of "this being" and argues that it is identical with God.[48] Descartes could have been similarly encouraged to write of "God (that is, a supremely perfect being)" (Cress translation), as if 'God' and 'a supremely perfect being' were co-referential singular terms.

4.2.2 That at least one thing than which nothing greater can be thought exists both in the mind and in reality. However, Anselm did not bequeath what merely can be mistaken for a proof of the existential generalization. It would be defamatory to suggest that he was himself 'played' by the amphiboly of

'a thing than which nothing greater can be thought exists both in the mind and in reality'. For there is plainly a proof, an impressive even if not perfectly sound proof, in *Proslogion* II of this existential generalization, and there is no evidence that Anselm supposed that he had at the same time proved the universal generalization.

The 'heart' of the proof, from material of *Proslogion* II, for the universal generalization is the informal subsidiary argument that proceeds from M_j and $\sim R_j$ to $\sim Gj$. Using the terms of this informal argument, one can construct a proof from the existential generalization that a thing than which nothing greater can be thought exists in a mind, $(\exists x)(Gx \,\&\, Mx)$, to the existential generalization conclusion, $(\exists x)[Gx \,\&\, (Mx \,\&\, Rx)]$. This is very relevant to Anselm's text, which first establishes that a thing than which nothing greater can be thought exists at least in a mind: "Even the Fool . . . is forced to agree that something-than-which-nothing-greater-can-be-thought **exists in the mind**, since he understands this when he hears it, and whatever is understood is in the mind." This, that a something-than-which-nothing-greater-can-be-thought exists in the mind, was for Anselm a *premise* for his proof of the existence of such a thing both in the mind and in reality. It was a premise established by a preliminary *a priori* argument encapsulated in the words 'since he understands this when he hears it, and whatever is understood is in the mind'. The proof in 4.3.1, which *could be by someone*, but, we should say, was not *by Anselm*, mistaken for a proof of existence in reality as well, does not use this premise. Now we come to a proof of that existence that does use it. It proceeds ***from this premise***, delivered by a preliminary argument, ***through*** *an indirect subsidiary argument* that, after its assumption, mocks the subsidiary argument of the previous proof, to the conclusion that that thing in the fool's mind exists also in reality, and beyond. *To be proved*: (1) At least one thing than which nothing greater can be thought exists both in the mind and in reality:

$$(\exists x)[Gx \,\&\, (Mx \,\&\, Rx)]$$

(2) At least one thing than which nothing greater can be thought exists in the mind: premise

$$(\exists x)(Gx \,\&\, Mx)$$

Preliminary argument. Even a fool who said in his heart, There is no God, could understand what (1) is about, and understand the words 'a thing than which nothing greater can be thought', and what he understood would be in his mind. *End of preliminary argument*.

Let j be such a thing.

(3) j is a thing than which nothing greater can be thought, and j exists in the mind:

$$Gj \ \& \ Mj.$$

(4) j is a thing than which nothing greater can be thought:

$$G_j.$$

(5) j exists in the mind: from (3)

$$M_j.$$

(6) j exists in reality:

$$R_j.$$

Subsidiary argument for (6) *from* (4) *and* (5). Suppose, for purposes of an indirect argument, that, (7) j does not exist in reality, $\sim$Rj. Then we can *think* of something j′ that is exactly like j, *except* that, though j by (5) and (7) exists in the mind but not in reality, j′ exists in reality as well as in the mind. ("[Even what] exists solely in the mind . . . can be thought to exist in reality also. . . . " Anselm 1965, p. 117.) This thing j′ of which we can think is greater than j, for existing both in the mind and reality is greater than existing only in the mind. Therefore j is not a thing than which nothing greater can be thought, $\sim\mathbf{G_j}$. But by (4) there is such a thing., $\mathbf{G_j}$. This contradiction completes our subsidiary argument for (6) from (4) and (5).[49]

(8) j is a thing than which nothing greater can be thought, and j exists both in the mind and in reality:

$$G_j \ \& \ (Mj \ \& \ Rj) \qquad \text{from (3) and (6).}$$

(9) At least one thing than which nothing greater can be thought exists both in the mind and in reality: Q.E.D.

$$(\exists x)[Gx \ \& \ (Mx \ \& \ Rx)] \qquad \text{from (3).}$$

4.3. The *burden* of the proof falls on its preliminary argument. Anselm's proof is in certain ways like the modal proof of Charles Hartshorne discussed in the next chapter. The premise that *at least one thing than which nothing greater can be thought exists in the mind*, $(\exists x)(Gx \ \& \ Mx)$, is like Hartshorne's Intuitive Postulate, which, adapted to greatness, is that *greatness is possible*, $\Diamond(\exists x)Gx$. That

modal premise, I argue in the next chapter, is strictly question-begging, in an argument that would prove that at least one thing is great, (∃x)Gx, since, given stipulations assumed concerning greatness, that premise is logically equivalent to □(∃x)Gx, which is even more than what is explicitly supposed to be proved. The premise (∃x)(Gx & Mx), without preliminary argument, would be similarly question-begging in Anselm's major argument that I have extended to prove that there is something than which nothing greater can be thought exists both in the mind and in reality, (∃x)[Gx & (Mx & Rx)]. Given Anselm's stipulation concerning greatness, that a thing that exists both in the mind and reality is greater than a thing like it except that it exists only in the mind, it is necessary that (x)[Gx ⊃ (Mx & Rx)]; see the demonstration in the previous section. That necessity entails that (∃x)(Gx & Mx) is logically equivalent to conclusion (∃x)[Gx & (Mx & Rx)].[50] From this equivalence we can gather that the main work of *Proslogion* II is done in what I have cast as its preliminary argument for (∃x)(Gx & Mx).

4.4. The 'charge' against the preliminary argument: 'It is a nonsequitur.' Anselm's efforts in *Proslogion* II may be for some readers abetted by an indeterminancy corresponding to existential/universal ambiguity of 'a' in 'a thing than which nothing greater can be thought exists in both the mind and in reality'. His efforts were for him served by the availability of what can *seem* to be an undeniable premise, which is however question-begging, unless well-argued, since it is *not undeniable*. His opponent Gaunilon maintains that it is *not* undeniable for anyone to whom the argument might be addressed *to persuade*, that is, anyone who is not already convinced that at least one thing than which nothing greater can be thought exists both in the mind and in reality. For any such person can say that, for all he knows, there is not such a thing in his mind *in the sense needed by the argument*. 'A being greater than all others that can be conceived,' I take Gaunilon to say, is *undeniably* in a doubter's mind when Anselm speaks those words to him, no more or less than would be 'an island greater than all others that can be conceived' when some dreamer speaks those words to someone who understands them is *undeniably* in the mind of this auditor. In neither case, I take Gaunilon to say, can the person who hears the words, but is not sure that they identify anything *in reality*, be sure that he has in mind an object, an unsurpassable being, or a blessed isle that must by its nature exist not only in his mind, but also in reality. He cannot deny that he has in mind *the words*, or that he has in mind what they mean, for he understands them. But he can wonder whether he has in mind a thing described by them, for he can understand that he does have a thing in mind, given how these words describe things, if and only if such things exist in reality.

Gaunilon does not say that there is not a thing than which nothing greater can be thought. One gathers that he personally thinks that there is such a thing. He certainly does not say that he does not have these words in mind

when he reads Anselm's inscriptions of them or that he does not then understand them and have what they mean in mind. We may gather, furthermore, that he finds no contradiction in these words. What he denies is that, from his understanding of these words, and we may add from there being no contradiction in them, it *follows* that *he has in his mind* something that must be all that these words entail, including, in particular, being, or existing, *in reality*. He denies this inference, the essential inference of Anselm's preliminary argument, that would place something than which nothing greater can be thought *in the mind*.

We must acknowledge, however, that he denies that inference *only with difficulty*, and not in a manner that makes clear his grounds or fully illuminates the problem of the inference. The problem with Anselm's preliminary argument for an unsurpassable being's *existence in the mind* is essentially the problem that spoils a very similar preliminary argument in the next chapter for an unsurpassable being's *logical possibility* (or 'existence in some possible world'). 'The more accurate explication' (adapting words from Hume 1902, p. 33n) by Charles Hartshorne of Anselm's argument in terms of logical modalities (Section 2 of the next chapter) and the more accurate explication of this objection of Gaunilon cast in these terms (Section 7 of the next chapter) will 'give additional evidence to' to this objection. Understandability and conceptual coherence do not entail logical possibility (Section 8 of the next chapter). Gaunilon, anticipating Leibniz, said in effect to Anselm, "You have not proved *the possibility* of it" (Section 2.4 above) and anticipating, beyond Leibniz, Rowe, wanted to say something like, "You have not proved the possibility of it, *because* it is not sufficient for the possibility of a kind of thing that there be understandable words for the kind, or even that there is no contradiction in the thought of it, and you have provided nothing more to the possibility of an unsurpassable being than that." The understandability and conceivability of a kind of thing and the absence of contradiction in its description are *particularly insufficient* for its possibility, if it is a kind of thing that by its defined natures would actually exist! Here, incidentally, is another reason why there can be no harm in including existence in the definition of a kind of thing. Anselm's argument had more going for it than Descartes's. This objection that I say that Gaunilon started against it is deeper and more important than the 'perfect island objection' for which he is famous. More of the argument and this objection is coming in the chapter.

5. PART FOUR: IMMANUEL KANT'S CRITIQUE OF DESCARTES'S ONTOLOGICAL ARGUMENT

My argument in the spirit of Descartes's dissolves when freed of quantifier equivocations, and the same is true of Spinoza's argument when freed as well of amphibolies of definite descriptions. In contrast, Plantinga's argument (Plantinga 1974a, pp. 108–12 [Plantinga 1974b, pp. 213–21]), arguments that David Lewis considers in Lewis (1970), and Kurt Gödel's argument (discussed

in the next chapter) are, like Hartshorne's, clear of the difficulties with which Parts One and Two of this chapter are concerned; and so, I have contended, is Anselm's argument in *Proslogion* II from which all of these descend. The best-known critic of ontological arguments is, in his criticisms of them, also clear of those difficulties.

5.1. It is sometimes said that Kant's criticism of ontological arguments is that they count existence as a predicate that can be included in concepts and definitions, whereas existence is *not* a predicate and cannot be included in any concept or definition. In fact, Kant does not say that existence is not a predicate. What he says is that, although a logical predicate, as is "[a]nything we please," it is not a real or determining predicate that when "added to the concept of [a] subject . . . enlarges it" (Kant 1958, CPR, p. 504 – A598/B626). The suggestion here is not that the predicate of existence cannot be added to a concept, but at most only that there can be no point to adding it to a concept. That would make it a peculiar predicate, but not without possible company.

G. E. Moore says of goodness that: "It is not . . . like most predicates which we ascribe to things, a part [or partly constitutive of] the thing to which we ascribe it" (Moore 1993, p. 175). Rather, it is a consequence of the constitutive predicates of things. The intrinsic values of things in his view supervene on their natures. But while this goes with Moore's conviction that ascriptions of goodness are not descriptive in the sense in which ascriptions of constitutive properties of things are descriptive, he thinks that what follows is that ascriptions of goodness are descriptive in another sense. 'Good' certainly is a predicate, but a predicate different in kind from those that constitute natures of good things on which it supervenes.

Leibniz can be read as including elements of Moore's ideas about the supervenient character of goodness in his idea of existence. (cf., Curley 1972, pp. 86–8). Leibniz regards existence as a possible property or condition of individuals. An individual concept, for Leibniz, in its fullness determines an entire possible world in which it would be instantiated. Which possible world is actual, and thus which possible individuals exist, depends on the descriptive properties of worlds and individuals. The possible world with the greatest 'quantity of descriptive essence,' that is, the best world, as a consequence of its greatness and value is actual. Possible individuals by their descriptive properties earn existence. 'Existence,' we can read Leibniz as saying, is a predicate, but a predicate different in kind from those that constitute the natures of existent things on which natures it supervenes: "When it is said that 'Adam a sinner exists,' there must be something in this possible notion, Adam a sinner, on account of which he is said to exist" (Leibniz, "On Contingency," quoted in Curley 1972, p. 87). A possible Adam and an actual Adam could not, in Leibniz's view, differ just in that, namely, in that the actual Adam exists. (There is a note on this view at the end of Appendix A to Chapter VI.)

Kant writes that 'existence' is a logical predicate different from those that constitute the natures of existent things. So far he agrees with Leibniz. Whether

Kant agrees further with Leibniz concerning 'the Creation,' and thus with what I am suggesting was Leibniz's view of the supervenient character of existence, I cannot say.

5.2. What is, however, presently more important than Kant's view about the peculiar character of the predicate of existence is that Kant does not suppose that including existence in a concept is tantamount to defining into existence an object answering to this concept. He was concerned to oppose this view, which he considered to be importantly mistaken: "So great, indeed, is the deluding influence exercised by this logical necessity that, by the simple device of forming an *a priori* concept of a thing in such a manner as to include existence within the scope of its meaning, we have supposed ourselves to have justified the conclusion that because existence necessarily belongs to the object of this concept – always under the condition that we posit the thing as given (as existing) – we are also of necessity, in accordance with the law of identity, required to posit the existence of its object, and that this being is therefore itself absolutely necessary" (CPR, p. 502 – A594/B622). Deluded, Kant says, we suppose that because, when we include existence in the scope of a thing's concept, existence then necessarily belongs to any object of this concept, we by this inclusion secure the existence of an object of this concept as something that is itself absolutely necessary. But, Kant says, including existence in the concept of a thing has no such effect. Of course, if existence is included in the concept of a G, then necessarily, for any thing that is a G, this thing exists. However, that is not to say, or to provide a reason, in contrast with an excuse, for thinking that there is a G that necessarily exists. Observe that in making these points Kant allows existence to be a predicate included in concepts of things. He says that it is only a confusion to think that such things must exist, a confusion I would add that is served by the uncertain identity of this thought: 'Such things must exist' can mean that any such thing would necessarily exist (for it is part of its concept that it should), and it can mean that necessarily some such thing does exist.

5.3. Plantinga bases his deconstruction of Kant's critique of 'the' ontological argument on quotations of two long passages other than the one I have quoted (Plantinga 1974a, pp. 92, 94) and thereby misses what I take to be the best of that critique. Also noteworthy is that though Kant claims explicitly only to deflate "the famous ontological argument of Descartes" (CPR, p. 507 – A603/B630), Plantinga assesses Kant's critique as if it were addressed to Anselm's ontological argument (there is no mention of Descartes's argument in Plantinga 1974a) and concludes that it is irrelevant to that argument, which Kant would not mind hearing and which is no surprise given the deep differences between Descartes's and Anslem's arguments. Plantinga forgets what he knows full well, "that the ontological argument comes in an enormous variety of versions" (p. 98) of different strengths and weaknesses and that insightful

criticisms of one version can be irrelevant to other versions. It is strange that Plantinga did not reflect on his own puzzlement with what he took to be the irrelevance of Kant's discussion to Anselm's argument: "But how is this even *relevant*?" (p. 93); "Now how, exactly is all this relevant to Anselm's argument?" (p. 95); "But is it relevant to the ontological argument?" (p. 97); "It is as if," Plantinga implies, "Kant was not even talking about Anselm's argument." "R-i-g-h-t," I say (as philosophers at Wayne State University used to say it, in the manner of Roderick Chisholm), "he wasn't."

5.4. Kant, in his role as critic of ontological arguments, is not part of a tradition I have been at pains to deplore in Parts One and Two. He is not concerned to ban existence from concepts. He does not think that one needs to do this to block ontological arguments. Kant seeks in his critique not to block these arguments, but to dissolve them. My criticisms of classical arguments such as Descartes's and Spinoza's are in the spirit of his. I detail 'the possible deluding influence' – that is, the potential for equivocation – of various generalizations and of negative definite description sentences in order that they should not delude. That complements Kant's animadversions on 'the deluding influence' of interplays between necessity and quantifiers when existence is included in a concept of a kind of thing.

5.5. There is no evidence of the Cartesian error I deplore in Kant's critique of Descartes's argument ("The Impossibility of an Ontological Proof of the Existence of God"), but, I regret, there is such evidence in his critique of cosmological arguments. Indeed we find him there making just this mistake in a comment on the 'precise assertion of the ontological proof'! Kant writes in "The Impossibility of a Cosmological Proof of the Existence of God" that, if a search of concepts turned up one that contained the condition of the possibility of a necessary being, we could in this way "establish its existence" (CPR, p. 510 – A608/B636). He writes: "If the proposition, that every absolutely necessary being is likewise the most real of all beings, is correct [merely in virtue of its concepts]. . . . I can convert . . . and say that ***every ens realissimum*** [*most real being*] is a necessary being. But since this proposition [would be] determined from its *a priori* concepts alone, the mere concept of the *ens realissimum* [would] carry with it the absolute necessity of that being; and this is precisely what the ontological proof has asserted. . . ." (CPR, pp. 510–11 – A608/B636; bold emphasis added). Kant seems here to have been self-deluded. He seems to have forgotten his point that, although we can make necessary that every most real being must exist, by including necessary existence in the concept of a most real being, this is not to say that we can, by that manoeuvre, ensure that there is a most real being that must exist. The point of Part One of this chapter includes that point. It includes that to say that every most real being is a necessary being, $(x)(Mx \supset$

□*E!*x), is not to say that there is a most real being that is a necessary being, (∃x)(Mx & □*E!*x).[51]

To begin a line from a different starting point – a line that leaves behind the confusions addressed in Part One of the present chapter, a line that goes back to Anselm (and so on to Part Three) and ahead to Charles Hartshorne (and so to the next chapter) – it is true that, from the proposition ***MrNb*** that necessarily every most real being would be a necessary being, □(x)(Mx ⊃ □*E!*x), and the proposition that it is possible that there is a most real being, ◊(∃x)Mx, it follows that there is a necessary being, (∃x)□*E!*x. Informally, if something is a most real being, (∃x)Mx, in some world, then, by ***MrNb***, there is something in that world that is most real and a necessary being in that world. Since it is a necessary being in that world, it is a necessary being in every world, including the actual world, in which, therefore, something is a necessary being, (∃x)□*E!*x.[52] But ◊(∃x)Mx and □(x)(Mx ⊃ □*E!*x) do not entail that there is something that is not only a necessary being but also a most real being – they do not entail that (∃x)(Mx & □*E!*x). For that, one might say that most real beings would be essentially most real, and so on, as in the next chapter, to the resistance of Rowe, which updates Gaunilon's.

APPENDIX A. SYMBOLS AND SYMBOLIZATIONS

Connectives	∼ : it is not the case that; ∨: or; &: and; ⊃ : only if; ≡: if and only if
Quantifiers	(x): for each x; (∃y): there is a y such that
Identity and existence	x = y; x is y: *E!x*: x exists
Description operators	℩xFx: the x such that Fx; ℩xFx: the x such that Fx

The difference is that '℩xFx' is a name whereas '℩xFx', Whitehead and Russell remind us, is an 'incomplete symbol.' It can be a proper part of an interpreted meaningful sentences, but it has no meaning in isolation. Only names and sentences can have 'meanings in isolation,' and an ℩-descriptions is neither of these. There is another sense in which '℩xFx' is an incomplete symbol: If it has an occurrence in a sentence, that occurrence 'goes with' a preceding occurrence in the sentence of '℩xFx'. To adapt words Sally addressed to Charlie Brown while 'practicing her brackets' in 1977, 'an ℩-description without a scope-indicator is up to no good.' (She said, "If you ever see a bracket by itself, you can be sure it's up to no good!")

Now come translations of several symbolic sentences using the abbreviations, F: *a* is a frog, G: *a* is green, and A: Jumping Jack.

(x) (Fx ⊃ Gx):
for each x, x is a frog only if x is green; any/every frog is green

(∃x)(Fx & Gx):
there is an x such that x is a frog and x is green; some frog is green

$(\exists y)(x)(Fx \equiv x = y)$:
there is a y such that for each x, x is a frog if and only if x is y;
there is exactly one frog

$\{\neg xFx\}$ $G\neg xFx$: the frog is green
'$\{\neg xFx\}$' is a 'scope-indicator.
It can be treated here as superfluous punctuation, for the simple translation
'the frog is green'.
<<Alternatively it can be read as a 'restricted quantifier' for the translation
'*the x such that x is frog is such that* it is green'>>
equivalently
$(\exists y)[(x)(Fx \equiv x = y)$ & $Gy]$:
there is a y such that for each x, x is a frog if and only if x is y, and y is green;
there is exactly one frog and it is green

$\sim\{\neg xFx\}G\neg xFx$: it is not the case that the frog is green
<<or better, it is not the case that the frog is such that it is green>>
equivalently
$\sim(\exists y)[(x)(Fx \equiv x = y)$ & $Gy]$:
it is not the case that there is exactly one frog and it is green

$\{\neg xFx\}$ $\sim G\neg xFx$: it is not the case that the frog is green
<<or better, the frog is such that it is not the case that it is green>>[53]
equivalently
$(\exists y)[(x)(Fx \equiv x = y)$ & $\sim Gy]$:
there is exactly one frog and it is not green

$\{\neg x(Gx$ & $Fx)\}$ $A = \neg x(Gx$ & $Fx)$: Jumping Jack is the green frog

*E!*A : Jumping Jack exists

$(\exists y)$ $A = y$: there is something to which Jumping Jack is identical

APPENDIX B. DERIVATIONS AND MODELS

B1 A model and a derivation for Section 2.8. The informal proof given in Section 2.2 thrives on (i) its being entirely correct when interpreted as a proof of the universal generalization that every supremely perfect being, if there be any, exists and (ii) its capacity – thanks to the amphiboly of 'a supremely perfect being exists'– to pass as a proof of the existence of a supremely perfect being. *No* proof could be correct from the premises of our argument to the existential generalization that at least one supremely perfect being exists, since those premises do not entail this generalization. Premises and this conclusion were symbolized in Section 2.8 for this last point thus:

$(x)(Sx \supset (y)[Py \supset H(x, y)])$. $P(E)$ $\therefore$ $(\exists x)[Sx$ & $H(x, E)]$

Here is a minimal invaliding model M: In it the premises are true and the conclusion is false:

U: {0}	
S: {}	S: {0}
H: {}	E: 0

M confines quantifiers to the one element 'universe' that contains exactly the number 0. Translating the first premise in terms of this model leads to 'for each x in {0}, if x is in {}, then for each y in {0}, if y is in {0}, then (x,y) is in {}', which is true in this model if and only if

$$(S(0) \supset [P(0) \supset H(0, 0)])$$

is true in this model. Translating the conclusion leads to 'there is at least one x in {0}, such that x is in {} and (x,0) is in {},' which is true in this model if and only if

$$[S(0) \,\&\, H(0, 0)]$$

is true in this model. Calculations establish that the premises are true and the conclusion is false in this model, so that this model does invalidate the argument:

$$\begin{array}{ccccccccc} S(0) & \supset & [P(0) & \supset & H(0,0)]) & P(0) & \therefore & [S(0) & \&\ H(0,0)] \\ f & & t & & f & \mathbf{T} & & f & f \\ & & & f & & & & & \mathbf{F} \\ & \mathbf{T} & & & & & & & \end{array}$$

Now comes a derivation for

$$(x)(Sx \supset (y)[Py \supset H(x, y)]) \,.\, P(E) \;\; \therefore \;\; (\exists x)[Sx \,\&\, H(x, E)]$$

from the premises of the universal generalization that tracks and expands the informal proof of Section 2.2:[54]

1. *SHOW* $(x)[Sx \supset H(x,E)]$
2. $\sim(x)(Sx \supset H(x,E))$ ID: assumption from indirect derivation
3. $(x)(Sx \supset (y)[Py \supset H(x,y)])$: a supremely perfect being has every perfection — premise
4. P(E): existence is a perfection — premise
5. $Sa \,\&\, \sim H(a,E)$ — 3, EI
6. *SHOW* $(x)[Sx \supset H(x,E)$

Since 'x' is not already on a line that gives information that is *specific* to it, to the derive the generalization it is sufficient to

7. *SHOW* $Sx \supset H(x,E)$.

To derive a conditional it is sufficient to assume its antecedent and on the strength of that derive its consequent:

8.	Sx	CD: assumption for conditional derivation
9.	(Sx ⊃ (y)[Py ⊃ H(x,y)]	3, UI
10.	(y)[Py ⊃ H(x,y)]	8, 9, MP
11.	P(E) ⊃ H(x,E)	10, UI
12.	H(x,E)	4, 11, MP

Line 13 completes the derivation for line 8, which once in hand completes the derivation for line 7. The contradiction then on lines 2 and 7 completes an indirect derivation for line 1:

1.	~~*SHOW*~~ (x)[Sx ⊃ H(x,E)]	ID
2.	∼(x)[Sx ⊃ H(x,E)]	ID: assumption for indirect derivation
3.	(x)(Sx ⊃ (y)[Py ⊃ H(x,y)])	premise
4.	P(E)	premise
5.	*SHOW* (x)(Sx ⊃ H(x,E)]	UD
6.	*SHOW* Sx ⊃ H(x,E)	CD
7.	Sx	CD: assumption for conditional derivation
8.	Sx ⊃ (y)[Py ⊃ H(x,y)]	3, UI
9.	(y)[Py ⊃ H(x,y)]	7, 8, MP
10.	P(E) ⊃ H(x,E)	9, UI
11.	H(x,E)]	4, 10, MP

B2 A model for Section 3.7. Symbolic premises sentences (1′), (2′), (4′), and (5′), spelled out below, *do not* entail (6′), which too is spelled out below. For they are true, and it is false in the following minimal model M*:

U:{0}

K:{0}	V:{}	X:{}
S:{}	I:{}	G:0

'G' names 0 in the model M*. So, in M*, X(D) is true only if 0 is in {}, the empty set. This means that X(G) is false in the model. Premise (1′), (x)(Kx ⊃

∼Vx), is true in M*: For this it is sufficient that, in M*,

$$\begin{array}{ccc} K(0) & \supset & \sim V(0) \\ t & & f \\ & t & \\ & \mathbf{T} & \end{array}$$

Premise (2′), (x)(Sx ⊃ Vx), is 'vacuously' true in M*: A '⊃-conditional' is true if its antecedent is false, so in M*:

$$\begin{array}{ccc} S(0) & \supset & V(0) \\ f & & f \\ & \mathbf{T} & \end{array}$$

Premise (4′), (∃x)(Ix & Sx) ⊃ {¬x(Ix & Sx)}[G = ¬x(Ix & Sx)], is true in M*, since its antecedent is false in M*: There is nothing in U that is in both I and S. Premise (5′), (x)[∼Kx ⊃ X(x)], is, like premise (2′), 'vacuously true' in the model: K(0) is true, and thus ∼K(0) is false in M*. Finally, notwithstanding the truth of all of its premises, (6′), {¬x(Ix & Sx)}[G = ¬x(Ix & Sx)] & X(G), *the argument's conclusion*, is false in M*, since X(G), its second conjunct, is false in M*. Its first conjunct is also false since it is equivalent to (∃y)((x)[(Ix & Sx) ≡ x = y] & G = y).[55]

B3. Derivations were promised in Section 3.8.2 to show that my symbolizations of (1) and (2) do entail my symbolizations of (3). These are delivered here. They spell out the informal derivation in Section 3.4.2 of (3) from (1) and (2), which I repeat for ready reference:

> Assume, for purposes of an indirect deduction of (3) – *the infinite substance* ***cannot*** *be conceived not to exist* – from (1) and (2) that to the contrary, (-3), *the infinite substance* ***can*** *be conceived not to exist*. Then, from (1) – *if a thing can be conceived not to exist, its essence or nature does not involve existence* – it follows that *the essence or nature of the infinite substance* ***does not*** *involve existence*. But, since the infinite substance is a substance, from (2) – *existence belongs to the nature of [a] substance* – it follows that *the essence or nature of the infinite substance* ***does*** *involve existence*. Contradiction!

I begin with a pair of indirect derivations in a standard quantifier logic with identity, Russellian descriptions, and Fregean descriptions.[56] This logic is for denoting terms. The first of this pair is addressed to ¬- symbolizations of various sentences and the second to identity theory equivalents of those symbolizations. The second uses only familiar logical principles. An advantage of the first is that it is somewhat closer to the language and natural logic of the informal derivation just recalled. Principles of inference used are explained

in Appendix C.

(i)	~~*SHOW*~~ $\sim\{\neg x(Ix \,\&\, Sx)\}K[\neg x(Ix \,\&\, Sx)]$	(this is (3′a))	ID
(ii)	$\{\neg x(Ix \,\&\, Sx)\}K[\neg x(Ix \,\&\, Sx)]$	ID: assumption for indirect argument (this is ((-3′))	
(iii)	$(\exists y)(x)[(Ix \,\&\, Sx) \equiv x = y]$	(ii), RD*	
(iv)	$K[\neg x(Ix \,\&\, Sx)]$	(ii), (iii), DesInt	
(v)	$K[\neg x(Ix \,\&\, Sx)] \supset \sim V\neg x(Ix \,\&\, Sx)]$	(1′), UI[$\neg$x(Ix & Sx)]	
(vi)	$\sim V[\neg x(Ix \,\&\, Sx)]$	(iv), (v), MP	
(vii)	$I[\neg x(Ix \,\&\, Sx)] \,\&\, S[\neg x(Ix \,\&\, Sx)]$	(iii), PD	
(viii)	$V[\neg x(Ix \,\&\, Sx)]$	(2′), UI[$\neg$x)(Ix & Sx)], (viii), S, MP	

(i)	~~*SHOW*~~ $\sim(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, Ky)$	(this is (3′a))	ID
(ii)	$(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, Ky)$	ID: assumption for indirect argument (this is (-3′))	
(iii)	$(x)[(Ix \,\&\, Sx) \equiv x = a] \,\&\, Ka$	(ii), EI	
(iv)	$\sim Va$	(iii), S, (1′), UI, MP	
(v)	$(Ia \,\&\, Sa) \equiv a = a$	(iii), S, UI	
(vi)	$a = a$	Id	
(vii)	Sa	(v), BC, (vi), MP, S	
(viii)	Va	(vii), (2′), UI, MP	

Next come essentially the same derivations, only this time in a *free* monadic quantifier logic with identity and both Russellian and Fregean descriptions. This logic countenances nondenoting terms:

(i)	~~*SHOW*~~ $\sim\{\neg x(Ix \,\&\, Sx)\}K[\neg x(Ix \,\&\, Sx)]$	(this is (3′a))	ID
(ii)	$\{\neg x(Ix \,\&\, Sx)\}K[\neg x(Ix \,\&\, Sx)]$	ID: assumption for indirect argument (this is ((−3′))	
(iii)	$(\exists y)(x)[(Ix \,\&\, Sx) \equiv x = y]$	(ii), RD*	
(iv)	$K[\neg x(Ix \,\&\, Sx)]$	(ii), (iii), DesInt	
(v)	$E![\neg x(Ix \,\&\, Sx)]$	(iii), FPD	
(vi)	$K[\neg x(Ix \,\&\, Sx)] \supset \sim V[\neg x(Ix \,\&\, Sx)]$	(1′), (v), FrUI[$\neg$x(Ix & Sx)]	
(vii)	$\sim V[\neg x(Ix \,\&\, Sx)]$	(iv), (vi), MP	
(viii)	$I[\neg x(Ix \,\&\, Sx)] \,\&\, S[x(Ix \,\&\, Sx)]$	(iii), PD	
(ix)	$V[\neg x(Ix \,\&\, Sx)]$	(2′), UI[$\neg$x) (Ix & Sx)], (viii), S, MP	

(i)	~~*SHOW*~~ $\sim(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, Ky)$	(this is (3′a))	ID
(ii)	$(\exists y((x)[Ix \,\&\, Sx) \equiv x = y] \,\&\, Ky)$		ID: assumption for indirect argument (this is ((-3′))
(iii)	$E!a \,\&\, [(x)[(Ix \,\&\, Sx) \equiv x = a] \,\&\, Ka]$		(ii), FrEI
(iv)	$E!a$		(iii), S
(v)	$\sim \boldsymbol{Va}$		(iv), (1′), FrUI, (iii), S. S. MP
(vi)	$(Ia \,\&\, Sa) \equiv a = a$		(iv), (iii), S, S, FrUI
(vii)	$a = a$		Id
(viii)	Sa		(vi), BC, (vii), MP, S
(ix)	$\boldsymbol{Va}$		(iv), (2′), FrUI, (viii), MP

B4. Now come the derivations promised in Section 3.8.3 for the entailment of symbolizations of (6), by symbolizations of (3), (4), and (5). These formal derivations spell out the informal derivation given in Section 3.4.2, which, for ready reference again, is

This follows by a sequence of inferences. From (3) – *the infinite substance cannot be conceived not to exist* – we have that the infinite substance exists, for otherwise it could not be true of it, as (3) says that it *is* true of it, that *it* cannot be conceived not to exist. So certainly *an* infinite substance exists. With that we can 'detach' from (4) – *if an infinite substance exists, then God is the infinite substance* – that *God is* the substance, from which, with (3) again, it follows that *God* cannot be conceived not to exist, for what is true of a thing is true of anything identical with it. So, by (5) – *what cannot be conceived not to exist, exists necessarily* – God necessarily exists. And there, folding the two into one, you have it: *God, the infinite substance, necessarily exists*.

First, two derivations in standard logic for denoting terms:

(i)	~~*SHOW*~~ $\{\neg x(Ix \,\&\, Sx)\}[G = \neg x(Ix \,\&\, Sx)] \,\&\, X(G)$	(this is (6′))	DD
(ii)	$\{\neg x(Ix \,\&\, Sx)\} \sim K[\neg x(Ix \,\&\, Sx)]$		(3′b)
(iii)	$(\exists y(x)[Ix \,\&\, Sx) \equiv x = y]$		(ii), RD*
(iv)	$I[\neg x(Ix \,\&\, Sx)] \,\&\, S[\neg x(Ix \,\&\, Sx)]$		(iii), PD
(v)	$(\exists x)(Ix \,\&\, Sx)$		(iv), EG
(vi)	$G = \neg x(Ix \,\&\, Sx)$		(v), (4′), MP, (iii), DesInt
(vii)	$\sim K[\neg x(Ix \,\&\, Sx)]$		(ii), (iii), DesInt
(viii)	$\sim K(G)$		(vi),(vii), Leibniz's Law
(ix)	$X(G)$		(5′), UI, (viii), MP
(x)	$\{\neg x(Ix \,\&\, Sx\}[G = \neg x(Ix \,\&\, Sx)]$		(iii), (vi), DesInt
(xi)	$\{\neg x(Ix \,\&\, Sx)\}[G = \neg x(Ix \,\&\, Sx)] \,\&\, X(G)$		(ix), (x), Adj: *QED*

Were (4′) replaced by – DfGod, 'G' =df '⌝x(Ix & Sx)' – this derivation could be somewhat simplified: Inference (v) could be deleted; the annotation for (vi) could be 'Id, DfGod'; and the annotation for (viii) could be '(vi), DfGod'.

(i)	~~*SHOW*~~ (∃y)((x)[(Ix & Sx ≡ x = y] & G = y) & X(G)	(this is (6′))	DD
(ii)	(∃y)((x)[(Ix & Sx) ≡ *x* = *y*] & ~Ky		(3′b)
(iii)	(x)[(Ix & Sx) ≡ x = a] & ~Ka		(ii), EI
(iv)	Ia & Sa		(iii), S, UI, BC, Id, MP
(v)	(∃x)(Ix & Sx)		(iv), EG
(vi)	(∃y)((x)[(Ix & Sx) ≡ x = y] & G = y)		(v), (4′), MP
(vii)	(x)[(Ix & Sx) ≡ x = b] & G = b		(vi), EI
(viii)	X(a)		(iii), S, (5′), UI, MP
(ix)	a = b		(vii), S, UI, BC, (iv), MP
(x)	G = a		(vii), S, (ix), T
(xi)	X(G)		(viii), (x), Leibniz's Law
(xii)	(∃y)((x)[(Ix & Sx) ≡ x = y] & G = y) & X(G)		(vi), (xi), Adj: *QED*

Next, We have similar derivations in a free logic that countenances nondenoting terms:

(i)	~~*SHOW*~~ {⌝x(Ix & Sx)}[G = ⌝x(Ix & Sx)] & X(G)	(this is (6′))	DD
(ii)	{⌝x(Ix & Sx)} ~K[⌝x(Ix & Sx)]		(3′b)
(iii)	(∃y)(x)[(Ix & Sx) ≡ x = y]		(ii), Rd*
(iv)	E!⌝x(Ix & Sx)]		(iii), FPD
(v)	I[⌝x(Ix & Sx)] & S[⌝x(Ix & Sx)]		(iii), PD
(vi)	(∃x)(Ix & Sx)		(iv), (v), FrEG
(vii)	G = ⌝x(Ix & Sx)		(vi), (4′), MP, (iii), DesInt
(viii)	~K[⌝x(Ix & Sx)]		(ii), (iii), DesInt
(ix)	~K(G)		(vii), (viii), Leibniz's Law
(x)	X(G)		(iv), (5′), FrUI, (ix), MP
(xi)	{⌝x(Ix & Sx)}[G = ⌝x(Ix & Sx)]		(iii), (vii), DesInt
(xii)	{⌝x(Ix & Sx)}[G = ⌝x(Ix & Sx)] & X(G)		(x), (xi), Adj: *QED*

(i)	~~*SHOW*~~ $(\exists y)((x)[Ix \,\&\, Sx) \equiv x = y] \,\&\, G = y) \,\&\, X(G)$	(this is (6′))	DD
(ii)	$(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, \sim Ky$		(3′b)
(iii)	$E!a \,\&\, [(x)[(Ix \,\&\, Sx) \equiv x = a] \,\&\, \sim Ka]$		(ii), FrEI
(iv)	$E!a$		(iii), S
(v)	$Ia \,\&\, Sa$		(iv), (iii), S, S, FrUI, BC, Id, MP
(vi)	$(\exists x)(Ix \,\&\, Sx)$		(iv), S, (v), FrEG
(vii)	$(\exists y)((x)[(Ix \,\&\, Sx) \equiv x = y] \,\&\, G = y)$		(vi), (4′), MP
(viii)	$E!b \,\&\, [(x)[(Ix \,\&\, Sx) \equiv x = b] \,\&\, G = b]$		(vii), FrEI
(ix)	$X(a)$		(iv), (5′), FrUI, (iii), S, S, MP
(x)	$a = b$		(iv), (viii), S, S, FrUI, BC, (v), MP
(xi)	$G = a$		(viii), S, S, (x), Leibniz's Law
(xii)	$X(G)$		(ix), (xi), Leibniz's Law
(xii)	$(\exists y)((x)[(Ix \,\&\, Sx \equiv x = y] \,\&\, G = y) \,\&\, X(G)$		(vii), (xii), Adj: *QED*

APPENDIX C. RULES OF INFERENCE AND FORMS OF DERIVATIONS

Rules of inference

First a definition: an occurrence of a variable α is *bound* in a formula ψ if and only if it stands in ψ in an occurrence of a formula $(\alpha)\phi$ or a formula $(\exists\alpha)\phi$ or an occurrence of a Fregean description $\text{⌝}\alpha\phi$ or a Russellian description $\text{⍱}\alpha\phi$; an occurrence of a variable in a formula is *free* in it if and only if it is not bound in it; and every occurrence of a name letter in a formula is free in it. Now the rules.

EI (existential instantiation): For variable α, formula ϕ, variable β, and formula ϕ' that comes from ϕ by replacing each free occurrence of α by a free occurrence of β, from an existential generalization $(\exists\alpha)\phi$ one may infer ϕ' in a derivation provided that β does not occur on a previous line of this derivation.

FrEI (free existential instantiation): For variable α, formula ϕ, variable β, and formula ϕ' that comes from ϕ by replacing each free occurrence of α by a free occurrence of β, from an existential generalization $(\exists\alpha)\phi$ one may infer $(E!\beta \,\&\, \phi')$ in a derivation provided that β does not occur on a previous line of this derivation.

UI (universal instantiation): For variable α, formula ϕ, term β, and formula ϕ' that comes from ϕ by replacing each free occurrence of α by a free occurrence of β, from a universal generalization$(\alpha)\phi$ one may infer ϕ'.

FrUI (free universal instantiation): For variable α, formula ϕ, term β, and formula ϕ' that comes from ϕ by replacing each free occurrence of α by a free occurrence of β, from a universal generalization $(\alpha)\phi$ together with the existence-formula $E!\beta$, one may infer ϕ'.

EG (existential generalization): For a formula ϕ and a formula ϕ' that come from ϕ by replacing each free occurrence in ϕ of α by a free occurrence of some term β, from ϕ' one may infer $(\exists\alpha)\phi$.

FrEG (free existential generalization): For a formula ϕ and a formula ϕ' that come from ϕ by replacing each free occurrence in ϕ of α by a free occurrence of some term β, from ϕ' together with the existence formula $E!\beta$ one may infer $(\exists\alpha)\phi$.

For a variable α and formula ϕ, let $\neg\alpha\phi$ be a *simple* $\neg$*-description* if no variable other than α is free in ϕ. Let ψ be a *simple* $\neg$*-formula* if and only if there are occurrences in ψ of exactly one $\neg$-description. An $\neg$-description $\neg\alpha\phi$ has *widest-scope* in an $\neg$-formula if and only if this formula begins with an occurrence of the scope-indicator $\{\neg\alpha\phi\}$. $\neg$-descriptions are not 'terms' for purposes of rules such as Universal Instantiation and Leibniz's Law. In *Principia Mathematica* they are 'incomplete symbols' that have "the logical properties" of terms precisely when they are 'proper' (Whitehead and Russell 1962, p. 180). But there corresponds to every $\neg$-description $\neg\alpha\phi$ the Fregean description $\iota\alpha\phi$ that, but for the dot, looks like it. Fregean descriptions are 'complete symbols': They are *bona fide* terms for all purposes whether or not they are 'proper'.

RD* (Russellian descriptions*): For a simple $\neg$-formula χ in which the $\neg$-description $\neg\alpha\phi$ has widest-scope, from χ one may infer the propriety-premise for this $\neg$-description, $(\exists\beta)(\alpha)(\phi \equiv \alpha = \beta)$, wherein β is a variable that is not free in ϕ, which 'says' that there is exactly one ϕ.

DesInt (descriptions interchange): For a simple $\neg$-formula χ in which a simple $\neg$-description $\neg\alpha\phi$ occurs, and formula χ' that comes from χ by deleting every occurrence of the scope-indicator $\{\neg\alpha\beta\}$ and then replacing every remaining occurrence of '$\neg$' by an occurrence of 'ι', from χ and the propriety-premise, $(\exists\beta)(\alpha)(\phi \equiv \alpha = \beta)$, one may infer χ'. Cf.:

> [P]rovided $(x)(\phi x)$ exists, it has (speaking formally) all the logical properties of symbols which directly represent objects. Hence when $(x)(\phi x)$ exists, the fact that it is an incomplete symbol becomes irrelevant. . . . (Ibid.)

PD (proper descriptions): For variable α, formula ϕ, variable β not free in ϕ, and formula ϕ' that comes from ϕ by replacing every free occurrence in ϕ of β by an occurrence of the Fregean description $\iota\alpha\phi$ that is free in ϕ', from the propriety-premise $(\exists\beta)(\alpha)(\phi \equiv \alpha = \beta)$ one may infer ϕ'. [This rule gives formal expression to the idea that, if there is exactly one thing that satisfies some condition, then it does indeed satisfy that condition.]

FPD (free proper descriptions): For variable α, formula ϕ, variable β not free in ϕ, and from the propriety-premise $(\exists\beta)(\alpha)(\phi \equiv \alpha = \beta)$, one may infer the existence-formula $E!\iota\alpha\phi$, and vice versa. [These rules correspond to what Whitehead and Russell cast as a definition of $E!\iota\alpha\phi$: Whitehead and Russell 1962, pp. 30–1.]

Leibniz's Law: For terms in which no variables are free, from $\delta = \delta'$ and ϕ, one may infer ϕ'; and from $\delta = \delta'$ and ϕ', one may infer ϕ, where δ and δ' are terms in which no variables are free, and formula ϕ' comes from ϕ by replacing one or more occurrences of δ by an occurrence or occurrences of δ'. [If $\iota\alpha\phi$ is a simple ι-description, then no variable is free in the Fregean description $\iota\alpha\phi$.]

Id (identity): For any term α, one may enter $\alpha = \alpha$. (This is a premiseless rule of inference.)

S (simplification) 'says' that one can infer from a conjunction either of its conjuncts; Adj (adjunction) 'says' that from two formulas one can infer their conjunction. MP is *modus ponens*, DN is double negation, and MTP is *modus tollendo ponens* or disjunctive syllogism.

Assumptions for, and Terminations of, Derivations (Major and Subsidiary)

ID assumptions for indirect derivations: For any sentence ϕ, if *SHOW* ϕ is on a line, then the assumption $\sim\phi$ may be entered on the next line.

CD assumptions for conditional derivations: For any sentences ϕ and ψ;, if SHOW $(\phi \rightarrow \psi)$ is on a line, then the assumption ϕ may be entered on the next line. (Annotation: assumption (CD).)

Definition. A sentence is *available* on a line at a stage of a derivation if and only if (i) this line does not at this stage contain an *un*canceled '*SHOW*', and (ii) this line is not at this stage in a box.

Terminations of derivations, major and subsidiary: Given a line containing *SHOW* ϕ, if there is no uncanceled *SHOW*-line below this line, then one may simultaneously box all lines under this line and cancel the '*SHOW*' on it, if either

(i) ϕ itself is an available line under this *SHOW*-line (DD – direct derivation);

(ii) for some sentence ϕ, both ϕ and $\sim\phi$ are on available lines under this *SHOW*-line (ID – indirect derivation); or

(iii) given a line containing $(\psi \rightarrow \chi)$, if its consequent, χ, is on an available line under this *SHOW*-line (CD – conditional derivation); and

(iv) given a line containing *SHOW* $(\alpha)\phi$, if there are no subsequent uncanceled *SHOW*-lines, ϕ occupies a subsequent line, and α is not free on a preceding available line, one may simultaneosly box all lines under the *SHOW*-line and cancel the '*SHOW*' on it (UD – universal derivation).

III

Modern Modal Ontological Arguments

Classical arguments of Descartes, Leibniz, and Spinoza, the logics of which are implicitly of quantifiers and descriptions, definite and indefinite, and not modalities, are not valid. These able philosophers were, it seems, taken in by equivocalities of Latin forms corresponding to ambiguities and amphibolies in English of definite and indefinite articles. An argument of Charles Hartshorne, which improves on one of Norman Malcolm and which agrees with one of Alvin Plantinga, stays clear of those difficulties, and indeed of every strictly logical difficulty, as the first ontological argument did. It is essentially the 'major part' of that first argument in clarifying modern dress. The preliminary argument of *Proslogion* II is similarly transcribed by the modern company, as is Gaunilon's main objection to it, which benefits most of all from the modern modal framework in which the debate can now be conducted.

1. NORMAN MALCOLM'S ARGUMENT

Norman Malcolm says that Anselm maintained not only that existence contributes to greatness, and is in Descartes's terms a perfection, but also, in *Proslogion* III, that 'necessary existence' is a perfection:

> Speaking of the being a greater than which cannot be conceived, he says: 'And it so truly exists that it cannot be conceived not to exist. For it is possible to conceive of a being which cannot be conceived not to exist; and this is greater than one which can be conceived not to exist.' (Malcolm 1960, p. 45)

This, Malcolm says, gives a proof of an unsurpassably great being, different from that of *Proslogion* II. It gives a proof from the mere possibility of a perfect, or unsurpassably great, being.

> [I]f God exists His existence is necessary. Thus God's existence is either impossible or necessary. . . . Assuming that [it is not impossible], it follows that He necessarily exists. (p. 50)

But surely it is not impossible that there is a perfect being. Even the fool who says that there is not a perfect being understands what he says and can see that there is no contradiction in it. Even this fool can see by reflection on his own understanding that God, a perfect being, is not impossible. So goes Malcolm's argument. Charles Hartshorne is sometimes represented as spelling out in formal terms Malcolm's Anselmian argument. That is how Plantinga positions him. In fact, as will be explained, there is a flaw in Malcolm's argument, identified by Plantinga, that is not present in Hartshorne's argument. Malcolm supposes that it is enough for his argument that necessary existence should be a perfection. Hartshorne sees that more is needed, and uses more.

. . . . . .

For the record on Anselm. Malcolm takes himself to be expounding an argument that can be found entirely in *Proslogion* III. Although this is not for us important, he is wrong about that. Anselm intended in *Proslogion* III not an independent argument for the existence of God, but a continuation of the argument of *Proslogion* II, to reach in two steps that conclusion, from the result reached in *Proslogion* II that (a certain) something-than-which-nothing-greater-can-be-thought *exists in reality*. The first step, the step taken in the first paragraph of *Proslogion* III, is to enhance that result: Anselm writes "and certainly this being so truly exists that it **cannot be even thought not to exist**" (Anselm 1965, p. 119; emphasis added). The second step identifies this something-than-which-nothing-greater-can-be-thought with God. This step presumed what had not been established, namely, that *this* something-than-which-nothing-greater-can-be-thought is the *only* such thing. Exactly this is never explicitly said let alone argued in the *Proslogion*. That is surprising, given Anselm's extreme acuity.

2. CHARLES HARTSHORNE'S ARGUMENT

After detailing Hartshorne's argument, I will relate it to the argument of *Proslogion* II. Each argument can be set out with two premises. With allowances made for their different philosophical/logical vocabularies, their first premises are identical and their second premises are logically equivalent.

2.1 Hartshorne's deduction. Hartshorne offers a deduction of the existence of a perfect being, $(\exists x)Px$, from two modalized premises (Hartshorne 1962, pp. 50–1). He uses 'N' for necessity and '$\sim$N$\sim$' for possibility. I use instead, as is standard, '$\Box$' and '$\Diamond$'. He dubs his first premise 'Anselm's Principle':

AP $\Box[(\exists x)Px \supset \Box(\exists x)Px]$,

where '$(\exists x)Px$' abbreviates the sentence 'there exists a perfect being'. Since he does not in his argument 'break into this sentence,' he could have used a

single letter for it, say 'P', and made Anselm's Principle

$$\Box(P \supset \Box P).$$

Hartshorne provides for ***AP*** the free translation "perfection could not exist contingently" (op. cit., p. 51) for

$$\sim\Diamond(P \,\&\, \sim\Box P).$$

That is logically equivalent to $\Box(P \supset \Box P)$. Hartshorne may have supposed that ***AP*** is essentially Anselm's finding that "this being [the one proved in *Proslogion* II to exist] so truly exists that it cannot be even thought not to exist" (Anselm 1965, p. 119). Where Anselm writes of *what cannot be thought*, Hartshorne 'reads' *what is logically impossible*. His second premise comes with the comment, "Intuitive postulate (or conclusion from other theistic arguments)" (op. cit., p. 51): it is the apparently innocuous proposition that perfection is possible:

IP $\Diamond(\exists x)Px.$

Hartshorne seems to have had in mind for ***IP*** something other than 'the preliminary argument' of *Proslogion* II for an unsurpassable being exists in the mind, which is not a 'theistic argument'.[1]

It follows from ***AP*** and ***IP*** that there is a perfect being, $(\exists x)Px$. Now comes an informal deduction that goes on to the necessity of $(\exists x)Px$, $\Box(\exists x)Px$. Formal derivations are given in Section B2 of Appendix B. Necessity and possibility are respectively 'truth in every' and 'truth in some' world, more exactly, for a sentence Φ: $\Box\Phi$ is true at a world if and only if it is true at every world, and $\Diamond\Phi$ is true at a world if and only if Φ is true at some possible world. A sentence Φ is true if and only if it is true at the actual world. To 'say' in the coming informal deduction that a sentence is true at a world or worlds, I enter it in a column for it or them.

	Actual world	Some possible world	All other possible worlds
(i) Premises			
IP	$\Diamond(\exists x)Px$		
AP	$\Box[(\exists x)Px \supset \Box(\exists x)Px]$		
(ii) From **IP**		$(\exists x)Px$	
(iii) From **AP**	$[(\exists x)Px \supset \Box(\exists x)Px]$	$[(\exists x)Px \supset \Box(\exists x)Px]$	$[(\exists x)Px \supset \Box(\exists x)Px]$
(iv) From (ii) and (iii) by *modus ponens*		$\Box(\exists x)Px$	
(v) From (iv)	$(\exists x)Px$ – Q.E.D.	$(\exists x)Px$	$(\exists x)Px$
(vi) From (v) – a bonus!	$\Box(\exists x)Px$		

2.2. This argument from ***IP*** and ***AP*** to (∃x)Px that Hartshorne builds at least in part on ideas he finds in *Proslogion* III is an update of 'the major argument' in *Proslogion* II that Hartshorne could have accomplished without recourse to *Proslogion* III, merely by certain translations of key terms.

2.2.1. The major argument of *Proslogion* II can be cast as from two premises, each delivered by a subsidiary argument. The first is that

a perfect being *exists in the mind*,

which is Anselmian speech for Hartshorne's

IP A perfect being *is possible*.
◊(∃x)Px

The second is that

a perfect being cannot exist in the mind, though no perfect being exists in reality

(cf., "that-than-which-a-greater-cannot-be-thought cannot exist in the mind alone" – Anselm 1965, p. 117) or

it cannot be that a perfect being exists in the mind, though no perfect being exits in reality,

which is Anselmian speech for

AnLP (Anselm's Leibnizian Principle) ∼◊[◊(∃x)Px & ∼ (∃x)Px]

or equivalently

□[◊(∃x)Px ⊃ (∃x)Px],

which says that, necessarily, if a perfect being is possible, then a perfect being exists or, in Anselmian speech, that if a perfect being exists in the mind, then a perfect being exists in reality. Hartshorne 'got' his second premise, ***AP***, somehow from the *Proslogion* III's idea that a perfect being would exist, and be perfect, necessarily. ***AnLP*** is rather plainly delivered in *Proslogion* II without anticipation of ideas in *Proslogion* III. Yet ***AP*** and ***AnLP*** are strictly equivalent. I come back to this after establishing that ***IP*** and ***AnLP*** entail (∃x)Px and more.

*2.2.2 An informal deduction that uses **AnLP** instead of **AP**.*

	Actual world	Some possible world	All other possible worlds
(i) Premises			
IP	$\Diamond(\exists \mathbf{x})\mathbf{Px}$		
AnLP	$\Box[\Diamond(\exists \mathbf{x})\mathbf{Px} \supset (\exists \mathbf{x})\mathbf{Px}]$		
(ii) From **IP**		$(\exists x)Px$	
(iii) From (ii)	$\Diamond(\exists x)Px$	$\Diamond(\exists x)Px$	$\Diamond(\exists x)Px$
(iv) From **AnLP**	$[\Diamond(\exists x)Px \supset (\exists x)Px]$	$[\Diamond(\exists x)Px \supset (\exists x)Px]$	$[\Diamond(\exists x)Px \supset (\exists x)Px]$
(v) From (iii) and (iv) by *modus ponens*	$(\exists \mathbf{x})\mathbf{Px}$ – Q.E.D.	$(\exists x)Px$	$(\exists x)Px$
(vi) From (v) – a bonus!	$\Box(\exists x)Px$		

*2.2.3 On the strict equivalence of **AP** and **AnLP**.* It has been established that the arguments

IP: $\Diamond(\exists x)Px$. ***AP***: $\Box[(\exists x)Px \supset \Box(\exists x)Px] \ \therefore \Box(\exists x)Px$,

and

IP: $\Diamond(\exists x)Px$. ***AnLP***: $\Box[\Diamond(\exists x)Px \supset (\exists x)Px] \ \therefore \Box(\exists x)Px$

are valid. It could be similarly established that $\Box(\exists x)Px$ entails each of ***AP*** and ***LP***. From these results it can be gathered that $\Diamond(\exists x)Px$ entails the equivalence of ***IP*** and ***AnLP***, indeed, their logical equivalence:

$$\Diamond(\exists x)Px \ \therefore \Box(\Box[(\exists x)Px \supset \Box(\exists x)Px] \equiv \Box[\Diamond(\exists x)Px \supset (\exists x)Px]).$$

It also can be established that this logical equivalence is entailed by $\sim\Diamond(\exists x)Px$,

$$\sim\Diamond(\exists x)Px \ \therefore \Box(\Box[(\exists x)Px \supset \Box(\exists x)Px] \equiv \Box[\Diamond(\exists x)Px \supset (\exists x)Px]),$$

for it can be establish that $\sim\Diamond(\exists x)Px$ entails both ***IP*** and ***AnLP***. Since the logical equivalence of ***IP*** and ***AnLP*** is entailed by both $\Diamond(\exists x)Px$ and $\sim\Diamond(\exists x)Px$, it holds unconditionally:

$$\therefore \Box(\Box[(\exists x)Px \supset \Box(\exists x)Px] \equiv \Box[\Diamond(\exists x)Px \supset (\exists x)Px])$$

Left as an exercise, if wanted, is the derivation of this sentence in the system of Section B1 of Appendix B. Other possible exercises are derivations of the equivalences to ***AP*** and ***AnLP*** of *Leibniz's Lemma*,

> Either it is not possible that there is a perfect being, or it is necessary: $\sim\Diamond(\exists x)Px \vee \Box(\exists x)Px$ (cf., Malcolm 1960, p. 50.)

3. A FLY IN THE OINTMENT?

3.1. Plantinga finds fault with an argument that he implies is the best that one can get from the ideas of "Professors Charles Hartshorne and Norman Malcolm" (Plantinga 1974a, p. 105). The argument uses the premises,

> It is possible that there is a perfect being: $\Diamond(\exists x)Px$,

which is Hartshorne's ***IP***, and ***PERFnecEx***,

> "A being . . . [is perfect] in a . . . possible world . . . only if [it] *exists in every possible world*" (Ibid.)
> $\Box(x)\Box(Px \supset \Box E!x)$

which can be taken to say that 'necessary existence is a perfection.' The trouble he identifies with this argument is that, though it 'seeks' the conclusion that *there is a perfect being*,

$$(\exists x)Px,$$

all it 'gets' is that *there is a necessary being that is possibly perfect*,

$$(\exists x)(\Box E\,!x \,\&\, \Diamond Px).^{2}$$

> "This is an interesting argument, but suffers from at least one annoying defect. What it shows is that if it is possible that there be a greatest possible being . . . and if that idea includes necessary existence, then in fact there is a being that exists in every world and in *some* world has a degree of greatness that is nowhere excelled." (Op. cit., p. 106.)

3.2 Unfair to Hartshorne. Plantinga may in his critique do justice to Norman Malcolm's idea, which seems to have been that an Anselmian argument is available as soon as one counts necessary existence as a perfection. But Hartshorne would see that ***PERFnecEx*** leaves out an important Anselmian enhancement, namely, that a perfect being would not only necessarily exist, but be necessarily perfect. Part of what lies behind Hartshorne's ***AP*** (Anselm's Principle) is that "anything exemplifying [perfection] merely contingently (so that it [perfection] could have been unexemplified) would be imperfect, and so would not exemplify it after all" (Hartshorne 1962, pp. 52–3).[3] Plantinga implies that though Hartshorne says that "necessary existence . . . is . . . a perfection" [Plantinga 1974b, p. 212], he, just as Malcolm, does not say that necessary or essential perfection is perfection. By the principle of charitable interpretation,

however, we should find that Hartshorne says both things. Plantinga might have come to this conclusion if he had asked himself what, for Hartshorne, were the grounds for ***AP***, and then given him the benefit of the doubt. To be fair to Hartshorne, Plantinga needs to amend the argument of the previous section, replacing ***PERFnecEx*** with ***PERFnecEx&necPerf***,

$$\Box(x)\Box[Px \supset \Box(E!x \,\&\, Px],$$

which can be taken to say that both necessary existence and necessary perfection are perfections. The conclusion sought, that $(\exists x)Px$, then follows. Indeed, it then follows that $(\exists x)[Px \,\&\, \Box(E!x \,\&\, Px)]$.[4] And Plantinga's own argument can be seen to have been anticipated by Hartshorne.

3.3. Hartshorne's response to Plantinga's complaint could have been, "There are no flies on me. For I count as perfections both necessary existence, and necessary perfection, and so I want from you as a second premise to go with the possibility of perfection as a first premise, if not my own ***AP***, then ***PERFnecEx&necPerf***." For a response that would end up the same, Hartshorne could have said (i) that perfect beings would need to be *essentially* perfect, made definite thus, $\Box(x)\Box[Px \supset \Box(E!x \supset Px)]$; and (ii) that perfect beings would necessarily exist, made definite thus, $\Box(x)\Box(Px \supset \Box E!x)$. Condition (i) is weaker than that perfect beings would be necessarily perfect, $\Box(x)\Box(Px \supset \Box Px)$. Conditions (i) and (ii) entail that, necessarily, perfect beings necessarily exist and are perfect, ***PERFnecEx&necPerf***, $\Box(x)\Box[Px \supset \Box(E!x \,\&\, Px)]$. Conversely, ***PERFnecEx&necPerf*** entails (i) and (ii).

Hartshorne's response, instead of leading to a request that Plantinga give him a better second premise, could have been that, properly understood, the possibility of perfection all by itself was premise enough for him. He could have explained that a perfect being would be essentially perfect and a necessary existent, so that to say that a perfect being is possible is to say that

> an essentially perfect necessary existent is possible:
> $\Diamond(\exists x)[\Box(E!x \supset Px) \,\&\, \Box E!x]$.

This possibility entails all by itself that

> a perfect being is necessary: $\Box(\exists x)Px$.

We leave as a challenging exercise the derivation of this entailment in the system of Section B3 of Appendix B.

4. OTHER ANSELMIAN ARGUMENTS

4.1. ***PERFnecEx&necPerf*** says how Hartshorne may have gotten ***AP***. For ***PERFnecEx&necPerf*** entails ***AP***. Indeed, the logically weaker principle

****PERFnecEx&necPerf****: □(x)[Px ⊃ □(E!x & Px)]

entails ***AP***. For this entailment we may reason that, if something exemplifies perfection (∃x)Px, then there is a thing that by ****PERFnecEx&necPerf**** is necessarily existent and perfect, from which it follows that in every world something is perfect □(∃x)(Px). Section B4 of Appendix B spells this reasoning out in a formal derivation. Given that it entails ***AP***, this principle can replace it for the following Anselmian argument:

IP: ◊(∃x)Px. ****PERFnecEx&necPerf****: □(x)[Px ⊃ □(E!x & Px)]
/∴ (∃x)Px

Hartshorne's premises ***AP*** and ***IP*** do not involve quantification into modal contexts. This allows his deduction to go forward in a sentential modal logic. ****PERFnecEx&necPerf**** involves quantification into a modal context, and the deduction from it and ***IP*** must proceed in a quantified modal logic. Similarly for the next argument.

4.2. The argument of the previous section 'goes behind' ***AP*** to a premise that explains it. Now comes an argument that, rather than replacing ***AP*** by a stronger premise that explains it, replaces it by a weaker premise while at the same time replacing ***IP*** with a stronger premise. The replacement for ***AP*** stipulates that perfection is an 'existence-entailing predicate.' One might say that only existents can be purple and pudgy. This replacement says that only existents can be perfect:

PrfExEntlng: □(x)□(Px ⊃ E!x).[5]

"But are not all predicates 'existence-entailing' – do not all predicates presuppose the existence of their subjects?" No. Certainly 'does not exist', as in 'Santa Claus does not exist,' is not existence-entailing: It is not a contradiction to say that Santa Claus, who does not exist, does not exist. Nor is 'does not exist necessarily', as in 'Jahweh does not exist necessarily', existence-entailing: It is not a contradiction to say that Jahweh, who does not exist necessarily, does not exist (simply). For another kind of example, 'is popular the world over' as in 'Santa Claus is popular the world over' is not existence-entailing. It is not a contradiction for a spoilsport to say that, though Santa Claus is popular the world over, there is no Santa Claus, and similarly for 'was only a dream' as the song says that Laura was (though she was not only a dream in the movie). Some predicates that are not existence-entailing might be labeled 'intensional predicates'; 'is loved' and 'is popular' are examples of these. But there is no natural sense in which,

for example, 'does not exist' is an intensional predicate.

From ***PrfExEntlng*** it follows not only that,

$$(\exists x)\Box Px \supset \Box(\exists x)Px,$$

but also that

$$\Box[(\exists x)\Box Px \supset \Box(\exists x)Px].$$

(See Section B5 of Appendix B for a derivation.) And from this latter sentence, and a strengthened version of ***IP***,

StrgIP: $\Diamond(\exists x)\Box Px$,

it again follows that $\Box(\exists x)Px$. ***StrgIP*** says not merely that perfection is possible, but that necessary perfection is possible in the sense that in some world there is a being that is necessarily perfect. In Hartshorne's arrangement, while ***IP*** can seem innocuous, ***AP*** depends on strong Anselmian stipulations concerning perfection. In the arrangement just established, ***StrgIP*** is somewhat more demanding, while ***PrfExEntlng*** seems innocuous.

5. 'IT'S THE POSSIBILITY!'[6]

The burden of Anselmian arguments falls on their possibility-premises. Their other premises are consequences of stipulations concerning what are perfections, and so are not proper objects of resistance, even if they cannot be – as it seems they can be (see Malcolm 1960, p. 46ff; Plantinga 1974a, pp. 106ff) – provided with theological and metaphysical motivation. In contrast, Hartshorne's ***IP***, and its recent strengthening, ***StrgIP***, are not stipulations. These possibility-premises, given in-force stipulations concerning the sense of perfection, make difficult substantive claims. Much of *Proslogion* II is an *argument* for a perfect being's existing at least 'in the mind'. "The postulate of logical possibility . . . is in my view the hardest to justify" (Hartshorne 1962, p. 52).

"But should not these possibility-premises be given the benefit of the doubt on the general ground that things should be judged possible until they are proved impossible?" Leibniz thought so: "We have the right to presume the possibility of every being, and especially that of God, until some one proves the contrary" (Leibnitz 1949, p. 504; cf., Section 2.4 of the previous chapter). Perhaps Leibniz thought that we not only may, but should, presume possible every being that has not been proved to be impossible. This principle is, however, certainly unacceptable. As van Inwagen points out, it would in many cases entail inconsistent 'presumptions.' For an example close to hand, it may be that we cannot prove that Hartshorne's perfect beings are impossible *and* that we cannot prove that "a 'knowno' . . . a being who knows there is no such

perfect being" (van Inwagen 1994, p. 92) is impossible. Suppose so. Then a principle that would have us presume that whatever we cannot prove to be impossible is possible would have us presume possible and, by Hartshorne's argument, necessary, that there is a Hartshorne perfect being, and presume also that a 'knowno' is possible so that it is **not** necessary that there is a Hartshorne perfect being.[7]

6. FOES OF ONTOLOGICAL ARGUMENTS SAY THAT THEIR POSSIBILITY-PREMISES BEG QUESTIONS

Foes of ontological arguments may say that ***IP***, the possibility of a perfect being, *given that being perfect entails existing in and being perfect in every world*, says *more* than the argument's 'extended' conclusion, which is merely that it is necessary that there exists a perfect being, and similarly for ***StrgIP***, the possibility of a being that is perfect in every world, *given that only existents can be perfect*. Why 'more'? Because there can exist a perfect being in every world – which is what $\Box(\exists x)Px$ says – even if each world has a different perfect being. Under italicized stipulations relevant to them, ***IP*** and ***StrgIP*** entail that it is necessary that there exists a being that exists and is perfect in every world, $\Box(\exists x)\Box(E!x \,\&\, Px)$.[8] The trouble, critics may say, is not that 'perfection' variously understood is not possible, though, depending on its exact terms, it may not be possible. The trouble, given how 'perfection' is elaborated in valid Hartshorne-Plantinga modal ontological arguments, is that these arguments use possibility-premises that are equivalent to their conclusions. Some, it is complained, use possibility-premises that are stronger than their conclusions.

To find this double trouble in Hartshorne's original argument for the existence of a perfect being, *(∃x)Px*, from ***AP***, $\Box[(\exists x)Px \supset \Box(\exists x)Px]$, and ***IP***, $\Diamond(\exists x)Px$, one need only observe that (i) ***AP*** is to be analytic of the Anselmian concept of perfection, and as such is not to make a substantive contentious claim, and (ii) that this analytic articulation of the meaning of perfection entails the equivalence,

$$[\boldsymbol{IP} : \Diamond(\exists x)Px] \equiv \Box(\exists x)\,Px.$$

For a derivation from ***AP*** of the first half, $[\Diamond(\exists x)Px] \supset \Box(\exists x)Px]$, of this biconditional one can adapt the derivation in Section B2 of Appendix III. The other half, $[\Box(\exists x)Px \supset \Diamond(\exists x)Px]$, of the biconditional conclusion is trivial without the aid of ***AP***.

Taking certain possibilities as premises in these arguments, given stipulations made in them concerning the nature of perfection, ***begs the question***, critics say, that these arguments pretend to address. It more than begs the question, they may say, when it entails not merely the existence of a perfect being, but the existence of a being that is necessarily perfect and existent. These

modal ontological arguments, according to their foes, have this in common with some classical nonmodal ontological arguments. The difference is that those nonmodal ontological arguments beg questions under cover of ambiguities and amphibolies, whereas these modal arguments proceed validly under covers of complicated, concealing fogs of complexity.

7. FRIENDS OF ONTOLOGICAL ARGUMENTS RESPOND

7.1 Some friends say question-begging is not a viable charge against these arguments. An advocate may say that these arguments are valid, that they have logically necessary premises since these affirm logical necessities and possibilities, and that they thus have logically necessary conclusions that, since logically necessary, are equivalent to their premise. This is a feature of every *demonstration* or valid deduction from premises all of which are necessarily true, including venerable mathematical demonstrations that no one would say were guilty of begging questions. Question-begging, if it is something bad that a demonstration can do, is not something that every demonstration does, and, this advocate of an ontological argument contends, his argument does not commit that offense, whatever exactly are its elements.

7.2 Friends may add that if this is 'question-begging', it is good question-begging. Friends may say that, though possibility-premises of our arguments do, given stipulations concerning perfections, entail by themselves necessary existence conclusions, they enjoy, over these conclusions, a heuristic advantage. By making the immediate issue not the truth of a necessity, $\Box(\exists x)Px$, but instead the truth of the related possibility, $\Diamond(\exists x)Px$, the argument, its friends may say, makes evident the applicability of *a priori* methods. Possibilities can sometimes be found out, and confidently believed, by thinking about concepts and 'seeing' that one is working with an idea that does not harbor a contradiction and is of an understandable, conceivable kind of thing. Such views of the epistemological advantages of possibilities are common to most, if not all, advocates of ontological arguments. Here is Malcolm at some length on the subject:

> The only . . . way of rejecting Anselm's claim that God's existence is necessary is to maintain that the concept of God, as a being greater than which cannot be conceived, is self-contradictory or nonsensical. . . . Gaunilo attacked Anselm's argument on this very point.[9] He would not concede that a being a greater than which cannot be conceived existed in his understanding. . . . Gaunilo's faith and conscience [Anselm replied] will attest that it is false that . . . 'He [God – that being] is not understood (*intelligitur*) or conceived (*cogitatur*)'. . . . Descartes also remarks [in his letter to Clerselier, in which he replies to some objections made by Gassendi] that one would go to 'strange extremes' who denied that we understand the words "*that thing which is most perfect that we can conceive . . .* ". . . . (Malcolm 1962, p. 49). God's

existence is either impossible or necessary. It can be the former only if the concept of such a being is self-contradictory or in some way logically absurd. (Malcolm 1962, p. 50)

Plantinga sometimes equates the coherence of an idea with the possibility of its objects: c.f., "What [this argument] shows is that if it is **possible** that there be a greatest possible being (if the idea of a greatest possible being is **coherent**). . . . " (Plantinga 1974a, p. 106; emphasis added.)

Present friends say that the possibility-premises of modal ontological arguments afford plain access of a kind not plainly afforded by their existence and necessary existence conclusions. According to these friends, this makes wrong-headed the castigation of these premises as question-begging. They show us advantaged ways to the conclusions that they anticipate. For, as one who was presumably not a friend of ontological arguments has said, " 'Tis an establish'd maxim in metaphysics" that **possibilities can be proved *a priori*, that whatever is *conceivable* is *possible*** (Hume 1888, p. 32 – see Section 8.4 for Hume's very words for this maxim).[10]

8. BUT THAT – THAT CONCEIVABILITY ENTAILS POSSIBILITY – IS SIMPLY NOT TRUE!

"'*A priori* self-consistency' does not entail logical possibility," respond foes, who see this negation as the heart of the matter. It can seem that possibilities are easier to reach than necessities because there is *a priori* access to possibilities. But it is not so. For there *are* no *a priori* possibilities!! It is a mistake to think that '*a priori* self-consistency' – which is to say, 'freedom from *a priori* discoverable contradiction' – is sufficient for the possibilities affirmed in premises of modal ontological arguments. This is *the* mistake of the best Anselmian arguments. (It is not relevant to the argument of Kurt Gödel of the next chapter.)

8.1 'Magicans' to the point

8.1.1. Rowe uses, to expose the error, "the idea of a ***magican*** [*sic*], an existing magician" and has us "[s]uppose . . . that no magicians have ever existed" (pp. 39–40; emphasis added). Regarding a *magican* or 'existing magician,' understand that something is a *magican* in a possible world if and only if it is a *magician* in the *actual world* (cf., p. 42n13), that is, this world in which we live and breathe. In this account or definition both '*magican*' and 'magician' are existence-entailing predicates in the sense of Section 4.2. The account, for greater explicitness, is thus that something x is a *magican* 'at' a possible world w if and only if (i) x exists in w and (ii) x exists and is a magician in the actual world. The predicate 'is a *magican*' in one way goes beyond the predicate 'is a magician': Being a *magican* in a world entails existing not only in that world,

but also in the actual world, if that world is not itself the actual world. In another way the predicate 'is a magician' goes beyond 'is a *magican*': Being a *magican* 'at' a world w does not entail being a magician in w.[11]

8.1.2. The phrase 'the actual world' is potentially ambiguous. Considering uses of this phrase by you and me, 'the actual world' can serve either as a 'rigid designator' at every possible world for this world of ours in which we live and breathe or as a 'nonrigid designator' that, at a possible world, designates that world. I have in the previous paragraph used 'the actual world' as I will sometimes, following (Lewis 1983), use '@' and as Plantinga sometimes uses 'Kronos' (Plantinga 1974a, p. 43). These symbols are used as proper names of this world of ours. Had I used 'the actual world' as a nonrigid designator, I would have, with the words 'something is a *magican* in a possible w if and only if it is an existent magician in the actual world' have said that something is a *magican* in a world if and only if it is an existent magician in that world, which, given that '*magican*' is existence-entailing, would have made '*magican*' a synonym of 'magician'. (The rigid/nonrigid ambiguity of 'the actual world' is somewhat like an ambiguity of 'today'. Today is January 18, 2002. Was it true on January 17, 2002, that today is January 18, 2002? An answer can begin with, "Yes, and no" or "That depends.")

8.1.3. Answers to two questions about *magicans* make the point that '*a priori* self-consistency' does *not* entail possibility. Similar points are made when the first question is varied to one about imaginability, conceivability, or understandability.

Question 1: Is the idea of a *magican* ***'a priori self-consistent***' or free of *a priori* inconsistency? Answer: YES, certainly. No contradiction follows simply by virtue of ideas or meanings of words from the assumption that there is a *magican*. After all, there *is* a *magican* if and only if there is a magician, and 'there is a magician' is not *a priori* inconsistent. It is not knowable *a priori* that there are no magicians.

Question 2: Is it ***possible*** that there is a *magican* – is this ***logically possible***? Answer: NO, not given the supposition under which we are operating that "no magicians have ever existed" (Ibid.). Though "[t]here is nothing contradictory in the idea of a magican . . . in asserting that a magican is possible we are . . . directly implying that some existing thing is a magician" (p. 41). So if, as supposed, "no existing thing is a magician" (p. 41), there is in no possible world a *magican*. Put otherwise, it is *possible* that there is a *magican* if and only if there *is* a magician. So, given that there is not a magician, it is not possible – it is not *possible* – that there is a *magican*.

8.2 'Unicrons' and 'dragoons' to the same point. Rowe needed to *assume* that there have been no magicians. To proceed without assumptions he might have used ***unicrons***, defined as things that exist in @, this actual world of ours, and

are unicorns in @, for it is a 'given' in current philosophical discourse that no unicorn has ever existed. Similarly for dragons. To rehearse his point in other terms, let 'D' say that *there are* DRAGONS, let (italicized) '*D*' say that *there are* DRAGOONS, and let something be a *dragoon* at a possible world if and only if (i) it exists in that possible world and (ii) it exists in and is a dragon in @. Then,

> It is possible that there is a *dragoon* if and only if there is actually a dragon: $\Diamond(\exists x)\ Dx \equiv (\exists x)Dx$.

Indeed, that is settled by the definition of '*dragoon*', and so necessary *a priori*.[12] And now to make again the point presently before us, we have that

> it is not possible that there is a *dragoon*: $\Diamond(\exists x)Dx$

since

> there is not actually a dragon $(\exists x)Dx$.

Dragoons are not *possible*, though they are imaginable and though there is no *a priori* contradiction in our concept of them, any more than there is in our concept of a dragon.[13]

8.3 Consistency, a priori and logical. Rowe's idea of a *magican* is *a priori* self-consistent or coherent. No contradiction follows merely by virtue of meanings of words from the proposition *M* that there is a magican. It does not matter to this *a priori* consistency whether there are in fact magicians, M. However, if there are no magicians, ~M, then *M* is not *logically* consistent. For if there are no magicians, ~M, then it is *necessary* that there is not a *magican*, $\Box \sim M$; so that, in this case, *M* entails both itself, *M* (every proposition entails itself), and its negation, $\sim M$ (every proposition entails every proposition that is necessary). Entailing this contradiction makes *M* logically inconsistent or, in other words, logically impossible.[14]

8.4 On Hume's 'maxim of metaphysics'. "Wherever ideas are adequate representations of objects, the relations, contradictions and agreements of the ideas are applicable," Hume wrote in the *Treatise* (Hume 1888, p. 29, first line of Section II, Book I, Part I). One consequence he drew was "that whatever *appears* impossible and contradictory upon comparison of . . . ideas, must be *really* impossible" (Ibid.). This is true if 'appears' means 'is evidently' and the claim is that *what can be seen* a priori *to harbor a contradiction is impossible*. However, Hume also maintained what he described as 'an establish'd maxim in metaphysics," namely, "*[t]hat whatever the mind clearly conceives includes the idea of possible existence*, or in other words, *that nothing we imagine is absolutely impossible*" (p. 32, same section), and this maxim is false. We have a clear conception of, and can imagine, *dragoons*. There is no contradiction in the idea. Yet *dragoons* are impossible, if, as we all believe, there are no dragons in this world of ours.

8.5 On the relevance of conceivability to possibility. Certainly "[c]onceivability is not proof of logical possibility" (Putnam 1975, p. 233), but perhaps it is *evidence* for it, somewhat as seeming to see something is evidence for its reality. Perhaps having 'conceived' a thing or situation, where that counts as 'conjuring an appearance of its possibility', entitles a *presumption* that this thing or situation is possible; here conjuring an 'appearance of its possibility' consists not merely in rehearsing words that would be descriptive of the thing or situation, or entertaining an idea of it that is, so far as one can see, not *a priori* inconsistent. This conceiving, let it be *Yablo-conceiving* (Yablo 1993), is rather like *imagining*, though it does not necessarily involve an image in the mind.[15] "Whatever you find conceivable [in this manner], you are," according to Stephen Yablo, "*prima facie* entitled to regard as metaphysically possible" (p. 34).[16] But it is very doubtful that even this is true. For we can conceive (that there are) *dragoons*, as we can conceive that there are dragons, and there is no contradiction *a priori* in the idea of a *dragoon*, if, as we all suppose, there is none in that of a dragon. But neither that we can conceive that there are *dragoons*, and even imagine it, nor that their idea harbors *a priori* no contradiction, would *prima facie* entitle us to regard them as possible, unless it did the same for regarding dragons to be *actual*. More fully stated: We have that the proposition that there is a *dragoon* is possible if and only if the proposition that there is a dragon is true, for short, $[\Diamond(D) \equiv \mathrm{D}]$. *This* – this equivalence – is *a priori* necessary. So something – some reasoning, some experience – provides us with evidence for the proposition that $\Diamond(D)$ is true or an entitlement to consider it true *pro tem* if and only if it provides us with evidence for the truth of proposition that D or such an entitlement in its regard. That I can conceive that there are *dragoons*, that is, that I can conceive of a dragon actually existing, is not evidence for the truth of D, that there exist dragons. So it is not evidence for the truth of $\Diamond(D)$, that *dragoons* are possible . . . that I can imagine a *dragoon* next door, is not 'evidence' that *prima facie* entitles me to regard true D, that there exists a dragon.

It is not true that "**[w]hatever** you find conceivable, you are *prima facie* entitled to regard as metaphysically possible" (Ibid.; emphasis added). Perhaps, however, if what one Yablo-conceives as possible is such that its possibility neither is *a priori* equivalent to a necessity nor *a priori* entails something contingent, then that you conceive it as possible provides evidence (how much depends on the clarity and distinctness of your conception) that it is possible.[17] That – though there are problems with it[18] – is a limited principle that I believe and upon which I sometimes rely (for example, in my belief that there could have been no conscious beings related in ways required for communication). This limited principle is, however, of no avail for Anselmian ontological arguments in order that they not be imperfect and merely assume the possibilities on which they depend. For these possibilities are *a priori* equivalent to necessities. That is the 'idea' of them in these arguments. Stipulations ensure that their beings are not possible unless they are necessary.

8.6. The possibility of a *magican* cannot be decided *a priori*, since this possibility entails the existence of magicians, and that cannot be decided *a priori*. Similarly, one *supposes*, for the possibility of an unsurpassably great or perfect being, if this possibility is, by stipulations regarding greatness and perfection, made to entail the actual existence of a great or perfect being. The *coherence of the concept* of such a being presumably can be so decided, but not its possibility. The possibility of such a being is it seems *precisely* a case of a possibility for which *a priori* consistency and Yablo-conceiving (supposing that makes sense here) can afford no evidence. Texts (see Section 7.2) say Anselm, Descartes, Malcolm, and Plantinga thought otherwise. They give as reasons for possibilities of their beings only that words for them are understandable, that we have coherent ideas of them that are free from *a priori* inconsistencies, and so on. But even if all of that, and more of its kind, is true of the words for, and concepts of, their beings, it seems no more to begin a case for their *possibilities* than similar showings would do for the possibilities of *magicans* and *dragoons*.

9. A DEMOTION OF THE ARGUMENT FROM A PROOF, TO A LICENSE TO BELIEVE

Plantinga is an unusual exponent of an ontological argument. He says that in the best form he has discovered for it, it is without a doubt 'question-begging' and thus not a 'proof' of its theistic conclusion for anyone who needs a proof to believe it. There is, he implies, no getting around that: "No one who didn't already accept [its] conclusion [that there is a 'maximally excellent' being] would accept [its] first premise [that there is a possible world in which there is a 'maximally great' being, which by definition of 'maximal greatness' is a being that exists in, and is maximally excellent in, every world]" (Plantinga 1974a, p. 112 [cf., Plantinga 1974b p. 220). But, he says, the argument is not without merit. "[I]t is evident . . . that there is nothing *contrary to reason* or *irrational* in accepting this premise. What I claim for this argument, therefore, is that it establishes, not the *truth* of theism, but its rational acceptability" (Plantinga 1974a, p. 112).[19]

It is surprising that Plantinga should say there that it is *evident* that there is nothing contrary to reason or irrational in accepting that the existence of a maximally great being is possible, given that he has said that it "isn't very clear," that is, that it is *not evident*, that "the idea of a greatest possible being" is *consistent* (Plantinga 1974a, p. 91). The problem he considers is one of the existence of *maxima* for the various dimensions in which a maximally excellent being would excel. He reports (all on this one page) that there are no problems with the ideas of maximal power, knowledge, and moral excellence, but that it "isn't very clear" whether there are relevant *maxima* to love. Bracketing this issue, Plantinga represents himself as confident that the idea of a maximally great being is consistent. And from this confidence he moves quickly to the confidence that accepting the possibility of a maximally great

being is not contrary to reason, and from that to its not being contrary to reason to accept the necessary existence of a maximally great being, and the truth of theism, and all of this merely from the consistency of that idea. However, even granting the consistency of the idea, this claim that "this [ontological] argument therefore . . . establishes . . . of theism . . . its rational acceptability" (op. cit., p. 112) was, Rowe might say, if not 'excessive' (Rowe 1994, p. 77b), at least premature.

It may, *when all is said*, be contrary to reason to accept that a necessarily existent, essentially perfect being is a possibility, *even if the idea of such a being is consistent*. The consistency of the idea of a *magican* does not entail the possibility of *magicans*, since this possibility would entail the reality of magicians. Presumably the same holds of the consistency of the idea of a maximally great being and that, since the possibility of a maximally great being would entail the reality of a maximally excellent being (a being that is omnipotent and the rest), the consistency of the idea of a maximally great being would not entail the possibility of one. That consistency would be of no *relevance* to this possibility. And since this possibility of a necessarily existent, essentially perfect being would entail the *reality* of such a being in this world of ours, there are *facts* that to most of the world *seem* to argue against it. There is the misery, there is the evil and suffering without apparent reason that would justify it to a perfect being of this world of ours. There is evidence against the possibility of a maximally great being – an omnipotent, omniscient, and perfectly good being, according to Plantinga – *if* it is allowed that facts of rampant evil and suffering are evidence against the reality of such a being. Plantinga's modal ontological argument does not *neutralize* this evidence (if it be such): It does not somehow dissolve its potency as a skeptical assault might pretend to do. It does not *counter* this evidence with other evidence, say of design or of 'miracles,' which would 'overweigh' it. *It* is thus *far* from *establishing* the rational acceptability of the possibility of a maximally perfect being and thus "the truth of [maximally great] theism" (p. 20). It is so far from that as to make quite strange that Plantinga should have thought that it did have this not inconsiderable merit. Perhaps, however, he thought that it did not do that alone, but only when taken together with what he considered to be his solutions to the so-called logical and evidential problems of evil (see pp. 12–59 and 59–63). If so, it remains strange to me – strange in the extreme – that he could have thought "it [was] ***evident*** . . . that there is nothing *contrary to reason* or *irrational* in accepting this premise" (p. 112); which says that maximal excellence is possible, and that *entails all by itself* that maximal greatness, is *real*. He had written (p. 41) that ***he*** **KNEW*****of no evidence*** against the proposition – 'Son of Dr. Pangloss', we may name it – that of the worlds God could have created, none is better than this world of ours. Could Plantinga have thought that ***it is*** **EVIDENT** ***that there is no evidence*** against this proposition, or that it is ***evident*** that there are no grounds in our lives, in events around us, and in history for doubting that a maximally great being could have made

things better? *Surely* not. Surely not then, and almost certainly not today, so soon after he has written that "the argument from evil . . . really does give the believer (some believers, anyway) something to worry about" (Plantinga 1998, p. 543).

CONCLUSIONS

There is we should say *nothing* for theism in Anselm's marvellous *Proslogion* II argument or in its Hartshorne/Plantinga modern update. There **is** something in *Proslogion* III – there is the idea that God would exist necessarily – but that has been something *negative* for philosophical theology. I return to this in the concluding section of the next chapter, after having said all I will about ontological arguments.

David Hume and St. Anselm in Conversation

DH: Whatever we can conceive as existent, we can conceive as nonexistent. There is no conceivable being whose nonexistence in reality implies a contradiction.

StA: Not true. For we can conceive of a being than which nothing greater can be conceived, and the nonexistence in reality of such a being implies a contradiction.

DH: That is so only in a sense that I did not intend to deny, and that does not give you what you want. That *there is a being than which nothing greater can be conceived that does not exist in reality*, $(\exists x)(Gx \,\&\, \sim Rx)$, *does* imply a contradiction. For such a being would exist in the mind only, and since existence in reality as well as in the mind is greater (if you say so), such a thing would be at once greater and not greater than anything that can be conceived, which is a contradiction. From this contradiction we can conclude that there is not a being than which nothing greater can be conceived that does not exist in reality, $\sim(\exists x)(Gx \,\&\, \sim Rx)$, but it does not follow that there is a being than which nothing greater can be conceived that exists in reality, $(\exists x)(Gx \,\&\, Rx)$, which is what you want. So I persist: "[T]here is no [kind] of being whose existence is demonstrable" or provable "by any arguments *a priori*" (Hume 1991, Part 9, p. 149).

StA: But consider a bit longer beings than which nothing greater can be conceived. You agree that such a being is conceivable. You understand me when I speak of it and can find no contradiction in my words. I pause for your correction. Good, you do agree. So you must agree that a being than which nothing greater can be conceived *is possible* and that such a being *exists as an object of thought*, or, as I like to say without meaning anything different, 'exists in the mind': $(\exists x)(Gx \,\&\, Mx)$. Furthermore, as you have allowed, a being than which nothing greater can be conceived

> cannot exist only in the mind, for to exist as well in reality is greater, and it is conceivable that a being than which nothing greater can be conceived should exist in reality. So we agree that any such being would exist in reality as well: $(x)[(Gx \ \& \ Mx) \supset Rx)]$, or, for greater accuracy to the grammar of our agreement, $(x)[(Gx \ \& \ Mx) \ \Box\!\!\rightarrow \ Rx]$. These things are demonstrable *a priori*, for we have just done that. But from them it follows *a priori* that there is a being than which a greater cannot be conceived *in reality*, $(\exists x)(Gx \ \& \ Rx)$. This we have established by argument *a priori*, which corrects one thing you said; and this clearly cannot be denied without contradiction, again *a priori*, which corrects another thing you said. There now, shall we pray together?

How might Hume have responded? Perhaps he would have said first that 'he needed time to weigh these Anselmian considerations' (to relate to the case words of his to Richard Price quoted in Section 11 of Chapter VIII). Taking time to weigh Anselm's argument *might* have brought Hume to see the error of the metaphysical maxim according to which nothing that the mind clearly conceives can be absolutely impossible and the related Anselmian maxim that whatever is conceivable exists at least as an object of thought in the mind. However, Hume could have reached this negative only with difficulty, which may explain the puzzling gap in Hume's discussion of proofs for the existence of God made by his never taking up ontological arguments *a priori* (as distinct from cosmological arguments, which are not purely *a priori*). Perhaps he devoted some time to them without reaching judgments that satisfied him. It would, after all, have been a stretch for him to find that what is conceivable without contradiction is not necessarily possible and that it *may*, indeed, be *impossible*, by an argument *a posteriori* of the kind that can persuade that there are not any unicorns.

APPENDIX A. 'POSSIBLE WORLDS'

A1 Worlds enough

A1.1 Ways in which things might have been. 'Possible worlds' are comprehensive ways that settle everything regarding both what things there are and how they are both in themselves and in relation one to another. This explains how I use this term of art. This world, our world, the actual world in which we live and breathe, is not a possible world in my sense; rather, it instantiates a possible world. It instantiates one comprehensive way in which things can be, for it instantiates the comprehensive way in which things are. Other possible worlds are like the comprehensive way of this world, but of course different, since they are other than it. While strictly speaking, this world of ours is not in my sense a possible world, everywhere, except in the present appendix, to avoid

circumlocutions I write as if this world of ours were a possible world. The fine point of my theory of possible worlds, according to which the actual world is not a possible world but only the instantiation of one, while of course relevant to this metaphysical theory, is never relevant to uses to which I put it when glossing various modal logical points. There is no question whether there are possible worlds. But what are they like? What sort of entities are they? Well, they are like their sort of thing and not exactly like any other sort. More light would be shed by studies, not pursued here, of their similarities to, and differences from, for example, numbers and other mathematical entities, colors and (many) universals, sentences and sets of sentences, and (many) propositions and sets of propositions. But this much can be said: Possible worlds, in that they are certain ways things might have been, are (at least) more of the nature of propositions than sentences and of universals than paradigmatic particulars such as this monitor at which I am peering. A key point is that there are not indiscernible but distinct ways things might have been, whether these are relatively general ways or the completely specific ways that are my 'possible worlds.'

My use of 'possible world' agrees with an early *dicta* of David Lewis: "I . . . believe in the existence of entities that might be called 'ways things could have been'. I prefer to call them 'possible worlds'" (Lewis 1973, p. 84). But I do not agree when he adds, with indexical reference to "our actual world," that "other worlds are more things of *that* sort. . . . Our actual world is . . . one world among others" (p. 85). That was a slip on his part, for our actual world is not a way things might have been, but an instantiation of a way things can be or might have been. Lewis now attends explicitly to the distinction between ways and their instantiations and identifies his possible worlds not with comprehensive ways that things might have been, but with great particulars that instantiate these ways. (See Lewis 1986, pp. 83, 86–7.) I stay with his early suggestion and identify possible worlds not with possible instantiations, but with the ways themselves. Whereas Lewis is "agnostic about whether there are indiscernible worlds" (Lewis 1986, p. 87), I am agnostic whether there are worlds other than the actual world and whether there are indiscernible instantiations of possible worlds. World-ways that Lewis-worlds would instantiate are possible worlds enough for me.

A1.2. The actual world exhibits or instantiates one possible world. It exhibits that way in which things might have been that is the ways things are. There are presumably many other possible worlds, since presumably there are many other ways that things might have been other than the way things are. Pursuing modal intuitions of this presumption: Distinct possible worlds are different in some ways, may be alike in others, and must be alike in some ways. For example, in some worlds, including ours, some but not all grass is green; in other possible

worlds, there is no grass; in one there is nothing at all except necessary things such as numbers. But in all possible worlds either there is some grass that is green or there is no grass that is green, anything that is triangular has three sides, and $2 + 2 = 4$; for these things are true by virtue of meanings without dependence on the vagaries of existence and on what merely happens to be the case.

Neither my possible worlds, nor what would be their instantiations, are 'other worlds' somewhere in space. They are not other solar systems or even other, inaccessible from our (but still actual and real), space–time systems, if there are such inaccessible systems. Other solar systems and other space–time systems, if there are any (contrary to Lewis 1973, pp. 71–2, I think there could be), are parts of the totality of what is the case and so parts of what is actual. They are neither complete ways that things might have been nor instantiations of such ways. Furthermore, it seems that not every possible world, that not every way that things might have been, is spatial and temporal. A world in which there were three unchanging and unmoving billiard balls, and nothing more other than necessary beings, would, I think, be spatial but not temporal. And a world in which there was nothing but sounds might be temporal but not spatial (cf., Chapter 2, "Sounds," in Strawson 1959). A world in which there was nothing at all other than necessary beings would, I think, be neither, and not "just some homogeneous unoccupied spacetime" (Lewis 1986, p. 73).

A2 Truth and actuality at possible worlds

A proposition is 'true at' a possible world if and only if,
if that way in which things might have been were the way things are,
then this proposition would be true.

Given this manner of speaking, there is a sense in which it is true at every world w, and not just at the actualized (actually instantiated) world, that it (the world w) is the actual world. There is a sense in which each possible world is 'actual by its own lights.' Suppose that the possible world w – that is, the way things might have been w – is not a way things are. Then world w is not a possible world that is actual (that is, that is actually instantiated) but, had it been instantiated, then its instantiation would be, from its standpoint, the actual world. So, given the manner of speaking that I have adopted, it is true at it that it is actual; it is true at it, this comprehensive way in which things might have been, that it is the comprehensive way in which things actually are. That is, one might say, 'just as true' (strange idiom) at it, as it is true at the possible world that is actual, that is, that is actually instantiated by this world of ours, that it is actual. Similar ways of speaking, though not relevant to our purposes, could, as more harmless metaphysical smoke, be employed in connection with

the reality and existence of things 'in worlds', so that the things that would exist and be real were a given possible world w to be a way things are, do exist and are real 'in this possible world', even if they do not exist in this world of ours or in the possible world it instantiates. (Nothing will turn on the 'ins and outs" of this idiom.)

A3 Modal realism without tears. While I believe in possible worlds, I do not believe everything about mine that David Lewis believes about his. As stressed, I take them to be of the nature of universals, whereas he takes them to be peculiar particulars. One can believe in the color red, triangularity, the number 2, and identity. One can believe, as I think we all do implicitly when we talk about them, that there are, that there really are (unless, boringly, this word 'really' is loaded to ensure otherwise), these things and countless others of their like without believing in Plato's interesting theories about them, and while believing, with all respect, that Plato was mistaken about their natures and relations to other things – that he got them wrong in some ways. Somewhat similarly of possible worlds, and Lewis's theory of them. I do not suggest that Lewis supposes that his modal realism is the only possible modal realism, as if it were protected by copyright. Nor, more importantly, do I mean that he supposed that every 'modal realism' properly so-called would encounter incredulous stares, though commentators have sometimes explicitly suggested at least this about all possible modal realisms properly so-called (cf., Rosen 1990, p. 329). I contend that if modal realism is the view that there are possible worlds, then (i) though Lewis's modal realism does, "modal realism does [*not necessarily*] disagree, to an extreme extent [and does not necessarily disagree to *any* extent], with firm common sense opinion about what there is" (Lewis 1986, p. 133; emphasis deleted, and reversing interpolations inserted), and (ii) though Lewis's modal realism does, modal realism does not necessarily tell you "that there are uncountable infinities of donkeys and protons and puddles and stars, and of planets very like Earth, and of cities very like Melbourne, and of people very like yourself" (p. 133). Possible worlds, for all for which they are generally *used*, can be comprehensive ways in which things might have been, and nothing more. My modal realism tells you (if common modal intuitions are to be trusted) that there are uncountable infinities of possible worlds – ways that things might have been – at which it is true, in the counterfactual sense explained above, that there are people very like you, and so on. That, however, can be seen to make sense that is uncommon and out of the way only in its somewhat strained and stylized terms. In contrast with Lewis's modal realism, mine, when properly understood, does not invite incredulous stares and has answers to those that it encounters, answers in the form of explanations of itself that confess its blandness.

"But what kind of modal realism is that? You propound not an exciting and amazing metaphysical thesis, but a banality." Sorry, but then boredom, when combined with truth and sufficiency for the elaborations of modal idioms, though possibly bad for controversy, polemics, and the 'business' of possible worlds metaphysics, has its compensations.

A4 This is not a story. Gideon Rosen makes "apparent quantification over possible worlds . . . an innocent *façon de parler*, involving no commitment to worlds of any sort" (Rosen 1990, p. 330). He seeks to avoid incredulous stares by a "a deflationist interpretation of the possible worlds framework" that does without a commitment to worlds (p. 330). In contrast, I offer a deflationist interpretation of this framework that includes a commitment to worlds of a sort. Starting where Lewis began (Lewis 1973, p. 84), I reach a modal realism that is at once *bona fide* and fully committed to the reality of many possible worlds and also an innocent *façon de parler* that would merely facilitate perspicuous systematization of our modal opinions (cf., Lewis 1973, p. 88). That is how, in 1973, I supposed it was intended in Lewis (1973). But he makes more of, and may well in the beginning have intended more by, his modal realism than a useful system of new-speak truisms. This is because he wants more out of it. My modal realism does not aspire to reductive analyses of modalities, featuring as it does the 'primitive' modal condition of 'a *possible* world' or 'a way that things *might* have been'. His has that aspiration.

The realism that I practice is not a story, but a way of literal and uncontentious speaking and thinking. Rosen says that "fictionalism may be thought to strain credulity by identifying [certain modal] facts . . . with facts about the content of an arcane [and, Rosen is convinced, incredible] story [specifically, the story, slightly elaborated, of *On the Plurality of Worlds*]" (p. 351). My 'prosaicism' aspires merely to be a useful way of speaking and thinking about modalities. It may disappoint in its metaphysical and semantic unambituousness, but then it has for it that it cannot strain credulity and is, in the manner of Lewis's more robust realism, sufficient for myriad useful applications in logic and philosophy.[20]

A5 A logic for possible worlds

A5.1. Possible worlds, now often simply 'worlds', are comprehensive ways in which things might have been that would settle everything. They figure in philosophical theories designed to illuminate dimensions of propositions, their modalities, probabilities, desirabilities, and so on. Theories, philosophical and logical, about possible worlds can enhance their utility in these theories of

propositions – necessary, probable, and so on. Now comes a theory that studies the 'logic' of worlds as *world-propositions*, where a world-proposition is a proposition that is true at exactly one world. The 'logic' of world-propositions organizes, by way of systematic adaptations, many principles for possible worlds themselves.

A5.2 Notation and semantics. Let '$\Diamond_2$' and 'W' abbreviate, respectively, 'it is true at at least two worlds that' and 'it is true at exactly one world that,' or, equivalently to the latter, 'it is a maximally specific possibility that'; grammatically, '$\Diamond_2$' and 'W' are modal operators, like '$\Box$' and '$\Diamond$'. If $W\Phi$ is true, Φ is a *world-sentence* and expresses a *world-proposition*.

A5.3 Axioms. The following valid principles can be gathered from Fine (1972). They imply, in the context of a complete system for S5 sentential modal logic such as SMC of the next appendix, all valid principles expressible in the language of standard sentential modal logic augmented by '$\Diamond_2$' and 'W' interpreted as above.

$\Diamond_2 1$:	$\Diamond_2\Phi \supset \Diamond\Phi$	(See 3 of Fine 1972, p. 517.)
$\Diamond_2 2$:	$\Diamond_2\Phi \equiv \Diamond_2(\Phi \,\&\, \sim\Psi)\,\mathrm{v}\,[\Diamond(\Phi\&\Psi)\,\&\,\Diamond(\Phi\&\sim\Psi)]\,\mathrm{v}\,\Diamond_2(\Phi\,\&\,\Psi)$	(See 4, p. 517, and abbreviations on p. 516, of Fine 1972.)
$\Diamond_2 3$:	$\Box(\Phi \supset \Psi) \supset (\Diamond_2\Phi \supset \Diamond_2\Psi)$	(See 4 of Fine 1972, p. 517.)
$\Diamond_2 4$:	$\Diamond_2\Phi \supset \Box\Diamond_2\Phi$	(See 9 of Fine 1972, p. 520.)
Def$W$:	$W\Phi \equiv \Diamond\Phi \,\&\, \sim\Diamond_2\Phi$	

From these can be derived the following W- principles.

$W1$:	$W\Phi \supset \Diamond\Phi$
$W2$:	$\Diamond W\Phi \supset \Box W\Phi$
$W3$:	$W\Phi \supset \Box(\Phi \supset \Psi)\,\mathrm{v}\,\Box(\Phi \supset \sim\Psi)$
$W4$:	$\Box(\Phi \equiv \Psi) \supset (W\Phi \equiv W\Psi)$
$Def\Diamond_2$:	$\Diamond_2\Phi \equiv \Diamond\phi \,\&\, \sim W\Phi$

$W1$ is Arthur Prior's Q1 (Prior 1967, p. 205) and $W2$ is stronger than his Q3, which is $W\Phi \supset \Box W\Phi$. $W4$ is equivalent to his Q4. $W1$ gives expression to the idea that a possible world is 'a way in which things might have been.' $W3$ gives expression to the idea that 'a possible world is a comprehensive way that settles everything.' For a world-sentence Φ, we can say that a sentence Ψ is 'true at Φ' if and only if Φ entails Ψ, $\Box(\Phi \supset \Psi)$, and 'false at Φ' if and only if Φ entails $\sim\Psi$, $\Box(\Phi \supset \sim\Psi)$. In these terms, $W3$ says that, at each world-sentence, every sentence is either true or false, which is to say that a world-sentence settles by entailment every sentence.

APPENDIX B. MODAL LOGIC

B1 Sentential modal logic. For a deduction of Hartshorne's conclusion, necessitated from his premises, I have used Sobel (1983a) **SMC**, the Sentential Modal Calculus, which is an extension for S5-modalities of the sentential calculus of Kalish et. al. (1980) that adds to its language the unary operators '□' and '◊', and to its logic six rules of inference and a form of proof.

B1.1 The primitive modal rules of SMC. For any sentence Φ:

Necessity (N): □Φ /∴ Φ.
Universal Necessity (UN): ◊□Φ /∴ □Φ.
Modal Negation (MdlNeg):∼◊Φ/∴ □∼Φ; □∼Φ/∴ ∼◊Φ; ∼□Φ/∴ ◊∼Φ; ◊∼Φ/∴ ∼□Φ.

N is valid for every kind of 'alethic' ('to do with being true') necessity.[21] UN is characteristic of S5-modal logic, the logic used by Hartshorne and Plantinga, and also of Gödel, in their ontological reasoning. It is the logic of necessity and possibility as, respectively, truth at all possible worlds and truth at some possible world. It is right for the strongest kind of necessity and the weakest kind of possibility. In place of N one could have

$$P : \Phi / \therefore \Diamond\Phi$$

and in place of UN one could have

$$UP : \Diamond\Phi / \therefore \Box\Diamond\Phi.$$

B1.2 The modal form of proof of SMC. To the forms of proof of SC, which are direct, indirect, and conditional (see Appendix C of the previous chapter), is added,

Necessity Derivation (ND). Given a line containing

$$SHOW\ \Box\Phi,$$

if there is no uncanceled *SHOW*-line under this line, then one may simultaneously box all lines under it and strike out the '*SHOW*' on it, if Φ is on an 'available' line (see Appendix C of the previous chapter for a definition) under this *SHOW-line* and (i) only necessity sentences have been 'entered under it from without' and (ii) every sentence 'entered under it from without' was 'entered under it *entirely* from without'.

A sentence has been *entered under a SHOW-line from without* if and only if it has no license other than that it is a premise of the argument whose conclusion

is being derived or an inference from lines including at least one that precedes this *SHOW*-line. A sentence has been *entered under a SHOW-line entirely from without* if and only if it was entered from without, and it has a justification in which it is not an inference from lines at least one of which is below that *SHOW*-line.

B1.3. Necessity Derivation (ND) codifies the intuitive principle that whatever follows from necessities is itself necessary. Restrictions (i) and (ii) block what, given the rest of system SMC, would otherwise be validations of patently invalid arguments. But for restriction (i), the argument P ∴ □P would have the derivation

1.	~~*SHOW*~~ □P	ND
2.	P	premise

Since P was entered from without on (2) and is not a necessity sentence, the box-and-cancellation is not licensed by clause (iv). But for restriction (ii), the invalid argument P ⊃ □Q ∴ □(P ⊃ Q) would have the derivation

1.	~~*SHOW*~~ □(P ⊃ Q)	DD
2.	P ⊃ □Q	premise
3.	~~*SHOW*~~ □(P ⊃ Q)	ND
4.	~~*SHOW*~~ P ⊃ Q	CD
5.	P	assumption for conditional proof
6.	□Q	2, 5, MP
7.	Q	6, N

Line 6 was not entered entirely from without. For the invalidity of the argument P ⊃ □Q ∴ □(P ⊃ Q) consider that there could be just two kinds of worlds, worlds in which both P and Q are false and worlds in which, though P is true, Q is false, with the actual world being of the first kind. The premise (P ⊃ □Q) would be true at the actual world, and the conclusion □(P ⊃ Q) would be false at the actual world, since (P ⊃ Q) would be false at all worlds of the second kind.

B1.4 Derived modal rules. In addition to its primitive modal rules, there are theorems that ground the following useful derived inference rules SMC, of

which we will make free use:

Possibility (P): Φ /∴ $\Diamond\Phi$.
Universal Possibility (UP): $\Diamond\Phi$ /∴ $\Box\Diamond\Phi$.
Interchange of Equivalents (IE): If Φ' comes from Φ by 'interchange of equivalents', then Φ /∴ Φ'.

For sentences Φ and Φ', Φ' *comes from* Φ *by interchange of equivalents*, if there are sentences Ψ and χ such that $\Psi \equiv \chi$ is a theorem of SMC, and Φ' comes from Φ by replacement(s) of some or all occurrences of Ψ by an occurrence or occurrences of χ.

B1.5. Rather than add both '$\Box$' and '$\Diamond$' to the language of the sentential calculus, one can add just '$\Box$' and use '$\sim\Box\sim$' for possibility and have instead of UN the primitive rule UN′ $\sim\Box\sim\Box\Phi$/∴ $\Box\Phi$. If '$\Diamond$' were then made informal for '$\sim\Box\sim$', the four forms of Modal Negation would be available as derived rules for informal sentences. For example, corresponding to the first form MdlNeg is the sentence '$(\sim\Diamond P \supset \Box\sim P)$,' which in this economical system would be informal for the theorem of this system '$(\sim\sim\Box\sim P \supset \Box\sim P)$', and similarly for the other forms of MdlNeg. Left as an exercise is confirmation of this.

B1.6 Nonmodal rules. Here is a set of nonmodal rules that is more than sufficient for all purposes.

Double Negation (DN) $\Phi/ \therefore \sim\sim \Phi; \sim\sim \Phi$ Repetition (R): $\Phi/ \therefore \Phi$

Modus Ponens (MP) $(\Phi \supset \Psi), \Phi/ \therefore \Psi$

Modus Tollens (MT) $(\Phi \supset \Psi), \sim \Psi/ \therefore \sim \Phi$

Adjunction (Adj) $\Phi, \Psi/ \therefore (\Phi \,\&\, \Psi)$

Simplification (S) $(\Phi \,\&\, \Psi) \quad / \therefore \Phi; (\Phi \,\&\, \Psi)/ \therefore \Psi$

Addition (ADD) $\Phi/ \therefore (\Phi \vee \Psi); \Psi/ \therefore (\Phi \vee \Psi)$

Modus Tollendo Ponens $(\Phi \vee \Psi), \sim \Phi/ \therefore \Psi; (\Phi \vee \Psi), \sim \Psi/ \therefore \Phi$

Biconditional Conditional (BC) $(\Phi \equiv \Psi)/ \therefore (\Phi \supset \Psi); (\Phi \equiv \Psi)/ \therefore (\Psi \supset \Phi)$

Conditional Biconditional (CB) $(\Phi \supset \Psi), (\Psi \supset \Phi)/ \therefore (\Phi \equiv \Psi)$

Hypothetical Syllogism(HypSyl) $(\Phi \supset \Psi), (\Psi \supset \chi)/ \therefore (\Phi \supset \chi)$

Separation of Cases (SepCs) $(\Phi \supset \Psi), ((\sim \Phi \supset \Psi)/ \therefore \Psi$

B2 Hartshorne's modal ontological argument. Now come the formal derivations promised in Section 2.1 that validate Hartshorne's argument, with conclusion enhanced:

IP: $\Diamond(\exists x)Px$,. ***AP***: $\Box[(\exists x)Px \supset \Box(\exists x)Px]$[22] $\therefore$ $\Box(\exists x)Px$.[23]

Following (Hartshorne 1962, pp. 50–1), to bring the case into the language of SMC, we may let 'Q' abbreviate '$(\exists x)Px$' and construct a derivation for the argument $\Diamond Q$. $\Box(Q \supset \Box Q) \therefore \Box Q$. First an unabbreviated derivation in which only primitive rules are used.

1.	~~*SHOW*~~ $\Box Q$	ID
2.	$\Diamond Q$	premise
3.	$\Box(Q \supset \Box Q)$	premise
4.	~~*SHOW*~~ $\sim \Diamond\Box Q \supset \sim \Diamond Q$	CD
5.	$\sim \Diamond\Box Q$	assumption
6.	~~*SHOW*~~ $\Box \sim Q$	ND
7.	~~*SHOW*~~ $\sim Q$	ID
8.	Q	assumption
9.	$\Box(Q \supset \Box Q)$	3, R
10.	$\Box \sim \Box Q$	5, MdlNeg
11.	$Q \supset \Box Q$	9, N
12.	$\sim \Box Q$	10, N
13.	$\sim Q$	11, 12, MT
14.	$\sim \Diamond Q$	6, MdlNeg
15.	$\sim\sim \Diamond Q$	2, DN
16.	$\sim\sim \Diamond\Box Q$	4, 15 MT
17.	$\Diamond\Box Q$	16, DN
18.	$\Box Q$	17, UN

Lines 7 and 8 do not get in the way, but they do no work. The derivation of 1 is direct: It is based on 18, so there are no restrictions on entries into it. The derivation of 6 is of the necessity kind: It is based on 7. Applicable restrictions on entries from without are met by entries on 9 and 10. Regarding the rationale for 6, in the system the best way to get a sentence $\sim\Diamond\Phi$ that

cannot be inferred is to derive $\Box\sim\Phi$, which provides an occasion for necessity derivation. Left as exercises are unabbreviated derivations for the distribution principles $[\Box(P \supset Q) \supset (\Box P \supset \Box Q)]$, and $[\Box(P \equiv Q) \supset (\Diamond P \equiv \Diamond Q)]$. The second exercise can present an occasion for the strategy just described for obtaining negations of possibilities.

Now comes an abbreviated derivation for the same argument. Abbreviations consist in combining some lines and, more importantly, in the use of the derived inference rule UP.

1.	~~*SHOW*~~ $\Box Q$	ID
2.	$\sim \Box Q$	assumption
3.	$\Diamond \sim Q$	2, MdlNeg

We have as a premise $\Diamond(\exists x)Px$ or, as we are abbreviating it, $\Diamond Q$. An idea is to get $\sim\Diamond Q$ for a contradiction.

4.	~~*SHOW*~~ $\Box \sim Q)$	ND
5.	~~*SHOW*~~ $\sim Q)$	ID
6.	Q	assumption
7.	$\Box(Q \supset \Box Q)$	***AP***
8.	$\Box Q$	7, N, 6, MP
9.	$\Box\Diamond \sim Q$	3, UP
10.	$\sim \Box Q$	9, N, MdlNeg
11.	$\Diamond Q$	***IP***
12.	$\sim \Diamond Q$	4, MdlNeg

Line 3 anticipates the coming necessity derivation for 4: A necessity can be inferred from 3 by using UP; none of our stated rules allows a necessity to be inferred from 2. Lines 1–3 could be replaced by,

1′.	~~*SHOW*~~ $\Box Q$	ND
2′.	~~*SHOW*~~ Q	ID
2″.	$\sim Q$	assumption
3′.	$\Diamond \sim Q$	

Whenever a necessity sentence is derivable indirectly, in the manner of ID, it is derivable in the manner of ND. (Why?) There is a trivial sense in which the converse is true: If one can, by using ND, derive $\Box\Phi$, then, having begun an indirect derivation for it with the assumption $\sim\Box\Phi$, one can use ND to derive $\Box\Phi$ for a contradiction. However, what is derivable by using ND is always derivable *quite* without it: The system minus ND is not 'complete.'

Establishing this is left as a nontrivial exercise in the 'metatheory' of sentential modal logic for S5-modalities. Derivations (abbreviated or unabbreviated) in SMC were left as possible exercises in notes 2 and 17 of Chapter II. For an additional somewhat more challenging exercise, one might derive the confinement principle, [◊(P ⊃ □Q) ⊃ (□P ⊃ □Q)]. Derived rules P and UP can be useful in the first and third of these exercises.

B3 Quantified modal logic

B3.1. I now extend SMC to a language with universal and existential quantifiers, monadic predicate letters, individual constants, the logical dyadic predicate '=' for identity, and the logical monadic predicate 'E!' for existence. Rules and a form of proof are added for completeness and correctness relative to a somewhat free semantics, which will now be detailed. It is free of the restriction on interpretations to terms that denote in domains of all worlds but not of the restriction to nonempty world-domains. Let this extension of SMC be the *Free Monadic Quantifier Modal Calculus* – ***FrMQMdlC***.

B3.2 Domains. In an interpretation of the language, domains of distinct worlds can be disjoint, identical, or neither, so that they merely overlap. But it is required that each world-domain in an interpretation should contain something (as in Bonevac 1987, p. 365). An interpretation specifies not only domains for its worlds, but also a universal domain that is the union of world-domains. When the universal domain is metaphysically complete and includes absolutely everything, whatever that comes to, the question – Why is there something rather than nothing? – since it is about contingent things, is not answered by a valid sentence that says there is something contingent; for example, '(∃x)(E!x & ∼□E!x)' is not valid. This semantical scheme does make valid the sentence '(∃x)E!x'. A less free scheme would require that there be something common to world-domains of an interpretation and would make valid the sentence '(∃x)□E!x.' A freer semantics would require only that in an interpretation at least one world's domain is nonempty and would make valid only '◊(∃x)E!x'. A completely free semantics would countenance interpretations in which all world-domains are empty and would not make valid even that sentence. I have settled on the somewhat free scheme in which no world-domain can be empty because it is familiar, and because, since we never talk about nothing, we have no practical interest in interpretations appropriate to such discourse. Nothing, however, is made in applications of the restriction of the semantic scheme and its logic to nonempty domains.

Individual constants are in interpretations fixed in their extensions in the universal domain, once for all worlds, which opens up the possibility of a given constant's not naming an existent in a particular world, that is, of its

not naming a member of this world's domain. It will name a member of some world's domain, however, since its 'extension' is in the universal domain, and that is the union of world-domains. In this semantic scheme $(\exists x)\ A = x$ is not valid, though $\Diamond(\exists x)\ A = x$ is valid.

Predicate letters have, in interpretations, worlds-to-individuals functions as their extensions. These assign to predicate letters 'world-specific extensions'. The world-specific extension of a predicate letter for a world is a subset of the universal domain that need not be a subset of this world's domain. This allows, for example, FA to be true at a world even though E!A is false at this world, which is to say, even though A is not in the domain of this world. The extensions of a predicate letter for different worlds can be different subsets of the universal domain.

Quantifiers, (α) and $(\exists\alpha)$, are confined here in their ranges to domains of particular worlds. It is, for example, to be true in a world that everything satisfies some condition if and only if everything that exists in this world, that is, everything that is in its domain, satisfies this condition.

B3.3 Rules and procedures of FrMQMdlC. Standard rules and procedures for quantifiers assume that all individual constants denote in the domains over which quantifiers range. These rules and procedures need to be modified for our world-domain quantifiers to make rules and procedures sensitive to the possibilities of nondenoting terms.[24] To illustrate, $(\exists x)\ A = x$ is not valid in our somewhat free semantics, though it is readily derivable using standard rules that make $A = A$ a theorem and allow $(\exists x)\ A = x$ to be derived from that by existential generalization or, indirectly, by using quantifier negation and universal instantiation.

B3.4 Rule and proofs in FrMQMdlC

B3.4.1 SENTENTIAL. Rules, primitive and derived, and proof procedures of SMC, the Sentential Modal Calculus, are now readdressed to all formulas of FrMQMdlC. For example, in the rule $\Box\Phi\ /\therefore\ \Phi$ we have that Φ is not merely any sentence of FrMQMdlC, such as '$(P \supset Q)$' and '$(\exists x)Fx$', but also any formula or expression that becomes a sentence when 'free occurrences of variables' (explained in Appendix C of the previous chapter) are replaced by name letters.

B3.4.2 QUANTIFICATIONAL. There are, for quantifiers, three rules inferences in FrMQMdlC. For variable α, term β, formula Φ_α, and 'formula Φ_β that comes from Φ_α by proper substitution of β for α' (definition below):

Free existential instantiation (FrEI): $(\exists\alpha)\Phi_\alpha\ /\therefore\ \Phi_\beta\ \&\ E!\beta$ provided that β is a variable that has not occurred on a preceding line of the derivation.
Free existential generalization (FrEG): $\Phi_\beta, E!\beta\ /\therefore\ (\exists\alpha)\Phi_\alpha$.
Free universal instantiation (FrUI): $(\alpha)\Phi_\alpha, E!\beta\ /\therefore\ \Phi_\beta$.

Definition. *Formula ϕ' comes from formula ϕ by* proper *substitution of a term β (i.e., a variable or name-letter) for a variable α* if and only if Φ' is like Φ except that, wherever α occurs free in Φ, β occurs free in Φ'.

B3.4.3 EXISTENCE AND IDENTITY. For any variable α, terms β and β', formula Φ, and formula Φ' that comes from Φ by replacing some or all free occurrences of β by free occurrences of β' –

Existence: $/\therefore (\exists\alpha)\mathrm{E}!\alpha$.
Identity (Id): $/\therefore \beta = \beta$.
Leibniz's Law (LL): $/\therefore \beta = \beta$ ', $\Phi /\therefore \Phi$'.

The rule Existence is not used in derivations in this book. It is present only to match the semantic restriction on interpretations to nonempty world-domains, of which, as said, 'nothing is made.' Regarding Necessity Derivation, entries by Existence and Id are *not* 'entries from without,' so it does not matter that they are not necessity-formulas.

B3.4.4 FREE UNIVERSAL DERIVATION. For universal generalizations we have the license that, given a line containing *SHOW* $(\alpha)\Phi$, if there are no subsequent uncancelled *SHOW*-lines, $(\mathrm{E}!\alpha \supset \Phi)$ occupies a subsequent line, and α is not free on a preceding available line, then one may simultaneosly box all lines under ths *SHOW*-line and cancel the '*SHOW*' on it; Free Universal Derivation – FrUD.[25]

B3.4.5 We have in addition to these primitive rules the following derived rules of quantifier negation (QN):

$$\sim(\alpha)\Phi/\therefore(\exists\alpha)\sim\Phi; \qquad \sim(\exists\alpha)\Phi/\therefore(\alpha)\sim\Phi;$$
$$(\exists\alpha)\sim\Phi/\therefore\sim(\alpha)\Phi; \qquad (\alpha)\sim\Phi/\therefore\sim(\exists\alpha)\Phi.$$

Derivations are left as exercises. Also left as an exercise is the derivation $\mathrm{E}!x \equiv (\exists y)\, x = y$, without using the rule Existence.

B4. Hartshorne's 'Anselm's Principle'

Here is the derivation promised in Section 4.1 of

$$\boldsymbol{AP}\text{: } \Box[(\exists x)\mathrm{P}x \supset \Box(\exists x)\mathrm{P}x],$$

from a principle weaker than

$$\boldsymbol{PERFnecEx\&necPerf}\text{: } \Box(x)\Box[\mathrm{P}x \supset \Box(\mathrm{E}!x\ \&\ \mathrm{P}x)],$$

namely,

$$^{*}\boldsymbol{PERFnecEx\&necPerf}^{*}\ \Box(x)[\mathrm{P}x \supset \Box(\mathrm{E}!x\ \&\ \mathrm{P}x)].$$

1.	~~*SHOW*~~ $\Box[(\exists x)Px \supset \Box(\exists x)Px]$	ND
2.	~~*SHOW*~~ $(\exists x)Px \supset \Box(\exists x)Px$	CD
3.	$(\exists x)Px$	assumption
4.	Pa & E!a	3, FrEI
5.	$\Box$ (x) [Px $\supset \Box$([E!x & Px)]	****PERFnecEx&necPerf****
6.	Pa$\supset \Box$(E!a & Pa)	5, N, 4, S, FrUI
7.	Pa	4, S
8.	~~*SHOW*~~ $\Box(\exists x)Px$	ND
9.	$\Box$(E!a & Pa)	6, 7, MP
10.	E!a & Pa	9, N
11.	$(\exists x)Px$	10, S, S, FrEG

B4. Derivation of $\Box[(\exists x)\Box Px \supset \Box(\exists x)Px]$ Now comes the derivation promised in Section 4.2 from ***PrfExEntlng*** ('perfection is existence-entailing), $\Box(x)\Box(Px \supset E!x)$.

1.	~~*SHOW*~~ $\Box[(\exists x)\Box Px \supset \Box(\exists x)Px]$	ND
2.	~~*SHOW*~~ $(\exists x)\Box Px \supset \Box(\exists x)Px$	CD
3.	$(\exists x)$ Px	assumption
4.	$\Box(x)\Box(Px \supset E!x)$	***PrfExEntlng***
5.	$\Box$Pa & E!a	3, FrEI
6.	$\Box(Pa \supset E!a)$	4, N, 5, S, FrUI
7.	~~*SHOW*~~ $\Box(\exists x)Px$	ND
8.	$\Box$Pa	5, S
9.	$\Box(Pa \supset E!a)$	6, R
10.	Pa	8, N
11.	E!a	9, N, 10, MP
12.	$(\exists x)Px$	10, 11, FrEG

As stated at the end of Section 4.2, the argument

StrgIP: $\Diamond(\exists x)\Box Px.$, $\Box[(\exists x)\Box Px \supset \Box(\exists x)Px] \therefore \Box(\exists x)Px$

is valid in SMC of Section B1. A confirming derivation for this is left as an exercise. To bring the problem into the sentential language of that system, one may abbreviate '$(\exists x)\Box Px$' and '$(\exists x)Px$' by 'Q' and 'R,' respectively. Derivation exercises in FrMQMdlC were left in Sections 2.4 and 3.3; notes 2, 4, and 8 of this chapter; and note 55 of the previous chapter.

IV

Kurt Gödel's *Ontologischer Beweis**

1. INTRODUCTION

1.1 Texts and style. Photocopies of three handwritten pages titled "Gödel's Ontological Proof" (Appendix B) began to circulate in the early 1980s. The handwriting is Dana Scott's; the ideas are Kurt Gödel's. They agree with ideas conveyed in two pages of notes in Gödel's own hand dated 10 February (Appendix A).[1] Scott's three pages, on which I concentrate here, contain a sketch of a theory of positive properties, individual essences, and necessary existence that culminates in a theorem that says that it is necessary that there is a being that has every positive property. The plan of the proof honors Leibniz. It goes through demonstrations of the possibility of such a being, and that such a being is either not possible or necessary.[2] Its style is Spinozistic but formal: Axioms and definitions are set, and theorems are proved in a formal language that is free of amphibolies that bother classical proofs. Its logic is quantified modal, not simply quantificational as classical proofs, nor only sentential modal as Hartshorne's. As said, it does not merely postulate the possibility of its God-like being but demonstrates it with no suggestion that it is forthcoming given merely the conceivability of a being that has every positive property, and that this definition of God-likeness harbors no contradiction.

1.2 What it meant to Gödel. Robert Merrihew Adams writes in his introductory note:

> Gödel showed [it] to Dana Scott, and discussed it with him, in February 1970. Gödel was very concerned about his health at that time, feared that his death was near, and evidently wished to insure that this proof would not perish with him. Later in 1970, however, he apparently [*sic*] told Oskar Morgensgtern that though he was 'satisfied' with the proof, he hesitated to publish it, for fear it would be thought 'that he actually believes in God, whereas he is only engaged in a logical investigation (that is, in showing that such a proof with classical assumptions, correspondingly

axiomatized, is possible)' [Morgenstern's diary, 29 August 1970]. (Gödel 1995, p. 388)

I doubt it. I doubt that the proof was for Gödel merely a logical/historical exercise, and I doubt that, as Morgenstern's diary implies, Gödel was a settled and comfortable nonbeliever. We can take from Morgenstern's entry that Gödel was satisfied with his proof, given classical assumptions, and add to this that there are no indications in our texts of points of dissatisfaction with those assumptions, let alone irremediable points of dissatisfaction, that would allow Gödel "to make nothing of it", notwithstanding his proof's validity. Regarding Gödel's spirituality, and whether he believed in God or "someone rather like Him", there is this ground for positive speculation to balance Morgenstern's contrary implications. "Two days after Gödel's death on 14 January 1978 I went to see Adele. . . . On this occasion Adele told me that Gödel, although he did not go to church, was religious and read the Bible in bed every Sunday morning" (Wang 1996, p. 51). Also we have it that "[i]n 1975 Gödel gave his own religion as 'baptized' Lutheran . . . and noted that his belief was *theistic*, not pantheistic, following Leibniz rather than Spinoza" (op. cit., p. 112).[3]

1.3 In this chapter. I begin with comments on the language and logic of the system. Then parts of it, to its culmination in the notes under discussion in a demonstration that there must be a God-like being, are reviewed. After that I argue, given the terms and conditions of the system, that no being that possessed all positive properties could reasonably be said to be God. Next it is demonstrated that there is a collapse of modalities in the system – that in it everything that is actual or true is so of logical necessity, and everything that is not actual or false is impossible. From this may be gathered – if one considers these theorems of the system in which its primitive does not occur to be false, as (to understate) one may reasonably do – that not all of its axioms can be true on any interpretation of that primitive, and that proving something in the system on an interpretation provides no grounds for thinking that it is true. One reaction to these difficulties is to withdraw the axiom that makes necessary existence a positive property. In conclusion, brief comments are made on this possibility and the ideas, with which I agree, that one should give up on deities that would be necessary beings, and on ontological arguments, and that one *can* do this without giving up on God. A postscript comments on changes in Gödel's system suggested by C. Anthony Anderson to avoid its modal collapse. Gödel's and Scott's notes are transcribed in Appendices A and B. From time to time I promise supporting derivations in an extension of the quantified modal calculus of Appendix B of the previous chapter. These are delivered in Appendix C. Symbols, with few exceptions, are as in Scott's notes. He uses '$\forall$' and '$\exists$' without parentheses as universal and existential quantifiers, '$\neg$' for negation, '$\rightarrow$' for material implication, and '$\leftrightarrow$' for material equivalence.

2. LANGUAGE AND LOGIC

2.1. The formal language of the system accommodates quantification over properties, and property-expressions that can occupy term and predicate positions. Consider, for example, the '$G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$' (the so-called 'definition' of 'G'): the property-variable 'ϕ' stands in a term position in '$P(\phi)$,' which says ϕ is P (ϕ is positive), and 'ϕ' stands in a predicate position in '$\phi(x)$,' which says that x is, or has, ϕ. Similarly, 'G' in that formula occupies a predicate position 'G(x),' which says that x is God-like, whereas it occupies a term position in 'P(G)' that says that God-likeness is a positive property. 'P' is for a property of properties. It occupies only predicate positions, never term positions.

The language features 'sorted' quantification. Variables 'x' and 'y' range over everything (or so I assume, because Gödel confined them to things that are not properties). The variables 'ϕ' and 'ψ' range over only properties (perhaps Gödel confined them to first-order properties). (**N.B.**: These Greek letters are here variables in the language not, as usual, metalinguistic variables for general talk about the language.) There is in the language an abstraction operator for a kind of property expression; these occur sometimes in its proofs, though never in axioms, definitions, or theorems. Gödel operates with a very 'generous' notion of a property that for one important thing does *not* make redundant the description 'possibly instantiated property.' This generosity of conception is important to Gödel's project not simply to assume, but to demonstrate, the possibility of a God-like being. Both '$\hat{x}[x = x]$' and '$\hat{x}[x \neq x]$' for the properties of self-identicalness and non-self-identicalness occur in our texts. The scheme '$\neg\phi = \hat{x}[\neg\phi(x)]$' is given without comment, evidently as an explanation of notation for properties that are 'the complements' of properties (Scott's notes, p. 1). To this explanation I add the scheme $\neg\phi = \neg\hat{x}[(\neg\phi)x]$, to say that a thing has the negation (or complement) of a property if and only if it does not have this property this way – '$\forall\phi\forall x[(\neg\phi)x^{\leftrightarrow}\ \neg\phi(x)]$': '$(\neg\phi)x$' says that x has the negation of property ϕ and '$\neg\phi(x)$' says that x does not have the property ϕ. As property expressions occupy both term and predicate positions in the notes, so the negation sign '$\neg$' applies sometimes to formulas and sometimes to property expressions; recently '$\neg$' first was applied to the property-term 'ϕ' to make the property-term '$\neg\phi$' for the complement of ϕ and then to the formula '$(\phi)x$' to make the negation-formula '$\neg\phi)x$'. One gathers that in general $\neg\hat{x}[\phi] = \hat{x}[\neg\phi(x)]$; here again '$\neg$' is applied first to a term and then to a formula. The property identity $\neg[x = x] = \hat{x}[x \neq x]$ plays a role in the proof of Theorem 1. Similar conventions are assumed at points below for disjunctive and conjunctive property-terms, though only negative property terms figure in our texts.

2.2. The system's logic can be third-order (third 'Russell-order' or 'Church-level' – Hacking 1977, p. 375, for there is in the language the property of

properties symbol 'P') quantified S5-modal with identity and abstraction. Quantification into modal-contexts is countenanced, but every case implicit in our texts of 'quantification in' is of '*property* quantification in'. 'Closed' property-terms of the system name properties, which I identify with functions from worlds to sets of things; intuitively, a 'closed' property-term names the function whose value for a world is the set of things that at that world has the property that the look-alike property predicate signifies. Properties, that is, these functions, exist in all worlds, so that subsets of world-domains of properties are the same. That makes property quantification into modal contexts unproblematic. The logic of the system is not elementary and, if pressed, might prove problematic. But at least as far as the conservative demands made in our texts – and the demands I make when discussing them – it is, I think, problem-free.

3. AXIOMS, DEFINITIONS, AND TWO THEOREMS

3.1 Positive properties

3.1.1. '***P***' is the primitive of the theory. It abbreviates sometimes 'positiveness' and sometimes 'is positive.' Its intended interpretation must be gleaned from cryptic explanatory sentence fragments in Gödel's hand, as there is nothing to it in Scott's notes. Gödel writes in *Ontologischer Beweis*, p. 2:

> Positive means positive in the moral aesthetic [This word is not perfectly legible. J. H. S.] sense (independently of the accidental structure of the world). Only then [is/are?] the ax. [axiom/axioms? – the reference may be specifically to something equivalent to Axiom 2, given below, or, more likely I think, to all the axioms] true. It may also mean pure 'attribution'* as opposed to 'privation' (or *containing* privation). This interpret. simpler proof. . . . *i.e., the 'disj' normal form in terms of elem. prop. contains [a? – Presumably, though I cannot decipher the mark here. Gödel (1995, p. 404) has 'a'.] member without negation.

Both 'pure' and 'disj.' are above-the-line insertions. For relevance to the object of proving the existence *God*, by proving the existence of a God-like being that has every positive property, only the **moral aesthetic** option is useful. In my view, for that relevance, 'positive' must be interpreted in a manner so related to the **religious or spiritual** that: (i) there are religiously or spiritually positive properties that *contribute to worshipfulness*, and, since it turns out that a God-like being has all and only positive properties, (ii) the religiously or spiritually positive properties that are positive in a sense that makes Gödel's axioms true, discounted by the religiously or spiritually negative properties (properties that *detract from worshipfulness*), if any, that are positive in that sense, *make a God-like being worshipful at least on balance*. For, 'bottom-line' in my view, God would, by His properties be made worthy of worship. Clearly there must *not*

be a property that is positive in *every* sense of positive that makes Gödel's axioms true, which property *makes a God-like being* NOT *worshipful*. Even if there is exactly one God-like being of the system, and it exists in every possible world, it is not a foregone conclusion that it is God, or that it *may* be God.

3.1.2 Platonism. Gödel was inclined to a Platonic equation of goodness (moral/aesthetic) with being (pure, without privation), notes of which are prominent in Leibniz's philosophy [cf., "Perfection . . . is nothing else but . . . quantity of essence" (Leibniz 1965, p. 86)]. As will be evident, his theory would be served by this combination. It can use a moral aesthetic interpretation of 'positive' for spiritual and religious *relevance*, while, for the *prima facie* truth of some of its axioms – for a "simpler proof," Gödel writes – it can use a logical/ontological interpretation of this primitive.[4] It is another matter whether he can *have* this combination. He seems not to have had in mind a settled sense for his primitive with which he was satisfied. Cf: "The evidence available from his notebooks suggests that he never found an interpretation of this notion that fully satisfied him, and it is perhaps best to assume that he thought of his ontological argument not as a conclusive proof of the existence of God, but as an attempt at a reconstruction of Leibniz's argument" (Hazen 1998, p. 364). Gödel hoped, I think, that his reconstruction could be turned into a conclusive proof of God's existence. I read it as a proof in search of an interpretation, a proof with the hopeful, "And what can we make of all this?"[5]

3.2 Initial axioms and a theorem[6]

3.2.1. It is announced that, of every property and its negation, exactly one is positive:

Axiom 1. $\mathbf{P(\neg\phi)\leftrightarrow\neg P(\phi)}$**,**

which is equivalent to

$$[P(\neg\phi) \vee P(\phi)] \;\&\; \sim[P(\neg\phi) \;\&\; P(\phi)]$$

and short for

$$\Box\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)].$$

(**N.B.** Axioms, definitions, theorems, and consequences thereof are all implicitly 'necessities of universal closures.') It is not clear why, on either an axiological or logical interpretation of 'positive', that this should be so. The first, 'if-part' – $P(\neg\phi) \vee P(\phi)$ – challenges "[e]ven the sympathetic reader" (Anderson 1990, p. 291) on axiological interpretations of 'positive', though not on Gödel's logical interpretation. If the disjunctive normal form in terms of elementary properties of a property does not have a member without negation, then its

negations does. It is next said that every property necessarily contained in a positive property is itself a positive property:

Axiom 2. $\mathbf{P(\phi)\& \Box\forall x[\phi(x) \rightarrow \psi(x)]\rightarrow P(\psi)}$

$$\Box\forall\phi\forall\psi[P(\phi) \,\&\, \Box\forall x[\phi(x) \rightarrow \psi(x)] \rightarrow P(\psi)]$$

This axiom is problematic for moral aesthetic positivity. A discussion could start with the problem of a Moore/Leibniz principle of the organic unity for intrinsically valuable properties and venture to 'perfect or through-and-through intrinsically valuable properties' (cf., Moore 1912, pp. 30–2),[7] *perhaps* to solve it. Gödel's generous interpretation of properties is at least awkward for an axiological interpretation of Axiom 2, since, according to it, if there is a positive property, then every necessarily universal property such as being self-identical, and being either red or not red, is a positive property.[8]

Now comes the theorem that each positive property is possibly instantiated:

Theorem 1. $\mathbf{P(\phi)\rightarrow \Diamond\exists x\phi(x)}$

$$\Box\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$$

The argument given for this theorem assumes that (i) $P(\phi)$ and (ii) $\neg\Diamond\exists x\phi(x)$ for an indirect proof. [The negation of the conditional theorem to be proved is equivalent to the conjunction of (i) and (ii).] From (ii), $\Box\forall x\neg\phi(x)$ is derived, and from that $\Box\forall x[\phi(x) \rightarrow x \neq x]$. From that, (i) $P(\phi)$, and Axiom 2, it follows that (iii) $\boldsymbol{P(\hat{x}[x \neq x])}$, since $x \neq x$ is equivalent to $(\hat{x}[x\neq x])x$ (!). We have as a theorem that $\Box\forall x[\phi(x) \rightarrow x = x]$, and so, by Axiom 2 and $P(\phi)$ again, $P(\hat{x}[x = x])$. It is then stated that "$\hat{x}[x \neq x] = \neg\hat{x}[x = x]$ which contradicts (half of) axiom 1." (p. 1) The idea could have been that from $P(\hat{x}[x = x])$ and $P(\hat{x}[x = x]) \rightarrow \neg P(\neg\hat{x}[x = x])$ – which conditional comes from the second 'only if-part' of Axiom 1, which part is equivalent to $\Box\forall\phi[P(\phi) \rightarrow \neg P(\neg\phi)]$ – it follows that $\neg P(\neg\hat{x}[x = x])$ and that, given the identity $\hat{x}[x \neq x] = \neg\hat{x}[x = x]$, it follows, 'putting equals for equals,' that $\neg\, \boldsymbol{P(\hat{x}[x \neq x])}$ for a contradiction with (iii).

Here is a somewhat simpler deduction that does not traffic in properties of self-identity and non-self-identity or use 'nice' principles for negations of properties. Assume (i) $\boldsymbol{P(\phi)}$ and (ii) $\neg\Diamond\exists x\phi(x)$. From (ii) it follows that (iii) for *every* property ψ, $\Box\forall x[\phi(x) \rightarrow \psi(x)]$; (ii) 'says', in C. Anthony Anderson's words, that "ϕ is inconsistent," and "an inconsistent property entails everything" (Anderson 1990, p. 292). See Section C2 of Appendix C for a formal derivation of (iii) from (ii). From (i), the case of (iii), $\Box\forall x[\phi(x) \rightarrow (\neg\phi)x]$, and the case of Axiom 2, $P(\phi) \,\&\, \Box\forall x[\phi(x) \rightarrow (\neg\phi)x] \rightarrow P(\neg\phi)$, it follows that (iv) $P(\neg\phi)$. But it follows from (iv) and the good half of Axiom 1 that $\neg\, P(\phi)$, which contradicts (i).

3.2.2. It is settled by a definition that "*x* is God-like if it possesses all positive properties" (Scott's notes, p. 1).

$$\textbf{Def G.}\quad \mathbf{G(x)} \leftrightarrow \forall \boldsymbol{\phi}[\mathrm{P}(\boldsymbol{\phi}) \rightarrow \boldsymbol{\phi}(\mathrm{x})]^{9} \qquad (\mathrm{God})^{*}$$

$$\Box\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$$

(*This parenthetical signal stands in Gödel's original notes, which contain no occurrence of 'God-like.' Scott's notes contain no occurrence of 'God.') Next comes

Axiom 3. P(G)

"Any such property ought also [itself] to be positive" (Ibid.)[10] What properties are here 'such' properties? An answer may be implicit in Gödel's notes, which have, instead of Axiom 3, $\mathrm{P}(\phi)\mathrm{P}(\psi)\ \mathrm{P}(\phi\cdot\psi)$, to which is appended the note "and for any number of summand" ("*Ontologischer Beweis*," p. 1). The axioms to which Gödel alludes are meant to include Scott's Axiom 3; 'any number' does not mean here only any *finite* number. The Leibniz-Moore doctrine of the organic unity of intrinsic values can again give pause when one is concentrating on the value-guise of positive, though when one concentrates on its logical/ontological guise, that 'intersections' of all positive properties should all be positive seems hardly remarkable. Gödel's disjunctive normal form gloss is less useful for this axiom, since there is the abstract possibility that the disjunctive normal forms into elementary properties of positive properties should differ in their members that are without negation, in which case their intersection would have no such member. This axiom may be best served by the axiological gloss according to which positiveness is 'perfection' or 'through and through goodness.'

That the possibility of God-likeness is largely assumed. It is stated immediately after Axiom 3 that the *possibility* of a God-like being, $\Diamond\exists\mathrm{xG(x)}$, holds as a corollary. This possibility is an easy consequence of Axiom 3 and Theorem 1. *Too easy*, Leibniz would think. He wrote, recall, that the argument that "M. Descartes borrowed from Anselm . . . is not fallacious, but it is an incomplete demonstration which assumes something which should also be proved . . . it is tacitly assumed that this idea of a wholly great or wholly perfect being is possible and does not imply a contradiction . . . it is desirable that able people should fill the demonstration out" (Leibniz 1981, pp. 437–8.) Gödel takes on this task. He does not, as Hartshorne and Plantinga do – and as Descartes did – merely assume the possibility of his God-to-be, or pretend that it follows from the coherence of our concept of it. But he takes it on half-heartedly. The possibility of God-likeness, that is, of a being's having every positive property, is not demonstrated in a manner that would satisfy Leibniz. Gödel does demonstrate that *each* positive property is possible; that Theorem 1 is derived from Axioms 1 and 2. But it is only *said* that the 'conglomeration' of positive properties is possible or, in Leibniz's word, that these properties

are *compossible*. This crucial bit is contributed by Axiom 3; indeed, it is the *only* thing that it contributes. Axiom 3 is not used in the proof of Theorem 2 and is used just once in the proof of Theorem 3. It is used there with Theorem 1 to infer that God-likeness is possibly instantiated, $\Diamond\exists xG(x)$, which possibility could, for all that P(G) contributes, have been assumed instead as an axiom. Indeed, given Axioms 1 and 2, Def G, and Axiom 4 below, the possibility of God-likeness and its positiveness are *equivalent*. From those four principles

$$\Diamond\exists xG(x) \leftrightarrow P(G)$$

is derivable. Left as an exercise is the derivation from those principles of this equivalence in the system of Appendix C. One already has, in Theorem 1, half of this equivalence.

The efforts of Leibniz and Gödel to prove the possibilities of their gods were very different. Gödel reaches the possibility of the 'conjunction' of all positive properties by way of its positiveness (Axiom 3), whereas Leibniz in his best attempt to establish the possibility of the 'conjunction' of all perfections argues from the simplicity of each. "By a *perfection* I mean every simple quality which is positive and absolute or which expresses whatever it expresses without any limits. But, because a quality of this kind is simple, it is unanalyzable and indefinable. . . . From this it is not difficult to show that *all perfections are compatible with each other* or can be in the same subject" (Leibniz 1969, p. 166).[11] Leibniz cannot argue, in Gödelian fashion, for the possibility of the conjunction of all perfections by way of the perfection of this conjunction. Since there are in his views several perfections, their conjunction, since it is not simple, is *not* itself a perfection. While Leibniz and Gödel agree that God would have only 'positive properties', Gödel alone rules that God would have *all* of them. Leibniz leaves open that God would not have all of them, since he leaves open that not all positive properties are 'simple,' and also that they are not all 'absolute and such as express what they express without any limits'.

3.2.3 A challenge to these beginnings. Difficulties for Axiom 2 and Axiom 3 considered singly have been noted. They may be found to pale in comparison with a difficulty for them taken together. Together they generate a problem for disjunctive properties. For a dramatic illustration from Hàjek (2001), let Devil-likeness be having all properties that are *not* positive. From Axioms 2 and 3 it follows that being either God-like or Devil-like is positive, $P(G \vee D)$. The difficulty has been said to be that the proposition that this disjunctive property is positive in either a moral/aesthetic or a logical/ontological sense "appears counterintuitive" (Hàjek 2001). The difficulty is worse. There is *prima facie* no more reason for saying that that disjunctive property is positive than there is for saying that it is not positive; it is entailed by a property that is positive according to Axiom 3 (and that had better be positive for the object of this exercise), and it is entailed by a property that is presumably 'equally negative'. Axioms 2

and 3 at least *challenge* any interpretation of 'positive' that can serve Gödel's purposes. This is serious for his Leibnizian strategy of working through the possibility of God-likeness to its necessity. The crucial possibility comes from Theorem 1 and *Axiom 3*. Theorem 1 comes from *Axiom 2* and the good part of Axiom 1.[12] It is an unintended virtue of Anderson's emendations, observed in Section PS2.3 of the Postscript below, that they meet this challenge.

3.2.4 Moving on. It is written that "[b]eing a positive property is logical, hence, necessary" (Scott's notes, p. 2):

$$\textbf{Axiom 4. } \mathbf{P}(\phi) \rightarrow \Box\mathbf{P}(\phi)$$

$$\Box(\phi)[\mathrm{P}(\phi) \rightarrow \Box\mathrm{P}(\phi)]$$

Gödel, in his own hand, justifies Axiom 4 in these terms: "because it follows from the nature of the property." (*Ontologischer Beweis*, p. 1). Axiom 4 is there half of the 'second' Axiom 2, the other half of which is $\sim\mathrm{P}(\phi) \rightarrow \Box \sim\mathrm{P}(\phi)$. The necessitated closures of the two halves are interderivable in the system of Appendix C, which may be why only one is stated as an axiom in Scott's notes. Gödel's cryptic comment can suggest that Axiom 4 is another point at which the logical/ontological interpretation of 'positive' was thought to give a simpler proof. However, he may here be writing from the moral aesthetic perspective in which positiveness attaches to properties 'independent of the accidental structure of the world.' Gödel may be saying that positiveness is in this way like Moorean *intrinsic* goodness: Properties that make things Good in Moore's sense entail that they are Good; such properties are 'necessarily Good-making'; they are by their natures 'Good-making.'

3.3 Essences and necessary existence

3.3.1. In Gödel's theory an ***essence*** of an individual is a property of it that entails each of its properties:

$$\textbf{Def Ess.} \quad \phi\ \textbf{Ess x} \leftrightarrow \phi\ (\mathrm{x})\textbf{\&}\ \forall\boldsymbol{\psi}[\boldsymbol{\psi}(\mathrm{x}) \rightarrow \Box\forall\mathbf{y}[\phi(\mathbf{y}) \rightarrow \boldsymbol{\psi}(\mathbf{y})]]$$

$$\Box\forall\phi\,\forall x(\phi\ Ess\ x \leftrightarrow \phi(x)\&\forall\psi[\psi(x) \rightarrow \Box\forall y[\phi(y) \rightarrow \psi(y)]]),$$

since, if ϕ is an essence of x, x has ϕ; ϕ is possibly instantiated and does not entail *every* property. Rather, it entails every property *of x*, and *only* these properties. Since for every property ψ, x has either ψ or its complement $(\neg\psi)$, if ϕ is an essence of x, ϕ entails for every property ψ exactly one of ψ and $(\neg\Psi)$. Essences are 'maximally consistent' properties.[13] It is shown that an essence of any God-like individual is God-likeness:

$$\textbf{Theorem 2. } \mathbf{G(x)} \rightarrow \mathbf{G\ Ess\ x}$$

$$\Box\forall\mathrm{x}[\mathrm{G}(\mathrm{x}) \rightarrow \mathrm{G\ Ess\ x}].$$

Now comes somewhat amplified the proof sketched by Scott. Suppose, for a conditional proof of Theorem 2, that (i) G(x). What needs to be shown additionally for G Ess x is that $\forall\psi[\psi(x) \rightarrow \Box\forall y[G(y) \rightarrow \psi(y)]$. To show that, it is sufficient to assume that (ii) $\psi(x)$, and derive $\Box\forall y[G(y) \rightarrow \psi(y)]$. Here we go. We 'have' that $P(\psi)$. [For that, suppose $\neg P(\psi)$. Then, by the *first* 'if-part' of Axiom 1 (the 'bad part'), $\Box\forall\phi[\neg P(\phi) \rightarrow P(\neg\phi)]$, it follows that $P(\neg\psi)$. So, from (i), G(x), and Def G, $\Box\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$, it follow that $(\neg\psi)x$. But that, by the principle of negations of properties, $\Box(\phi)\forall x[(\neg\phi)x \leftrightarrow \neg(\phi)x]$, is equivalent to $\neg(\psi)x$. And that contradicts (ii), $(\psi)x$.] We have, from $P(\psi)$ by Axiom 4, that (iii) $\Box P(\psi)$. Def G is again $\Box\forall x[G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$. From it and a little modal logic, it follows that $\Box(P(\psi) \rightarrow \forall x[G(x) \rightarrow \psi(x)])$; a formal derivation of this entailment is given in Section C2 of Appendix C. It follows from that, by the modal distribution principle $\Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$, that $\Box P(\psi) \rightarrow \Box\forall x[G(x) \rightarrow \psi(x)]$. *Modus ponens* yields from that and (iii), $\Box P(\psi)$, that $\Box\forall x[G(x) \rightarrow \psi(x)]$. That has been derived on the strength of the assumption, $\psi(x)$, which, as said, amounts to a derivation of $\forall\psi[\psi(x) \rightarrow \Box\forall x[G(x) \rightarrow \psi(x)]]$ from the primary assumption, (i), G(x). Conjoining we have $G(x) \,\&\, \forall\psi[\psi(x) \rightarrow \Box\forall x[G(x) \rightarrow \psi(x)]]$. That, by Def Ess, implies G Ess x, the consequent of Theorem 2, thereby completing a conditional proof of that theorem.

3.3.2. ***Necessary existence*** is defined in the system in terms of essences. It is explained that

$$\textbf{Def NE.} \quad \mathbf{NE(x)} \leftrightarrow \forall\boldsymbol{\phi}[\boldsymbol{\phi}\mathbf{Ess\ x} \rightarrow \Box\exists\boldsymbol{\phi}(\mathbf{x})]^{14}$$

$$\Box\forall x(NE(x) \leftrightarrow \forall\phi[\phi Ess\ x \rightarrow \Box\exists x\phi(x)]).$$

For Gödel, Def NE comes to this: If ϕ is an essence of x,[15] and x has Necessary Existence, then it is necessary that x itself exists and has ϕ. Here is why. From Def NE it follows that, if ϕ is an essence of x, and x has necessary existence, then it is necessary that something has ϕ. Scott notes that in the system things cannot *share* a property that is an essence of one of them: "ϕ Ess $x \rightarrow \Box\forall y[\phi(y) \rightarrow y = x]$." This is necessary, given that a thing's essence contains *all* of its properties; for each thing has the 'property' of being identical with itself, and nothing else can have that property. What this principle 'says' is that *there cannot be two things in the same world*, ***or different worlds****, that share a property that is an essence of one of them in a world*. Furthermore, it must be that *if ϕ is an essence of something in a world, and something has ϕ in another world, then ϕ is an essence of that thing in that world*. This must be, given the *completeness* of essences; each is a maximally consistent conjunctive property; if something has such a property, it is an essence of this thing. From these italicized principles for essences it follows that a thing that has an essence ϕ has Necessary Existence by Def NE if and only if it is necessary that it itself

exists and has ϕ, or, in other words, if and only if it itself exists and has ϕ in every world:

$$\phi \text{ Ess } x \,\&\, \text{NE}(x) \rightarrow \Box(\text{E!}x \,\&\, \phi x)$$

or equivalently

$$\phi \text{ Ess } x \,\&\, \text{NE}(x) \rightarrow \Box(\text{E}y\ x = y \,\&\, \phi x).$$

Necessary Existence for an x with essence ϕ comes to more than existing necessarily, $\Box$E!x; Necessary Existence adds to that, being necessarily ϕ, $\Box\phi x$.

It is remarked that "NE(x) means that *x* necessarily exists if it has an essential property" (Scott's notes, p. 3). The point intended is not that if x has an essence, $\exists\phi\phi$ Ess x, then x has Necessary Existence, NE(x). Indeed, it is a consequence of Def NE that if x does *not* have an essence, then x has Necessary Existence! The point intended is that, for an x that has an essence, NE(x) 'means' or implies that x necessarily exists. To make sure that there are not essenceless necessary existents that do not necessarily exist (!!), one could either lay it down that every existent has an essence or modify Def NE along the lines of Def Ess by requiring for NE(x) that $\exists\phi\phi$ Ess x. I follow the first course in Section 6.1.

Necessary existence is by the last axiom of the system ruled a 'positive' property as Anselmians would have it be:

Axiom 5. P(NE)

"Being logically defined in this way necessary existence is a positive property" (Scott's notes, p. 3). This may be another point at which a logical/ontological take on 'positive' might serve. Even if, as I think, NE may *well* not be positive in any 'moral aesthetic' sense, there does seem to be 'no privation' about it.

4. THAT IT IS NECESSARY THAT THERE IS A GOD-LIKE BEING

4.1. Here is the theorem and proof, more or less as they stand in Scott's notes:

Theorem 3. $\Box\exists x\text{G}(x)$

Proof $\text{G}(x) \rightarrow \text{NE}(x)$ & G Ess $x \rightarrow \Box\exists x\text{G}(x)$

$$\exists x\text{G}(x) \rightarrow \Box\exists x\text{G}(x)$$

$$\Diamond\exists x\text{G}(x) \rightarrow \Diamond\Box\exists x G(x) \rightarrow \Box\exists x\text{G}(x)$$

But $\Diamond\exists x\text{G}(x)$ by Theorem 1.

$$\Box\exists x\text{G}(x) \quad \text{Q.E.D.}$$

When scrutinized, this proof can be seen to execute the 'Leibnizian strategy,' which is – to show that God is necessary, show that God is possible, and that

if God is possible, then God is necessary – or equivalently, that God is either impossible or necessary. The conclusion that God is necessary then follows by double negation and disjunctive syllogism. Let us scrutinize. Lines of Scott's sketch, teased out, are

$$\text{(i) } G(x) \rightarrow NE(x) \,\&\, G \text{ Ess } x;$$

$$\text{(ii) } G(x) \rightarrow \Box\exists x G(x)$$

$$\text{(iii) } \exists x G(x) \rightarrow \Box\exists x G(x)$$

$$\text{(iv) } \Diamond\exists x G(x) \rightarrow \Diamond\Box\exists x G(x)$$

$$\text{(v) } \Diamond\exists x G(x) \rightarrow \Box\exists x G(x)$$

$$\text{(vi) } \Diamond\exists x G(x)$$

$$\text{(vii) } \Box\exists x G(x).$$

Now for how these lines are reached. We have,

$$\begin{aligned} \text{Def G} \qquad & G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)], \\ \text{Axiom 5} \qquad & P(NE), \\ \text{and Theorem 2} \qquad & G(x) \rightarrow G \text{ Ess } x. \end{aligned}$$

From these it follow that

$$\textbf{(i)}\ G(x) \rightarrow NE(x) \,\&\, G \text{ Ess } x.$$

[Here is how. Assume G(x). From this and Def G, infer [P(NE) → NE(x)]. From that and Axiom 5, infer NE(x). From the assumption G(x) and Theorem 2, infer G Ess x. Conjoin for NE(x) & G Ess x.]

From (i), and Def NE

$$NE(x) \leftrightarrow \forall\phi[\phi \text{Ess } x \rightarrow \Box\exists x\phi(x)],$$

it follows that

$$\textbf{(ii)}\ G(x) \rightarrow \Box\exists x G(x)$$

[For this, assume G(x). From that and (i), infer NE(x) & G Ess x. From NE(x) and Def NE, infer G Ess x → □∃xG(x). From that and G Ess x, infer □∃xG(x).]

and indeed

$$\text{(ii}') \ \forall x[G(x) \rightarrow \Box\exists x G(x)],$$

for definitions, axioms, and theorems, and all consequences of these, are implicitly necessities of universal closures of stated formulas. From (ii′), by quantifier confinement, it follows that

$$\textbf{(iii)}\ \exists x Gx \rightarrow \Box\exists x Gx,$$

and indeed

$$(\text{iii}')\ \Box(\exists \text{xGx} \rightarrow \Box\exists \text{xGx}),$$

for the reason recently stated. From (iii′), by the modal distribution principle $\Box(\text{P} \rightarrow \text{Q}) \rightarrow (\Diamond\text{P} \rightarrow \Diamond\text{Q})$,

$$\textbf{(iv)}\ \Diamond\exists \text{xGx} \rightarrow \Diamond\Box\exists \text{xGx},$$

follows, from which, by the S5 reduction principle $\Diamond\Box\text{P} \rightarrow \Box\text{P}$, it follows that

$$\textbf{(v)}\ \Diamond\exists \text{xGx} \rightarrow \Box\exists \text{xGx}$$

or equivalently (shades of Leibniz)

$$\neg\Diamond\exists \boldsymbol{xGx} \ \vee \Box\exists \boldsymbol{xGx}.$$

But from Axiom 3

$$\text{P(G)},$$

and Theorem 1

$$\Box(\phi)[\text{P}(\phi) \rightarrow \Diamond\exists \text{x}\phi(\text{x})]$$

it follows that

$$\textbf{(vi)}\ \Diamond\exists x\, G(x),$$

from which boldly italicized consequences it follows by double negation and disjunctive syllogism that

$$\textbf{(vii)}\ \Box\exists \boldsymbol{xGx}. \qquad \text{Q.E.D.}$$

Nice, huh? Well, no surprise. This is Gödel's proof, tidied by Scott, for goodness sake! Section C3 of Appendix C has a formal derivation of Theorem 3. Gödel could have added the corollary that,

It is necessary that there is exactly one God-like being, and *that This One has Necessary Existence*, or equivalently, *that This One necessarily exists and is God-like*:

$$\Box\exists \text{x}[\forall \text{y}(\text{Gy} \leftrightarrow \text{y} = \text{x}) \ \&\ \textit{NE(x)}]$$

or equivalently

$$\Box\exists \text{x}[\forall \text{y}(\text{Gy} \leftrightarrow \text{y} = \text{x}) \ \&\ \Box\textit{(E!x}\ \&\ \textit{Gx)}].$$

'E!x' can be short for '$\exists \text{z}\ \text{z} = \text{x}$'. Now comes a deduction of this corollary: One may gather from Theorem 3 that there exists a being that has G, and, considering a particular one, *g*, we may gather from Theorem 2 that G is an essence of *g*. According to the principle for unshared essences (Section 3.2.2), there exists in no world something other than *g* that has property G. Since, by Theorem 3 again, in every world there exists something that has this property G, *g* is the thing that, in every world, exists and has property G. This God-like

being is in every world *the* God-like being. Gödel had ready a name for it, and Theorem 3 had the substantive corollary,

God exists!

reached by identifying *God* with *the God-like being*. (See Section 3.1.1 for the issue of this identity.)

4.2 Our texts stop here with the demonstration that it is necessary that there is a God-like being. Perhaps Gödel would have said either, 'that's enough for now' or, in the spirit of Hume's *Cleanthes*, 'that's enough for me': "I have found my Deity; and here I stop my enquiry" (Hume 1991, p. 127). Hume's *Dialogues*, however, continued, and reasonably so. Given the character of *Cleanthes'* argument, there was the possibility of its being upset by the production of better explanations of the facts at that point in evidence, and by the introduction into evidence of additional facts. Gödel's *a priori* argument for the necessary existence of a God-like being was not in danger of being similarly upset, but in its case too discussion might reasonably have continued, for further developments can raise questions concerning the theological significance of the theorem with which Gödel stopped. They can raise questions concerning every significance of every theorem of the system. The next section develops the first possibility, the section after it the second.

5. WOULD THAT BE GOD, *COULD* IT BE GOD?

5.1. There is a proof in the system that there is a God-like being, a being that has every positive property. Before rejoicing, however, we should ask what such a being would be like and what properties, designated in ordinary terms used in ordinary ways, it would have and not have. It is obvious that, although we can by various devices include existence in definitions of kinds of things, we cannot thereby define into existence things of any kinds we please. A God-like being has been defined into existence. This production begs that we ask: What would such a being be like? Would it be anything like a *god* properly so-termed? Answers can be argued from theorems that are not set out by Scott or Gödel. The first of these says that a God-like being would have *only* positive properties:

Theorem 4. $\mathbf{G(x) \rightarrow \forall \phi [\phi(x) \rightarrow P(\phi)]}$

$$\Box \forall x (G(x) \rightarrow \forall \phi [\phi(x) \rightarrow P(\phi)]).$$

This follows from Axiom 1 (from *the 'only if', bad half* of Axiom 1) and Def G by a small argument. Section C3 of Appendix C contains a formal derivation. A second theorem of present relevance says that in the system every positive

property is necessarily instantiated:

Theorem 5. $\mathbf{P(\phi) \rightarrow \Box\exists x \phi(x)}$

$\Box\forall\phi[P(\phi)\rightarrow\Box\exists x\phi(x)]$.

This theorem, given that G contains every positive property, is a corollary of Theorem 3; Section C3 of Appendix C includes its formal derivation. It is an easy consequence of Theorems 4 and 5 that a God-like being would have only necessarily instantiated properties,

Theorem 6. $\mathbf{G(x)\rightarrow \forall\phi[\phi(x)\rightarrow \Box\exists x\phi(x)]}$

$\Box\forall x(G(x)\rightarrow\forall\phi[\phi(x)\rightarrow\Box\exists x\phi(x)])$.[16]

Indeed, in the system, every property of a God-like being would be necessarily instantiated by a God-like being:

Theorem 7. $\mathbf{G(x)\rightarrow \forall\phi[\phi(x)\rightarrow \Box\exists x[G(x)\&\phi(x)])}$

$\Box\forall x(G(x)\rightarrow\forall\phi(\phi(x)\rightarrow\Box\exists x G(x)\&\phi(x)])$.

This theorem (derived in Section C3 of Appendix C) provides a frame for problems with an identification of Gödel's God-like being with God.

Interpreting the system requires that substance be given to its primitive 'P'. If the system is to have religious significance, that substance must make plausible that a being possessed of precisely positive properties would be, if not *God*, then at any rate *a god*, that is, *a worshipful being*. 'Gx' must say that x is a worshipful being and that this 'God-like being' is in this crucial respect *properly so termed*. Theorem 7, read with this in mind, is a trouble-maker. It should, given this theorem, be obvious and beyond reasonable question that no God-like being that could be a *god*, that is, that no 'God-like' being that could be *worshipful*, would be red, blue, or indeed, any color at all. For it should be obvious that it is not logically necessary that there exists a thing that has the property of being both worshipful and red – that there could have been no such red thing – that it is not the case that there is in every possible world *such* a red thing, a *worshipful* red thing. Perhaps the stoplight exists and is red in every world, and similarly for Santa Claus's suit. I will assume so, somewhat as the number 2 exists and is even in every possible world. However – now comes the difficult heart of the matter to which I will return in Section 6, and, in a manner, again in Chapter VI – *such existents would not be worshipful*. Just as it would be absurd to worship the anti-log of 105 (which happens to four places to be 2.0212), and The North Star, since they are *incapable of hearing prayers and devotions*, so it would be absurd to worship the stoplight or Santa Claus's suit. It is not logically necessary that there exists a *worshipful* red thing, if it is not logically necessary that there exists a red thing that is not an 'abstract entity', that is, if it is not logically necessary that there is a red thing in the world *with*

us, and capable of causal relations with us, a red thing that might, for example, *hear or otherwise be affected by our prayers and worshipful devotion*. Surely, however, it is not logically necessary that there should be *such* a red thing. We can conjure 'appearances of possibilities' of worlds in which there are none of that sort, and nothing counters the evidence of these appearances for the possibilities they present. In this case, I assume, concrete clear and distinct 'conception' is sufficient for possibility. If it is possible that there should have been no red things related to us as worshipful things must be, then it is not logically necessary that there is such a red thing.

It is not disturbing that no God-like being could be a red thing in the world with us. But it should be nearly as obvious, and may be somewhat disturbing, that no God-like being that was in the world with us, and capable of interacting with us, would be conscious, knowledgeable, powerful, loving, or wise. For a *god* would be *worshipful* and thus would be in the world with us, and it would be worshipful *because of its properties*; and the list of the last sentence threatens to be extendable to all properties that anyone could suppose would make something worshipful. The problem is that it seems to many to be a firm modal intuition that there are possible worlds in which there are not only no *red* things capable of interacting with us, but no *such* powerful, loving, or wise beings. Similarly for beings who might have stood in certain relations to us in history, for example, beings who 'were there' for our ancestors in their times of trouble, who sacrificed themselves for us, and so on.

5.2. "Sobel's intuitions on this point are shared by many philosophers, but consciously rejected by virtually all partisans of the ontological argument" (Adams 1995, p. 399). "Friends of the ontological argument are bound to see [this objection of Sobel's] as merely a repackaging of a familiar empiricist objection, based on the claim (consciously rejected by them) that a being possessing the sort of reality generally ascribed to God could not exist necessarily" (Ibid.). Perhaps, though I am not reporting *here* the intuition that no logically necessary existent could be worshipful. I am reporting here the intuition that it is not logically necessary that there should exist a being who is both worshipful *and* . . . feel free to enter any property or relation that you think might contribute to a being's worshipfulness. This intuition comes from reflection on *worshipfulness* and the sense that to be worshipful a being would need to be 'reachable' by worshippers and 'touched' – *causal notion* – by their devotion. Such a being would, it seems, need to be 'in the world' in a manner in which it is not *necessary* that any loving, powerful, or knowing being is 'in the world'. The not *so* simple intuition I am confessing, framed in terms that relate it to Theorem 7, is that if 'G' – 'God-likeness' – includes being at least *possibly* worshipful, then for every property ϕ that might be thought to tend to *make* a being worshipful no 'G-being' has ϕ, *because* it is not *necessary* that there is something that has both G and ϕ, $\neg\Box\exists x[G(x) \,\&\, \phi(x)]$.

5.3. There *could* have been nothing 'real', in the world with us, other than earth, air, fire, and water, along with necessary things such as the number 2, the color blue, and the stoplight. At least that seems *to me* to be a logical possibility. Such a world has been said to be not even *conceivable*.

> *Phil*. How say you, Hylas, can you see a thing which is at the same time unseen? *Hyl*. No, that were a contradiction. *Phil*. Is it not as great a contradiction to talk of conceiving a thing which is unconceived? (Berkeley 1965, p. 162.)

But to Philonious's question *I* answer, "No, it is *not* impossible to conceive of things in this world that never have been and never will be thought about or conceived by anyone, though I of course can give as examples only things that *might* never have been thought about or conceived of by anyone." More pertinently to our present subject, one can conceive of a world in which nothing is thought about or conceived and in which there is in fact no conscious being to do any thinking or conceiving. Berkeley has not given, nor can one find suggested in *anyone's* writings, including those of ontological arguers, a good argument for doubting this possibility of conceiving, 'Yablo-conceiving' (Section 8.5 of the previous chapter), which involves an *appearance of the possibility* of what is conceived. That, in the present case, is the same as a *plain appearance* that it is no more necessary, *logically* necessary, that there is a sentient or cognizant 'real' being, a being to which we could relate in 'meaningful' accidental ways (I have in mind ways of benefaction and communication) – or, in other words, a possibly *worshipful* being that is sentient or cognizant – than that there is such a being that is red. And on modal issues we have at least initially only such appearances or 'intuitions' to go by.

These are not *arguments*, but they are *reasons for believing*, that operate to raise presumptions and to establish burdens of argument. For example, amongst those who share the compelling intuition just reported, a burden of argument falls on one who claims that even so it *is* logically necessary that there is a red thing that just might be worshipful, whereas no argument is called for in defense of these compelling intuitions. For those who share the just previous intuition, the burden would be no less on one who claimed that even so it is logically necessary that there is a possibly worshipful sentient or cognizant being, or indeed a possibly worshipful being of any mode of consciousness. Lastly, to broadcast an intuition that Adams may tell us he does not share, it seems ever so obvious to me that there *could* have been, as far as things related to us in *possibly* worshipful way manners, nothing at all other than quite unconscious things, and thus *unworshipful* things. This seems so to me just as clearly as it seems that there *could* have been nothing in the world with us that is blue, as it is sometimes said (falsely) that there are no blue foods. I can in a way imagine it.[17]

5.4. A God-like being, indeed every being, has for every property either it or its negation. But, subject to arguments sufficiently powerful to reverse very firm

intuitions conjured as relatively vivid 'appearances of possibilities,' at least some of us may now see that a possibly worshipful being that was God-like in the sense of the system would, in connection with many religiously important properties, have not them, but their negations. It may now be evident, subject to the availability of good counterintuitive arguments that we have no reason to believe exist, that a *God-like* being of Gödel's system, if possibly worshipful, would not be omniscient, omnipotent, just or benevolent, the creator . . . and so on for every worshipful making property. That is, a God-like being of the system, if *possibly* worshipful, for want of every property that would make that one worshipful, *would not be* worshipful. Which is to say that no God-like being of the system would be a *god*.[18]

6. MODAL COLLAPSE

If widespread clear and firm modal intuitions are to be trusted, a God-like being of the system would lack many if not every religiously important property. To that may be added that it is demonstrable in the system (perhaps slightly augmented) that such a being would *have* properties, the having of which would be embarrassing – *logically* embarrassing, even if not religiously so. Given the generous interpretation of 'property' that is in force for the system, a God-like being would have properties that entailed the existence of every existent and the truth of every truth. Since a God-like being would have only necessarily instantiated properties, it follows that in the system every truth is a necessary truth and every existent is a necessary existent. If one thinks that these theorems of the system are *false* – and who doesn't? – then one must think that proving something *in the system* is not to establish the *truth* of this thing. Put otherwise, if you think these coming theorems are false, then, since they follow validly from the axioms, these cannot all be true *under any interpretation of the primitive P*. What is entailed by false premises, while it need not be false, of course need not be true.[19]

6.1 Essences and properties. To show *formally* that every existent is a necessary existent, which is done in Section C3 of Appendix C, I use the principle that every individual has an essence:

Essences. $\forall x \exists \phi\, \phi \mathbf{Ess}\ x$.

This principle, if not, as I think it is, already implicit in the system, is only a slight addition. A reason for thinking that *Essences* is implicit in the system is that, whereas certain *contingent* beings, for example, people such as Adam who do not necessarily exist, would be prime candidates for things that lacked essences and 'complete concepts,' in the system any being that lacked an essence would have Necessary Existence, and so one is tempted to say would be in the system's terms a '*necessary* being' even if in ordinary terms it is a contingent being. This is by Def NE. [Suppose that *x* lacks an essence, so that $\forall\phi\neg\phi$ Ess x. Then

$\forall\phi[\phi\ \text{Ess}\ x \rightarrow \Box\exists x\phi(x)]$, and so, by Def NE, NE(x).] *Essences* avoids such awkward beings. Incidentally, one reason for resisting Essences – namely, that its imposition would alone leave no room for free agents – is not afforded by the system under discussion, this for reasons in the last paragraph of Section 6.3. Also used in formal derivations are 'instances' of the following schema for abstraction principles for properties of individuals:

Properties. $\alpha[F](\beta)\leftrightarrow\mathbf{F}'$,

α a variable, β a term, F a formula, and F' is a formula that comes from F by 'proper substitution' of β for α (i.e., F' is like F except that wherever α occurs free in F, β occurs free in F'). Here is a case of *Properties* with words matched to it under the assumptions that 'Rx' and 'Ra' abbreviate 'x is red' and 'a is red':

($\hat{x}$[Rx]a $\leftrightarrow$ Ra): *a* has the property that exactly things that are red have if and only if *a* is red.

The principle *Properties*, which in its scope is essential to demonstrations of coming Theorems 8 and 9, expresses the generous interpretation of 'properties' that is in evidence in our texts. Cf.:

> It is worth noticing that there is here [the reference is to Def G and Axiom 3] an implicit assumption: if we have defined a predicate [as in Def G], then we can straight-away form a name of the property it expresses [and use if as in Axiom 3]. (The technically minded will thus wish to note that it is in effect assumed that anything is counted as a property which can be defined by "abstraction on a formula.") (Anderson 1990, p. 292.)

Something like *Properties* is in ample evidence notwithstanding that it "is not a part of Gödel's argument" (Adams in Gödel 1995, p. 402), which uses only a few instances of it. Another principle used is that things have unique essences,

Unique of Essences. $\forall\phi\forall\psi\forall\mathbf{x}(\boldsymbol{\phi}\mathbf{Ess\ x}\ \&\ \boldsymbol{\psi}\mathbf{Ess\ x} \rightarrow\Box\forall y[\boldsymbol{\phi}(y)\leftrightarrow\boldsymbol{\psi}(y)])$,

which is presumably equivalent to ϕ Ess x & Ψ Ess x $\rightarrow \Box\phi = \Psi$, a principle mentioned previously in Section 3.2.1 that is endorsed in our texts (Scott's notes, p. 3).

6.2 That there exist ***only*** *necessary beings, or that everything that exists, exists necessarily.* This unwanted result is part of something stronger that is provable in the system (or in the system as augmented by *Essences*), namely,

Theorem 8. $\forall y\text{NE}(y)$,

or equivalently

$\forall y[\exists y\, x = y \rightarrow \text{NE}(y)]$.

This theorem says, of each thing that exists, that *it exists, with the essence it has, in every world*. (See Section 3.2.2 for this gloss on Def NE.) A formal derivation of this theorem is given in Section C3 of Appendix C. The idea of the proof – very roughly – is that, for any existent distinct from a God-like being, a God-like being *g* would have *the property that there is something x different from g the essence of which is E*. Since that property, as every property of a God-like being, is necessarily instantiated, Theorem 6, the essence *E* is necessarily instantiated, which is to say that x necessarily exists. In short, in the system, since there is necessarily a God-like being, Theorem 3, every existent is a necessary existent.

6.3 That there are only necessary truths. It is provable in the system that whatever is true is necessarily true:

Theorem 9. $Q \rightarrow \Box Q$.

The idea of a formal derivation given in Section C9 of Appendix C – again only roughly – is that, for any truth P, a God-like being would have *the property of being identical with itself in the presence of the truth that P*. This, as every property of a God-like being, would be necessarily instantiated. And so, since there is necessarily a God-like being, every truth is a necessary truth. Theorem 9 has, incidentally, in the language of our somewhat free quantified modal logic, an 'instance' a universal closure of which says exactly the thing that I have observed that Theorem 8 implies, namely, that everything that exists exists necessarily:

$$\forall x(E!x \rightarrow \Box E!x)$$

or equivalently

$$\forall x \Box E!x.$$

Equivalent theorems that replace 'E!x' by '$\exists y\ x = y$' use no symbols not in evidence in Scott's notes.

The formal derivation for Theorem 9 uses *Properties* and so depends on the scope of the generous interpretation of 'property' in evidence in the notes of Gödel and Scott, as does the derivation for Theorem 8. *Essences* is not used in the deduction of Theorem 9 (nor is *Unique of Essences*). This means that a reason for resisting *Essences* – namely, that omitting it would make room for freedom in the world, and for free individuals – is not available to one who accepts the system. The necessity that Leibniz was so concerned to avoid *obtains* in Gödel's system. In it, of everything that is true, that it is true is necessary. And this necessity obtains whether or *not* it is stipulated that every individual has an essence in the Leibnizian sense, that is, a complete individual concept. (Appendix A to Chapter VI says more about Leibniz's individual concepts.)

6.4 The system would 'collapse modalities' into just two. It is a corollary of Theorem 9 that, in the system categories of actuality (truth, is-the-caseness), possibility and necessity are one:

Theorem 10. $(\mathbf{P} \leftrightarrow \Diamond\mathbf{P}) \,\&\, (\Diamond P \leftrightarrow \Box\mathbf{P})$ **&** $(\mathbf{P} \leftrightarrow \Box\mathbf{P})$.

For Theorem 10 it is sufficient that $(\Diamond P \rightarrow P)$ is derivable from Theorem 9, for $(P \rightarrow \Diamond P)$ is a theorem in every logic for alethic modalities. Also, given that $(\Diamond P \rightarrow P)$, it is an easy consequence of Theorem 9 that $(\Diamond P \rightarrow \Box P)$, whereas $(\Box P \rightarrow \Diamond P)$ is a theorem in every such logic. For the last conjunct of Theorem 10, we have Theorem 9 itself and that $(\Box P \rightarrow P)$ is of course a theorem of every such logic. Going back to the beginning, to derive $(\Diamond P \rightarrow P)$ from Theorem 9 one may: Assume for a conditional proof $\Diamond P$, and then $\neg P$ for an indirect proof of the consequent; from that it follows by Theorem 9 that $\Box\neg P$, from which it follows that $\neg\Diamond P$, which contradicts the initial assumption $\Diamond P$. That completes the indirect proof that, given Theorem 9, it follows from $\Diamond P$ that P, which completes the conditional proof that $(\Diamond P \rightarrow P)$ follows from Theorem 9. Theorem 10, 'contrapositized,'

$$(\neg\mathbf{P} \leftrightarrow \neg\Diamond\mathbf{P})\mathbf{\&}\ (\neg\Diamond\mathbf{P} \leftrightarrow \neg\Box\mathbf{P})\mathbf{\&}\ (\neg\mathbf{P} \leftrightarrow \neg\Box\mathbf{P}),$$

says that nonactuality (falsity, is-not-the-caseness), impossibility, and non-necessity are one.

Almost everyone believes that Theorem 10 is false. Almost everyone believes that there are truths that are not necessarily true. And so almost everyone should believe that the axioms of the system are not all true under any interpretation of 'P' and that, to derive something in the system – for example, to derive Theorem 3 in the system that says that it is necessary that there is a God-like being – is not to *prove* that this, *whatever* one makes of it, is *true* or indeed to provide *any* grounds for thinking that.

7. CONCLUDING RECOMMENDATIONS

7.1. The solution to the difficulties of the last two sections – the best and smallest change that would obviate them – can seem obvious: Stop counting necessary existence as a positive property that a 'God-like being' properly so termed would have, give up on the idea of ontological arguments, and concede that no worshipful being could be demonstrable.[20] It has been said (Section 3.1.1) that, necessary for the identification of 'the God-like being' of Gödel's system with God, is that there *not* be a property that is 'positive' in *every* sense of 'positive' that makes his axioms true, which property *makes a God-like being* NOT *worshipful*. I am now saying straight out what was implied plainly enough in Section 5, that Axiom 5, according to which Necessary Existence is positive, is for this reason too bad for his system the object of which is that identification.

It seems to me obvious that no being that had properties that made it an object worthy of worship *could* also have necessary existence. But then, on reflection, it can also seem, for it *has* seemed to some, obvious that no being that *lacked* necessary existence could be a proper object of worship. These two perspectives taken together and refined can be made to yield a kind of ontological argument for the impossibility of a God – an argument like one that J. N. Findlay presented in 1948. He summed up his case against the possibility of a God in these words:

> The religious frame of mind . . . desires the Divine . . . both to have an inescapable character . . . and also the character of "making a real difference". . . . if God is to satisfy religious claims and needs, he must be a being in every way inescapable, One whose existence and whose possession of certain excellences we cannot possibly conceive away. . . . It was indeed an ill day for Anselm when he hit upon his famous proof. For on that day he not only laid bare something that is of the essence of an adequate religious object, but also something that entails its necessary non-existence. (Findlay 1955[1948], p. 54[182])

Findlay changed his mind about his 1948 disproof of the existence of God. "I have moved," he wrote in 1970, "to a position where, by a change of attitude to a single premiss, the disproof has swung over into something that may, if it betrays no inward, logical flaw, converge towards a proof" (Findlay 1970, p. 13). Findlay was still convinced that an Absolute would be a necessary existent (p. 39), which would have a necessary nature: "If there is an Absolute of a certain essential sort, then there cannot not be an Absolute of that essential sort" (p. 24). But he was in 1970 no longer prepared to say that nothing that was by its nature worthy of the name 'Absolute' – or, presumably, worthy by its nature (as distinct from its possibly 'contingent side') of worship – could have a nature that was in all ways necessary. I confess, however, that the 1948 proposition on this point still seems right to me, and that I am in fact confirmed in my conviction by the *last* things set down in Findlay's 1970 book. For it *does* seem to me that a necessary being would need to be of the nature of a Form or of an abstract entity such as a number or the stoplight, and that there is no way in which the existence of any other kind of being could be necessary. Findlay, I believe, agreed in 1970. In any case he held that God, that *that* would-be necessary being, would have to be of the nature of a Form, though Findlay gave as his reason for this, not that His existence would be necessary, but, somewhat surprisingly, that only a Form can be truly worthy of worship:

> I give it as the verdict of my feeling that only a Form, something basically universal, can be truly adorable, can in any way deserve the name of 'God'. One cannot rationally worship this or that excellent thing or person, however eminent and August: only Goodness Itself, Beauty Itself, Truth Itself, and so on are rationally venerable, and to bow one's knee to an instance is to commit idolatry. (From *Towards a Neo-neo-Platonism*, Findlay 1970, p. 267, last paragraph of the book.)

But, to go back to a line of Findlay (1948), such things can 'make no real difference'. Nor can we make any real difference to them. And while there is, I think, nothing irrational about liking a Form or abstract entity, having a favorite Form or abstract entity, being thrilled by the details an abstract entity such as The Perfect Being understood or The Ford Prefect, or even *adoring* such a thing, though that is something of a stretch,[21] I draw a line this side of worship and 'give as the verdict of my feeling' that to *worship* a Form or abstract entity would be like declaring one's gratitude, one's love and devotion, one's awe and amazement to a recorded announcement *after recognizing it as such* (!), and similarly *absurd*.[22]

Were Gaunilon to offer to redirect Gödel's system to prove the necessary existence of a blessed isle, Anselmians could be expected to say, "No, it does not work. For the re-direction would make Axiom 5 say that Necessary Existence is a positive property of isles, whereas it is not a *possible* property of isles, where these would be real islands and not notional ones such as this Blessed Isle itself of philosophic lore." I agree, but, turning this Anselmian retort, say that *similarly* Necessary Existence is not a possible property of a *god*. It is, in my modal opinion, not a possible property of a being who could 'make a difference', to whom one could reasonably be *grateful*, to whom one could reasonably turn for assurance, assistance, or guidance. It is not, to cut to the main thing, a possible property of a being whom one could without absurdity *worship*.

7.2 'It was *an ill day.'* Having proved to his satisfaction in *Proslogion* II that a being than which none greater can be conceived exists, Anselm, proceeded in *Proslogion* III to argue in a similar manner that such a being exists 'so truly' that it cannot be conceived not to exist. Rather than find in this corollary of his proof a reason for doubting its soundness, he made a virtue of it. That is how the idea was born that necessary existence is a perfection appropriate to The One whom Anselm worshipped and adored and would have everyone worship and adore. Aquinas liked the idea, which is too bad, though he did not like the proof, which is good. Leibniz made the idea central to his cosmology, and moderns from Hartshorne to Gödel have taken to the idea for the sake of valid ontological arguments. But it was a bad idea, ill-born. The argument of *Proslogion* II from which it emerged, though valid, begs its question and runs with what only *seemed* to be good grounds for its possibility-premise. Necessary existence, even if it *had* the *philosophical* advantages that some claim for it, far from contributing to proper greatness and *worshipfulness*, is *prima facie* and, I think, not only *prima facie*, at odds with that, and disadvantaged in the extreme religiously.

There is no possibility of coercive demonstration here. But it does seem to many that only things such as numbers and Forms have necessary existence, and that no such things qualify for what would be God's office. Be eternally grateful to infinite goodness itself? *Worship* it? There are limits to nonabsurd

orientations of religious attitudes and behavior, and things such as that, that is, all necessarily existent things certainly seem to lie beyond. The words 'worshipful necessarily existing being' harbor no contradiction, but even so they do not describe a possibility, and when we hear and understand them we do not have in our minds a worshipful necessarily existing being. And that should in my view put an end to the arguments that Anselm began in *Proslogion* II, though knowing something of those who see something in them – as many perfect being philosophical theologists still do – I am confident that it will not do that.[23,24]

POSTSCRIPT: ANDERSON'S EMENDATIONS

Exposition

C. Anthony Anderson, in a careful and valuable discussion, suggests changes in Gödel's system. These changes avoid modal collapse and have several other advantages.

PS1.1 A new primitive in terms of which Gödel's is defined, and deletion of the 'bad part' of Axiom 1. Perfection is the primitive of the emended system, in terms of which 'positive' is defined. For this defined notion I use 'positive*,' and in formulas I use Pos instead of P. Except for the use of Pos rather than P, Axiom 1* looks the same as the 'good part' of Axiom 1 that says that the negation of a positive property is not positive. Left out is the part that says that the negation of a nonpositive property is positive. This 'bad part' of Axiom 1 is used in the proof of Theorem 2 in Section 3.2. Retained is the part that plays a role in the proof of Theorem 1 in Section 3.1.2. Axioms 1* and 2* are derived from the definition of 'positive*': "A property is positive if its absence in an entity entails that the entity is imperfect *and* its presence does not entail [that]" (Anderson 1990, p. 297). The second conjunct, for compatibility with perfection, entails that positive* properties are possible and "is needed to prove Axiom 1*" (Ibid.). This definition is, from the perspective of perfect-being theology, exactly right. "*Of course*," an Anselmian might say, " 'positive' properties are properties a thing needs to have in order to be perfect, and can have if perfect!!"

It is an unvoiced part of the perspective of perfect-being theology that *perfection is possible* or, equivalently, that imperfection is not necessary, and Anderson's emendation 'wants' this as a 'pre-axiom.' Were perfection not a possible condition, then by his definition there would not be any positive* properties. Section C.9.1of Appendix C includes some elaboration. But would not such a pre-axiom beg a question that Leibniz would want addressed? No, not if the question is whether or not a being possessed of every positive* property or perfection is possible. Settling by an axiom that *perfection* is

possible leaves open its relation to the intersection of positive* properties or *perfections* and so leaves open whether this composite property is possible, which is what Leibniz took to be the issue.

There is a problem with the plausibility of Axiom 1, which says that, of any property and its complement, exactly one is positive in an evaluative sense. This "seems to overlook a possibility: that both . . . should be *indifferent*" (p. 295). Anderson's Axiom 1* does not have this problem. And Anderson's definition of positiveness in terms of imperfection resolves the *prima facie* problem of Axiom 2 when taken to be of 'moral aesthetic positiveness.' His definition also enables the derivation of Axiom 4*, which 'looks like' Axiom 4. Statements and derivations of his Axioms 1*, 2*, and 4* from his definition of positiveness are given in Section C4 of Appendix C.[25] He implies that neither his Axiom 3* nor Axiom 5* are derivable from his definition of positiveness (p. 301n16).

PS1.2 A new definition of a thing's essence and, in terms of it, of God-likeness. "Another change which seems advisable is"[26] to define the essence* of an entity as "a property which entails all and only [its] *essential* properties" (p. 295).[27] The definition of necessary existence is re-addressed to essences* for a definition of necessary existence*. "Finally," Anderson advocates equating God-likeness* with the higher order property of having an essence* that entails "those and only those properties which are positive" (p. 296). Anderson loses Theorem 4, according to which a God-like being has only positive properties. That opens up the possibility of a God-like being's positive* properties not coinciding with its 'god-making' properties, that is, with the properties that make it worshipful. "It is not," an Anderson/Gödel can say, "only for His essential and 'positive*,' but also for some of His accidental, properties that we should worship Him."

Anderson shows that Theorems 2* and 3*, which are 'lookalikes' of Theorems 2 and 3, concerned however with essences* and God-likeness* rather than with essences and God-likeness, are provable in his system. His proof of Theorem 2* is somewhat different from our demonstration of Theorem 2. His proof of Theorem 3* goes through "much as before" (p. 296), that is, much as his demonstration, which is like mine, of Theorem 3. A derivation of Theorem 2* somewhat like Anderson's informal demonstration, and corroboration that a formal derivation of Theorem 3* can be 'much as before,' are given in Section C4.3 of Appendix C.

Anderson has performed a nontrivial task of a kind at which I vaguely waved in 1987 (see note 19). His changes avoid the modal collapse demonstrated in Section 6,[28] and undercut arguments of Section 5. Those arguments, which say that Gödel's god could hardly be God and worthy of worship, do not work against the god (or gods) of Anderson/Gödel. Anderson sees that his emendations "preserve at least some of the essentials of Gödel's proof"

(Anderson 1990, p. 297) and concludes, "If Kurt Gödel thought that the matter can be settled in the affirmative by proof, perhaps those of us who are interested in the question ought to see what merit we can find in his line of reasoning" (p. 298).

Discussion

PS2.1 'But it might not be God'. However, though Anderson's emendations block arguments that would show that no God-like* being *could* be God, they invite analogous arguments that would show that no God-like* being *need* be God. According to Anderson, "x is *God-like** if and only if x has as **essential** properties those and only those properties which are positive" (p. 296; bold emphasis added). But,

$$\text{Theorem } 5^*. \quad \text{Pos}(\phi) \rightarrow \Box\exists x\phi(x)$$

is demonstrable in the Anderson/Gödel system and – I now, without repeating the arguments of Section 5, adapt its concluding words – subject to the availability of good counterintuitive arguments that we have no reason to believe exist – a God-like* being related to us in a manner that made it possibly worshipful, *need* not be, that is, it is not *essential* that it be omniscient, omnipotent, just, or benevolent. A possibly worshipful God-like* being *can* lack every attribute that might recommend it as an object of worship. Every God-like* being *can*, for anyone who would speak in ordinary terms, be an inappropriate subject for the name 'God'. There *need* not be anything 'God-like' *properly so-termed* about one. To earn that designation for a particular God-like* being, one would have to establish that this being, though it need not, does in fact have properties that make it an appropriate object of worship. For few, if any, such properties will be parts of its *essence*, if it is 'in the world' and so possibly worshipful. It is possible, as far as I can tell, that a God-like* being that was worshipful would need to be made so entirely by properties *not* essential to it. If that is right, than establishing that there must be a God-like* would not be even 'getting started here' on establishing that there is a *god*.

PS2.2. Anderson's useful definition of perfections or positive* properties in terms of the condition of perfection places not only Axiom 1* but also Axioms 2* and 4* in favorable lights. They are derivable from this correct for perfect-being theology definition. That leaves Axioms 3* (that God-likeness* is a positive* property) and Axiom 5* (that Necessary Existence* is a positive* property) for scrutiny.

PS2.3. A bonus for Anderson's emendations is that they 'solve' a problem for disjunctive properties generated by Gödel's Axioms 2 and 3 (Section 3.2.3),

though this problem was not on the agenda of problems in 1990. By Axioms 2* and 3*, the disjunctive property [D ∨ G], either Devil-likeness or God-likeness, is positive*. The definition of 'positive*' explains this: According to that definition, Pos(x) if and only if (here '⇒' is for strict implication, or entailment)

$$(\sim[\mathrm{D} \vee \mathrm{G}](\mathrm{x}) \Longrightarrow \mathrm{Imp}(\mathrm{x})) \;\&\; \sim([\mathrm{D}\ \mathrm{G}](\mathrm{x}) \Longrightarrow \mathrm{Imp}(\mathrm{x})).$$

Each conjunct is plausible. Regarding the first, the absence of [D ∨ G] entails the absence of G and thus of all positive* properties that should entail imperfection. Regarding the second conjunct, the presence of [D ∨ G] could be by the presence of G alone, which should not entail imperfection.

PS2.4 That Axiom 3 does not do all that it can seem to do.* Anderson's definition of positiveness leaves a job that Axiom 3* can fill. Suppose that ϕ and ψ are positive* in Anderson's sense: The absence of each entails 'imperfection,' while the presence of neither also does that. Then it follows that the absence of (ϕ & ψ) also entails 'imperfection,' since, if this conjunctive property is absent, at least one of its conjuncts is absent, and the absence of each entails imperfection. But it does *not* follow formally that the presence (ϕ & ψ) does not also do that. The formal definition leaves open the possibility of two conditions of 'excellence,' one featuring ϕ but not ψ, the other featuring ψ and not ϕ. There is the possibility that ϕ and ψ do not go well together, so that (ϕ & ψ) entails imperfection and less than 'excellence.' Anderson's Axiom 3* says 'No' to this possibility. It lays down that this 'imperfection' of which, through the definition of positiveness in its terms the axioms speak, is not like that. However, that may be only because this 'imperfection' is not imperfection as naturally understood and, as it needs to be understood if 'God-likeness*,' as defined in terms ultimately of this 'imperfection,' is to be understood to be the essence* of a *god*. To establish that 'positive* properties' are not only compossible, but, in terms of religious or spiritual aspirations, that they go well all together, or equivalently to establish that 'excellence' relative to those aspirations does not come in incompatible varieties, we need to think *substantively* about these aspirations. Anderson cites, with a nod to Lewis (1970), the dictum that "the ontological arguer is entitled to whatever standards of greatness (or positiveness . . .) he wants" (Anderson 1990, pp. 294–5). But this is in general true not true if the object is a *worshipful* being, and it would not be true for Anderson if he wanted out of his system exactly one such being.

PS2.5 The emended system contains something 'approaching' a proper proof of the possibility. One 'reason' for Axiom 3*, one job that it has in the argument, is with Theorem 1* to reach the possibility of God-likeness or, equivalently, the compossibility of all positive* properties. That, supposing only 'finitistic'

logical means, is not derivable straightaway from Def Pos, though it is indefinitely approachable from that definition. What is derivable is that members of finite sets of positive* properties are compossible or, equivalently, that every finite conjunction of positive* properties is possible. What is derivable from Def Pos is an infinite sequence of possibility-theorems starting with Theorem 1* and including, for every finite n, a theorem for sets of size n of positive* properties. (There is a plan for these derivations at the end of Section C4.3 of Appendix C.) That does not say that all positive* properties are compossible, for it is not given that there is only a finite number of positive* properties. To settle that it is best not to stipulate that there are only finitely many positive* properties, but as Anderson following Gödel does, to stipulate, Axiom 3*, that G* is positive*, which with Def G* and Theorem 1* entails that positive* properties are compossible. This is best, since Axiom 3* performs another job in the argument (see the derivation of Theorem 2*, line 10), where what is required is precisely what this axiom says, that God-likeness* is positive*. There is here a contrast with Axiom 3 in Gödel's system that serves only an inference to the possibility of God-likeness. The possibility and positiveness* of God-likeness* are not equivalent in Anderson's system, in the manner in which their analogs are equivalent in Gödel's system.[29]

PS2.6 These emendations make no advance, however, on the issue of the 'theological/axiological compatibility' *of 'positive*' properties.* Anderson goes some way towards satisfying Leibniz's demand that the several positive properties all of which would be God's be *shown* to be compossible. While Anderson's definition of positiveness does not enable a demonstration of this compossibility, it does enhance its plausibility by making demonstrable the compossibility of members of all finite sets of positive* properties. *However*, Anderson's definition does not enhance the plausibility of a burden of ontological reasoning that Leibniz failed to notice and that is nicely isolated in Anderson's system, namely, that the composite property of all positive* properties should itself be 'positive*' or, in other loose words, that we should want God to have all positive* properties.

Left open by Anderson's definition of positiveness* is that having all positive* properties – that indeed, contrary to Gödel's, $P(\phi) \cdot P(\psi) \supset P(\phi \cdot \psi)$, having any two positive* properties – would be too much of a good thing. Left open, as already observed, is that some positive* properties do not go well together and that perfection comes in several incompatible varieties. Until *that* is refuted – until it is *demonstrated* that having them all would be a good thing indeed – arguments such as Descartes's, Leibniz's, Gödel"s, and Anderson's are, in Leibnizian words, 'imperfect demonstrations which assume something that must still be proved,' even after satisfactory demonstrations that positive* properties are compossible and that a being can have them all. It is somewhat surprising that Leibniz did not notice and remark on the problem of the possible imperfection of a being who had all perfections, given his use of something

like the principle of the organic unity of intrinsic value to solve a logical problem of evil. As that principle provides space for compositions of goods and evils to be better than pure goods, it would seem to provide space for compositions of a few goods to be better than compositions in which added to them are several other goods.

PS2.7. It has been explained that necessary existence for Gödel comes, for a thing that has an essence, to existing necessarily, and necessarily having this essence. That follows from his definition, given two principles for essences: that *there cannot be two things in the same or different worlds that share a property that is an essence of one of them*; and that *if ϕ is an essence of something in a world, and something has ϕ in another world, then ϕ is an essence of that thing in that world* (see Section 3.3.2 above). 'Necessary existence' for a thing that has an essence comes to existing necessarily, *and something more*, which makes this Gödelian terminology *somewhat* misleading. The terminological situation may be worse for Anderson's 'necessary existence*', the definition of which is like Gödel's of 'necessary existence', *except* that Anderson's definition runs in terms of *Essences**, where a thing's essence is a property that entails exactly its essential properties. Analogs for essences* of the italicized principles are not very plausible. But, pending arguments for them, there is room for a thing x with an essence* ϕ having necessary existence*, even though x exists only contingently. Pending these arguments, NE*(x) implies only that at each world *something* has ϕ.

PS2.8. There are related consequences of Anderson's Theorem 3*, □∃xG*x. Pending those arguments, Theorem 3* leaves open that there are, at different worlds, different God-like* beings. Left open, pending those arguments, is the possibility of *several* God-like* beings in this world of ours. Left *open* is that the God-like* being or beings that exist is or are all *contingent* beings! Which means that these conditional, pending-further-arguments consequences of Anderson's redefinition of the essences of things are not all *bad* for Anderson's 'ontological system'. For if he can *stop* with them, then my main objection to Gödel's system interpreted as concerned with the existence of God-like beings, *appropriately so-called*, does not convert to an objection to Anderson's system. That main objection is to Axiom 5, which says that necessary existence is a positive property: My objection to this axiom is that, far from being 'positive' in the sense of *being a property that would contribute to a thing's being worthy of worship*, this necessary existence is in that sense 'negative,' since all things that have it are *necessary existents* and thus, in my modal opinion, of the nature of Forms, and numbers, and 'abstract objects' such as the stoplight, so that worshipping them, praying to them, 'taking counsel' from them, and so on would be absurd. Anderson's Axiom 5* is, so far, not similarly challenged, for so far *contingent* existents can have necessary existence*! If he can stop with this license, he can claim for his system,

subject to other problems that it may have, that 'it is a beginning', that it delivers that it is necessary that there exists a God-like* being. What could come next, after fixing the system's terminology and getting away from what would then be a *very* misleading use of 'necessary existence*', would be arguments to show that God is a God-like* being, and which one is God, if there are several.[30]

APPENDIX A. NOTES IN KURT GÖDEL'S HAND

[Transcribed with the permission of John Milnor on behalf of the custodians of Kurt Gödel's *Nachlass*. Lettered footnotes are Gödel's, though he uses not letters but the symbols 'ϕ ', 'x', and '.'.]

Feb 10, 1970

Ontologischer Bewis

P(ϕ)	ϕis positive	(e $\phi \epsilon$P)		
Ax 1	$P(\Phi) \cdot P(\psi) \supset P(\Phi \cdot \psi)$[a]	Ax 2	$P(\Phi) v^{b} P(\sim \Phi)$	
Df 1	$G(x) \equiv (\Phi)[P(\Phi \supset \Phi(x)]$	(God)		
Df 2	$\Phi Ess.x \equiv (\psi)[\psi(x) \supset N(y)[\Phi(y) \supset \psi(y)]]$			(Essence of x)[c]
	$p \supset_N q = N(p \supset q)$	Necessity		
Ax 2	$P(\Phi) \supset NP(\Phi)$]			because it follows from
	$\sim P(\Phi) \supset N \sim P(\Phi)$]			the nature of the property
Th.	$G(x) \supset G$ Ess. x			
Df	$E(x) \equiv (\Phi)[\Phi Ess.x \supset N(\exists x)\Phi(x)$			Necessary Existence
Ax3	P(E)			
Th	$G(x) \supset N(\exists y)G(y)$			
	hence $(\exists x)G(x) \supset N(\exists y)G(y)$			
	"M $(\exists x)G(x) \supset MN(\exists y)G(y)$	M = possibility		
	"$\supset N(\exists y)G(y)$			

[a] and for any number of summand

[b] exclusive or

[c] any two essences of x are nec. equivalent [It is not clear from my copy exactly where this footnote belongs.

J. H. S.]

[page two]

M(– x)G(x) means the system of all pos. props. is compatible. This istrue because of:

Ax4 : $P(\phi) \cdot \phi \supset_N \psi : P(\psi)$ which implies

x = x	is positive
x = x	is negative

But if a system S of pos. props. were incompatible, it would mean that the sum prop. s (which is positive) would bex ≠ x.

Positive means positive in the moral aesthetic sense (independently of theaccidental structure of the world). Only then the ax. true. It may also mean pure"attribution"[d] as opposed to "privation"(or *containing* privation). This interpret. simpler proof.

.....

If ϕ pos. then *not*: $(x)N \sim \phi\ (x)$. Otherwise: $\phi(x) \supset_N x \neq x$ hence $x \neq x$ positive so $x = x$ neg contrary Ax 4 on the exist. of pos. prop.

---

[d]i.e., the 'disj' normal form in terms of elem. prop. contain [a? only? – I cannot decipher the mark here. (Gödel 1995, p. 404) has 'a'. J. H. S.] member without negation

APPENDIX B. NOTES IN DANA SCOTT'S HAND

[Transcribed by J. H. Sobel with permission of Dana Scott, and John Milnor on behalf of the custodians of Kurt Gödel's *Nachlass*.]

GÖDEL'S ONTOLOGICAL PROOF

$P(\phi)$ means ϕ is a *positive* property

$\neg\phi = \hat{x}[\neg\phi(x)]$

AXIOM 1. $P(\neg\phi) \leftrightarrow \neg P(\phi)$
That is, either the property or its negation is positive, but not both.

AXIOM 2. $P(\phi)\ \&\ \Box\forall\, x[\phi(x) \rightarrow \psi(x)] \rightarrow P(\psi)$
A property is positive if it necessarily contains a positive property.

THEOREM 1. $P(\phi) \rightarrow \Diamond\exists\, x\phi(x)$

Proof Suppose $P(\phi)$ and $\neg\Diamond\exists\, x\phi(x)$

$\therefore \Box\forall\, x\neg\phi(x)$ $\quad\therefore \Box\forall\, x[\phi(x) \rightarrow x \neq x]$

By Axiom 2, $P(\hat{x}[x \neq x])$

But $\Box\forall x[\phi(x) \rightarrow x = x]$

By Axiom 2, $P(\hat{x}[x = x]$

But $\hat{x}[x \neq x] = \hat{x}[x = x]$

which contradicts (half of) Axiom 1.

[page two]

DEF. $G(x) \leftrightarrow \forall\phi\ [P(\phi) \rightarrow \phi(x)]$
x is God-like if it possesses all positive properties.

AXIOM 3. $P(G)$
Indeed, $P(\phi)$ is a logical property and G is defined logically as an intersection of positive properties.
Any such property ought also to be positive.

CORO. $\Diamond\exists x G(x)$

AXIOM 4. $P(\phi)\rightarrow\Box P(\phi)$

Being a positive property is logical, hence, necessary.

DEF. ϕ Ess. $x \leftrightarrow \phi(x) \,\&\, \forall\psi[\psi(x)\rightarrow\Box\forall y[\phi(y)\rightarrow\psi(y)]]$

ϕ is the *essence* of x iff x has ϕ and this property is necessarily minimal.

THEOREM 2. $G(x)\rightarrow G$ Ess. x

Proof. Suppose G(x). $\therefore$ G Ess. x

Suppose $\psi(x)$. If $\neg P(\psi)$, then $P(\neg\psi)$; then

$\neg\psi(x)$. $\therefore P(\psi)$

But $P(\psi) \rightarrow \forall x[G(x)\rightarrow\psi(x)]$ logically, by def.

[page three]

$\therefore$ $\Box P(x)\rightarrow\Box\forall x[G(x)\rightarrow\psi(x)]$ by Modal logic.

But $\Box P(\psi)$ by Axiom 4.

Hence $\Box\forall x[G(x)\rightarrow\psi(x)]$

Thus G Ess. x.

NOTE. ϕ Ess. x & ψ Ess. $x\rightarrow\Box\ \phi = \psi$

ϕ Ess. $x\rightarrow\Box\forall y[\phi(y)\rightarrow y = x]$

DEF. $NE(x) \leftrightarrow \forall\ \phi[\phi$ Ess. $x\rightarrow\Box\exists\ x\phi(x)]$

NE(x) means that x necessarily exists if it has an essential property.

AXIOM 5. P(NE)

Being logically defined in this way, necessary existence is a positive property.

THEOREM 3. $\Box\exists x G(x)$

Proof. $G(x)\rightarrow NE(x)$ & G Ess. $x\rightarrow\Box\exists x G(x)$

$\therefore$ $\exists x G(x)\rightarrow\Box\exists\ x G(x)$

$\therefore$ $\Diamond\exists x G(x) \rightarrow \Diamond\Box\exists x G(x)\rightarrow\Box\exists x G(x)$

But $\Diamond\exists\ x G(x)$ by Theorem 1.

$\therefore \Box\exists\ x G(x)$. QED

APPENDIX C. MAINLY DERIVATIONS

C1 A logic for Gödel's system. Derivations shall be in an *extension* of the FMQMC explained in Section B3 of Appendix B of the previous chapter. Now come comments on its interpretations of this extension, and on its rules and procedures specifically for property generalizations.

C1.1 Domains in interpretations. The universal domain of an interpretation consists of a set of *individuals*, a nonempty set of *properties*, and *the property of properties* P. *The domain of a world* in an interpretation consists of a set

of the individuals and all of the properties in this interpretation's universal domain.

C1.2 Properties in an interpretation. The properties of individuals of an interpretation are complete functions from worlds to subsets of the set of individuals in this interpretation's universal domain. The subset assigned by a property to a world is, intuitively, the things in the universal domain that have this property at this world. It is not required that these things be members of this world's domain – it is not required that they exist in this world. This characterization of properties allows for there to be properties in an interpretation that are not 'existence-entailing' such as nonexistence and non-self-identity.[31] Things that have such properties at a world do not have them *in* this world or exemplify or instantiate them *in* this world's domain. The important point about properties of an interpretation is not their natures as functions from worlds to subsets of the universal domain, but their position in this domain and in world-domains. *The* PROPERTIES *of an interpretation are members of each world's domain*. The property subsets of world-domains of an interpretation are identical with one another and with the universal domain. Additionally, there is in an interpretation at least one property.[32] Given these conditions, *standard rules and procedures for quantifiers – the rules and procedures for quantifiers UI, EG*, *EI*, *and UD of Appendix C of Chapter II – are right for, and will be used for, property-quantifiers*. To match the requirement that there is in every interpretation a property, the rule Existence of FrMQMdlC, /∴ (∃α) E!α, α variable, is replaced by Existence of Properties, /∴(∃*Pr*)(E!*Pr*), *Pr* a property variable, and to reflect the requirement that properties are the same in world-domains, the rule Necessary Existence of Properties, /∴□E!*Pr*, *Pr* a property term, is added, though nothing will be made of these two rules.

For every property ϕ in an interpretation there is the complement property ¬ϕ that is had at a world by exactly the things that do not have ϕ at this world. So there are in an interpretation at least *two* properties. Also, for any properties ϕ and ψ in an interpretation, there are the conjunctive and disjunctive properties (ϕ & ψ) and (ϕ∨ ψ) that, intuitively, are had at a world by exactly the things that have both ϕ and ψ or exactly the things that have either ϕ or ψ. So there are in an interpretation at least *four* properties. Completeness requires rules of inference that reflect the relations of complement properties to the properties of which they are complements, and of conjunctive and disjunctive properties to the properties of which they are conjunctive and disjunctive properties. There should also be a rule that equates properties if and only if, intuitively, they are logically equivalent. These rules are not formulated here, as they are not needed for coming derivations. There is for every formula *F* an abstraction property $\hat{\alpha}$ [*F*] that is had at a world by exactly the individuals

that 'satisfy' F at this world when free occurrences in F of α are replaced by their names.

The property P of properties of an interpretation is a subset of the properties of this interpretation, subject to specifications framed in Axioms 1–5.

C2 *Two promised derivations*. For the deduction of

$$\text{(iii) } \forall\psi\Box\forall x[\phi(x) \rightarrow \psi(x)]$$

from

$$\text{(ii) } \neg\Diamond\exists x\phi(x)$$

made in Section 3.1.2, we have the following derivation.

1.	~~SHOW~~ $\forall\psi(\Box\forall x[\phi(x) \rightarrow \psi(x)])$	UD
2.	$\Box\neg\exists x\phi(x)$	premise, MdlNeg
3.	~~SHOW~~ $\Box\forall x[\phi(x) \rightarrow \psi(x)]$	ND
4.	~~SHOW~~ $\forall x[\phi(x) \rightarrow \psi(x)]$	FUD
5.	~~SHOW~~ $E!x \rightarrow [\phi(x) \rightarrow \psi(x)]$	CD
6.	$E!x$	assumption
7.	~~SHOW~~ $\phi(x) \rightarrow \psi(x)$	CD
8.	$\phi(x)$	assumption
9.	$\Box\neg\exists x\phi(x)$	2, R
10.	$\neg\phi(x)$	9, N, QN, 6, FUI(x)

Line 9 is the only 'entry from without' into the necessity derivation for line 3. As required, it is a necessity, and it is entered entirely from without.

Here is a derivation for the deduction, made in Section 3.2.1, of

$$\text{(ii) } \Box P(\psi) \rightarrow \Box\forall x[G(x) \rightarrow \psi(x)]$$

from

$$\text{Def G. } \Box\forall x(G(x) \leftrightarrow \forall\psi[P(\psi) \rightarrow \psi(x)]).$$

1.	~~*SHOW*~~ $\Box P(\psi) \rightarrow \Box\forall x[G(x) \rightarrow \psi(x)]$	CD
2.	$\Box P(\psi)$	assumption
3.	~~*SHOW*~~ $\Box\forall x[G(x) \rightarrow \psi(x)]$	ND
4.	~~*SHOW*~~ $\forall x[G(x) \rightarrow \psi(x)]$	FUD
5.	~~*SHOW*~~ $E!x \rightarrow [G(x) \rightarrow \psi(x)]$	CD
6.	$E!x$	assumption
7.	~~*SHOW*~~ $G(x) \rightarrow \psi(x)$	CD
8.	$G(x)$	assumption
9.	$\Box\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$	DefG
10.	$G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$	9, N, 6, FUI(x)
11.	$P(\psi) \rightarrow \psi(x)$	10, BC, 8 MP, UI(ψ)
12.	$\Box P(\psi)$	2, R
13.	$\psi(x)$	11, N, 12, MP

C3 Derivations of theorems in Gödel's system

C3.1. Here is an 'articulation' of Gödel's (Scott's) proof of Theorem 3. Principles cited are necessities of closures of principles of Appendix A: Axiom 3, $\Box P(G)$; Axiom 5, $\Box P(NE)$; Theorem 1, $\Box\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$; Theorem 2, $\Box\forall x[G(x) \rightarrow G \text{ Ess } x]$; and Def G, $\Box\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$.

1.	~~*SHOW*~~ $\Box\exists xG(x)$	DD
2.	~~*SHOW*~~ $\Box\forall x[G(x) \rightarrow \Box\exists xG(x)]$	ND
3.	~~*SHOW*~~ $\forall x[G(x) \rightarrow \Box\exists xG(x)]$	FUD
4.	~~*SHOW*~~ $E!x \rightarrow [G(x) \rightarrow \Box\exists xG(x)]$	CD
5.	$E!x$	assumption
6.	~~*SHOW*~~ $G(x) \rightarrow NE(x)$ & G Ess x	CD
7.	$G(x)$	assumption
8.	$P(NE)$	Axiom 5, N
9.	$P(NE) \rightarrow NE(x)$	Def G, N, BC, 7, MP, UI(NE)
10.	$NE(x)$ & G Ess x	8,9,MP,7,Theorem 2,N,5,FUI(x),MP,Adj
11.	~~*SHOW*~~ $NE(x)$ & G Ess $x \rightarrow \Box\exists xG(x)$	CD
12.	$NE(x)$ & G Ess x	assumption
13.	G Ess $x \rightarrow \Box\exists xG(x)$	12, S, Def NE, N, BC, MP, UI(G)
14.	$\Box\exists xG(x)$	12, S, 13, MP
15.	$G(x) \rightarrow \Box\exists$ $xG(x)$	6, 11, Hypothetical Syllogism $[(\phi \rightarrow \psi), (\psi \rightarrow \chi) / \therefore (\phi \rightarrow \chi)]$
16.	$\Box[\exists xG(x) \rightarrow \Box\exists xG(x)]$	2, IE(Q-conf)
17.	~~*SHOW*~~ $\Diamond\exists xG(x) \rightarrow \Diamond\Box\exists xG(x)$	CD
18.	$\Diamond\exists xGx$	assumption
19.	~~*SHOW*~~ $\Diamond\Box\exists xG(x)$	ID
20.	$\neg\Diamond\Box\exists xG(x)$	assumption
21.	~~*SHOW*~~ $\Box\neg\exists xG(x)$	ND
22.	$\Box[\exists xG(x) \rightarrow \Box\exists xG(x)]$	16, R
23.	$\Box\neg\Box\exists xG(x)$	20, MdlNeg
24.	$\neg\exists xG(x)$	22, N, 23, N, MT
25.	$\neg\Diamond\exists xG(x)$	21, MdlNeg
26.	$\Diamond\exists xG(x)$	18, R
27.	$\Diamond\Box\exists xG(x) \rightarrow \Box\exists xG(x)$	S5 Reduction Theorem
28.	$\Diamond\exists xG(x) \rightarrow \Box\exists xG(x)$	17, 27, Hypothetical Syllogism
29.	$\Diamond\exists xG(x)$	Axiom 3, N, Theorem 1, N, UI(G) MP
30.	$\Box\exists xG(x)$	28, 29, MP

Lines 27–30 can be replaced by the following lines for a *Necessity Derivation*

of 1. That way the proof of Theorem 3 uses a Brower reduction principle – /∴ $\Diamond\Box\phi \rightarrow \phi$ – rather than an S5 reduction principle such as /∴ $\Diamond\Box\phi \rightarrow \Box\phi$. (Here I have used ϕ as a metalinguistic variable ranging over formulas, not as a property-of-individuals variable as it is used by Gödel and Scott.)

27.	$\Diamond\Box\exists xG(x) \rightarrow \exists xG(x)$	Brower Reduction Theorem
28.	$\Diamond\exists xG(x) \rightarrow \exists xG(x)$	17, 27, MdlNeg
29.	$\Diamond\exists xG(x)$	Axiom 3, N, Theorem 1, N, UI(G) MP
30.	$\exists xG(x)$	28, 29, MP

Except for S5 reduction, my derivations in Gödel's system use no modal procedures and principles other than ND (necessity derivation – that what follows from necessities is itself necessary), N (the principle, $\Box P \rightarrow P$), MdlNeg (principles such as $\Box\neg P \rightarrow \neg\Diamond P$), and the distribution principle $\Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$, which is derivable using ND and N. I use that principle in a derivation of Theorem 9. The derivation in Section D4.3 of Axiom 4* in Anderson's emendation of Gödel's system uses the S5 expansion principle, $\Diamond P \rightarrow \Box\Diamond P$.

. . . . . .

ON USING A 'BROWER SYSTEM'. Most derivations in this appendix use the form of proof ND, which form is not available in a Brower system. With ND one can derive the principle $\Box P \rightarrow \Box\Box P$, which is not B-valid. 'B-accessibility' is reflexive and symmetric. To show $\Box P \rightarrow \Box\Box P$ is false for a truth assignment in a B-model, consider a model of three worlds, @, w′, and w″. Let @ and w′ be accessible from @, while not only @ and w′ but also w″ are accessible from w′. Let P be true in @ and w′ but false in w″. Then $\Box P$ is true in @ but false in w′. P is true in every world accessible from @ but not in every world accessible from w′. So $\Box\Box P$ is false in @: $\Box P$ is not true in every world accessible from @. Since $\Box P$ is true and $\Box\Box P$ is false in @, $\Box P \rightarrow \Box\Box P$ false in @.

	P	$\Box P$	$\Box\Box P$	$(\Box P \rightarrow \Box\Box P)$
@	t	t	f	f
↓↑				
w′	t	f		
↓				
w″	f			

For a Brower deductive system B, one can replace ND by a proof procedure BND like it that, however, restricts entries from without to *entries*

by N-inferences and that replaces the rule $\Diamond\Box\phi \;/\!\therefore\; \Box\phi$ by the rule $\phi \;\therefore\; \Box\Diamond\phi$. BND is a weaker proof procedure. Anderson writes: "For purposes of proving Theorem 3, it would actually suffice to use the weaker modal logic B" (Anderson 1990, p. 298n5). I have observed that a B-*principle* is sufficient for a proof of Theorem 3. Inspection of my derivation for this theorem confirms that it could be modified to work in modal *logic* B: Formulas 'entered from without' into the ND-derivation for 21 are then 'reduced' by N-inferences. Those inferences could have been made in advance for 'entries from without' into BND-derivations. Modal logic B would be similarly sufficient for derivations of Theorems 1 and 2. Anderson doubts "that the logical weakening [from S5 to B] corresponds to any epistemic advance" (Anderson 1990, p. 291). Perhaps he means that reasons found in thoughts about possible worlds that recommend that B-principle for 'the logical or metaphysical modalities' of philosophy also recommend the S5-principle. That in any case is my view.

C3.2. Now comes the derivation promised for

Theorem 4. $G(x) \rightarrow \forall\phi[\phi(x) \rightarrow P(\phi)]$

from Axiom 1, $P(\neg\phi) \leftrightarrow \neg P(\phi)$ and Def G, $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$.

1.	~~*SHOW*~~ $G(x) \rightarrow \forall\phi[\phi(x)\rightarrow P(\phi)]$	CD
2.	$G(x)$	assumption
3.	~~*SHOW*~~ $\forall\phi[\phi(x) \rightarrow P(\phi)]$	assertion (UD)
4.	~~*SHOW*~~ $\phi(x)\rightarrow P(\phi)$	assertion (CD)
5.	$\phi(x)$	assumption (CD)
6.	~~*SHOW*~~ $P(\phi)$	assertion (ID)
7.	$\neg P(\phi)$	assumption (ID)
8.	$P(\neg\phi)$	Axiom 1, BC, 7, MP
9.	$P(\neg\phi) \rightarrow \neg\phi(x)$	Def G, 2, MP, UI($\neg\phi$)
10.	$\neg\phi(x)$	8, 9, MP
11.	$\phi(x)$	5, R

C3.3. Here is the derivation promised for

Theorem 5. $P(\phi) \rightarrow \Box\exists x\phi(x)$.

from Axiom 4, $P(\phi) \rightarrow \Box P(\phi)$; Def G, $\Box\forall x(G(x) \leftrightarrow \forall\phi\,[P(\phi) \rightarrow \phi(x)])$; and Theorem 3, $\Box\exists xG(x)$.

1.	~~*SHOW*~~ $P(\phi) \rightarrow \Box\exists x\phi(x)$	CD
2.	$P(\phi)$	assumption
3.	$\Box P(\phi)$	2, Axiom 4, MP
4.	~~*SHOW*~~ $\Box\exists x\phi(x)$	ND
5.	~~*SHOW*~~ $\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$	FUD
6.	~~*SHOW*~~ $E!x \rightarrow (G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$	CD
7.	$E!x$	assumption
8.	$G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$	Def G, N, 7, FUI
9.	$\Box\exists xGx$	Theorem 3
10.	$E!a \,\&\, G(a)$	9, N, FEI
11.	$P(\phi) \rightarrow \phi(a)$	10, S, 5, FUI(a), BC, 10, S, MP, UI(ϕ)
12.	$\exists x\phi(x)$	3,R, N, 11, MP, 10, S, FEG

The entry from without of Def G leading to line 8 is of a necessity, as required for the necessity-derivation of line 4.

C3.4. Here is the derivation that was promised for

Theorem 7. $G(x) \rightarrow \forall\phi(\phi(x) \rightarrow \Box\exists x[G(x)\&\phi(x)]$.

To be cited: Theorem 4, $G(x) \rightarrow \forall\phi[\phi(x) \rightarrow P(\phi)]$; Axiom 4, $P(\phi) \rightarrow \Box P(\phi)$; Theorem 3, $\Box\exists x G(x)$; and Def G, $\Box\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$.

1.	~~*SHOW*~~ $Gx \rightarrow \forall\phi(\phi(x) \rightarrow \Box\exists x[G(x) \,\&\, \phi(x)])$	CD
2.	Gx	assumption
3.	~~*SHOW*~~ $\forall\phi(\phi(x) \rightarrow \Box\exists x[G(x) \,\&\, \phi(x)])$	UD
4.	~~*SHOW*~~ $\phi(x) \rightarrow \Box\exists x[G(x)\&\phi(x)]$	CD
5.	$\phi(x)$	assumption
6.	$\Box P(\phi)$	2, Theorem 4, MP, UI(ϕ), 5, MP, Axiom 4, MP
7.	~~*SHOW*~~ $\Box\exists x[G(x) \,\&\, \phi(x)]$	ND
8.	$\Box P(\phi)$	7, R
9.	$\Box\exists x G(x)$	Theorem 3
10.	$G(a)$ & E!a	9, N, FEI
11.	$\Box\forall x(G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)])$	Def G
12.	$\phi(a)$	11, N, 10, S, FUI(a), BC, 10, S, MP, UI(ϕ), 9, N, MP
13.	$\exists x[G(x) \,\&\, \phi(x)]$	10, S, 12, Adj, 10, S, FEG

C3.5. Next comes the derivation promised for

Theorem 8. $\forall y NE(y)$.

Cited shall be: Axiom 3, P(G); Axiom 2, $\Box\forall\phi\forall\psi[P(\phi) \,\&\, \Box\forall x[\phi(x) \rightarrow \psi(x)] \rightarrow P(\psi)]$; Theorem 2, $\Box\forall x[G(x) \rightarrow G \text{ Ess } x]$; Theorem 3, $\Box\exists x G(x)$; Theorem 5, $P(\phi) \rightarrow \Box\exists x\phi(x)$; *Essences*, $\forall x\exists\phi\phi \text{ Ess } x$; *Properties*, $\hat{\alpha}[F](\beta) \leftrightarrow F'$ wherein α is an individual variable, β a term, F a formula, and F' is a formula that comes from F by proper substitution of β for α; *Unique of Essences*, $\forall\phi\forall\psi\forall x(\phi \text{ Ess } x \,\&\, \psi \text{ Ess } x \rightarrow \Box\forall y[\phi(y) \leftrightarrow \psi(y)])$; Def Ess, $\Box\forall\phi\forall x(\phi \text{ Ess } x \leftrightarrow \phi(x) \,\&\, \forall\psi[\psi(x) \rightarrow \Box\forall y[\phi(y) \rightarrow \psi(y)]])$; Def G, $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$; and Axiom 5, P(NE).

1.	~~*SHOW*~~ $\forall y NE(y)$	assertion (FUD)
2.	~~*SHOW*~~ $E!y \rightarrow NE(y)$	assertion (CD)
3.	$E!y$	assumption (CD)
4.	$\Box\exists x G(x)$	Theorem 3
5.	$E!a \,\&\, G(a)$	2, N, FEI(a)
6.	~~*SHOW*~~ $y \neq a \rightarrow NE(y)$	assertion (CD)
7.	$y \neq a$	assumption (CD)
8.	ψ Ess y	*Essences*, 3, FUI(y), UI(ψ)
9.	$\exists y(y \neq a \,\&\, \psi \text{ Ess } y)$	7, 8, Adj, 3, FEG
10.	$\hat{a}[\exists y(y \neq a \,\&\, \psi \text{ Ess } y)](a)$	*Properties*, N, BC, 9, MP
11.	G Ess a	Theorem 2, N, 5, S, FUI, 5, S, MP
12.	$G \text{ Ess } a \leftrightarrow G(a) \,\&\, \forall \psi\,[\psi(a) \rightarrow \Box\forall z[G(z) \rightarrow \psi(z))]$	Def Ess, AV(y to z), N, UI(G), 5, S, FUI(a)
13.	$\hat{a}[\exists y(y \neq a \,\&\, \psi \text{Ess } y)](a) \rightarrow \Box\forall z[G(z) \rightarrow \hat{a}[\exists y(y \neq a \,\&\, \psi \text{Essy})](z)]$	12, BC, 11, MP, S, 5, S, UI($\hat{a}[\exists y(y \neq a \,\&\, \psi \text{ Ess } y)]$)
14.	$\Box\forall z[G(z) \rightarrow \hat{a}[\exists y(y \neq a \,\&\, \psi \text{Ess } y)](z)]$	10, 13, MP
15.	$P(\hat{a}[\exists y(y \neq a \,\&\, \psi \text{Ess } y])$	Axiom 3, 14, Adj, Axiom 2,
16.	$\Box\exists x\,\hat{a}[\exists y(y \neq a \,\&\, \psi \text{Ess } y)](x)$	15, Theorem 5, MP AV(x to z), N, UI(G), UI($\hat{a}[\exists\, y(y \neq a \,\&\, \psi \text{ Ess } y)]$), MP
17.	~~*SHOW*~~ $\Box\exists y\psi(y)$	ND
18.	$\Box\exists x\,\hat{a}[\exists y(y \neq a \,\&\, \psi \text{Ess } y)](x)$	16, R
19.	$E!b \,\&\, \hat{a}[\exists y(y \neq a \,\&\, \psi \text{ Ess } y)](b)$ $\exists y(y\, b \,\&\, \psi \text{Ess } y)$	18, N, FEI S, *Properties*, N, BC, MP
20.	$E!c \,\&\, (y \neq b \,\&\, \psi \text{ Ess } c)$	16, R, FEI(c)
21.	$E!c$	20, S
22.	ψ Ess c	20, S, S
23.	$\exists y\psi(y)$	Def Ess, N, AV(ψ to χ), UI(ψ), 21, FUI(c), BC, 22, MP, S, 21, FEG
24.	~~*SHOW*~~ $\forall\phi[\phi \text{Ess } y \rightarrow \Box\exists x\phi(x)]$	UD
25.	~~*SHOW*~~ $\phi \text{Ess } y \rightarrow \Box\exists x\phi(x)$	CD
26.	ϕ Ess y	CD
27.	ψ Ess y ϕEss y	8, 26, Adj

28.	ψ Ess y & ϕEss y $\rightarrow$ $\Box\forall y[\phi(y) \leftrightarrow \psi(y)]$	*Unique of Essences*, N, AV(ϕ to ψ, and ψ to ϕ) UI(ψ), UI(ϕ), 3, FUI(y)
29.	~~*SHOW*~~ $\Box\exists x\phi(x)$	ND
30.	$\Box\exists y\psi(y)$	17, R
31.	$\Box\forall y[\phi(y) \leftrightarrow \psi(y)$	27, 28, MP
32.	$\exists x\phi(y) \leftrightarrow \exists y\psi(y)$	31, N, Q-dist
33.	$\exists x\phi(x)$	32, BC, 30, N, MP, AV
34.	NE(y)	Def NE, N, 3, FUI(y), BC, 24, MP
35.	~~*SHOW*~~ $y = a \rightarrow$ NE(y)	CD
36.	y = a	
37.	$\Box\forall\phi[P(\phi) \rightarrow \phi(a)]$	Def G, N, 5, S, FUI(a), BC, 5, S, MP
38.	NE(y)	37, N, UI(NE), Axiom 5, MP, 36,LL
39.	NE(y)	6, 35, Separation of Cases [($\phi \rightarrow \chi$), ($\sim\phi \rightarrow \chi$)/ $\therefore \chi$]

C3.6. Now the derivation promised for

Theorem 9. $Q \rightarrow \Box Q$

from Theorem 2, $\forall x[G(x) \rightarrow G$ Ess x] and Theorem 3, $\Box\exists xG(x)$, Def Ess. $\forall\phi\forall x(\phi$ Ess x $\leftrightarrow \phi(x)$ & $\forall\psi[\psi(x) \rightarrow \Box\forall y[\phi(y) \rightarrow \psi(y)]])$, and *Properties* (recently recalled).

1.	~~*SHOW*~~ $Q \rightarrow \Box Q$	CD
2.	Q	assumption
3.	E!a & G(a)	Theorem 3, N, FEI(a)
4.	a = a & Q	Identity, 2, Adj
5.	â[a = a & Q](a)	4, *Properties*, N, BC, MP
6.	G Ess a	Theorem 2, 3, S, FUI(a), 3, S, MP
7.	G Ess a $\leftrightarrow$ G(a) & $\forall\psi[\psi(a) \rightarrow \Box\forall y[G(y) \rightarrow \psi(y)]]$	Des Ess, UI(G), 3, S, FUI(a)
8.	$\Box\forall y[G(y) \rightarrow$ â[a = a & Q](y)]	7, BC, 6, MP, S, UI(â[a = a & Q]), 5, MP
9.	~~*SHOW*~~ $\Box[\exists yG(y) \rightarrow \exists y$ â [a = a & Q](y)]	ND
10.	$\Box\forall y[G(y) \rightarrow$ â[a = a & Q](y)]	8, R
11.	$\exists yG(y) \rightarrow \exists y$â[a = a & Q](y)	10, N, Q-dist
12.	~~*SHOW*~~ $\Box Q$	ND
13.	$\Box[\exists yG(y) \rightarrow \exists y$â[a = a & Q](y)]	9, R
14.	$\Box\exists yG(y) \rightarrow \Box\exists y$â[a = a&Q](y)	13, Modal-dist
15.	$\Box\exists xG(x)$	Theorem 3
16.	$\Box\exists y$â[a = a & Q](y)	15, AV, 14, MP
17.	â[a = a & Q](b)	16, N, FEI(b),S
18.	Q	*Properties*, N, BC, 17, MP, S

Instead of lines 4 and 5 there could be a line containing 'â[Q](a)' from line 2 by *Properties*. Then, throughout, instead of 'â[a = a & Q]' we could have 'â[Q]'. (Cf., Anderson 1990, p. 294.)

C4 Derivations for Anderson's emendation of Gödel's system

C4.1 Anderson's emendations in formal dress. My formal statements of Anderson's informally stated definitions, axioms, and theorems are framed in terms that suit them to the deductive system being used. Two of my formalizations of informal definitions he states depart not merely notationally from his.

(Def Pos) $\Box\forall\boldsymbol{\phi}(\mathbf{Pos}(\boldsymbol{\phi}) \leftrightarrow \Box\forall\mathbf{x}[\neg(\boldsymbol{\phi})\mathbf{x} \rightarrow \mathbf{Imp(x)}] \,\&\, \neg\Box\forall\mathbf{x}[(\boldsymbol{\phi})\mathbf{x} \rightarrow \mathbf{Imp(x)}])$.

Anderson's more compact formal definition is $\mathrm{Pos}(\phi) =_{\mathrm{df}} (\sim\phi \Rightarrow \Delta). \sim(\phi \Rightarrow \Delta)$. '$\Delta$' is for *defect*. Anderson writes that "[t]he idea . . . is based on that

of Alan R. Anderson . . . (1958)" (Anderson 1990, p. 301n15). '($\phi \Rightarrow \Delta$) ', for example, is short for '$\Box\forall x[\phi(x) \rightarrow \Delta(x)]$'.

(Def Ess*) $\Box\forall\phi\forall \mathbf{x}(\phi\ \mathbf{Ess}^*\ \mathbf{x} \leftrightarrow [\phi\mathbf{(x)}\ \&\ \forall\psi(\Box\forall\ \mathbf{y}[\phi\ \mathbf{(y)} \rightarrow \psi\ \mathbf{(y)}] \leftrightarrow \Box\ \mathbf{[E!x} \rightarrow \psi\ \mathbf{(x)}])])$.

This improves on Anderson's ϕ Ess* x $=_{df}$ $(\psi)[\Box\psi(x) \equiv (\phi \Rightarrow \psi)]$, which leaves out the requirement that $\phi(x)$. Also, departing from his definition, I interpret 'ψ is an *essential* property of x' and, when this is to say the same thing, 'x *necessarily has* property ψ,' to mean not that x has ψ at every possible world but to mean that x has ψ *at every world in which x exists*.

(Def NE*) $\Box\forall\ \mathbf{x}(\mathbf{NE}^*\mathbf{(x)} \leftrightarrow\ \forall\ \boldsymbol{\phi}[\boldsymbol{\phi}\mathbf{Ess}^*\mathbf{x} \rightarrow\ \Box\exists\mathbf{x}\boldsymbol{\phi}\mathbf{(x)}])$.

This is a '*-lookalike' of Def NE.

(Def NE*) $\Box\forall\,\mathbf{x}\,[\mathbf{G}^*\,\mathbf{x} \leftrightarrow\ \exists\,\boldsymbol{\phi}\,[\boldsymbol{\phi}\ \ \mathbf{Ess}^*\,\mathbf{x}\,\&\,\forall\,\boldsymbol{\psi}\,(\Box\ \forall\,\mathbf{y}\,[\boldsymbol{\phi}\,\mathbf{(y)} \rightarrow \boldsymbol{\psi}\,\mathbf{(y)}] \leftrightarrow\ \mathbf{Pos}\,(\boldsymbol{\psi}))]]$.

This is from "having all and only positive properties as essential properties is plausibly definitive of divinity" (p. 296), and to go with Def Ess. So it differs somewhat from Anderson's G(x) $=_{df}$ $(\phi)[\Box\phi(x) \equiv$ Pos($\phi)]$.

Axiom 1*. $\Box\forall\boldsymbol{\phi}[\mathrm{Pos}(\boldsymbol{\phi}) \rightarrow \neg\mathrm{Pos}(\neg\boldsymbol{\phi})]$.

This is the only Anderson axiom or theorem that is not a 'Pos lookalike' of the corresponding Gödel principle.

Axiom 2*. $\Box\forall\boldsymbol{\phi}\forall\boldsymbol{\psi}\ [\mathbf{Pos}(\boldsymbol{\phi})\ \&\ \Box\forall\mathbf{x}[\boldsymbol{\phi}\mathbf{(x)} \rightarrow\ \boldsymbol{\psi}\mathbf{(x)}] \rightarrow \mathbf{Pos}(\boldsymbol{\psi})]$.
Axiom 3*. $\mathbf{Pos}(\mathbf{G}^*)$.
Axiom 4*. $\Box\forall\boldsymbol{\phi}[\mathbf{Pos}(\boldsymbol{\phi}) \rightarrow\ \Box\mathbf{Pos}(\boldsymbol{\phi})]$.
Axiom 5*. $\mathbf{Pos}(\mathbf{NE}^*)$.
Theorem 1*. $\Box\forall\boldsymbol{\phi}[\mathbf{Pos}(\boldsymbol{\phi}) \rightarrow \Diamond\exists\mathbf{x}\boldsymbol{\phi}\mathbf{(x)}]$.
Theorem 2*. $\Box[\mathbf{G}^*(x) \rightarrow \mathbf{G}^*\mathbf{Ess}^*\mathbf{x}]$.
Theorem 3*. $\Box\exists\mathbf{x}\mathbf{G}^*\mathbf{(x)}$.

*C4.2 Pre-axiom 0**. It is a consequence Gödel's Axiom 1 that there is a positive property, $\exists\phi$P(ϕ). Anderson's Axiom 1* does not entail that there is a positive* property $\exists\phi$Pos(ϕ), nor does his Def Pos. (This definition entails $[\exists\phi\mathrm{Pos}(\phi) \leftrightarrow \Box\forall\mathrm{xImp(x)}]$ or, equivalently – letting 'Perf' be short for '($\neg$Imp)' – $[\exists\phi\mathrm{Pos}(\phi) \leftrightarrow \Diamond\exists\mathrm{xPerf(x)}]$.) It is for these reasons that I make explicit in Anderson's 'perfect being system,'

Axiom 0*. $\Diamond\exists$xPerf(x),

which says that it is possible that there is a perfect being, even though this axiom is not independent of the axioms and definitions he states. [Axiom 0* is a consequence of Def Pos and Axiom 3*, Pos(G*), as well as of Def Pos and his Axiom 5*, Pos(NE*). Each of these axioms of course entails that $\exists\phi Pos(\phi)$, and from that it follows by Def Pos, through the recently displayed equivalence, that $\Diamond\exists xPerf(x)$.] Now comes a derivation of the first equivalence from Def Pos and

Negations of Properties. $\Box\forall\phi\forall x[(\neg\phi)x \leftrightarrow \neg(\phi)x]$.

It will be evident that a similar derivation works for the related equivalence

$$Pos(Perf) \equiv \Diamond\exists xPerf(x).$$

In Anderson's system, $\exists xPerf(x)$, $\exists xPos(\phi)$, and Pos(Perf) are equivalent.

1.	~~*SHOW*~~ $\exists\phi Pos(\phi) \leftrightarrow \neg\Box\forall$ xImp(x)	DD
2.	~~*SHOW*~~ $\exists\phi Pos(\phi) \rightarrow \neg\Box\forall$ xImp(x)	CD
3.	$\exists\phi Pos(\phi)$	assumption
4.	~~*SHOW*~~ $\neg\Box\forall$ xImp(x)	ID
5.	$\Box\forall$ xImp(x)	assumption
6.	$Pos(\phi)$	3, EI (Variable 'ϕ'' is evidently different from ϕ and new to the derivation.)
7.	$\neg\Box\forall x[(\phi')x \rightarrow Imp(x)]$	Def Pos, N, UI ('ϕ')", BC, 6, MP, S
8.	~~*SHOW*~~ $\Box\forall x[(\phi')x \rightarrow Imp(x)]$	ND
9.	~~*SHOW*~~ $\forall x[(\phi')x \rightarrow Imp(x)]$	FUD
10.	~~*SHOW*~~ $E!x \rightarrow [(\phi')x \rightarrow Imp(x)]$	CD
11.	E!x	assumption
12.	~~*SHOW*~~ $(\phi')x \rightarrow$ Imp x)	CD
13.	$\Box\forall$xImp(x)	5,R
14.	Imp(x)	13,N,11,FUI

15.	~~*SHOW*~~ $\neg\Box\forall x\text{Imp}(x) \rightarrow \exists\phi\text{Pos}(\phi)$	CD
16.	$\neg\Box\forall$ xImp(x)	assumption
17.	~~*SHOW*~~ $\Box\forall x[\neg(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	ND
18.	~~*SHOW*~~ $\forall x[\neg(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	FUD
19.	~~*SHOW*~~ $\text{E!}x \rightarrow [\neg(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	CD
20.	$\text{Imp}(x) \rightarrow \text{Imp}(x)$	sentential logic theorem
21.	$\neg(\neg\text{Imp})x \rightarrow \text{Imp}(x)$	20,IE(DblNeg),Negations of Properties
22.	~~*SHOW*~~ $\neg\Box\forall x[(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	ID
23.	$\Box\forall x[(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	assumption
24.	~~*SHOW*~~ $\Box\forall x\text{Imp}(x)$	ND
24.	$\Box\forall x[(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	23,R
25.	~~*SHOW*~~ $\forall x\text{Imp}(x)$	FUD
26.	~~*SHOW*~~ $\text{E!}x \rightarrow \text{Imp}(x)$	CD
27.	E!x	assumption
28.	~~*SHOW*~~ Imp(x)	ID
29.	$\neg\text{Imp}(x)$	assumption
30.	$\neg\text{Imp}(x) \rightarrow$ Imp (x)	24,N,27,FUI,Negation of Properties
31.	Imp(x)	29, 30,MP
32.	$\neg\Box\forall x\text{Imp}(x)$	16,R
33.	$\Box\forall x[\neg(\neg\text{Imp})x \rightarrow \text{Imp}(x)] \,\&\, \neg\Box\forall x[(\neg\text{Imp})x \rightarrow \text{Imp}(x)]$	17,22,Adj
34.	$\text{Pos}(\neg\text{Imp})x$	Def Pos, N, UI(¬ Imp), BC,33,MP
35.	$\exists\phi\text{pos}(\phi)$	34,EG
36.	$\exists\phi\text{Pos}(\phi) \leftrightarrow \sim\Box\forall x\text{Imp}(x)$	2,15,CB

C4.3 Derivations of axioms and theorems. Axiom 1 from Def Pos and negation of properties.*

1.	~~*SHOW*~~ $\Box\forall\phi[\mathrm{Pos}(\phi) \rightarrow \neg\mathrm{Pos}(\neg\phi)]$	ND
2.	~~*SHOW*~~ $\forall\phi[\mathrm{Pos}(\phi) \rightarrow \neg\mathrm{Pos}(\neg\phi)]$	UD
3.	~~*SHOW*~~ $\mathrm{Pos}(\phi) \rightarrow \neg\mathrm{Pos}(\neg\phi)$	CD
4.	$\mathrm{Pos}(\phi)$	assumption
5.	~~*SHOW*~~ $\neg\mathrm{Pos}(\neg\phi)$	ID
6.	$\mathrm{Pos}(\neg\phi)$	assumption
7.	$\Box\forall x[\neg(\neg\phi)x \rightarrow \mathrm{Imp}(x)]$	Def Pos, N, UI($\neg\phi$), BC, 4, MP, S
8.	~~*SHOW*~~ $\Box\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)]$	ND
9.	~~*SHOW*~~ $\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)]$	FUD
10.	~~*SHOW*~~ $\mathrm{E!}x \rightarrow [(\phi)x \rightarrow \mathrm{Imp}(x)]$	CD
11.	$\mathrm{E!}x$	assumption
12.	~~*SHOW*~~ $(\phi)x \rightarrow \mathrm{Imp}(x)$	CD
13.	$(\phi)x$	assumption
14.	$\Box\forall x[\neg(\neg\phi)x \rightarrow \mathrm{Imp}(x)]$	7, R
15.	$\Box\forall\phi\forall x[(\neg\phi)x \leftrightarrow \neg(\phi)x]$	Negations of Properties
16.	$\neg(\neg\phi)x$	15, N, UI, 11, FUI, BC, 13, DN, MT
17.	$\mathrm{Imp}(x)$	14, N, 11, FUI, BC, 16, MP
18.	$\neg\Box\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)]$	Def Pos, N, UI, BC, 4, MP, S

Axiom 2 from Def Pos*

1.	~~*SHOW*~~ $\Box\forall\phi\forall\psi[\mathrm{Pos}(\phi) \,\&\, \Box\forall x[\phi(x)\rightarrow \psi(x)]\rightarrow \mathrm{Pos}(\psi)]$	ND
2.	~~*SHOW*~~ $\forall\phi\forall\psi[\mathrm{Pos}(\phi) \,\&\, \Box\forall x[\phi(x) \rightarrow \psi(x)] \rightarrow \mathrm{Pos}(\psi)]$	UD
3.	~~*SHOW*~~ $\forall\psi[\mathrm{Pos}(\phi) \,\&\, \Box\forall x\,[\phi(x)\rightarrow \psi(x)] \rightarrow \mathrm{Pos}(\psi)]$	UD
4.	$\mathrm{Pos}(\phi) \,\&\, \Box\forall x[\phi(x) \rightarrow \psi(x)] \rightarrow \mathrm{Pos}(\psi)$	CD
5.	$\mathrm{Pos}(\phi) \,\&\, \Box\forall x[\phi(x) \rightarrow \psi(x)]$	assumption
6.	$\Box\forall x[\neg(\phi)x\rightarrow \mathrm{Imp}(x)] \,\&\, \neg\Box\forall x\,[(\phi)x\rightarrow \mathrm{Imp}(x)]$	Def Pos, N, UI, BC, 5, S, MP
7.	$\Box\Diamond\neg\forall x[(\phi(x) \rightarrow \mathrm{Imp}(x)]$	6, S, MdlNeg, UP
8.	~~*SHOW*~~ $\Box\forall x[\neg(\psi)x \rightarrow \mathrm{Imp}(x)]$	ND
9.	~~*SHOW*~~ $\forall x[\neg(\psi)x \rightarrow \mathrm{Imp}(x)]$	FUD
10.	~~*SHOW*~~ $\mathrm{E!}x \rightarrow [\neg(\psi)x \rightarrow \mathrm{Imp}(x)]$	CD
11.	$\mathrm{E!}x$	assumption
12.	~~*SHOW*~~ $\neg(\psi)x \rightarrow \mathrm{Imp}(x)$	CD
13.	$\neg(\psi)x$	assumption
14.	$\Box\forall x[\phi(x) \rightarrow \psi(x)]$	5, S
15.	$\Box\forall x[\neg(\phi)x \rightarrow \mathrm{Imp}(x)]$	6,S
16.	$\mathrm{Imp}(x)$	14, N, 11, FUI, 13, MT, 15, N, 11, FUI, MP
17.	~~*SHOW*~~ $\neg\Box\forall x[(\psi)x \rightarrow \mathrm{Imp}(x)]$	ID
18.	$\Box\forall x[(\psi)x \rightarrow \mathrm{Imp}(x)]$	assumption
19.	~~*SHOW*~~ $\Box\forall x[\phi(x) \rightarrow \mathrm{Imp}(x)]$	ND
20.	~~*SHOW*~~ $\forall x[\phi(x) \rightarrow \mathrm{Imp}(x)]$	FUD
21.	~~*SHOW*~~ $\mathrm{E!}x \rightarrow [\phi(x) \rightarrow \mathrm{Imp}(x)]$	CD
22.	$\mathrm{E!}x$	assumption
23.	~~*SHOW*~~ $\phi(x) \rightarrow \mathrm{Imp}(x)$	CD
24.	$\phi(x)$	assumption
25.	$\Box\forall x[\phi(x) \rightarrow \psi(x)]$	5, S
26.	$\Box\forall x[(\psi)x \rightarrow \mathrm{Imp}(x)]$	18, R

27.	Imp(x)	25, N, 22, FUI, 24, MP, 26, N, 22, FUI, MP
28.	$\neg\Box\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)]$	6, S
29.	$\mathrm{Pos}(\psi)$	Def Pos, N, UI(ψ), BC, 8, 17, Adj, MP

THEOREM 1*, $\Box\forall\phi[Pos(\phi) \rightarrow \Diamond\exists x\phi(x)]$. This theorem can be derived from Axioms 1* and 2* along the lines drawn in Section 3.1.2. But that would be a long way to it. For a more direct derivation, once one has assumed $\mathrm{Pos}(\phi)$, one can with Def Pos go to $\neg\Box\forall x[\phi(x) \rightarrow \mathrm{Imp}(x)]$, which entails $\Diamond\exists x\phi(x)$.

Axiom 4 from Def Pos*

1.	~~SHOW~~ $\Box\forall\phi[\mathrm{Pos}(\phi) \rightarrow \Box\mathrm{Pos}(\phi)]$	ND
2.	~~SHOW~~ $\forall\phi[\mathrm{Pos}(\phi) \rightarrow \Box\mathrm{Pos}(\phi)]$	UD
3.	~~SHOW~~ $\mathrm{Pos}(\phi) \rightarrow \Box\mathrm{Pos}(\phi)$	CD
4.	$\mathrm{Pos}(\phi)$	assumption
5.	$\Box\forall\phi(\mathrm{Pos}(\phi) \leftrightarrow \Box\forall x[\neg(\phi)x \rightarrow \mathrm{Imp}(x)]$ & $\neg\Box\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)])$	Def Pos
6.	$\Box\forall x[\neg(\phi)x \rightarrow \mathrm{Imp}(x)]$	5, N, UI, BC, 4, MP, S
7.	$\Box\Diamond\neg\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)]$	5, N, UI, BC, 4, MP, S, MdlNeg, UP
8.	~~SHOW~~ $\Box\mathrm{Pos}(\phi)$	ND
9.	$\Box\forall x[\neg(\phi)x \rightarrow \mathrm{Imp}(x)]$	6,R
10.	$\Box\Diamond\neg\forall x[(\phi)x \rightarrow \mathrm{Imp}(x)]$	7,R
11.	$\mathrm{Pos}(\phi)$	Def Pos, N, UI, BC, 9, 10, Adj, MP

A LEMMA. It is convenient for derivations of Theorems 2* and 3* to derive first, from Def G* and Def Ess*, a lemma that says that a God-like* being has every positive* property. Necessitated and with explicit quantifiers this is,

$$\Box\forall x(G^*(x) \rightarrow \forall\psi[\mathrm{Pos}(\psi) \rightarrow \psi(x)])$$

or, equivalently,

Lemma. $\Box\forall\psi\forall x[\mathrm{Pos}(\psi)\ \&\ G^*(x)\rightarrow \psi(x)]$.

1.	~~*SHOW*~~ $\Box\forall\psi\forall x[\mathrm{Pos}(\psi)\ \&\ G^*(x) \rightarrow \psi(x)]$	ND
2.	~~*SHOW*~~ $\forall\psi\forall x[\mathrm{Pos}(\psi)\ \&\ G^*(x) \rightarrow \psi(x)]$	UD
3.	~~*SHOW*~~ $\forall x[\mathrm{Pos}(\psi)\ \&\ G^*(x) \rightarrow \psi(x)]$	FUD
4.	~~*SHOW*~~ $\mathrm{E!x} \rightarrow [\mathrm{Pos}(\psi)\ \&\ G^*(x) \rightarrow \psi(x)]$	CD
5.	$\mathrm{E!x}$	assumption
6.	~~*SHOW*~~ $\mathrm{Pos}(\psi)\ \&\ G^*(x) \rightarrow \psi(x)$	CD
7.	$\mathrm{Pos}(\psi)\ \&\ G^*(x)$	assumption
8.	$\phi'\ \mathrm{Ess}^*\ x\ \&\ \forall\psi(\Box\forall y[\phi'(y) \rightarrow \psi(y)] \leftrightarrow \mathrm{Pos}(\psi))$	Def G^*, N, 5, FUI, BC, 7, S, MP, EI
9.	$\phi'(x) \rightarrow \psi(x)$	8, S, UI, BC, 7, S, MP, N, 5, FUI
10.	$\phi'(x)$	Def Ess^*, N, UI(ϕ'), 5, FUI, BC, 8, S, MP, S
11.	$\psi(x)$	9,10,MP

Theorem 2 from Axioms 2* and 3*, Def G*, Def Ess*, and the lemma*

1.	~~*SHOW*~~ $\Box\forall x[G^*(x) \rightarrow \mathrm{G\ Ess}^*\ x]$	ND
2.	~~*SHOW*~~ $\forall x[G^*(x) \rightarrow \mathrm{G\ Ess}^*\ x]$	FUI
3.	~~*SHOW*~~ $\mathrm{E!x} \rightarrow [G^*(x) \rightarrow \mathrm{G\ Ess}^*\ x]$	CD
4.	$\mathrm{E!x}$	assumption
5.	~~*SHOW*~~ $G^*(x) \rightarrow \mathrm{G\ Ess}^*\ x$	CD
6.	$G^*(x)$	assumption
7.	~~*SHOW*~~ $\forall\psi(\Box\forall y[G(y) \rightarrow \psi(y)] \leftrightarrow \Box[\mathrm{E!x} \rightarrow \psi(x)])$	UD
8.	~~*SHOW*~~ $\Box\forall y[G(y) \rightarrow \psi(y)] \rightarrow \Box[\mathrm{E!x} \rightarrow \psi(x)]$	CD
9.	$\Box\forall y[G(y) \rightarrow \psi(y)]$	assumption
10.	$\mathrm{Pos}\ \psi$	Axiom 2^*, N, UI(G), UI, Axiom 3^*, 9, Adj, MP

11.	ϕ' Ess*x & $\forall\psi(\Box\forall y[\phi'(y) \rightarrow \psi(y)] \leftrightarrow \text{Pos}(\psi))$	Def G*, N, 4, FUI, BC, 6, MP, EI
12.	$\Box\forall y[\phi'(y) \rightarrow \psi(y)]$	11, S, UI, BC, 10, MP
13.	$\Box[\text{E!}x \rightarrow \psi(x)]$	Def Ess*,N, UI(ϕ'), 4, FUI, BC, 11, S, MP, UI, BC, 12, MP
14.	~~*SHOW*~~ $\Box[\text{E!}x \rightarrow \psi(x)] \rightarrow \Box\forall y [G(y) \rightarrow \psi(y)]$	CD
15.	$\Box[\text{E!}x \rightarrow \psi(x)]$	assumption
16.	ϕ'' Ess* x & $\forall\psi(\Box\forall y[\phi''(y) \rightarrow \psi(y)] \leftrightarrow \text{Pos}(\psi))$	Def G*, N, 4, FUI, BC, 6 , MP, EI
17.	$\phi''(x)$ & $\forall\psi(\Box\forall y[\phi''(y) \rightarrow \psi(y)] \leftrightarrow \Box[\text{E!}x \leftrightarrow \psi(x)])$	Def Ess*, N, UI(ϕ''), 4, FUI, BC, 16, S, MP
18.	$\Box\forall y[\phi''(y) \rightarrow \psi(y)]$	17, S, UI, 15, MP
19.	$\Box$ Pos(ψ)	16 , S, UI, 18, MP, Axiom 4*, N, UI(ψ), MP
20.	~~*SHOW*~~ $\Box\forall y[G(y) \rightarrow \psi(y)]$	ND
21.	~~*SHOW*~~ $\forall y[G(y) \rightarrow \psi(y)]$	FUD
21.	~~*SHOW*~~ E!$y \rightarrow [G(y) \rightarrow \psi(y)]$	CD
22.	E!y	assumption
23.	~~*SHOW*~~ $G(y) \rightarrow \psi(y)$	CD
24.	$G(y)$	assumption
25.	$\Box$Pos(ψ)	19,R
26.	$\Box\forall\psi\forall x[\text{Pos}(\psi)$ & $G^*(x) \rightarrow \psi(x)]$	*Lemma*
27.	$\psi(y)$	26, N, 22, FUI, 25, N, 24, Adj, MP
28.	$\Box\forall y[G(y) \rightarrow \psi(y)] \leftrightarrow \Box[\text{E!}x \rightarrow \psi(x)]$	8, 14, CB
29.	G Ess* x	Def Ess*, N, UI(G), 4, FUI, BC, 6, 7, Adj, MP

THEOREM 3*. A derivation from Axioms 3* and 5*, Theorems 1* and 2*, and Def G* can, as Anderson would have it do, go 'much as' the derivation in Section D3 of Theorem 3, for which only Def G is not a 'P lookalike' of its '*-correspondent.' Def G is used just once in that derivation, for line 9, which is,

$$P(NE) \rightarrow NE(x)$$

Line 9*, in a similar derivation for Theorem 3*, can be reached thus:

5*.	E!x	assumption
6*.	*SHOW* $E!x \rightarrow [G^*x \rightarrow \Box\exists xG^*(x)]$	
7*.	$G^*(x)$	assumption
8*.	$Pos(NE^*)$	Axiom 5*, N
9*.	~~*SHOW*~~ $Pos(NE^*) \rightarrow NE^*(x)$	CD
10*.	$\Box\forall\psi\forall x[Pos(\psi) \,\&\, G^*(x) \rightarrow \psi(x)]$	*Lemma*
11*.	$NE^*(x)$	10*, N, UI(NE), 5, FUI(x), 8*, 7*, Adj, MP

THEOREM 1* '*generalized*'. Now comes the 'plan' that was promised in Section *PS*3.2.12 for derivations from Def Pos of the possibility of any finite conjunction ($\phi_1 \,\&\, \ldots \,\&\, \phi_n$) of Pos-properties. For this plan I use some of Anderson's compact notation, '$\Box(\boldsymbol{\phi} \rightarrow \boldsymbol{\Delta})$', for example, instead of, '$\Box\forall x[\phi(x) \rightarrow Imp(x)]$'. Also used are the rules

General Simplification. $p_1 \& \cdots p_n \;/\therefore p_i \Delta (i > 0, \text{and} \leq n)$.

General DeMorgan. $\neg(p_1 \& \cdots \& p_n) \;/\therefore (\neg p_1 \vee \cdots \vee \neg(p_n)$.

General Separation of Cases. $(p_1 \vee \cdots \vee p_n), (p_1 \rightarrow q), \cdots, (p_n \rightarrow q) \,/\therefore q$.

*Theorem 1** is for $n = 1$. Here is a plan to derive, for any finite n equal to or greater than 1,

$$\text{Theorem } 1^*_n. \quad Pos(\phi_1) \,\&\, \cdots \,\&\, Pos(\phi_n) \rightarrow \Diamond(\phi_1 \& \cdots \& (\phi_n).$$

1.	~~*SHOW*~~ $\mathrm{Pos}(\phi_1)\&\cdots\&\mathrm{Pos}\phi_n) \rightarrow \Diamond(\phi_1\&\cdots\&(\phi_n)$	CD
2.	$\mathrm{Pos}(\phi_1)\&\cdots\&\mathrm{Pos}(\phi_n)$	assumption
3.	$\Box(\neg\phi_n \rightarrow \boldsymbol{\Delta})\&\neg\Box(\phi_1 \rightarrow \boldsymbol{\Delta})$	2, GenrealSimplification, Def Pos, BC, MP
.	.	.
3_n	$\Box(\neg\phi_n \rightarrow \boldsymbol{\Delta})\&\neg\Box(\phi_n \rightarrow \boldsymbol{\Delta})$	2, Genreal Simplification, Def Pos, BC, MP
4.	~~*SHOW*~~ $\Diamond(\phi_1\&\cdots\&\phi_n)$	ID
5.	$\neg\Diamond(\phi_1\&\cdots\&\phi_n)$	assumption
6.	~~*SHOW*~~ $\Box(\phi_1 \rightarrow \Delta)$	ND
7.	~~*SHOW*~~ $\phi_1 \rightarrow \Delta$	CD
8.	ϕ	assumption
9.	$\Box\neg(\phi_1\&\cdots\&\phi_n)$	5, MdlNeg
10.	$\neg(\phi_1 \vee \cdots \vee \neg(\phi_n)$	9, N, GeneralDeMorgan
11_1	$\Box(\neg\phi_1 \rightarrow \boldsymbol{\Delta})$	3_1, S
.	.	.
11_n.	$\Box(\neg\phi_n \rightarrow \boldsymbol{\Delta})$	3_n, S
12.	$\boldsymbol{\Delta}$	$10, 11_1, N, \ldots, 11_{n,}, N,$ GeneralSeparationCases
13.	$\neg\Box(\phi_1 \rightarrow \boldsymbol{\Delta})$	3_1,S

V

First Causes
"*The Second Way*"

> It is not conceivable that successions of causes and effects leading to now should have sprung from nowhere. Sclauzero of Toronto[1]

These words can inspire 'nondemonstrative' arguments for the existence of God that would proceed from very general facts about the world. Never mind, they say, details and ways in which the world is special. It and that is enough. For causes and reasons are required for that, causes and reasons need causes and reasons, and, eventually, if these systems are to be grounded and not to have sprung from nowhere, they must relate all to first and ultimate causes and reasons. Two such arguments – two rather different 'takes' on Sclauzero's words – will be examined. The argument of the next chapter, for an ultimate reason for The World, can be seen as a response to problems of the other, the argument of the present chapter, which is for a first cause for things.

1. PART 1, QUESTION 2, OF *SUMMA THEOLOGICA* – "*THE EXISTENCE OF GOD*"

1.1. The question of the existence of God is resolved by St. Thomas Aquinas into three questions.

Article 1: "Whether the Existence of God Is Self-evident?" He says, Yes, but not to us. (ST I q2,a1 pp. 18–19.)[2] That God exists is self-evident in itself, since its predicate is in fact contained in the essence of its subject. But it is not self-evident for us since this essence, the essence of God, is not known to us.[3]

Article 2: "Whether It Can Be Demonstrated That God Exists?" The question is whether God's existence can be *proved from premises available to us*; that is what Aquinas means by 'demonstrated.' Aquinas's answer is, Yes, but only *quia*, by arguments to God as cause, from effects of This One. (ST I q2,a2 pp. 20–1.) Aquinas contrasts demonstration *quia* from effects with

"[d]emonstration through the cause . . . called *propter quid*" (ST I q2,a3 p. 20). "[T]he existence of God, in so far as it is not self-evident to us" (ST I q2,a2 p. 21) is not demonstrable *propter quid*. It is not provable from the nature of the cause, for this nature is not sufficiently known to us. This existence is demonstrable only *quia*, that is, from effects we know better and well enough.

Article 3: "Whether God exists?" Aquinas's answer is, "The existence of God can be proved in five ways" (ST I q2,a2 p. 22). He gives five brief demonstrations *quia* from well-known effects to God as their cause. Of these, only the second is in so many words from 'effects' to a 'cause.' "The first . . . way is . . . from motion [to a mover]. . . . The third . . . from . . . things that are possible to be and not to be [to something the existence of which is necessary]. . . . The fourth . . . from the gradation to be found in things. Among beings there are some more and some less good, true, noble, and the like . . . [to] something which is the maximum . . . truest, best, noblest, and . . . most being. . . . The fifth . . . from . . . things which lack knowledge, such as natural bodies, act[ing] . . . to obtain a best result . . . [to] some intelligent being . . . by whom all natural things are directed . . . the governance of the world [which evidences design and direction by an intelligent being]." And now back to "the second way."

1.2 Verbatim *(in translation)*

The second way is from the nature of efficient cause. In the world of sensible things we find there is an order of efficient causes. There is no case known (neither is it, indeed, possible) in which a thing is found to be the efficient cause of itself; for so it would be prior to itself, which is impossible. Now in efficient causes it is not possible to go on to infinity, because in all efficient causes following in order, the first is the cause of the intermediate cause, and the intermediate cause is the cause of the ultimate cause, whether the intermediate cause be several, or one only. Now to take away the cause is to take away the effect. Therefore, if there be no first cause among efficient causes, there will be no ultimate, nor any intermediate, cause. But if in efficient causes it is possible to go on to infinity, there will be no first efficient cause, neither will there be an ultimate effect, nor any intermediate efficient causes; all of which is plainly false. Therefore it is necessary to admit a first efficient cause, to which everyone gives the name of God. (ST I q2,a3 p. 22)

1.3 The 'idea' of this way. We find in the world sensible things that have causes. We find that there is in the world of these things an order of causes, of causes that have causes. But this order cannot 'go on to infinity.' There cannot be an *infinite* regress of causes of sensible things without a beginning or first cause. Think about it! Try really to *think* about it. The mind rejects the thought. It is not conceivable that the successions of causes and effects leading to sensible things – to our chairs and tables, to *us* – should have 'sprung from nowhere' or, more accurately, should not have *sprung or begun*. The successions

of causes and effects to sensible things must be 'grounded.' Take that away and you take away every sensible thing – were there no first cause for sensible things, there could be nothing sensible, which is plainly false. Therefore, there is a first cause for sensible things. To this source and universal ground "everyone gives the name of God" (loc. cit.) – this awesome cause to which we, along with all sensible things, owe our existence and thus everything is that which we *should* honor with that name and to which we should worshipfully bend in boundless gratitude.

2. AN ARTICULATION OF THE *SECOND WAY*

To facilitate a discussion of ideas suggested by Aquinas's Second Way, I spell out an argument in the spirit of his words and comment directly on it. My argument reads between the lines of his and may not be exactly the argument he intended, though departures, if any, are made in the spirit of generous interpretation for the best argument that can be made of his words. Present comments are mainly expository. Critical discussion is in later sections. Here first are the 'lines' of my articulation: *Seven Premises*: (1) There exist sensible things that have efficient causes. (2) If a thing has an efficient cause, it has exactly one efficient cause. (3) Efficient causes of things are prior to them. (4) The priority relation is irreflexive: If x is prior to y, then x is distinct from y. (5) The priority relation is transitive: If x is prior to y, and y is prior to z, then x is prior to z. (6) Every sensible thing that is an efficient cause of some sensible thing, itself has an efficient cause. (7) "[I]n efficient causes it is not possible to go to infinity" (ST Iq2,3 p. 22). *Preliminary Conclusion*: (8) "[There is] a first cause among efficient causes" (ST I q2,a3 p. 22) – more fully, there is, for all sensible things that have efficient causes, a first cause that does not have an efficient cause and is not itself a sensible being. *Additional Premise*: (9) "[T]o which everyone [correctly] gives the name of God" (loc. cit.). *Ultimate Conclusion*: (10) God exists!

SEVEN PREMISES

2.1

(1) *There exist sensible things that have efficient causes.*
$(\exists x)[Sx \ \& \ (\exists y(yCx)]$

('Sx' and 'yCx' abbreviate, respectively, 'x is a sensible thing' and 'y is an efficient cause of x'.)

2.1.1. This premise does not claim that *every* sensible thing has an efficient cause, but only that *some* do. It is of course consistent with stronger claims. I take (1) for my starting point for several reasons. The first is that Aquinas starts his First Way with a claim that is weak in a similar fashion, viz., "[i]t is certain,

and evident to our senses, that in the world some things are in motion" (loc. cit.). A connected reason is that (1) is weaker than the universal claim 'with existential import' that would simplify the argument of the Second Way. Being weaker, it is at least as plausible. That stronger claim would be acceptable to Aquinas, but it lacks the immediate evidence that should attend points of departure for demonstrations *quia* and that do attend the starting points of the other four Ways. A third reason is that taking off from this relatively weak premise allows my articulation at least to put off encounters with certain thorny, specifically Christian, theological difficulties. Now comes comment on the acceptability for Aquinas of a stronger first premise that 'tables' an important issue of interpretation of the Second Way. Then, still before proceeding to premise (2), come brief comments on those theological difficulties.

2.1.2. The stronger universal-with-existential-import claim that,

(1′) There exist sensible things, and *every* sensible thing has an efficient cause.

$$(\exists x)Sx \;\&\; (x)[Sx \supset (\exists y)(yCx)]$$

would simplify the argument, and it is a claim that Aquinas could accept. There are two lines of thought that could lead him to principles expressible by sentence (1′). Each approach starts with the principle that *sensible things* (the observables from which all his demonstrations *quia* proceed) are *contingent things*, which is to say, "things that are possible to be and not to be, since they are found to be generated and corrupted" (ST I q2,a3 p. 22 – these are words from the Third Way). *The first approach* adds to this the principle that every contingent thing comes into existence: "[I]t is impossible for these always to exist, for that which can not-be some time is not" (ST I q2,a3 p. 23). Aquinas writes that "that which does not exist begins to exist only through something already existing" (ST I q2,a3 p. 23), that is, things that come into existence must have PRODUCTIVE OR GENERATING efficient causes. *The second approach* from the contingency of sensible things to a proposition expressible by (1′) is simpler. Contingent things, this approach says, need at every moment SUSTAINING efficient causes. It is not of their natures and essences to exist. They do not exist 'simply of themselves' or 'by themselves'. Each, Aquinas would say, depends every moment on another thing or on an 'efficient ground'. So it seems that in place of (1) Aquinas could have the stronger premise (1′). As said, that would simplify the argument that I am assembling for him. Even so, for reasons stated, and another coming, I start for him with (1).

2.1.3. Which efficient causation is it to be in the Second Way? There are two questions here: First, what kind of efficient causes is the Second Way about, that is, what kind does Aquinas intend as he sketches the map of this way? Second, what kind of causes would it *best* be about, that is, of what kind does

it make the better proof for a first cause? These questions, to which I return several times in this section and at length in Section 3, do not have entirely satisfactory answers. A generating-efficient-cause interpretation would best serve premise (1) and goes best with the text of the Second Way: I think it is the causation that Aquinas intends in the *Second Way*. But only a sustaining-efficient-cause interpretation (i) can serve the central premise of the argument that says that there cannot be infinite regresses of causes; (ii) is consistent with *other* texts of Aquinas's that are concerned with the infinite, and with infinite series of causes and effect; and (iii) allows the conclusion of the argument that God exists *now* to be drawn. Further to (ii), Aquinas in a later passage of the *Summa* flat-out contradicts the premise against infinite regresses of causes taken to be about generating efficient causes. He indicates at (ST I q46,a3 p. 455), in words of the Second Way adapted to this point, that in *generating* efficient causes it is *not impossible* to go on to infinity. F. C. Copleston, largely for reason (ii), endorses a sustaining-efficient-cause interpretation of the present text: "[W]hen Aquinas talks [in the Second Way] about an 'order' of efficient causes he is not thinking of a series stretching back into the past, but of a hierarchy of causes, in which a subordinate member is here and now dependent on . . . the present causal activity of the member above it" (Copleston 1955, p. 118).

2.1.4 "*But what about The Trinity?*" This theological difficulty, which must come up at some point for the argument, would be raised immediately by the strong universal (1′), according to which every sensible thing has efficient causes. That, as said, is one reason why Aquinas could have preferred what I take to be his weaker starting point. Jesus was a man who was seen and heard. He was a sensible being. And so, according to (1′), he had an efficient cause. But, we are told, Jesus was God, which means that if Jesus had an efficient cause, then so did God. But the argument will say that nothing can be an efficient cause of itself and concludes that there is a first efficient cause, which is God. Premise (1′) would require that The Trinity be dealt with without delay.[4] Jesus, however, though a sensible being who was born and died, is, according to The Trinity, none other than God. So Aquinas must say that He, Jesus, is not a contingent being. For Aquinas holds that God's existence, though not knowable by us *a priori*, is necessary: It is "self-evident [in itself] . . . for . . . God is His own existence. . . . [But] because we do not know the essence of God, the proposition is not self-evident to us" (ST I q2,a1 p. 19).

Henceforth, unqualified uses of 'sensible things' shall be short for 'sensible things *other than God the Son*.' This, while not required to make how The Trinity works irrelevant to premise (1), is required for its irrelevance to premise (6) below, which calls for efficient causes for all sensible things that are efficient causes. For Jesus was a carpenter, and in that capacity a generating cause of several things, and He was, the doctrine of The Trinity says, God, and so, Aquinas of course wants to say, a sustaining efficient cause, indeed the deep and first sustaining cause, of all contingent existents. My usage 'brackets off'

The Trinity from the penultimate conclusion, (8), that there is among efficient causes a first cause that is not itself a sensible thing.

2.2

(2) *If a thing has an efficient cause, it has exactly one efficient cause.*
$$(x)(y)(z)[(yCx \ \& \ zCx) \supset y = z]$$

Aquinas writes that it is not possible for "a thing . . . to be **the** efficient cause of itself" (loc. cit.; emphasis added), from which I gather that things that have efficient causes are to have unique ones. He envisions for a sensible thing x a sequence of efficient causes that lead to it. If, for example, there are three causes in such a sequence to x, then I read him as saying that exactly "the ultimate cause" is a cause of x while "the first is the cause [only] of the intermediate cause, and the intermediate is the cause [only] of the ultimate cause" (ST q2,a3 p. 22).

2.3

(3) *Efficient causes of things are prior to them.*
$$(x)(y)(xCy \supset xPy)$$
('xPy' abbreviates 'x is prior to y')

The kind of order of the Second Way depends on its kind of efficient causes. If they are productive or generating efficient causes, the order is temporal. Such causes preexist the things they bring into existence: "[T]hat which does not exist **begins** to exist only through something **already** existing" (ST I q2,a3 p. 23; emphasis added).[5] If, on the other hand, the efficient causes are sustaining, then the difficult idea of the relevant 'priority' would be, Copleston implies, 'ontological': "The word 'first' does not mean first in the temporal order, supreme or first in the ontological order" (op. cit., p. 119). Things would depend on their sustaining causes to be, but not vice versa: The sustaining cause for a thing would be something that was somehow of a more fundamental **"grade** in the order of efficient causes" (ST q46,a3 p. 455; emphasis added); it would be *somehow* more real and superior.

2.4

(4) *The priority relation is irreflexive: nothing is prior to itself.*
$$\sim(\exists x(xPx)$$

Premises (3) and (4) entail that no "thing is . . . the efficient cause of itself" (ST I q2,a3 p. 22).

2.5

(5) *The priority relation is transitive: if x is prior to y, and y is prior to z, then x is prior to z.*
$$(x)(y)(z)[(xCy \ \& \ yCx) \supset xCz]$$

2.6

(6) *Every sensible thing that is an efficient cause of some sensible thing, has itself an efficient cause.*

$$(x)([Sx \,\&\, (\exists y)(Sy \,\&\, xCy)] \subset (\exists w)(wCx)]$$

There is no textual basis for this premise, though it may be part of Aquinas's idea of an order of efficient causes "[i]n the world of sensible things" (loc. cit.). This premise serves in the context of the others to rule out the possibility that first causes might be fecund self-energizing sensible things. Aquinas would have the First Cause of sensible things to be The Creator of 'the world of them'. Given the weak nonuniversal character of (1), (6) is needed for this result. Premise (6), if about generating causes, says what is plausibly 'confirmed in experience' and 'something we find'. Aquinas could say there is no case known in which a sensible thing that is a generating cause of a sensible thing is itself without a generating cause. In all cases known we find, or reasonably presume, generating causes for generating causes. The case is otherwise if premise (6) is about sustaining efficient causes, which is part of the problem for that interpretation of the 'demonstration quia' of the Second Way.

2.7

(7) "*[I]n efficient causes it is not possible to go to infinity.*" (ST I q2,a3 p. 22).[6]

This is the heart of the argument and the main thing of continuing philosophic interest in it.

2.8 Preliminary conclusion

(8) *"[There is] a first cause among efficient causes" (ST I q2,a3 p. 22) – more fully, there is, for all sensible things that have efficient causes, a first cause that does not have an efficient cause and is not itself a sensible being.*

Deducing (8). From premises (1)–(6) it follows that there is, extending back or down from say this computer, a single series of efficient causes, each member of which is prior to all previous members, and so different from each of them. Given premise (7), that efficient causes cannot 'go to infinity', there is in this series a last member that is naturally termed its *first* cause. This, we can say, is the first cause of every cause in the series, as well as of the effect from which it started 'back or down,' this computer. This first cause of the regress of causes from my computer does not itself have an efficient cause. From premise (6) it follows that this first cause is not a sensible thing. Had I begun my construction not with premise (1), but with (1′), not only would premise (6) be redundant, but a stronger preliminary conclusion (8′) could be gathered for which one could delete from (8) the words 'that have efficient causes.' Does

the conclusion as it stands cover us? Does it say that there is a first cause for us? It does if we have efficient causes, and we certainly do have generating efficient causes as well, I think, as sustaining efficient causes, if any sensible beings have such causes.

2.9 Additional premise

(9) "*[T]o which everyone [correctly] gives the name of God*" (loc. cit.).

Aquinas's words for this premise are grammatically dependent on preliminary conclusion (8): Premise (9) looks to (8) for its subject, the 'to which' of which it speaks.

2.10 Ultimate conclusion

(10) GOD EXISTS!

This, though not stated, is the intended conclusion: this Way is one answer to the question, "Whether God exists?"

Critical discussion
There are philosophical and scholarly difficulties with the Second Way. Most that are of philosophic interest go to the anti-infinite-regress premise; others of more scholarly concern relate mainly to the question of which kind of efficient causes the Second Way is about. Section 3 worries the issue of interpretation. Section 4 is mainly about philosophic matters. Section 5 concerns the form of preliminary conclusion (8), if it is to follow from (1) through (7). Section 6 is about the form of (8), if it is to provide (9) with the 'target' that it grammatically requires, and that this is a *different* form. Section 7 is about whether the conclusion (10) – that God exists – follows without further ado, when premises are interpreted as they need to be, for truth in Aquinas's opinion.

3. 'EFFICIENT CAUSES' IN THE ARGUMENT – SUSTAINING, OR GENERATING?

3.1 Premise (1) – There exist sensible things that have efficient causes

3.1.1. While weaker than (1′), premise (1) is not entirely innocuous. It is at any rate not as solid as its counterpart in the First Way, which is that *some things move*. That is as obvious as anything can be. "It is **certain**, and **evident to our senses**, that in the world some things are in motion" (First Way, ST I q2,a3 p. 22; emphasis added). One cannot do better for "a demonstration *quia* . . . [from] an effect better known to us than its cause . . . to . . . knowledge of the cause" (ST I q2,a2 p. 21). The initial premises of the other three Ways are similarly advantaged. "[W]e **find** [that we need only look around to see] in nature things . . . generated, and . . . corrupted" (Third Way; emphasis added).

There is no doubt that "*more* and *less* are predicated of . . . things" (Fourth Way). Some roads are longer, some coffee cooler. And we cannot but be struck by appearances everywhere of design in nature (Fifth Way). Perhaps, however, while not being *as* obvious, Aquinas's starting point in the Second Way is *nearly* as obvious, and so clearly qualifies as "better known to us than its [ultimate] cause" (Ibid.) for a demonstration *quia*. The issue turns on whether the causes of Way would be generating or sustaining.

3.1.2. Aquinas writes, "In the world of sensible things we **find** there is an order of efficient causes" (ST q2,a3 p. 22; emphasis added). "In other words," according to Copleston, "in our experience of things and of their relations to one another we are aware of efficient causality" (Copleston 1955, p. 111). And this is so – we do find an order of efficient causes, of which one could can hardly fail to be aware of it – *if* what is meant is that we find sensible things generating and producing other sensible things. We are aware of chickens laying eggs and eggs becoming chickens, of persons having children, who in turn have children, and so on. There certainly are – without any possibility of philosophical (for example, Humean) challenge – *generating or producing* efficient causes of many things. It is evident that there are 'agents,' broadly understood, that from time to time *bring things into existence* by making them (sculptors), are *responsible* for things coming into existence (parents), or *out of which* things come into existence (eggs and seeds). There certainly are efficient causes of these sorts. We are aware of *temporal* orders of efficient *generating* causes. *Sustaining* efficient causes, however, makes another case that argues against their being the causes that Aquinas had in mind when he said that we find in the world of sensible things an order of efficient causes. This is in part because it is not obvious how such causes are to be understood, how they would be related to their effects, and how they might be seen to be at work.

3.1.3. We have an *inkling* of what *sustaining* causes would be like. Whatever else a sustaining cause of a thing would be, it would be something whose 'action' at a time was necessary for the thing's existence at this time and perhaps for its 'activity' of sustaining something in turn.[7] A *present* sustaining cause of X would be something Y upon the present existence and action of which the present existence of X and perhaps action depended, something without which it would not be, let alone act. Similarly for the past and future sustaining causes of a thing. Y would be a sustaining cause for X at a time if and only if the being, and perhaps certain causal activities of X at this time are dependent on causal activity, and thus the being, of Y at this time. With this partial idea to go, we may wonder whether sensible things, any of them, have sustaining efficient causes. Consider me and my existence at this moment. Do I have a sustaining efficient cause? Is there any 'thing' that is no part of me (for nothing can be prior to itself, so nothing can be a sustaining cause of itself) without which 'thing' I could not now exist and act? I doubt it, and am not bragging.[8] I doubt

that there is anything separate from you the present existence of which is necessary for your present existence, or that there is anything separate from *any* sensible thing that is necessary for its existence. "But we and everything depend every moment on God, the ground of all being!!" I doubt it, though what is more to the point are, (i) *this* dependence is certainly not evident in the manner required for the factual basis of a demonstration *quia* from effects better known than the nature of the cause, and (ii) this dependence is not available as a premise for a 'proof' of the existence of God or an argument that might reasonably persuade someone who needed to be persuaded.

William Rowe, for a sense of the causality in question, suggests that the activities of "oxygen, heat, etc." (Rowe 1975, p. 30) are necessary for my present existence. Copleston implies that I am "dependent here and now on . . . the activity of the air, and the life-preserving activity of the air is itself dependent here and now on other factors, and they in turn on other factors . . . [that] the activity of the pen tracing these words on this page is here and now dependent on the activity of my hand, which in turn is here and now dependent on other factors [the activity of my arm?]" (Copleston 1955, p. 118). Aquinas implies something along the first line when he writes of a particular man's depending "on an elementary body [heat?], and [this] on the sun, and so on [but not] to infinity" (St I q46,a3 p. 455). But I am not for my present existence or activity dependent on oxygen, heat, or air. I am dependent on these things only eventually for my future existence. I am dependent on them after a short time for my *persistence*, for my *continuing* existence. Take away oxygen and I am dead, not now, however, but only shortly. Take away heat from my environment, plunge it to absolute zero, and I am gone more quickly, but again not immediately. Take away the sun, and the heat, most of it hereabouts continues for eight minutes or so, so the sun is no part of its efficient sustaining cause. Oxygen and the like are at best not sustaining, but *perpetuating*, and so not necessarily *concurrent* efficient causes of people. The activity of a pen in my grasp is concurrent with the activity of my hand with which I am moving the pen, but, while suggestive, this causality is not any kind of 'efficient' causality: My hand is nothing to the *existence* of the pen.

Suppose, however, that my present existence did depend on oxygen now. Suppose, indeed, that this was certain and evident to our senses. Is there something similar that can be said with *any* plausibility to be similarly related to the existence here and now of this oxygen? Even supposing that it were obvious that I am presently sustained by oxygen, though there would be clear evidence that a sensible thing had a sustaining cause, it would not yet be evident to our senses that there was "an **order** of [sustaining] efficient causes" (loc. cit.; emphasis added) that just might, as envisioned in the Second Way, need to go on, or better 'down,' to infinity unless it gets to a ground *extraordinaire* that while sustaining is not itself sustained by anything. Certainly, dropping all pretence, we do *not* find such an order of sustaining causes, nor is one by any stretch "evident to our senses." It is not obvious that sensible things have sustaining

efficient causes. It is not obvious that *any* sensible thing has such a cause. As far as I know, no uncontentious examples of such causes are anywhere to the 'found in the literature'.

Anthony Kenny "rejects outright such . . . series [as] . . . 'vouched for only by medieval astrology'" [Anthony Kenny, *The Five Ways*, New York: Schocken Books 1969, pp. 43–4] and Aristotelian spheres-within-spheres cosmologies of kinds endorsed by some medievals (Rowe 1975, p. 31). Rowe suggests less harshly that this whole business of an order in the world of sensible things of sustaining efficient causes was, for Aquinas, a "basic belief" founded neither on naive experience nor on medieval science, but on "metaphysical analysis and argument" (Rowe 1975, p. 30). And so it may have been. It may be that the best that can be said about Aquinas's belief in this order in the world of sensible things is that it was an integral part of his metaphysics of the world. But that would not be, for its use in the Second Way, *any* good for him. Rowe's suggestion would make it an order Aquinas 'found' not by looking around, but by analysis and metaphysics. It would not be a starting point for the demonstration *quia* that the Second Way was to be.

3.1.4. Sustaining-cause versions of the Second Way are 'nonstarters' for *what Aquinas had in mind when he set out this Way*. And this notwithstanding advantages they will be observed to have over generating-cause constructions, advantages at points other than their starting points. That is not to say that a sustaining-cause version of the argument I have assembled to Aquinas is a better argument than the argument that he had in mind, for the point at which the argument he intended has the advantage is premise (1), which understood as about generating causes is certainly true. Understood in so far as we can manage that, as about sustaining causes, *this* premise is, I have argued, *false*, and the argument is over, unless (1) can be saved by the argument's conclusion, in which case the argument is lost as a 'proof' of that conclusion.

3.2 Premise (2) – If a thing has an efficient cause, it has exactly one efficient cause. This premise makes problems for some common examples of efficient causes. Persons have children, but at least until recently it has taken two persons to do that. What then shall we say is the efficient generating cause of a child? Aquinas would, I think, have said, the father, for in his view the mother provides only an incubating vessel for a child delivered by the father, 'complete in the small.' Perhaps some would say, the mother, albeit with a little outside help. Others might say, extending the idea of a sensible 'thing,' the mother–father pair. Still others might take that to be the efficient generating cause of the fertilized egg 'out of which' the child developed, and count this egg as the child's proper efficient cause. Similarly for chickens and their eggs, its taking two chickens to make a fertilized egg, and one of these to make a chicken. Sculptors do not pose problems for premise (2), since even if two sculptors working independently produce identical statues, they do not produce just one

physical statue. As for two sculptors working together on a single statue, we can say that they, the pair of them, produce this statue. If poets were independently to make the very same poem, word for word, then, though each of them made this poem, it would not have them as its efficient team-cause. But that is not a problem for premise (2) since this poem that each would have made, in contrast with inscriptions of it, if any, would not be a sensible thing.

While not a problem for premise (2), countenancing pairs and other collections of individuals as generating causes of single individuals would considerably complicate the 'tree-structure' of orders of generating causes and recommend an elaboration of premise (6), which could be changed to say *that every sensible thing* THAT IS A MEMBER OF AN EFFICIENT CAUSE OF SOME SENSIBLE THING *must itself have an efficient cause.* To keep things simple, I will assume that efficient causes of individuals are never several individuals acting together, and in particular that the generating efficient cause of an egg is a chicken, and, following Aquinas for an example he uses, that the generating efficient cause of a child is its father.

3.3 Premise (3) – Efficient causes of things are prior to them. As has been said, the priority of generating efficient causes is *temporal* priority at least in the case of efficient causes that are themselves sensible things. For example, the sculptor who brings a statue into existence preexists it. Even makers of shadows preexist the shadows that they make. If a tree casts a shadow, it is the tree at some hardly measurable time just before t that is responsible, in conjunction with a light source, for its shadow at t. The elapsed times for the shadows that the moon casts on earth are measurable. They are in the neighbourhood of 1 second. If the moon were suddenly to cease to exist, there would still be, for a period of about 1 second, shadows of it on earth. The idea of nontemporal ontological priority – which Aquinas would find he wanted for sustaining efficient causation – is difficult. This difficulty could recommend replacing premise (3) along with premises (4) and (5) by a simpler premise that is at least as plausible as any of them, specifically, that *if something x stands in a series of efficient causes that lead to y, then x is other y*. The essential point served by these premises is that series of efficient causes of causes of causes are linear and do not loop back on themselves, which is plausible on any natural understanding of causes.

3.4 Properties of efficient causation according to our premises. Putting together their uniqueness (Premise 2),

$$(x)(y)(z)[(yCx \,\&\, zCx) \supset y = z],$$

their prioriy to their effects (Premise 3),

$$(x)(y)(xCy \supset xPy),$$

that nothing is prior to itself (Premise 4),

$$\sim(\exists x)(xPx),$$

and that this priority is transitive (Premise 5),

$$(x)(y)(z)[(xPy \ \& \ yPx) \supset xPz],$$

and thus it is asymmetric (every transitive irreflexive relation is asymmetric)

$$(x)(y)(xPy \supset \sim yPx);$$

we may draw several consequences for the relation of efficient causation. This relation is like the priority relation in being irreflexive and asymmetric but, unlike the priority relation, that of efficient causation is not transitive, indeed it is intransitive,

$$(x)(y)(z)[(xCy \ \& \ yCx) \supset \sim xCz].$$

None of the 'ancestors' of the efficient cause of a thing can be an efficient cause of this thing. There is, however, no harm in casting them as 'ancestral efficient causes' or as 'efficient causes somewhat removed.' It is natural to speak of the first member of a series of efficient causes as a cause, as indeed 'the first efficient cause' of everything that is causally posterior to it, notwithstanding that, strictly speaking, it is an 'efficient cause' only of the member of the series that is immediately posterior to it. It is also natural to speak redundantly of efficient causes proper as 'immediate or proximate efficient causes.'

3.5 Generating and sustaining causes – similarities and differences. Whatever else they would be, sustaining causes would be *necessary* for the existence of things they sustained. But that is not all they would be, for they would be *sustaining* these things and so engaged in 'activity' somehow sufficient for their existence. What about generating causes? Clearly they would be engaged in activity sufficient for the generation of things, and thus of their existence. Would they, in contrast with sustaining causes, *not* be *necessary* for the existence of these things? I think not. When Aquinas writes in the Second Way, "Now to take away the cause is to take away the effect," I do not think he has in mind only sustaining, and not also generating efficient causes. "Take away the parents, and you take away the child," I think he would say. "Take away Michelangelo, and you take away his David." Again, suppose that 'a man were to be generated by a man to infinity.' Then an infinite series of fathers and sons would lead to this son, and this series would be "an infinite multitude . . . **necessary** [for this son to] exist" (ST I q7,a4 p. 61). I think that for Aquinas all efficient causes would be engaged in 'activity' sufficient for the existence of their effects and would be necessary for their effects. What distinguishes sustaining from generating efficient causation are, I assume, two

things only: (i) Sustaining causation would be *concurrent* causation – E would be sustained at a time by 'activity' of C at this time; and (ii) sustaining causation would be *per se* causation – if first cause F sustains intermediate cause I, which sustains Ultimate Effect U, then the 'activity' by which F sustains U 'comes from' the 'activity' by which F sustains I, somewhat as, when a hand moves a stick that moves a stone, the shove that the stick inflicts on the stone comes from the shove that the hand exerts on it, the stick.

4. THE INFINITE AND INFINITE REGRESSES

4.1 Premise (7)

"[*I*]*n efficient causes it is not possible to go to infinity*"
(ST I q2,a3 p. 22)

This premise, taken as it is natural to do to be about generating causes, is in the spirit of one interpretation of the words that head the present chapter: "It is not conceivable that successions of causes and effects leading to now should have sprung from nowhere," that is, that they should have not ever have been begun. But *is* this inconceivable? Is it *inconceivable* that there should be causes of causes of causes, without a beginning or first cause, each cause bringing into being its successor? Let me put the question differently. Did you understand that question?! If so, you may well have 'conceived' the very thing whose conceivability is at issue! Which is to say you may have an idea of it, a consistent idea that harbors no contradictions. Such ideas seem *easy* to come by. Here is an explanation for the idea of *infinitely many* past generations of *chickens*. Let the present generation be the (0)-generation, the just previous generation be the (−1)-generation, the generation previous to it be the (−2)-generation, *and so on for every negative integer (−n)*. We now have the idea of infinitely many generations of chickens; there is the current one for 0, and there is one for every negative integer of which there are of course infinitely many. So we have an idea of an *infinite regress* of generations of chickens. And since the chickens (perhaps in pairs) of each generation are efficient causes of the chickens of the next generation, we have the idea of *infinite regresses* of efficient causes! Though we do not *believe* in that regress, there is no question but that we have an *idea* of it. "Ah, but is it a *consistent* idea that harbors no *a priori* contradictions?" There *is* this question, but unless and until it is backed up with reasons for doubting the consistency of this idea, it is *only* a question, for the idea detailed certainly *seems* consistent, and, to *adapt* 'Leibniz's license,' we have the right to presume the consistency of every *prima facie* consistent idea until someone proves the contrary by deriving *a priori* a contradiction from it. That is *not* 'Leibniz's license' itself, which would be not only for the consistencies of ideas of things, but for the *possibilities* of things of which we have consistent ideas. He did not separate the

conditions of conceptual consistency and logical possibility as the conceptual consistency of *magicans* requires, given that there are no magicians (Section 8.1 of chapter III).

4.2. There are *prima facie* counterexamples to Aquinas's anti-infinite regress premise. It seems possible that there should have been infinitely many past generations of chickens, and not only of chickens. But tilting against apparent counterexamples are *arguments* for that premise. These want to be examined. Also, even if there are counterexamples, to that premise there can be something in it. There can be *kinds* of infinite regresses of causes that are *not* conceivable and that are indeed impossible. I think there are. Before getting on with these matters, there is to be considered briefly a popular argument against infinite regresses of generating-causes that Aquinas knows how to deflate. It goes like this: "If there have always been chickens who have reached productive maturity in not less than a year, then an infinite number of generations of chickens, each responsible for the one after it, preceded the present one, and chickens have lived in an infinite number of years preceding the present year. But it is impossible to traverse what is infinite. Therefore there would never have arrived these present chickens. Which is manifestly false!" This objection to infinitely many generations of chickens is modeled on an objection to the possibility that the world has always been that Aquinas takes up (ST I q46,a2 p. 452).[9] His reply when elaborated is sufficient to that objection and the present one. He writes: "Passage is always understood as being from term to term. Whatever by-gone day we choose, from it to the present day there is a finite number of days which can be traversed" (ST I q46,a2 p. 455). Aquinas suggests that, for openers, the question, "If there have been infinitely many previous days, how did the world get to this day?" should be met with the question, "Get to this day *from when*?" When this question is answered and the initial question *completed* as required by the fact that 'passage is always understood as from term to term,' the answer to the initial question, however exactly it is completed, can be, in the manner of sportscaster-speech, "One day at a-time." I leave to the reader further elaboration of Aquinas's reply to that popular consideration against infinite temporal regresses of causes.[10]

4.3. Aquinas's argument for the impossibility of an infinity of efficient causes of efficient causes. Aquinas does not believe that it is *necessary* that the world began some finite time ago. "That the world did not always exist we hold by faith alone: it cannot be proved demonstratively" (ST q46,a2 p. 453). Perhaps it was partly for this reason that he felt the need to demonstrate that infinite regresses of efficient causes are impossible. In any case, he does offer a demonstration for this, albeit a rather bad demonstration.

Now in efficient causes it is not possible to go on to infinity, because in all efficient causes following in order, the first is the cause of the intermediate, and the intermediate

cause the cause of the ultimate cause. . . . Now to take away the cause is to take away the effect. . . . Therefore, if there be no first cause among efficient causes, there will be no ultimate, nor any intermediate, cause. But if in efficient causes it is possible to go on to infinity there will be no first efficient cause, nor any intermediate efficient causes; all of which is plainly false. (ST I q2,a3 p. 22)

The main trouble with this passage is that it begs the question at issue. If one is seriously entertaining, in order to refute, the idea that efficient causes can 'go on back to infinity,' then one is entertaining the possibility that there was no first cause to 'take away.' Aquinas addresses the question of whether there must be first causes and begins by assuming that there is a first cause, as if there *must* be. His reference to 'the first' *presupposes* that "in all efficient causes following in order" there *is* a first, and it is no surprise that, while maintaining this presupposition by continued use of 'the first,' he derives an absurdity supposing there is not a first!! That is trouble enough for Aquinas's demonstration against infinite regresses of efficient causes, but there is more. When he gets into his demonstration, having fastened on the fact that, by allowing for argument an order of efficient causes that does not go to infinity to go to infinity, one 'takes away from it a first cause', Aquinas slides to the idea that, by exercising that allowance, one 'takes away from that order what was its first cause'! Suppose, however, that you had, as Frank Ramsey once wrote, 'the power of the almighty,' and you were actually to allow a series S of efficient causes that begins with A not to begin with A or anything, but 'to go on from A to infinity'. Suppose you were to 'allow' that, by prefixing to that series a beginningless series of efficient causes leading to the first cause C of S. Then you would have taken away from that cause C the role of first cause, without assigning this role to some other cause in the now much longer series. But you would not have 'taken away' C. You would not, by taking it away, and thereby, since it is needed for its effect, and that for its effect, and so on, have precipitated a 'collapse of the order' and the elimination of all effects that had been subsequent to it. You would not have *deleted* a from the series, but rather, in allowing series S to go to infinity, you would *added* to it, so that in the now much longer series there was a cause for every cause, whereas in S there had been a cause only for every cause other than C. "The [maker of this] infinite series is not 'taking [C] away.' He is [only] taking away the privileged status of [C] . . . its 'first causiness'" (Edwards 1967, p. 105).

4.4 A remarkable inconsistency in the Summa. Notwithstanding his argument in the Second Way that would show that "in efficient causes it is not possible to go on to infinity" (ST I q2,a3 p. 22), Aquinas in another place in the *Summa* takes up the suggestion that Aristotle proved that 'in efficient causes there cannot be an infinite series' (St q46,a2 p. 453) and says (I now paraphrase), 'No he did not, for it is not true' (ST q46,a2 pp. 454–5). He says that though "[I]n efficient causes it is impossible to proceed to infinity *per se* . . . for instance,

that a stone be moved by a stick, the stick by the hand, and so on to infinity . . . it is **not** impossible for a man to be **generated** by [a] man to infinity" (loc. cit.; bold emphasis added). What would be impossible, he elaborates, would be for "the generation of this man [to depend] upon this man, and on an elementary body, and on the sun, and so on to infinity" (loc. cit.). This would be a series that begins with a generating cause and goes on into an unending series of sustaining causes. The impossibility, Aquinas suggests, is with the sustaining-cause section. The trouble for Aquinas is that: (1) he intends in his Second Way a proof that, if good, would establish that infinite regresses of efficient causes of *all* kinds are impossible; though (2) he maintains elsewhere that infinite regresses of *some* kinds of efficient causes are *not* impossible. Aquinas's problems with infinite regresses may be compounded, for though he says in one place that it is not impossible that there should be infinitely many generations of men (ST q46,a3 p. 455), he implies in another that *precisely this* regress *is* impossible (ST I q7,a4 p. 61).

4.5 Aquinas against actually infinite multitudes. Aquinas had an argument that he could, with some plausibility, have directed specifically against infinite regresses of *concurrent* causes, as in 'the hand that moves the staff that moves the stone'. It is an argument that Cantor might have considered not so bad. If there were an infinite regress of concurrent causes leading to some effect, there would be **the infinite multitude of them** and, according to Aquinas, "it is impossible that there be an actually infinite multitude' (ST I q7,a4, p. 61).

> [T]hey [Avicenna and Algazel, and others] said there can be actually an . . . infinite multitude. This, however, is impossible, since every kind of multitude must belong to a species of multitude. **Now the species of multitude are to be reckoned by the species of numbers**. **But no species of number is infinite**, for every number is multitude [quantity?] measured by one. **Hence it is impossible that there be an actually infinite multitude**. . . . Furthermore, multitude in the world is created, and everything created is comprehended under some definite intention of the Creator. . . . Hence [again] everything created must be comprehended under a certain number. Therefore it is impossible for an actually infinite multitude to exist." (ST I q7,a4 pp. 61–2; emphasis added)[11]

Cantor quotes these words (Cantor 1932, p. 403) and, we are told, "states repeatedly . . . [that their passage contains] the only . . . really significant objections that have ever been raised against the actual infinite" (Rucker 1984, p. 49). Aquinas argues here against the possibility of infinite multitudes of either created things, such as sensible efficient causes and hotel rooms, or uncreated things, such as the numbers themselves. He then, for good measure, uses the premise that there is not an infinite number to argue *specifically* against the possibility of infinite multitudes of *created* things. After that comes an argument for the nature of 'definite intentions'.

"Cantor [we are told] remarks in his discussion . . . [that Aquinas's argument] against the existence of actually infinite collections is to be met *positively* by exhibiting a theory of infinite numbers" (Rucker 1984, pp. 49–50). There is, however, another possible response to that argument, since it is not after all obvious that there needs to be for every multitude a 'number.' "What is in the name 'number'?" Cantor has made us want to speak of transfinite numbers. Suppose, however, that, notwithstanding his persuasion, we were, for 'broadly pragmatic reasons' (Rudolph Carnap),[12] moved to desist from the practice and take back the word to its 'home' in the finite, the numbers 'measured by one,' the numbers $1, 2 = 1 + 1, 3 = (1+ 1) + 1$, and so on. 'Would not an erstwhile infinite multitude by any other name, or even by no distinguishing name, remain as multitudinous?' 'Would not an erstwhile multitude that is not finite remain *a multitude*?!' The species of multitudes are not *necessarily* reckoned by the species of number. There is *prima facie* the possibility of 'multitudes beyond number'. Whether it is realized depends on what multitudes there are and what numbers there are. Cantor himself believed in the existence of absolutely infinite and unlimited multitudes that are not subject to further increase. It is a good guess that he would have said, or proposed, that at least these multitudes of absolute infinity are not 'numbered' or made up of numbers of things. What we want, therefore, is a direct demonstration of the impossibility of some or all **actually infinite multitudes** that does not run in terms of what kinds of **numbers** there are and are not. We want to see how, if infinite multitudes are impossible, the supposition of one leads, without regard to possible limitations on numbers, to a contradiction or 'evident absurdity'. Galileo can seem to have inadvertently obliged.

4.6 'Galileo's Paradox'. Consider some possible infinite regress of causes and effects. Let these be, starting with the latest, which is an effect but not yet a cause, c_1, c_2, and so on. Suppose there is a *multitude* C of these. Then there is the multitude C′ that is C less the latest member c_1. *There* ARE *more causes in C than in C′;* there is *one* more, namely, c_1. However, *there* ARE NOT *more causes in C than in C′*, for their causes can be paired one to one.

$$\begin{array}{lccccc} \mathrm{C} & & c_1 & c_2 & c_3 & \cdots \\ & & \updownarrow & \updownarrow & \updownarrow & \\ \mathrm{C}' & & c_2 & c_3 & c_4 & \cdots \end{array}$$

From this contradiction we may conclude that, contrary to our supposition, there is not a multitude of these causes, *or of any things*, for nothing in this little *reductio* depended on the c's being causes. For another *reductio*, there would be, on the supposition that there is the infinite multitude C, the multitude C″ of exactly the odd-numbered causes in C: There would evidently be more causes in C than in C″, namely, all the even-numbered causes in the regress, and there

would not be more given that causes in C and C′′ can be paired one to one.

C	c_1	c_2	c_3	⋯
	↕	↕	↕	
C′′	c_1	c_3	c_5	⋯

Similarly, also, for C and the multitude C′′′ of the 'square numbered causes' c_1, c_4, c_9, and so on. This case corresponds to Galileo's own example (Rucker 1984, p. 5). These examples can persuade that *no* multitude is infinite. For any multitude that was would not be finite. And every multitude of things that is not finite includes, for every finite number, a distinct thing.[13] But then, the arguments go, there would be another multitude that both included more things and did not include more things, which is impossible.

. . . . . .

RESOLUTIONS. Given that C is a multitude, each of C′ and C′′ would be a 'proper submultitude' of it that included precisely some but not all of the things in C. Each, though a proper submultitude of C, would be such that there is a 1–1 correspondence of its members and the members of C. So far no contradiction. A contradiction emerges only when seemingly innocuous principles for multitudes are consulted, according to which (i) there are not more things in multitude M than there are in multitude M′, if *there is a 1–1 correspondence* of their members, and (ii) there are more things in M than there are in M′, if M′ is a *proper submultitude* of M. We cannot have both of these principles for all multitudes *and* have infinite multitudes, which means that we *can* have infinite multitudes if we are prepared to restrict one or the other of these principles to *finite* multitudes. Aquinas could himself have been persuaded, for he believed in the finite numbers and, if challenged, could have felt compelled to say there is the multitude of them, which would be demonstrably *not finite*: It would be for every finite number n, of a greater size than n, for it would include not only the finite numbers through n, but also the next one (n + 1). Unless he was prepared to say that though there *are* finite numbers, 'ever so many' of them, there is not a *multitude* of them, Aquinas would need to say that at least one of those principles has to go when we get to infinite multitudes.

Galileo took from his 'paradox' not that there are no infinite quantities, but that "we cannot speak of infinite quantities as being the one greater or less than or equal to another" (Galileo 1914, p. 26, as quoted in Rucker 1984, p. 5). The minimal truth he demonstrated was, however, that we cannot speak of them in these respects *in all of the ways in which we speak of finite quantities* or subject them to all of the constraints in place then. To speak of them as greater, less, or equal, we must choose for them between (i) and (ii). Galileo was not prepared to choose one and concluded that infinite multitudes, the *reality* of which he did not impugn, are *incomparable*. The choice we have taken from Cantor is

to hold on to (i) while restricting the proper submultiplicity condition to finite multiplicities. In this way we can 'have' comparable infinite multitudes.

4.7 Actual infinities of concurrent real things. There are no prospects for persuasive demonstrations of the impossibility of actually infinite multitudes, for there are the infinitely many numbers 1, 2, 3, and so on, there *is* this 'many', which it is so easy to term a 'multitude'. Perhaps, however, some would-be infinite multitudes of *real* things, of 'created' things, Aquinas would say, are to be rejected as *impossible*. I consider an unlikely candidate for this office, and then a family of more likely candidates.

4.7.1 Hilbert's Hotel

> [L]et us imagine a hotel with an infinite number of rooms, and let us assume that . . . all the rooms are occupied. . . . [W]hen a new guest arrives and asks for a room, the proprietor exclaims, "But of course!" and [all at once] shifts the person in room 1 to room 2, the person in room 2 to room 3, the person in room 3 to room 4, and so on [thereby freeing up a room – specifically, room 1 – of the previously fully occupied hotel, and this without sending a single guest packing]. . . . [S]uppose an *infinite* number of new guests arrive asking for rooms. "Certainly, certainly!" says the proprietor and he proceeds [all at once again] to move the person in room 1 into room 2, the person in room 2 into room 4, the person in room 3 into room 6, the person in room 4 into room 8, and so on. . . . In this way, [by merely moving guests from one room to another in the fully occupied hotel] all the odd-numbered rooms become free, and the infinity of new guests can . . . be accommodated. (Craig 1979, pp. 84–5)

What a marvellously accommodating hotel![14]

Hilbert's Hotel is Galileo's Paradox 'in concrete.' Contrary to William Craig, it does *not* show how "a basic exposition of the Cantorian system . . . [can] make . . . obvious that it is impossible for an actual infinite to exist in reality" (op. cit., p. 72). Difficulties with the hotel are practical and physical. Where could it be? Where is there room for it? Of what could it be made? Vast forests would be needed to build it of wood. *Really* big ones if its rooms are of one size. How much time would be needed for all of the room changes required to accommodate infinitely many new guests to take place? Would not some need to be to rooms very far removed from guests' old rooms? Such questions invite science fiction responses, which is to say they bring out the physical impossibility of this particular infinity of concurrent real things, not its logical impossibility.[15]

4.7.2 Infinite regresses of per se causes. More likely candidates for impossible infinities of real things are what would be infinite regresses of what Aquinas termed 'per se causes.' Whatever exactly was his idea of this causation (if he had an exact idea of it), it affords examples of infinite regresses that can seem to be impossible on *a priori* grounds found in this idea.

4.7.2.1. Per se efficient causation would, I assume, include as a case concurrent sustaining efficient causation of things by more fundamental things, though, from what little Aquinas says, one gathers that per se efficient causation would not be confined to sustaining efficient causation. He gives as an illustration of an infinite regress of per se efficient causes that he considered impossible, "that a stone be moved by a stick, the stick by the hand, and so on to infinity" (St q46,a3, p. 455). The causation involved here is *neither* generating *nor* sustaining, though, as in sustaining causation, the cause, the moving hand, is here *contemporaneous* with the effect, the moving stick, and their 'actions,' their relevant motions, are simultaneous. The causation here is not efficient for the existence of things, but is for changes in things: This case of per se causation is more relevant to Aquinas's First Way of 'motion' than to his Second Way of 'efficient causes.' Aquinas's only other illustration of an impossible regress per se is that "if the generation of this man depended upon this man, and on an elementary body, and on the sun, and so on to infinity" (ST q46,a2 p. 455). The causation would be first generating, this man by this man, and then sustaining, this man on an elementary body, that body on the sun, and so on to infinity.

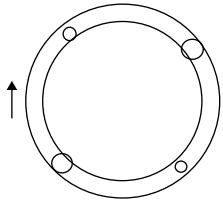
Rowe speculates plausibly – reading an account due to Duns Scotus (c1265–c1308) back into Aquinas (c1225–1274) – that, for Aquinas, per se efficient causation is causation that includes the condition that "'the second depends upon the first precisely in its act of causation' [John Duns Scotus, *Philosophical Writings*, edited and translated by Allan Wolter, New York: Nelson and Sons, 1962]" (Rowe 1975, p. 23). It is relevant to Rowe's suggestion that Aquinas contrasts "causes that are *per se* required" with 'accidental causes' (ST I q46,a3 p. 455).

4.7.2.2 INFINITE REGRESSES OF CONCURRENT PER SE EFFICIENT CAUSATION SEEM TO BE IMPOSSIBLE. In per se causation an effect is to be dependent precisely for its own causal activity, if any, on its cause's activity. This contrasts with generating causation, in which the effect is in its own generating activity, if any, typically entirely *independent* of its cause, to which it may owe only its production, its coming into existence. Cf.: "[I]t is . . . accidental to this particular man as generator to be generated by another man; for he generates as a man, and not as the son of another man" (ST I q46,a3 p. 455). It is plausible that, on account of this peculiarity of per se causation, *when concurrent* it 'cannot go on to infinity.' To persuade I can only present cases in which, for not entirely clear reasons, judgments of impossibility seem right to me.

4.7.2.3 CONCURRENT PER SE PULLERS AND PUSHERS. Consider a finite line of persons, each with his arms locked around the one in front of him. The first in this line is pulling the person behind him. Every other person, *merely because he is being pulled*, is pulling the person behind him – each other person is merely *transmitting* the pull upon him made by the person directly in front of him. Suppose now that the first person, the only person who is contributing any pulling, were literally 'taken away,' and in his place was put an infinite line

of persons without a lead member, each of whom, like all those in the original line other than its lead member, pulls merely because pulled. Is that possible? Is it possible that there should be all that pulling, given that it would all be merely *transmitted* pulling? From whence, I ask to persuade, would come the pulling being transmitted?![16] For a similar example, instead of pullers, one can have pushers.

4.7.2.4 NONCONCURRENT PER SE KNOCKERS. Bertil Strömberg has suggested that infinite lines of simultaneous pushers and pullers, each of which in its push-



ing is merely transmitting the push it receives, might be compared with a ring of finitely many 'knockers that knock only because they have been knocked,' with the knocking being, of course, *not* simultaneous. We have finitely many shiny smooth metal balls, say four of them, in a very smooth circular channel. Each is in turn knocked or 'kissed' by the ball counterclockwise to it, which stops dead in the track as the knocked ball rolls clockwise to knock the ball 'ahead' of it, which then stops dead while that ball rolls ahead, and so on and around and around. And when did all of this knocking begin? It didn't. It has been going on forever with a constant velocity. Indeed, this channel and balls cascading in it is all there is, and ever has been, in this world I am imagining. This seems possible. Imagination conjures 'an appearance of possibility' that is evidence for the reality of the possibility that is, as far as I can see, unopposed by arguments. Why couldn't this cascading have been going on forever and be all that there is and ever has been? Of course there are problems with what would be the perfectly frictionless track of it, and I suppose with the physics of the system, but these are problems not for the logical, but the physical, possibility of it, which is not at issue.

If a series of knocks by balls that knock only because they have been knocked, without one having ever 'knocked of its own accord', so that each knock is a merely 'transmitted knock', *is* possible, then why is a line of pushers each of whom pushes only because pushed, so that each push is a merely transmitted push, *not* possible? What is the difference that makes one possible and the other not? To my mind the '*foremost*' difference is that I can *imagine* the whole ring of metal balls conducting themselves as described for a period of time, *any* period of time, but I cannot imagine the whole line of pushers present at a time. The '*responsible*' difference, however, is the *nonconcurrency* of the knocking, and what would be the *concurrency* of the pushing, that would be merely transmitted knocking and pushing. That gives an edge to the question,

"From whence all this current pushing?" that the question, "From whence all this knocking?", seems to me to lack.

4.7.2.5 'THE CLASSICAL CASE' OF CONCURRENT PER SE 'HOLDER UPPERS.' Suppose a finite stack of elephants on elephants with the earth on the back of the top elephant and the bottom elephant standing on some ever so *terra firma*. There is a downward force on these elephants. But that 'ground' holds up against this force the bottom elephant, who, being held up, holds up the elephant on his back, who, being held up, holds up the elephant on his back, and so on to the top elephant, and then to the earth, which holds up nothing. Now take away that unearthly *terra firma*, put in its place an infinite stack of elephants, and look out! Look out, for there is no longer the stack of elephants, now 'ungrounded' but still under a downward force, must surely fall in an accelerating rush.

4.7.2.6. That – tentative argument by examples and appeals to intuition, and not by analysis and deductions of contradictions – is the best I can do to make plausible Aquinas's idea that "there cannot be an infinite number of causes that are *per se* required for a certain effect" (ST I q46,a3 p. 455). I am persuaded by it that *sustaining causes* could not "go on to infinity" (ST I q2,a3 p. 21), for I suppose that sustaining causation would be a species of per se causation, that it would be *concurrent* causation of existence in which the sustaining causality of an effect was in every case from the sustaining causality of its cause. A *disadvantage* of sustaining-cause versions of the Second Way was detailed in Section 3.1. Understood as concerned with sustaining causes, this way could not be cast as from a better-known effect to The Cause. Present discussion has forwarded an *advantage* of sustaining-cause versions. Infinite regresses of these causes would seem to *be* impossible as the argument requires of its efficient causes. Another advantage turns up at the end of this chapter.

5. THE PRELIMINARY CONCLUSION (8)

"[There is] a first cause efficient causes" (ST I q2,a3 p. 22) – more fully, there is, for all sensible things that have efficient causes, a first cause that does not have an efficient cause, and that is not itself a sensible thing.

5.1. These words are open to several interpretations. It is sufficient for present purposes to attend to two of these. What these words say, according to one of these interpretations, does follow from premises (1) through (7), but interpreted this manner, the Second Way stops here, *well* short of the conclusion that God exists. Interpreted in the other manner, what these words say allows the argument to proceed to its penultimate step, which is the introduction by the words,

"*[T]o which everyone [correctly] gives the name of God*" (ST I q2,a3 p. 22).

of an additional premise 'on line 9'. However, interpreted in a manner that prepares for these words, what the words for (8) say does *not* follow from premises (1) through (7). It is as if Aquinas thought that he could have (8) both ways. I think he did not see the difference, and that, to get without equivocation what he wants by valid inferences, he needed yet another premise, for which there is no support in sight.

5.2 Two interpretations of the words for (8). These words, out of context, can express either

> For every sensible thing that has an efficient cause, there is at least one thing that is not a sensible thing, does not have an efficient cause, and is a first cause of that thing.
> $(y)[Sy \,\&\, Hy \supset (\exists x)(\sim Sx \,\&\, \sim Hx \,\&\, F(xy)]$

or

> There is at least one thing that is not a sensible thing, does not have an efficient cause, and is a first cause of every sensible thing that has an efficient cause.
> $(\exists x)[\sim Sx \,\&\, \sim Hx \,\&\, (y)(Sy \supset F(xy)]$

In the context of the Second Way, more likely interpretations of our words for (8) are

> (8a) For every sensible thing that has an efficient cause *there is exactly one* thing that is not a sensible thing, does not have an efficient cause, and is a first cause of that thing.

and

> (8b) *There is exactly one* thing that is not a sensible thing, does not have an efficient cause, and is a first cause *of every* sensible thing that has an efficient cause.

For any variables α, β and formula ϕ in which α but not β or any other variable is free, $(\exists\alpha)\ (\beta)(\phi \equiv \beta = \alpha)$ 'says,' as compactly as possible, that there is exactly one thing that satisfies ϕ. So (8a) and (8b) can be symbolized respectively by

$$(y)[(Sy \,\&\, Hy) \supset (\exists x)(z)([\sim Sz \,\&\, \sim Hx \,\&\, F(zy)] \equiv z = x)]$$

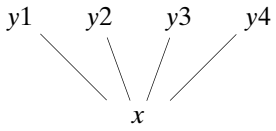
and

$$(\exists x)(z)[(\sim Sz \,\&\, \sim Hx \,\&\, (y)[(Sy \,\&\, Hy) \supset F(zy)]) \equiv z = x].$$

Other interpretations come from (8a) and (8b) by putting '*the* first cause' for '*a* first cause' in them. To study the 'play' of (8) in Aquinas's argument, it is sufficient to attend to (8a) and (8b), which are simpler.[17]

To see the difference between (8a) and (8b), suppose there are only four sensible things that have efficient causes. Then (8b) 'says'

sensible things that have efficient causes



a first cause for all of them

and (8a) leaves open that

sensible things that have efficient causes

y1	y2	y3	y4
|	|	|	|
x1	x2	x3	x4

first causes for them

5.3. A path of informaldeduction from (1) through (7) of *precisely* (8a) was given in Section 2.8.

From premises (1) through (6) it follows that there is, extending back or down from say this computer, a single series of efficient causes, each member of which is prior to all previous members, and so different from each of them. Given premise (7), that efficient causes cannot 'go to infinity, there is in this series a last cause that – remember that this series is of causes of causes back or down from – is naturally termed its *first* cause.

It is reasonably obvious that no deduction is possible for (8b) from premises (1) through (7). Those premises leave open that distinct sensible things should have distinct first causes, that there should be as many first causes for sensible things as there are sensible things that have efficient causes. Premise (1) says that there are sensible things that have efficient *causes*, leaving open that their efficient causes are not the same; that *needs* to be left open in the beginning by a demonstration *quia*. That suggestion is similarly, and fortunately for plausibility and a demonstration *quia*, not made by (6), which says that sensible things that are efficient causes of sensible things have efficient causes, not that they must have one and the same efficient cause. So, when one gets in the Second Way to "**a** first efficient cause" (ST I q2a3, p. 22; bold emphasis added), one may get to *several*.[18]

6. THERE IS A GAP IN THE ARGUMENT

Premise (9) 'look backs' to (8) for its subject and is *admissible* only if it finds one there. To be admissible it is necessary that (8) should identify a unique thing to which the words 'to which' in (9) can refer. Inference (8b) does that,

but (8a), which is all that follows from previous premises, does not. So (9) is *not* admissible. This problem could of course be remedied by adding as a premise what we might term *Fellini*, or *8½*.

If (8a) – for every sensible thing that has an efficient cause there is exactly one thing that is not a sensible thing, does not have an efficient cause, and is its first cause, then (8b) – there is exactly one thing that is not a sensible thing, does not have an efficient cause, and is a first cause of every sensible thing that has an efficient cause.

It is not obvious that (8½) is true. Aquinas provides no grounds for it. It bridges a *gap* in his reasoning. There are not that I know of any passably plausible premises that, when added to (1) through (7), entail (8½). Cf.: "[T]here does not seem to be any good ground for supposing that the various causal series in the universe ultimately merge. . . . [T]he possibility of a plurality of first members has not been ruled out" (Edwards 1967, p. 106).

7. ON THE ULTIMATE CONCLUSION, **THAT GOD EXISTS**: WHETHER THIS WOULD FOLLOW EVEN IF ALL WAS WELL IN THE ARGUMENT TO IT

Even given (8½), while (9) would be 'admissible', its entry would remain at best *premature*. We should want to know more about this first cause of all sensible things before we were prepared to give it the name 'God.' What if it were not a 'being' but an unconscious something, say an infinitely small, infinitely potent, something that simply *exploded* eventually into all sensible things? No one would give *it* the name of God.

Suppose, however, that we not only reached in a manner that satisfied us (8b) as well as (8a), that *there is exactly one* thing that is not a sensible thing, does not have an efficient cause, and is a first cause *of every* sensible thing that has an efficient cause, but also, with reference to (8b), that we accepted and were entirely satisfied with (9),

[T]o which everyone [correctly] gives the name of God. (ST I q2,a3 p. 22)

It can seem that there would at least *then* be no room for wondering whether *God exists*, but this is not true. In saying that there *is* exactly one first cause in that sense, (8b) *may* not be saying that this one is *now*. By way of contrast, ultimate conclusion (10) is to be *precisely* that God exists *now*. The Second Way is a putative demonstration by which Aquinas would settle the question, "Whether God **exists**?" (ST I q2,a3 p. 21; emphasis added), not to the question, "Whether God **has at some time existed**?"

So (10) may not, without further ado, follow from (8b) and (9). Whether it does depends (we are back to that!) on what kind of efficient causing (8b) is about. Conclusion (10) does *not* follow without considerable further ado (more premises, more argument), *if* (8b) says only that there is exactly

one first *generating* cause of all sensible things that have efficient generating causes. For then a premise leading to (8b), and thus (8b) itself, needs to be in the popular-amongst-logicians-and-philosophers 'atemporal present tense.' While we 'find' that, (1), there exist sensible things that have efficient causes *that exist now*, we do *not* 'find' that, (6), every sensible thing that has been an efficient cause of some sensible thing has itself had an efficient cause *that exists now*. Generating causes of present sensible things have all done their causing for these things. Past generating causes of many past sensible things did their causing 'generations ago.' Many of these are, alas, long gone. "[E]xperience clearly shows that an effect may exist long after its [generating] cause has been destroyed" (Edwards 1967, p. 106). "But the *first* cause in series of generating causes leading to sensible things that we find have efficient caused would need to be still be around. *It* could not do its work, and depart." Perhaps not, but I know of no good *reason* why not, and no reason good or bad for this is hinted in the Second Way, which would remain at best radically incomplete even after being helped to the preliminary conclusion (8b), if this is interpreted as for there having been exactly one first generating cause.

On the other hand, (10) *does* follow from (8b) and (9), if (8b) says that there is exactly one first *sustaining* cause for sensible things that have efficient sustaining causes, since sustaining causes are by definition *concurrent* causes. For presently existing sensible things, there would be presently existing sustaining causes. That (10) follows from (8b) and (9) in a sustaining-cause version of Aquinas's Second Way is the second advantage promised some time ago for this version. Cf.: "If the argument were [good] in this form it would . . . prove the present . . . existence of a first cause" (Edwards 1967, p. 69). But then there is that first considerable disadvantage.

SUMMING UP

One can read the Second Way as a *generating-efficient cause* argument, and so read it gets off to a decent start, for we certainly do find in the world of sensible things an order of such causes. However, so read it soon runs into trouble at three points, for (i) there are no good reasons for thinking that infinite regresses of efficient generating causes are impossible; (ii) even if one could demonstrate that there has been for things that have generating causes, unique first generating causes, there are no obvious reasons why each should have had the same first generating cause as every other; and (iii) even if *that* were well-established, there are no obvious reasons why this unique first generating cause of sensible things should still exist.

The Second Way can be read instead as a *sustaining-efficient cause* argument. So read, the argument does better at two points. It is at least plausible that there could not be infinite regresses of such causes, and the transition from

(8b) and (9) to (10) would be smooth: Given that there was exactly one first sustaining cause for things that now exist that have sustaining causes, since such causes would be concurrent causes, this first sustaining cause would exist now. *The trouble* with a sustaining-efficient-cause take on the Second Way is that it makes this Way a nonstarter as a demonstration *quia* for the existence of God from effects better known than what would be His nature.[19] An order in the world of sensible things of sustaining-efficient causes is not better known to us than hardly *anything*. Insofar as we can gather from its sponsors an idea of this kind of causation, it is very doubtful that there is any of it for any sensible thing, since it is very doubtful that there is, for even one sensible thing that exists at a time, anything separate from it, the existence of which at that time is necessary for the existence at that time of that sensible thing. I, for example, am not "dependent here and now on . . . the activity of the air" (Copleston 1955, p. 118) or "on an elementary body" (St I q46,a3 p. 455) unless there is one that is a part of me. The Second Way, if it is about sustaining causes, is not a way from the familiar to God, but a God-centered world-view that is far more challenged than is its God-center.

It would be nice to combine the virtues, while losing the faults, of these two versions of the Second Way in another version that ran in terms of a kind of causation that is both familiar and suited to the course. Not knowing how to do that, and suspecting that it cannot be done, I propose instead to go to a cosmological 'way of reasons' upgrade of Aquinas's Second Way of causes.

APPENDIX A. NOTES ON AQUINAS'S OTHER WAYS

A1. "The first and more manifest way is the argument from motion" (ST I q2, a3 p. 22).

> [It is manifest that things are in motion and that since nothing moves itself, each must be moved by another thing.] But this cannot go on to infinity, because then there would be no first mover, and consequently, no other mover [and thus nothing in motion]. Therefore it is necessary to arrive at a first mover, moved by no other [and indeed not itself in motion]; and this everyone understands to be God. (ST I q2,a3 p. 22)

Aquinas's 'argument' against an infinite regress of movers is here the bare assertion that such regresses are impossible and that there must be for moving things a first mover.[20] He does not entertain – to show what is wrong with it – the putative idea of an infinite regress of moving movers in which each is "as the staff [that] moves [the stone] only because it is moved by the hand" (ST I q2,a3 p. 22) moved by a predecessor and (unless it is the last in the series) a mover of its successor. Furthermore, as in his Second Way taken as for generating causes, the argument of the First Way, even if good, would show

only that there is, in the 'logical present,' "**a** first mover" (ST I q2,a3 p. 22). It neither makes a case for there being a unique first mover nor for there existing still today any things that on days past were first movers.

The First Way of movers has these analogs of problems of the Second Way of generating efficient causes. And the First Way is vulnerable to the apparent possibility of self-moving things that have always been in motion, for Aquinas's premise that "whatever is moved is moved by another" (ST I q2,a3 p. 22), is presumably meant to be a statement of strict metaphysical necessity. But it seems that at least things in perpetual motion could be self-movers.[21] It seems, in Aquinas's Aristotelian terms, that they could be at every moment things actually in motion and potentially in motion in the immediate future, their changing potentialities being continuously actualized by the action of their immediately antecedent actualities. This conception of a self-moving perpetual mover does not involve its being "**in the same respect and in the same way** . . . both mover and moved" (ST I q2,a3, p. 22; emphasis added).[22]

A2. "The third way is taken from possibility and necessity" (ST I q2,a3 p. 22).

[Here is a paraphrase.] Given that there are possibles, it follows that there is a necessity. For what is merely possible and not necessary is sometimes not, so that if there were only possibles, there would once have been nothing. But then there would still be nothing, since there would have been nothing through which anything could begin to exist. So, since there are possibles, there is something that is necessary, and indeed (for it is impossible to go on to infinity in causes of necessities) something that is necessary in itself, something whose necessity is not caused by another logically anterior necessary thing.

Having found that there must be a necessary thing, and conceded that the necessity of necessary things can be 'caused' by other necessary things, Aquinas relies on his nonargument of the Second Way: He writes in the Third Way "as has already been proved in regard to efficient causes" (ST I q2,a3 p. 23). With an anti-infinite-regress premise in hand, Aquinas infers correctly that there is "some being" (ST I q2,a3 p. 23) that has its necessity of itself and not (somehow) from another necessary being, but, as *in the Second Way, Aquinas mistakes the proposition he has reached for the conclusion that there is some one* such being, that there is *exactly one* such being.

The Third Way runs into other difficulties peculiar to it, difficulties encountered before it 'finds' its necessary beings. For one, this way depends on the impossibility of a merely possible thing existing always and gives for this impossibility the inadequate grounds that "that which can not be at some time is not" (ST I q2,a3 p. 23). Even if, as a matter of fact, no contingent being exists always, it does not follow that no contingent being can exist always. There is no difficulty, far less is there absurdity, in the thought of an everlasting billiard

ball cruising the universe, or of the world of contingent things itself, which I assume Aquinas would have considered the comprehensive contingent thing, having always been.

For a second early difficulty, this Way may depend on an illicit inference of its own from "everything [in nature] can not be," through everything in nature at some time was not, to "at one time there was [in nature, in the world] nothing in existence" (ST I q2,a3, p. 23). But even if everything at some time was not, $(x)(\exists t) \sim$ (x was at t), it does not follow that at some time everything was not, $(\exists t)(x) \sim$ (x was at t), or, equivalently, that at some time nothing was, $(\exists t) \sim (\exists x)$(x was at t). *Perhaps*, however, we should say that Aquinas has merely not highlighted a cosmic principle for the inference from 'everything can not be' in one sense to 'everything can not be' in another: It is after all plausible that the sum total of the several things, each of which can not be, must itself be a thing that can not be. This 'creative take' on the Third Way moves it toward cosmological arguments of the next chapter.

A3. "The fourth way is taken from the gradation to be found in things" (ST I q2,a3 p. 23). This Way goes immediately astray. Having observed that some beings are more and some less good, true, noble, and the like, Aquinas says that "*more* and less are predicated of different things according as they resemble in there different ways something which is the maximum" (ST I q2,a3 p. 23). In fact, more and less are sometimes predicated when it is known that there is no maximum, as in the case of greater and smaller cardinal numbers, and often without regard to whether there is a maximum, as when one object is said to be heavier, hotter, or better than another. Also, this Way, even supposing it got somewhere, would not reach its intended destination. Even if it led to "there is something which is truest, something best, something noblest," it would not follow from that that there is something that is all of these things and thus is "most being" (ST I q2,a3 p. 23).

A4. "The fifth way is taken from the governance of the world" (ST I q2,a3 p. 23). This Way is from what appear to be "designedly" end-directed "to obtain the best result" actions of natural things that lack intelligence, to an intelligent being "by whom all natural things are directed to their end[s]" (ST I q2,a3 p. 23). This is an 'argument from design' that distinguishes itself from later 'teleological arguments' by being deductive. It uses the no-probabilities-about-it premise that "whatever lacks knowledge cannot [*pace*, Charles Darwin] move towards an end unless it be directed by some being endowed with knowledge and intelligence" (ST I q2,a3 p. 23). Richard Swinburne makes this point about the Fifth Way when he observes, in his discussion of arguments from design, that Aquinas's argument moves "rather quickly" to its conclusion (Swinburne

1994a, p. 49). "It seems to me fairly clear that no argument from temporal order – whether Aquinas's Fifth Way or any other can be a good deductive argument. . . . [as distinct from] a good inductive argument" (Ibid.).

APPENDIX B. BANGS AND INFINITE REGRESSES OF CAUSES

B1 Big Bang!! Aquinas gives a bad argument against the possibility of infinite regresses of all kinds of efficient causes and, at times, inconsistently with that bad argument, says that infinite regresses of generating efficient causes are not impossible. "But what if the world of sensible things began a finite time ago in a Big Bang? Could there then be infinite temporal regresses of sensible-thing efficient causes for today's sensible things?" It is likely that Aquinas would say, "No, for then there would not have been enough time." I say, "Yes," though given the constraint that such causes could not recede to or beyond that Big Bang time after which 'all hell broke loose,' infinite regresses of such causes, considered in reverse temporal order, would at some time be from then back temporally squeezed more or more closely. One manner of regular squeezing would be, for every time t subsequent to the time of the Big Bang, to have the immediate sensible-thing cause of a sensible thing's coming to be at t, come to be itself at t/2. An infinite regress of the first-appearance-times of a sensible thing's more and more remote sensible-thing causes could be: t, t/2, (t/2)/2 = t/4, [(t/2)/2]/2 = t/8, and so on *ad infinitum*.[23] It is true that "if there was no first event, then there must have been an event prior to any given **event**" (p. 157; emphasis added), but it is not true that there must have been an event prior to any given *time*.

B2 A blast from the recent past – William Lane Craig's kalām[24] *causal argument*. There can be infinite beginningless temporal regresses of sensible-thing efficient causes in history, even if there was a beginning to history in the sense of a time at and before which there were no sensible things and nothing happened. This makes some trouble for Craig's 'basic *kalām* cosmological argument,' which is "(1) Everything that begins to exist has a cause of its existence. (2) The universe began to exist. Therefore – (3) The universe has a cause of its existence [that did not begin to exist]."[25] A problem for this argument is that 'the universe of sensible things' could have begun to exist *in the sense that* there is time at and before which there were no sensible things and after which there were 'fast-starting' *beginningless* series of sensible-thing causes in which series each sensible thing begins to exist and is caused by a member of the series that began to exist earlier. In this scenario, though "the universe began to exist" in a sense, nothing happened when it did; nothing came into existence then, and in particular The Universe did not come into existence then. In this story it was only later that things came into existence, and they all had causes in the universe of sensible things that themselves came into existence at earlier times, but of course at times subsequent to the 'beginning of history'. Relating this possibility to Craig's argument, it is 'philosophically

plausible' that everything that begins to exist, in the sense that there is an earliest time when it exists, has a cause of its existence. But it is 'philosophically contentious' that everything that begins to exist, *either* in that sense *or* in the sense described in which 'the universe of sensible things' could have come into existence, has a cause of its existence. The argument, to cover the possibility that it is in only the second sense that the universe began to exist, needs the contentious principle of this pair.

It may be complained that that scenario supposes that "the temporal series of past events could be **actually** infinite" (Craig 1979, p. ix; emphasis added). "Proponents of the *kalām* argument contended that it could not, and that the universe **therefore** had an absolute beginning [*when its first event or events took place*]" (Ibid.; bold emphasis added). This *kalām* arument, developed, would include an argument for Craig's second premise understood as required for an inference from (1) to the conclusion (3). This second premise would say that there is an earliest time when something in the universe existed. The issue – hotly disputed by Arabic and western non-Arabic medieval theologians – that comes up in this argument in connection with the second premise is "whether the temporal series of past events could be actually infinite" (Ibid.). I asked rhetorically in Section 4.6, "Why not?" Aquinas said that it *could* be actually infinite. He believed only in faith, and without reasons, that it is not actually infinite and that there was a time when "the world began to exist" (ST q46,a2 p. 453).

VI

Ultimate Reasons
Proofs a contingentia mundi

First sustaining-cause arguments are 'nonstarters.' The most serious problems with first-cause arguments, for example, generating-cause and moving-cause arguments, are (i) the apparent possibility that first generating and moving causes should no longer exist and (ii) the apparent possibility, conceded by Aquinas and believed in by many, of infinite regresses of generating and moving causes. Cosmological arguments could come from first-cause arguments by adjustments designed to deal with these difficulties while reinterpreting and deepening the insight that inspires first-cause thinking. Suppose one allows that it is possible that there should have been sensible generating causes of causes of causes *ad infinitum*, without beginnings or first sensible generating causes. Suppose, that is, that one rejects the words that head the previous chapter, *it is not conceivable that successions of causes and effects leading to now should have sprung from nowhere*, under one interpretation. Suppose one allows that there can be infinite beginningless series of generating and moving causes. One may still accept these words under another interpretation. For one may feel that, even if generating and moving causes can 'go to infinity' in beginningless regressive series, these series themselves need to have, if not causes, then reasons and grounds of some sort for being and that they, the several beginningless infinite series, cannot simply be, so that to, "Why all of this?", the answer is, "For no reason, for no reason at all." It can seem inconceivable, unthinkable, one may huff, that *successions* of causes and effects in the world may be without any cause and for no reason.

1. CLASSICAL SOURCES

Ideas for the main argument to be examined come largely from Leibniz, whose argument most clearly qualifies as 'cosmological' in the sense coined, I think, by Kant to pick out a kind of argument for the existence of God (Section 3.3.5).

According to William Craig, "it is [Leibniz's] version of the argument that is the basic form of the argument discussed today" (*The Cosmological Argument from Plato to Leibniz*, 1980, p. 257). I think that it should be, but that in fact arguments discussed today under the rubric 'cosmological' more often draw not from Leibniz but from arguments such as one advocated in Hume's *Dialogues* by Demea, who has been said to speak then for Samuel Clarke. These arguments of recent discussion do not use Leibniz's principle of sufficient reasons. The main argument of this chapter is, in contrast, avowedly and explicitly the metaphysician's. It uses his 'great principle' that there is for everything a '*sufficient reason*' why it is, and why it is as it is, in the very *special* sense that Leibniz assigned to this term that was first his term.

The present assembly of classical texts with comments begins with the mentioned passage from Hume's *Dialogues*, proceeds to several passages from Leibniz, and then comments briefly on relations between the arguments of these two sources. Then come the text of Clarke's argument and comments thereon that relate it to Demea's and explain why my main argument takes nothing from Clarke's. Other texts (not here produced) of interest *and quality*, but less influence, are those of Spinoza's second and third proofs for Proposition XI of *Ethics*. His first proof has been discussed at length in Chapter II. His other two proofs are in Kant's term 'cosmological'. The first works with a principle of causes or reasons, as does Demea's argument coming up; the second does not and is in this respect like Clarke's coming up.

1.1 Demea's intervention

But if so many difficulties attend the argument *a posteriori* [Cleanthes's inductive argument from facts of apparent design] said Demea; had we not better adhere to . . . argument *a priori* . . . ? Whatever exists must have a cause or reason of its existence; it being absolutely impossible for any thing to produce itself, or be the cause of its own existence. In mounting up, therefore, from effects to causes, we must either go on in tracing an infinite succession, without any ultimate cause at all, or must at last have recourse to some ultimate cause that is *necessarily* existent: Now that the first supposition is absurd may be thus proved. In the infinite chain or succession of causes and effects, each single effect is determined to exist by the power and efficacy of that cause, which immediately preceded; but the whole eternal chain or succession, taken together, is not determined or caused by anything: And yet it is evident that it requires a cause or reason, as much as any particular object which begins to exist in time. The question is still reasonable why this particular succession of causes existed from eternity, and not any other succession, or no succession at all. If there be no necessarily existent being, any supposition which can be formed is equally possible; nor is there any more absurdity in nothing's having existed from eternity, than there is in that succession of causes, which constitutes the universe. What was it, then, which determined something to exist rather than nothing, and bestowed being on a particular possibility, exclusive of the rest? *External causes*, there are supposed to be none. *Chance* is a word without meaning. Was it *nothing*? But that can never

produce anything.[1] We must, therefore have recourse to a necessarily existent Being, who carries the REASON of his existence in himself; and who cannot be supposed not to exist without an express contradiction. There is, consequently, such a Being – that is, there is a Deity. (Hume 1991, Part IX, pp. 148–9.)

The first sentences quoted want to be expanded for what comes after them. Demea's opinion, somewhat more carefully stated, is that whatever exists *contingently and not necessarily*[2] must have a cause or reason of its existence, it being absolutely impossible for any *such* thing to produce itself or be its own cause or reason. In Demea's opinion a necessary being, though it does not produce itself, does 'carry in itself the reason for its existence'.

The argument concedes the possibility of infinite regresses of causes, and this argument delivers a necessarily existent being. So it *presumably* gets around the problem that generating- and moving-cause arguments have with the possibility that their first causes and movers no longer exist. For it *seems* to follow from its necessity that a necessary existent being is eternal, so that there is no *question* whether it still exists. All relatively uncontroversial examples of necessary things are like that. For example, the number 4 exists 'timelessly' – 'exists' when predicated of it is not in the 'present *temporal* tense'. I doubt, on the basis of what Peter van Inwagen would consider a remarkably weak quasi-inductive argument, the possibility of temporal necessary beings (cf., note 19 in Chapter IV) and so think that Demea's argument is not challenged by the possibility that its necessarily existent being 'is dead'.

1.2 Gottfried Wilhelm von Leibniz on the reason for it all – his 'cosmological argument' Kant would say[3]

1.2.1 From "On the Ultimate Origination of the Universe" of 1697

Besides the World, that is, besides the aggregate of finite things, there is some dominant unit . . . [that] not only rules the world, [but] also makes or creates it. It is superior to the world and, so to speak, beyond the world, and is therefore the **ultimate reason** for things. Neither in any single thing, nor in the total aggregate and series of things, can the **sufficient reason** for their existence be discovered. Let us suppose a book . . . to have existed eternally, one edition having always been copied from the preceding: it is evident then that, although you can account for the present copy by reference to a past copy which it reproduces, yet, however far back you go . . . you can never arrive at a **complete [explanation]**[4], since you always will have to ask why at all times these books have existed, that is, why there have been any books at all and why this book in particular. What is true concerning these books is equally true concerning the diverse states of the world, for here too the following state is in some way a copy of the preceding one (although changed according to certain laws). However far you turn back . . . you will never discover in any or all of these states the **full reason** why there is a world rather than nothing, nor why it is such as it is.

You may well suppose the world to be eternal; yet what you thus posit is nothing but the succession of its states, and you will not find the sufficient reason in any

one of them, nor will you get any nearer to accounting rationally for the world by taking any number of them together: the reason must therefore be sought elsewhere. Things eternal may have no cause of existence, yet a reason for their existence must be conceived. Such a reason is, for immutable things, their very necessity or essence; while in the series of changing things, even though this series itself may be supposed *a priori* to be eternal, this reason would consist in the very prevailing of inclinations. For in this case reasons do not necessitate (that is, operate with absolute or metaphysical necessity, so that the contrary would imply contradiction), but only incline. Hence it is evident that even by supposing the world to be eternal, the recourse to an ultimate cause of the universe beyond this world, that is, to God, cannot be avoided.

The reasons [sufficient, full, complete] for the world are therefore concealed in some entity outside the world. . . . Thus we must pass from the physical or hypothetical necessity, which determines the later states of the world by the earlier, to something endowed with absolute or metaphysical necessity, for which no reason can be given. For the actually existing world is necessary only physically or hypothetically, but not absolutely or metaphysically. . . . Since therefore the ultimate root of the world must be something which exists of metaphysical necessity, and since furthermore the reason for any existent can be only another existent, it follows that a unique entity must exist of metaphysical necessity, that is, there is a being whose essence implies existence. Hence there exists a being which is different from the plurality of beings, that is, from the world; for it has been granted and proved that the world does not exist of metaphysical necessity. (Leibniz 1965, pp. 84–6; emphasis added.)

1.2.2 From "The Monadology" of 1714

There is an infinity of figures . . . of minute inclinations. . . . Now, all of this detail implies previous or more particular contingents, each of which again stands in need of similar analysis to be accounted for, so that nothing is gained by such analysis. The **sufficient or ultimate reason** must therefore exist outside the succession of series of contingent particulars, infinite though this series be. Consequently, the ultimate reason of all things must subsist in a necessary substance, in which all particular changes may exist only virtually as in its source: this substance is what we call *God*. (Leibniz 1965, *Monadology* 36–8, pp. 153–4; emphasis added.)

1.2.3 From "The Principles of Nature and Grace,Based on Reason" of 1714

7. . . . now we . . . make use of **the great . . . principle that nothing takes place without a sufficient reason**; in other words, that nothing occurs for which it would be impossible for someone who has enough knowledge of things to give **a reason adequate to determine why the thing is as it is and not otherwise**. This principle having been stated, the first question which we have a right to ask will be, "Why is there something rather than nothing?". . . . Further, assuming that things must exist, it must be possible to give a reason *why they should exist as they do* and not otherwise.

8. Now this sufficient reason for the existence of the universe cannot be found in the series of contingent things. . . . Although the present motion . . . arises from preceding motion, and that in turn from motion which preceded it, we do not get further however far we may go, for the same question always remains. The sufficient

reason, therefore, **which needs not further reason**, must be outside of this series of contingent things and is found in a substance which . . . is a necessary being bearing the reason for its existence within itself; otherwise we should not yet have **a sufficient reason with which to stop**. This **final reason** for things is called *God*. (Leibniz 1969, *Nature and Grace*, pp. 638–9; bold emphasis added.)

1.3 The arguments of Demea and Leibniz compared. Unlike Leibniz's argumentation, in which reasons of a special kind are demanded, Demea's argument turns simply on a demand that there be reasons, that for every existent, whether it is 'in time' or runs through all time or is outside of time, there is required a cause or reason. It is, however, a fair guess that the reason demanded for the particular succession of contingent things in time would be deductive. The guess is that in the light of demanded reasons it *should not be* that every *supposition is possible*, and that, in their light, *nothing's* having existed from eternity *should be* absurd. These *desiderata* are pretty clearly the ones intended by Demea's talk of the 'equally possible' and 'absurd.' He presumably does not mean merely 'equally probable' and 'most improbable.' He is presumably arguing not only for an ultimate cause or reason that is necessary, so that *it* cannot be supposed not to be without contradiction, but one that is also, to paraphrase Leibniz's words, *necessitating*, a cause or reason that 'operates with necessity,' so that, given it, its effects or consequences cannot be supposed not to be without contradiction.

1.4 Samuel Clark. Demea is sometimes said to speak in the quoted lines of the *Dialogues* for Samuel Clarke, and simply to retail succinctly Clarke's argument. "The argument which Demea has in view . . . is . . . a brief restatement of an argument formulated by Clarke. . . . " (Smith 1947, p. 115). William Rowe, when articulating to discuss Clarke's argument, writes of his decision to depart at a point from its terms, "This way of developing the argument . . . has the advantage of fitting nicely with the way in which Hume has Demea summarize Clarke's argument," (Rowe 1976, p. 117n3). However, while Hume had Clarke in mind when he wrote that speech for Demea, I do not believe that Hume thought that he was merely restating or summarizing Clarke's argument. Though Demea is evidently 'speaking for Clarke', he is, no surprise given the author of his lines, made to speak better and more clearly than Clarke. So that you may be the judge, here comes Clarke to speak for himself.

1.4.1 Samuel Clarke's 'cosmological argument'. Stage one of this argument is for Proposition I, "Something has existed from eternity" (Clarke 1738, p. 8). The central stage, which we have here, is for Proposition II, "There has existed from eternity, some one unchangeable and independent being." Left for the third stage is to establish Proposition III, "That unchangeable and independent being, which has existed from eternity, without any external cause of its existence; must be self-existent, that is, necessarily existing." Now comes the text

of the argument for Proposition II in the ninth edition of 1738 of *A Discourse Concerning the Being and Attributes of God* . . . , lectures of 1704 and 1705 first published in 1705.[5]

II. *There has existed from eternity*,* *some one unchangeable and Independent Being*. For since some thing must needs have been from eternity, as has been already proved, and is granted on all hands: Either there has always [*sic*] existed some one unchangeable and *independent* being, from which all other beings that are or ever were in the universe, have received their original; or else there has been an infinite succession of changeable and *dependent* beings produced one from another in an endless progression, *without* any original cause at all. Now this latter supposition is so very absurd that tho' all atheism must in its account of most things (as shall be shown hereafter) terminate in it, yet I think very few atheists ever were so weak as openly and directly to defend it. For it's plainly impossible, and *contradictory* to itself. I shall not argue against it from the *supposed* impossibility of infinite succession, *barely and absolutely considered in itself*; for a reason which shall be mentioned hereafter. But, if we consider such an infinite progression, as *one* entire endless *series* of *dependent* beings; 'tis plain this *whole series of beings* can have no cause *from without*, of its existence; because in it are supposed to be included *all things* that are or ever were in the universe. And 'tis plain it can have no reason *within itself* of its existence; because no one being in this infinite succession is supposed to be self-existent or *necessary*, (which is the only ground or reason of existence of any thing that can be imagined *within the thing itself*, as will presently more fully appear), but every one *dependent* on the foregoing: And where *no part* is necessary, 'tis manifest *the whole* cannot be necessary: Absolute necessity of existence, not being an *extrinsic*, *relative*, and *accidental* denomination; but an *inward and essential* property of the nature of the things which so exists. An infinite succession therefore of merely *dependent* beings, without any original independent cause; is a *series* of beings, that has neither necessity, nor cause, nor any reason or ground *at all* of its existence, either *within itself* or *from without*: That is, 'tis an express contradiction and impossibility; 'tis supposing *something* to be *caused*, (because it is granted in every one of its states of succession, *not to be* necessary and *of itself*); and yet that, in the whole, 'tis caused *absolutely* by *nothing*. Which every man knows is a contradiction to imagine done *in time*; and, because duration in this case makes no difference, 'tis equally a contradiction to suppose it done *from eternity*. And consequently there must, on the contrary, of necessity have existed from eternity, *some one* immutable and *independent being*.

*The meaning of this proposition (and all that the argument requires), is that there must needs have always been *some independent being*, some one at least. To show that there can be *no more than one*, is not the design of this proposition, but of the Seventh.

To suppose an *infinite succession* of changeable and *dependent* beings produced one from another in an endless progression, *without* any original cause at all; is only[6] a driving back from one step to another . . . the question concerning the ground or

reason of the existence of things. 'Tis in reality, and in point of argument, the very same supposition; as it would be to suppose *one continued being*, of *beginningless* and endless duration, neither self-existent and necessary itself, nor having its existence founded in any self-existent cause. Which is directly absurd and contradictory.

Otherwise, thus. . . . According to [the supposition that there has been an infinite succession of changeable and *dependent* beings, produced one from another in an endless progression, without any original cause at all], there is nothing, in the universe, self-existent or necessarily-existing. And if so; then it was originally *equally possible*, that from eternity there should never have existed any thing at all; as that there should from eternity have existed a succession of changeable and dependent beings. Which being supposed; then *What* is it that has from eternity determined such a succession of beings to exist, rather than that from eternity there should never have existed any thing at all? *Necessity* it was not. . . . *Chance*, is nothing but a mere word. And *other being* 'tis supposed there was none, to determine the existence of these. Their existence therefore was determined by *nothing*. . . . That is to say; of two *equally possible* things . . . the one is determined, rather than the other, *absolutely by nothing*: Which is an express contradiction. And consequently, as before, there must on the contrary, of necessity have existed from eternity *some one* immutable and *independent* being. Which *what* it is, remains in the next place to be inquired. (Clarke 1738, pp. 11–15.)

1.4.2 Clarke's argument is based on the exclusive disjunction:

Either there has always existed some one unchangeable and *independent* being, **from which all other beings that are or ever were in the universe**, have received their original; or else [that is, or not that but] there has been an infinite succession of changeable and *dependent* beings produced one from another in an endless progression, *without* any original cause at all. (Emphasis added)

Rowe "alters this passage" for an argument based instead on the disjunction:

Either there exists a being that has the reason of its existence within its own nature or there has been an infinite succession of beings, each having the reason of its existence in the causal efficacy of some other being. (Rowe 1976, p. 117.)

The argument that Rowe develops and discusses, while "fitting nicely with the way in which Hume has Demea summarize [sic] Clarke's argument" (Rowe 1976, p. 117n3], is different from Clarke's. One difference is that Clarke's argument, though 'officially' for the conclusion that "there must . . . of necessity have existed from eternity some one [at least one – see Clarke's 'marginal note' and note 3 above] immutable and *independent* being," supports as well the stronger conclusion that, "there has always existed some one [*at least one*] unchangeable and *independent* being, **from which all other beings that are or ever were in the universe**, have received their original." Clarke's argument for the existence of *at least one* immutable independent being thus contains a very short argument for the existence of *exactly* one such being: The short 'argument' for this corollary is that 'manifestly', Clarke could say, there cannot be two beings neither of which depends upon or has its original from any

other, upon each of which all beings other than it depend and have received their original! The argument that Rowe puts in place of Clarke's lacks that 'resource.' A reason for altering Clarke's argument in the way in which Rowe does – a reason other than those that Rowe gives – is that, in its original, Clarke's argument gets off to a very bad start from which it never recovers. Clarke says that the disjunction on which his argument is based is entailed by his Proposition I, which says that something has existed from eternity. But even supposing as an additional premise a strong principle of reason, though Rowe's basic disjunction may be made to follow with the help of deft readings and without further premises, Clarke's disjunction, in so far as it goes to things on which all other things would depend, does not follow without additional premises.

1.4.3. Another, and perhaps more important difference between Clarke's argument and Demea's argument, as well as the argument that Rowe discusses in place of Clarke's argument, is that **Clarke does not use a principle of reasons or causes**. No such principle is stated or implicitly in play in the 'primary' argument of the first paragraph of Proposition II. Clarke there pretends to show that the supposition of "an infinite succession of changeable and *dependent* beings . . . *without* any original cause at all" is "impossible, and *contradictory* to itself." How so? Initially with the aid of a fallacy of composition: "'[T]is supposing *something* to be *caused* [for each of its states is dependent and thus caused]; and yet that . . . 'tis caused absolutely by *nothing*."

Nor does a principle of reasons figure as a premise in his second paragraph, which ends with the mistaken view that it is "absurd and contradictory" to suppose that a being is "neither self-existent and necessary itself, nor having its existence found in any self-existent cause." Perhaps Clarke was encouraged to view this supposition as contradictory by its constituting for him, in other words, the supposition of a being that is neither *independent* nor *dependent*, which does 'sound' contradictory. Clarke's 'principle of reasons', if he had thought he needed one, could have been that **every being is either a dependent or independent being**.[7] However, it did not occur to him that he needed a principle of reasons, because the just-emphasized sentence, which would have expressed his principle, 'seemed' to him (without I suspect his giving much thought to it) a tautology.[8]

The third paragraph of Clarke's argument – the 'otherwise thus' paragraph – culminates in the line: "That is to say; of two equally possible things . . . the one is determined, rather than the other, *absolutely by nothing*: Which is an express contradiction." This line is embarrassed by its ambiguity. Clarke has reached the noncontradiction that there is nothing at all that determined there to be something rather than nothing. He seems to have confused that with the express contradiction that, though the fact that there is something rather than nothing is determined, it is determined by nothing, that is, not determined!

Clarke did not bring a principle of reasons to his argument for Proposition II. He supposed that 'logic alone' was sufficient for it. The principle of reasons that can be said to be implicit in his reasoning, though not used in it, would have seemed to him, before he had given it much thought, trivial. Perhaps, if he had made it explicit, and thought about it, he would have realized that it is not trivial. Perhaps he would have realized that to say, in his terms, that every being is either 'independent' ['containing the reason for its existence in itself'] or 'dependent' ['having the reason for its existence not in itself but in another thing'], was *not*, for him, to say that every being is either 'independent' or 'not independent.' And perhaps this intelligence would have started revisions that cleared away the confusions of his argument for Proposition II, to reach something like the 'cosmological arguments' that are discussed under his name.

2. A LEIBNIZIAN COSMOLOGICAL ARGUMENT

> (1) ***The World*** – the *Cosmos* – exists. (2) The World is contingent, it is a ***contingent entity***. (3) For everything that exists – for every fact and every existent entity – there is a ***sufficient reason*** for its existence. (4) The sufficient reason for the existence of any contingent entity runs in the end in terms of an existent being. ∴ (5) There exists an ***ultimate reason*** for the World, which reason is itself a ***necessary being***.

Important for the validity of the argument is that, by Leibnizian definitions, one of which will be elaborated,

> (*) *a sufficient reason*, since it would be a reason 'with which to stop,' *starts with necessities only*,

and

> (**) an *entity* with which a sufficient reason for a thing starts is *an ultimate reason* for that thing.

It follows from (1) and (3) that there is a sufficient reason for the World; then from (2) and (4) that this reason runs in the end to, or that begins with, a being; and then from (*) and (**) that 'this being', which I introduce by 'existential instantiation', is a necessary being that is an ultimate reason for the World. From that (5) follows by 'existential generalization'. The next section includes a defense of the attribution of this argument to Leibniz. Regarding general terminology: I use the word '*entity*' to cover things such as chairs; places such as The Scarborough Town Center; beings such as dogs and cats and people; and the World, a really big entity; and also such things as the number 4. *Propositions*, including true ones or in other words *facts*, and also states of affairs, are not covered by 'entity'. Nor are properties, universals, or forms such as redness and triangularity. Such things, while not entities, are for

me (as is already evident in this sentence) '*things*'. I use that word, as it is generally used, not for a special kind of thing (!), but as a most comprehensive term that would collect absolutely everything there is.[9] '*Beings*' are for me special entities, such as you and I are, and God would be.

3. ON THE PREMISES, AND TERMINOLOGY

3.1 Premise (1): ***The World*** *– the Cosmos – exists*

3.1.1. The World – this "aggregate of finite things" – is the totality of all contingent entities other than itself that there have been, are, and will be, along with every contingent truth 'concerning them', which is to say, every contingent truth that would not be true, if some one or several of them did not exist. 'Aggregates', as Leibniz understood them, and as intended here, are necessarily *nonempty*, as are 'multitudes,' 'collections,' and 'sets' as ordinarily understood. For an aggregate of X's to be, is for there to be X's. Premise (1), that 'the World' exists, thus entails (1′) that here exists at least one contingent entity, but not vice versa. For there could have been only contingent entities other than those that there are, and, if there were, then the World, *this* aggregate of contingent entities, would not have existed. Similarly if, though there were the same entities as there are, somewhat different contingent propositions had been true of them, there would have been a different world.

3.1.2. The World is not the aggregate of everything. Leibniz would say that it most certainly does not encompass everything. For it includes precisely existent *contingent* entities other than itself along with truths concerning them, and so it includes neither necessary truths nor necessary entities. In particular, this world of ours does not include God, the necessary being toward which this argument headed, though it stops two steps short of it. To go on it needs to be supplemented to show that, if there is an entity that is an ultimate reason for the World, there is exactly one, and that the unique ultimate reason for the World is "what we call *God*" (*Monadology* 38) and not the number 47.

Nor, as has been observed, though this is not important for the argument, does the World, this aggregate of finite entities, encompass itself. It more or less *explicitly* does not do that, if, as Leibniz supposed, it itself is an *infinite* thing, and it should, I think, be understood as not doing that even if it is finite. "Only 'should'? Is it not necessary, if the idea of it is not to harbour an inconsistency, that this aggregate or multiplicity of existent contingent things not include itself." No, all that would follow if it did include itself is that it was not a kind of aggregate or multiplicity for which self-membership was a problem! But there is no need to go into the general business of aggregates and multitudes here. (Sections 4.5 and 4.6 of the previous chapter were into it, as will be Sections 6 and 8.1.2 of Chapter X.)

3.1.3. The World – this aggregate of finite things – is not a possible world of contemporary modal logic and metaphysics. Also, the World does not 'answer precisely to' what might be termed 'the comprehensive contingent truth,' where that would be the contingent truth C that entails every contingent truth. This proposition C, just as every proposition, entails not only every contingent truth, but also every necessary truth, and so is, in its propositional way, more comprehensive, than the World of our Leibnizian argument. Also, this world of our argument encompasses only contingent truths that would not be true if entities in it, certain ones or all, did not exist. So it does not include all contingent truths, if, for example, it is a contingent truth that there does not exist a golden mountain, and that either there are no rubies, or all rubies are red (I think this is true).

3.2 Premise (2): The World is a ***contingent entity***. Contingent entities are possible entities the existences of which are not entailed by necessary truths.[10] Necessary entities are entities the existences of which are so entailed. Every existent is of one or the other of these kinds. The number 3 necessarily exists and is a necessary being. There are necessarily true propositions that affirm its existence, for example, the proposition that there exists an odd number, indeed exactly one odd number, between 2 and 4. The number 3 exists in every possible world in the modern modal logic sense of a 'possible world.' The Scarborough Town Centre, for contrast – if firm modal intuitions are to be trusted – is not a necessary entity. Nor are we. It and we are contingent entities. According to firm modal intuitions, no necessary truths affirm either its existence or ours.

Premise (2) says that the World, this aggregate of existent contingent things, this *cosmos*, is itself contingent. It is not merely through and through contingent, but is itself contingent. This aggregate did not need to exist. I have explained why this is true. There could have been different contingent entities, and different contingent propositions could have been true of the contingent entities there are. Does premise (2) 'say' that it is not necessary that anything contingent should have existed, that it is not necessary that there should have been any *cosmos*, let alone this particular one? No, though Leibniz believed that too. It is not an 'explicit part' of premise (2) that (2′) *it is not necessary that there is a world*, that there is a *cosmos*, or that there is something contingent, meaning some contingent entity or contingent truth about an existent contingent entity. Premise (2) is entailed by, but does not entail, (2′), except in so far as (2′), if true, is, because it is necessary and entailed by every proposition. A somewhat different Leibnizian argument from ours could, as far as validity goes, use (2′) instead of (2). Leibniz himself uses both, though each is sufficient given his other premises.

The World, according to premise (2), is *contingent*, and according to this premise it is an *entity*. This is not an idle ornamentation. For while premise (3), the principle of sufficient reasons of the argument is general for all existent

things, premise (4), to provide a way to what could be a god, is specific to existent ***entities***.

3.3 Premise (3): For everything that exists – for every fact and every existent being – there is a ***sufficient reason*** *for its existence.* "The great . . . principle that *nothing takes place without a sufficient reason*" (*Nature and Grace*), that "no fact can be real or existent, no statement true, unless there be a sufficient reason why it is so and not otherwise, although most often these reasons cannot be known to us" (*Monadology*).[11]

3.3.1 'Sufficient reasons.' We may begin with sufficient reasons for existent entities, *that is*, for *facts* concerning them. A 'sufficient reason' for one would be a 'full reason' for and a 'complete explanation' of it, a 'final reason' that 'needs no further reason'. A sufficient reason for an entity would explain everything about it, and say both why it is and why it is just so 'and not otherwise'. A *sufficient* reason would not itself give rise to further questions, nor would it provide occasions for further reasons. *A sufficient reason would be a reason* '**with which to stop**'. A *sufficient reason* would *not* be sufficient merely in the ordinary sense of *good enough for purposes at hand*. Nor would it explain merely by making *probable* as do citations of natural causes. A *sufficient reason* would not merely *incline to,* but would *necessitate*, what it would explain; it would, when it was a reason for a proposition, be a *deductive reason* that *entailed* it. Otherwise one could want a reason why it had operated to full effect, since what only inclines leaves open the *possibility* that that to which it inclines should fail to take place; otherwise, that is, it would not be a reason *with which to stop*.

3.3.2 "How are these reasons possible?" Sufficient reasons are possible because there are *necessary* truths and entities, for reasons can stop with these. And a reason can stop *only with these*. Sufficient reasons thus run eventually entirely in terms of necessary truths and necessary entities. A 'last entity' in a sufficient reason will bear "the reason for its existence within itself," which, in order to be "a sufficient reason with which to stop," must also be "a reason . . . why the thing is as it is and not otherwise" (*Nature and Grace* 1714). For illustrations, a sufficient reason for the World could, in Leibniz's view, terminate in the proposition amongst others that God exists, that He chooses among worlds, that is, among possible Leibnizian *cosmoi*, for the best, and that this one of ours, enter here a complete description of it (!), is the best of all possible worlds. That could be the main outline of a sufficient reason for the World *if* this several-part proposition is necessarily true. Or a sufficient reason might feature the proposition "that there is a God who ensures that the world must accommodate itself to our ways of thinking about it" (Walker 1997, p. 113), if that is a necessary truth.

The core of a sufficient reason, the necessary truths and entities that eventually are all that occur in it, would explain all other truths and entities it deploys. For economy, therefore, sufficient reasons can be identified with their necessary cores. I will practice this identification, drop the fancy of reasons as explanatory stories, and collect 'cores' of reasons into single propositions. That makes a sufficient reason for an entity E a one-premise *deductive* argument, the premise of which is a necessarily true proposition and the conclusion of which is a proposition about E that requires for its truth that E exist. It will also require that E is 'just so' as it is, though this aspect of sufficient reasons is not important to my discussion and will generally be left implicit. Regarding sufficient reasons for facts (which are not entities), a sufficient reason for a fact shall be a deductive argument the premise of which is a necessary truth and the conclusion of which is this fact.

3.3.3 Two Taylors follow Leibniz

> [O]ur knowledge of any event in Nature is not complete until we know **the full reason** for the event. So long as you only know that A is so because B is so, but cannot tell why B is so, your knowledge is incomplete. It only becomes complete when you are in a position to say that ultimately A is so because Z is so, Z being something which is its own *raison d'être*, and therefore that it would be senseless to ask *why* Z is so. (A. E. Taylor 1970, p. 69; emphasis added.)
>
> [To] say . . . that the world depends for its existence upon something else, which in turn depends on still another thing, this depending upon still another, *ad infinitum*. . . . does not render a **sufficient reason** why anything should exist in the first place. Instead of supplying a reason why any world should exist, it repeatedly begs off giving a reason. . . . Ultimately . . . the world, or the totality of contingent . . . things . . . must depend on something that is necessary. (Richard Taylor 1963, p. 92; emphasis added.)

A. E. Taylor's full and complete reasons are, as far as one can tell from what Taylor says, the same as Leibniz's sufficient reasons. This Taylor does not say they are the only reasons there are. Richard Taylor may make Leibniz's sufficient reasons the only *proper* reasons. At any rate, a 'run-on account', in which each thing was said to depend on an antecedent thing without end (and so without a beginning to the reason), would, in this Taylor's view, be no reason at all, properly speaking. Neither says explicitly that his reasons are *deductive*, but this can be gathered from the demands upon them that they make explicit. These two Taylors, terminological differences aside, are in agreement with each other and with Leibniz. For a sufficient reason for the world one must go to something that is necessary, something that contains within itself its reason for being, to something that is thus very different from every part of the world, that *deductively entails* the world.

3.3.4 Other 'principles of sufficient reason'. In his discussion of something like Clarke's argument, Rowe considers several forms of 'The Principle of

Sufficient Reason', some form of which he maintains operates in arguments like Clarke's. These principles are, however, none of them principles of specifically *sufficient* reasons or of reasons of any kind more special than *deductive* or (Leibniz's term) 'necessitating'. Rowe's "most general version of PSR" is, "Every actual state of affairs has a reason either within itself or in some other state of affairs" (Rowe 1976, p. 113). Rowe's PSRs are all like this one in calling not for reasons of any special kind other than deductive, for which Rowe understands without saying, that they call. They are all either universal or somewhat restricted demands for reasons of no particular sort beyond that, whereas it is only part of *Leibniz's* idea of a 'sufficient reason' that they necessitate or entail what they rationalize. Leibniz has 'rights' to the label 'sufficient reason' and even more clearly to the name 'The Principle of Sufficient Reason,' which he might have 'registered'. No version of a principle meriting this name figures in Rowe's surrogates for Clarke's argument. It is a minor curiosity of modern treatments of 'the cosmological argument', on which label Leibniz does not have rights (see note 3), that so many of them infringe upon his rights to 'The Principle of Sufficient Reason'.

3.4 Premise (4): The sufficient reason for the existence of any contingent entity must run in the end in terms of an existent being.

3.4.1. This premise amplifies Leibniz's line that "since the reason for any existent can only be another **existent**" (*Ultimate Origination*; emphasis added) for the move he makes from "the ultimate root of the world must be **something** that exists of metaphysical necessity," to "there is a **being**" that is this root and that exists of metaphysical necessity (*Ultimate Origination*; bold emphasis added). That is required if the argument is to lead to something that could be God, for whatever else God would be, God would be a *being*, and to that extent like us.

This premise says that to explain the existence of a contingent entity it is not enough to assemble a reason composed entirely of principles of geometry concerning the amazing shapes within shapes of the World, or of metaphysics concerning the 'quantity of being' of the World, or of value concerning the *excellence* of the World and of how much it *ought* to be.[12] Any such explanatory story must, according to Leibniz, assign a leading role to some creative *being*. Aquinas, in a somewhat similar vein, had written that "that which does not exist begins to exist only through **something** already existing" (ST I q2,a3, p. 23; emphasis added): Things that come into existence are contingent things, and such things cannot come into existence 'by their own metaphysical ponderance' and 'because of their excellence' without the involvement of an *agent*. Leibniz extends this idea to contingent entities that exist always, if any such there be, and says, speaking not only of 'causal reasons' but of all 'reasons', that a contingent entity finds its full account and reason only in some existing *being*. "But why?" Is it true that to explain the existence of any contingent entity

one must have recourse to the 'agency', broadly understood, of some existent entity, indeed, *being*? Not everyone thinks so.

3.4.2. John Leslie is against premise (4). Leslie, and perhaps John Findlay, though possibly not Plotinus, eschew 'creative entities' and are Platonists of sorts. "I call myself a neo-platonist. I think that The Good is responsible for the world's existence" (Leslie 1997, p. 218), which is, for Leslie, a picaresque way of saying that in his cosmology "***the world exists because it should***" (Leslie 1970; bold emphasis added).[13]

> Suppose one explained the world through this alleged reality: that it ought to exist. Would an ethical right to exist be a separate existent? It is better called a status which existents perhaps have. The sum of all existing objects might owe its presence to the ethical reason for existence which such a status provides. *Axiarchism* is my label for all theories picturing the world as ruled by value. One theory stands out: that the world's existence and detailed nature are products of a directly active ethical necessity. This rejects reliance on the . . . creative prowess of an . . . existing . . . deity.[14] However, it permits belief in God, who may himself be ethically required [and the agent of all else that is ethically required]; or 'God' may be the name of the principle that ethical requirements are creatively powerful [or of the ethical necessity of the world].[15] (Leslie 1970, p. 286)

John Leslie Mackie concedes that "[i]f, with Leibniz and others, you demand an ultimate explanation, then this may well be a better one than the postulation of a divine mind or spirit" (p. 235), avoiding as it does "postulation of a directly efficacious will" (p. 235). He rejects Leslie's godless Platonic cosmology: (i) because of its problems not only with apparently unnecessary evils, but with apparently unnecessary neutral beings and details (pp. 236–7); (ii) because of the implausibility of its "central principle . . . that objective ethical requiredness is creative" (p. 237); and (iii) "finally" mainly because it "rests essentially upon the assumption that there are objectively prescriptive values. . . . [which] assumption is false*" (pp. 237–8). [*That is argued in Chapter 1 of *Ethics: Inventing Right and Wrong* [1977] and throughout *Hume's Moral Theory* [1980] (p. 238n10)]. There is more on Leslie's cosmology in Appendix B.

4. COMPARISONS WITH ONTOLOGICAL, AND AGAIN WITH FIRST CAUSE, ARGUMENTS

4.1. Cosmological arguments differ from ontological arguments in that, though they too would establish the existence of necessary beings, they do not purport to do so from premises none of which are contingent. Indeed, premise (2) of our Leibnizian argument 'says' that premise (1) is contingent. A further difference is that, unlike ontological arguments, cosmological arguments do not, if successful, immediately establish a necessary being of a character – for example, that of a perfect being or an unsurpassably great being – that puts it well on the way to being religiously adequate, if any entity can be. Leibniz's

words, "that is, to God" (1697) were premature. He needed to convince that details of the sufficient reason for the World would establish that its ultimate reason was properly worshipful, and similarly for Clarke's and Demea's necessary causes. Like ontological arguments, cosmological arguments do not immediately yield *unique* entities. Leibniz addresses a sentence to this further requirement: "This reality can emanate only from a single source, in view of the interconnection of all these possibles and existences" (Leibniz 1965, *Ultimate Origination*, p. 90). Clarke, having to his satisfaction established the existence of an independent and unchangeable (Proposition II) and thus self-existent and necessary (Proposition III) being, argues its uniqueness (Proposition VII). He argues first from its necessity and then from its independence that there can be at most one such being. These arguments (Clarke 1738, pp. 47–8) would, however, if good, show that the number 3 and the color yellow, if necessary and independent existents, are identical and the same as the being of Proposition II.[16] Anselm's and Descartes's ontological arguments could have easily elaborated their argument to secure uniqueness, for, if existence is a perfection and mark of greatness, then why not uniqueness? One may wonder why they didn't.

4.2. Unlike first-cause arguments, cosmological arguments can embrace infinite regresses of causes. They say that, even if the World is infinite and eternal and contains infinite regresses of causes, it needs a reason or ultimate cause outside and beyond itself. And cosmological arguments need not worry that the beings for which they argue *no longer* exist. These entities would be necessary beings, and though it is not entirely clear *why* they are therefore not the sort of things that can exist at a time, and at a later time not exist, that this is not possible for them, since they are necessary beings, has, I think, never been challenged.

5. CLEANTHES' OBJECTION

> Did I show you the particular causes of each individual in a collection of twenty particles of matter, I should think it very unreasonable should you afterwards ask me what was the cause of the whole twenty. This is sufficiently explained in explaining the cause of the parts. (Hume 1991, Part IX, p. 150)[17]

5.1. "True," Demea could have said, "but the causes of these twenty particles could not all be members of this collection, for if they were, then these particles would, *per impossibile*, all be caused either by themselves, or by causal descendant's of themselves. And if this collection of particles were the only potential causes there were in the world, then you, Cleanthes, would be into necessities outside the world. For I assume that all contingents and contingencies are in the world." To which Cleanthes might have conceded, "That's right, but it misses my point. I spoke of twenty particles only in order

not to boggle the mind with the infinite. My point, at which I have only gestured, is that given *infinitely* many particles spread back through time, each can have as a cause another particle, without any particle's being, what we agree to be impossible, in any way a cause of itself. My point is that you need for your argument what you have been concerned to do without, namely, the assumption that the world is finite."[18] "But," Demea might then 'win' by saying, "though, if the successions of causes are infinite, there could be in that way a cause for each part, there would plainly *not* yet have been revealed a cause for the infinite, beginningless successions, contrary to your claim that **it** [that succession or successions] is sufficiently in explaining the causes of its parts' (Ibid.). For that, for a cause of the *whole*, you would need to go out of the world to necessities, just as is required in the finite case. And there are only these two cases for causes *in* the world: there could be finitely many; or there could be infinitely many."

5.2. Cleanthes does not challenge Demea's principle that "[w]hatever exists must have a cause or reason of its existence" (p. 148). And so Demea would have been right to insist that, even if 'shown' the cause of each cause in infinite successions without going outside them, one needs to go outside them to find causes or reasons for these successions. I do not understand why Demea was not given this line or why the impression was left that he had no comeback to Cleanthes's contention that "the cause of the whole would be sufficiently explained in the explaining of the causes of the parts." It is also remarkable that Cleanthes did not challenge Demea's principle that there is a cause or reason for everything that exists. Could it be that Hume accepted it? Well, yes, for Demea says it without demurrer from Cleanthes. And Philo has said it in a 'case-closed' statement for the being of a deity: "[W]here reasonable men treat these subjects, the question can never be concerning the *being* but only the *nature* of the Deity. The former truth, as you will observe, is unquestionable and self-evident. **Nothing exists without a cause**, and the original cause of this universe (whatever it be) we call God; and piously ascribe to him every species of perfection" (Part 2, p. 108; bold emphasis added).[19]

5.3. Demea had a good response to Cleanthes' objection. A proponent of our *Leibnizian* argument could make the same response, as well as another that differs from any open to Demea, by capitalizing on features of the *sufficient* reasons or explanations it demands. If the World – the *cosmos* – is *finite*, that is, if it encompasses only finitely many things, then sufficient explanations of its every part would sufficiently explain the whole. But sufficient explanations of the parts would need, given the transitivity of sufficient explanations, to go outside a finite *cosmos* into the 'larger world' of necessities, on pain otherwise of contingent things being *per impossibile* self-explaining. If, on the other hand, the *cosmos* is *infinite*, then, though explanations of its individual parts need not

go outside it to avoid self-explanations, *the explanations of its parts* must still go outside it for finality and *sufficiency*. Sufficient explanations of the parts of the *cosmos* must in any case, whether it is finite or infinite, run into that 'larger world' where there are necessary things, if the parts of the *cosmos* considered either individually or collectively are to be, in Hume's words, understood in Leibniz's way, *sufficiently* explained. It is true that once one has for each of the World's particular parts a Leibnizian sufficient explanation, then one has a Leibnizian sufficient explanation of the World as a whole, but it is false that any *such* explanation, of a part or successions of parts or the whole, can be given without going outside the World to necessary things. It makes no difference to that whether the *cosmos* is finite.[20]

6. A 'SMALL PROBLEM' WITH OUR LEIBNIZIAN ARGUMENT

6.1 Our Leibnizian argument's premises are not consistent! The main point for this criticism is that 'sufficient reasons' would be demonstrations, valid deductive arguments from necessarily true premises. Given that, the inconsistency follows from the easier point that demonstrations have, one and all, necessarily true conclusions. I take up first the easier point and then revisit the main one, which was made in Section 3.3.2.

First Point – *only necessary truths can be demonstrated*. A demonstration is a deductively valid argument each premise of which is necessarily true. But then the conclusion of any demonstration is itself not merely true, but necessarily true. Hume has Cleanthes make something like this point: "[T]here is an evident absurdity in pretending to demonstrate a matter of fact, or to prove it by any arguments *a priori*. Nothing is demonstrable unless the contrary implies a contradiction" (Hume 1991, Part 10, p. 149). Proof of this first point: The premises of a demonstration cannot be false. It is not possible for the conclusion of a demonstration to be false unless at least one of its premises is false, for a demonstration is a deductively valid argument, and an argument is deductively valid only if it is impossible for its conclusion to be false unless at least one of its premises is false. So it is not possible for the conclusion of a demonstration to be false. But the conclusion of a demonstration is true, so it must be a necessary truth.[21] Second Point – *sufficient reasons for truths and existents are all demonstrations*. As has been explained, Leibniz's idea of a sufficient reason has two parts: The premise of a sufficient reason would be a *necessary truth*; and the conclusion of a sufficient reason is *necessitated* or *deductively entailed* by its premise. That makes sufficient reasons demonstrations.

From these two points follows The Principle of Restricted Domain for Sufficient Reasons: *Only necessities, necessary truths, and entities, have sufficient reasons*. That the premises of our Leibnizian cosmological argument are not consistent is an easy consequence of this principle. These premises include (1) that the World exists, and (3) that for everything that exists there is a

sufficient reason for its existence. It follows from these premises that *the World has a sufficient reason*. From that and Restricted Domains for Sufficient Reasons, it follows that *the World* ***is*** *a necessary entity*. But that is, of course, not consistent with the second premise which is (2) that *the World is a contingent entity*, which entails that *the World* ***is not*** *a necessary entity*.

7. THAT 'SMALL PROBLEM' WITH THE ARGUMENT GOES INTO BIGGER PROBLEMS FOR ITS 'AMBITION'

7.1 The Principle of Sufficient Reasons would banish contingency. Premise (3) – the Principle of Sufficient Reason, henceforth PrSuffRsns – precipitates all by itself a 'modal collapse'. If anything that is, is contingent, then PrSuffRsns is false, since, as we have seen, only necessary entities and truths can have sufficient reasons. Turning this proposition around, we have that *if PrSuffRsns is true, every existent entity and every true proposition, is necessary*. PrSuffRsns entails, regarding propositions, that there are just two kinds, necessarily true propositions and necessarily false or impossible propositions. PrSuffRsns banishes contingent propositions, for these would be neither necessarily true nor necessarily false.[22] That is more remarkable than is the failure, on account of the inconsistency of its premises, of our Leibnizian cosmological argument. On the fair assumption that some things are contingent, PrSuffRsns, which expresses 'the ambition' of the argument for sufficient reasons all around, is false. And that is not the worst of it.

7.2 That 'deductive explanations all around' is also inconsistent with anything's being contingent. On the fair assumption that an explanation of something contingent cannot be circular, if anything is contingent, then not even 'deductive explanations' of everything are possible, where a deductive explanation is a valid deductive argument that has only true premises.

7.2.1. Let the *Principle of Deductive Explanations*, PrDedExpl, say that there is a deductive explanation that has only true premises for every contingent truth. PrDedExpl 'banishes' contingencies, just as surely as does the more demanding PrSuffRsns. Now comes an adaptation of an argument of Hud Hudson "modeled," he says "on Peter van Inwagen's objection to (PSR)" (Hudson 1997, p. 78). The argument will be explicit for truths, but its result, that *PrDedExpl* banishes contingent truths, entails that there are no contingent entities, since an entity is contingent only if its existence is entailed by a contingent truth.

7.2.2. To bring the definition of a deductive explanation into our argument, we have the premises,

(1) A deductive explanation for a proposition is a deductively valid argument with only true premises.

and

(2) If P is a deductive explanation for a contingent proposition c, then, if p is the sole premise of P and p is contingent, c does not entail p.

Since p entails c, if c entailed p, P would be circular. Premise (2) corresponds to Hudson's (iv), which is "that contingently true propositions do not contain their own sufficient reasons" (Hudson 1997, p. 76). So every deductive explanation is a valid deductive argument with true premises that, if for a contingent proposition, is not circular. The only other 'assumptions' I make are not about deductive explanations, but about propositions:

(3) If there is a true contingent proposition, then there is a true contingent proposition that entails every true contingent proposition.[23]

and

(4) For any true propositions, there is a true proposition that entails exactly these propositions, itself, and every necessary proposition.

I use for inferences only unproblematic principles such as that *every true proposition is either necessary or contingent*. "That is not unproblematic for David Lewis," Hudson says (see note 20 in Chapter III). No matter, however, for the principle is unproblematic whether it is consistent with there being many 'possible worlds' in Lewis's sense.

To show that PrDedExpl banishes contingency, I suppose for argument that it does not do that. That is, to show that

if PrDedExpl, then there is *not* a contingent truth,

which is to say, I suppose,

(5) PrDedExpl.

and, for some proposition c,

(6) c is a contingent truth.

A contradiction can be derived from (1) through (4) together with (5) and (6). This means that, given (1) through (4), if PrDedExpl, then there is not a contingent truth. Here we go for the contradiction. From (6) and (3) it follows that, for some proposition C,

(7) C is a contingent truth that entails every contingent truth.

It follows by PrDedExpl that C has a deductive explanation. Let DX be such a deductive explanation. 'Assumption' (4) says there is a deductive explanation for C that has a single premise, which premise entails contingent premises, if any, of DX and every necessary proposition. Let DX′ be such a deductive explanation for C, and let Pr be its sole premise:

(8) Pr is the premise of a deductive explanation of C.
(9) Pr is true.

It follows that,

(10) *Pr is not necessary*.
Subsidiary argument. DX′ is, by (1), a valid deductive argument. Suppose its premise Pr is necessary. Then, since Pr is true, (9), the conclusion, C, of DX′ is not only true but necessary. So, since C entails c, c is a necessary truth. So c is not contingent: Contingency and necessity are contrary conditions. That contradicts (6).

It also follows that,

(11) Pr is not contingent.
Subsidiary argument. Suppose (i), Pr is contingent. We have from (7) that (ii), C is contingent. It follows, by (2), from (8), (i), and (ii), that (iii), Pr is *not* entailed by C. But, for a contradiction, it follows from (7) and (i) that Pr *is* entailed by C.

We have from (9) and (11), by the unproblematic principle mentioned above according to which every true proposition is either contingent or necessary, that

(12) *Pr is necessary.*

Inference (12) contradicts (10), which contradiction completes the indirect proof, begun at (5), to show that (1) through (4) entail that, *if PrDedExpl, there is not a contingent truth*.[24]

7.3. Ralph Walker writes that "the Principle of Sufficient Reason does not demand the impossible" (Walker 1997). We have seen that, on the contrary, if this Principle of Sufficient Reason is Leibniz's principle, then it certainly does demand the impossible, *if*, as almost everyone believes, there is a contingent truth. Indeed, any principle that is like Leibniz's in demanding for everything a *deductive explanation*, demands the impossible, if there is a contingent truth. Walker, to distance himself sufficiently from Leibniz's sufficient reasons, argues that "Logical sufficiency is not what is required [for a 'sufficient reason']: we cannot say that the presence of a sheep in the room is explained by the fact that there are two sheep in the room" (Walker 1997, p. 112). But Walker's explanation shows only that logical sufficiency is not *all* that is required for a sufficient reason or, indeed, we can agree, any kind of reason properly

so termed. Walker, who considers himself friendly to the general thrust of Leibniz's theological cosmology, nevertheless locates his main disagreement with Leibniz "in his confidence that for everything that exists, and for everything that happens, there is a sufficient reason to be found" (Walker 1997, p. 113). A more important disagreement, however, is, as said, that for Walker's 'sufficient reasons' there would not be 'necessitating reasons'. So they would not be reasons "with which to stop" (*Nature and Grace*).

7.4. PrSuffRsns bans contingency; so does PrDedExpl. Rowe makes a similar point against a 'principle of sufficient reason' that he attributes to Clarke. It is the principle that, for everything that is, and every circumstance of it, there is a deductive reason why it is, rather than not (see Rowe 1975, pp. 90-1n25, 99). Rowe considers this principle to be approximately equivalent to,

PSR_1: Every actual state of affairs has a reason either within itself or in some other state of affairs. (p. 149)

He shows that if there are contingent states of affairs, then PSR_1 is false (pp. 103–7).[25] Rowe's response is to retrench to a more secure principle that is adequate to the work that this "much more general" principle does in 'Clarke's cosmological argument' (p. 112; see p. 114). Rowe protects the argument he discusses by assigning to it a weaker principle than he takes Clarke to have accepted, notwithstanding that "[i]t can . . . be argued that the only reason someone would have for accepting [this weaker principle] is whatever reason he might have for believing the more general principle" (p. 112). The weaker principle that Rowe uses combines the following:

PSR_2: Every existing thing has a reason for its existence either in the necessity of its own nature or in the causal efficacy of some other being. (p. 149)

PSR_4: For every set whose members are existing beings (which can be caused to exist or which can cause the existence of other beings) there must be an explanation of the fact that it has members. (p. 150)

There is no mention in these principles of reasons for the *circumstances* of existing things or of reasons for their *natures* and *characters*. Rowe, in this somewhat ad hoc manner, avoids precipitating a collapse of contingencies into necessities. He shows how one can work to the conclusion that there is a necessary being upon which contingent entities depend for their existence with something less than the completely general Principle of Deductive Reasons upon which Rowe thinks Clarke depended.

My work on Leibniz's reasoning has a different objective. Instead of seeking ways in which, by curtailing his Principle of Sufficient Reason, modal collapse can be avoided, and much of his conclusion retained, I seek to bring out problems made by the principles that actually inform cosmological reasoning, such as the Principle of Deductive Reasons, that Clarke did not trumpet and the

Principle of Sufficient Reason, of which Leibniz made so much. I want to advertise problems with the 'ambition' of cosmological reasoning, that everything should have a reason, and the related idea that contingencies, that at least that there is something rather than nothing, contingent, needs to be explained and can be explained by necessities.

8. PROOFS *A CONTINGENTIA MUNDI* – WHAT A NICE BAD IDEA

8.1. The problem with cosmological arguments that would brook no brute facts, that demand explanations for everything including the sum total of everything, is radical and profound. The problem of our Leibnizian argument is not merely that its first three premises are inconsistent. Nor is it the demand that premise (3) makes for extraordinary 'sufficient reasons', which demand is inconsistent with anything's being consistent. We have seen that the problem would remain if the demand were only for 'deductive explanations' all around. Indeed, to plumb the depth of the problem, it would remain even if the cosmologist's demand were diminished all the way to the demand merely for reasons of one sort or another all around, without any restriction on the kinds of reasons or explanations that would satisfy, so that they could be sometimes deductive, sometimes inductive, sometimes factual, sometimes normative (as would be the Platonic reason that 'the world exists because it *should*, or because it is *good*'), and so on for every variety of possible reason or explanation. The problem is that *reasons all around* is already inconsistent with anything's being contingent. That you and I exist necessarily are in all ways *necessarily* as we are, *do* in all ways necessarily as we do, is entailed by,

The Principle of Complete Reasons. Everything has a complete reason of some sort.

or equivalently,

***The Principle of Reasons.* For everything there is a reason of some sort or other.**

Regarding this, if there is a reason for everything, then there is a reason *for everything about everything*, and there is for everything a complete reason; and conversely, if there is for everything a complete reason, then there is for everything a reason. *These* principles banish contingencies. The *problem* for cosmological arguments that want *reasons all around* is that, If anything is contingent, *there cannot* BE *reasons all around*: If anything is contingent, then it is not possible that, for *every* fact or entity, x, there is a reason of some sort or other for x.

8.2 The main argument for brute *fact.* If anything is contingent, then there is the 'aggregate' of all contingent entities other than it itself, together with all

contingent truths other than those concerned with it. To give it a name, there is to conjure with it, ***The World of Limited Contingencies***, if there is anything that is contingent ('if' because 'aggregates' are by definition nonempty). This world is not exactly the World of our cosmological argument, for that world includes no facts other than those about entities in it, while this world excludes only facts about itself. It follows from the Principle of Complete Reasons that, if The World of Limited Contingencies exists, it *has* a complete reason. But, as will be explained in the next section, if this world exists, that is, if something is contingent, then this world *does not have* a complete reason. Therefore, if anything is contingent, not everything has a *complete* reason, which, as explained above, means that if there is anything that is contingent, then not *everything* has a reason *of any sort*.

8.3 A two-point argument to show that The World of Limited Contingencies could not have a complete reason. First Point:

The World of Limited Contingencies would itself be contingent.

This aggregate would be contingent on the general grounds that whatever includes or entails contingencies, some or, as in this case, all other than itself and contingencies concerned with it, is itself contingent. *Second Point:*

***A complete reason for The World of Limited Contingencies would be a* demonstrative *reason for it*.**

This follows from two considerations. FIRST, *a complete reason for The World of Limited Contingencies could have as premises only necessities.* Proof: A complete reason for the aggregate of all contingencies other than itself and contingencies concerned with it would explain absolutely *all* contingencies, since a complete reason that explained this aggregate (i) would explain it, (ii) would explain all contingencies concerned with it, and (iii) would explain all contingencies in it. That, however – The World of Limited Contingencies, all contingencies concerned with it, and all contingencies in it – is all the contingencies there are. (This world has been defined to include all contingencies other than itself and contingencies concerned with it.) But then, a *complete reason* for this world, *on pain otherwise of circularity and being no reason at all for it*, would have to go outside the realm of contingencies into that of necessities *for every one of its premises*: No contingency can be any part of an explanation, complete or otherwise, of itself, even if there is a sense in which necessities can be self-explaining. So *a complete reason for The World of Limited Contingencies could have as premises only necessities*, which is what was to be proved here. SECOND, *necessities alone can be reasons for things only by necessitating, or deductively entailing, them.* Why? Because the alternative to that mode of relevance, that they should be merely *inductive reasons* that

'incline to' or 'make probable' things for which they are reasons, is not open. *Necessary* truths, since they are 'true no matter what', cannot matter to probabilities of other truths. Truths other than necessary truths are either logically entailed by or completely independent of them.[26] It is, for example, necessarily true that it will either snow or not snow in this area later this year, which truth has absolutely no bearing on or relevance to whether it will snow this month. Nor is the problem the obviousness of this necessary truth. It is its 'vacuity', its 'lack of contingent content', that is the problem. Only truths that are *not* 'true no matter what' can be *inductively or probabilistically* relevant things that are not true no matter what. From this I conclude that necessary truths can be only *deductively or necessitatingly* relevant. For I know of no *candidate* for the relevance of necessities other than inductive 'inclining' and deductive necessitating.

Putting together these two considerations, we have the Second Point of the argument of this section, that *a complete reason for The World of Limited Contingencies would be a* demonstrative *reason for it*. From this and the First Point of the argument, that *the World of limited Contingencies would itself be contingent*, what was to be explained in this section, that *the World of Limited Contingencies could not have a complete reason*, follows, since only necessities have demonstrative reasons.

8.4. Collecting from Section 8.1, that everything has a complete reason if and only if everything has a reason 'full stop', from Section 8.2, that if anything is contingent, then there is the World of Limited Contingencies, and from Section 8.3, that the World of Limited Contingencies could not have a complete reason, we may pass to the grand conclusion that

if anything is contingent, then there is not for EVERYTHING ***x a reason of some sort or other for x***

or, equivalently,

if there is a contingent existent or truth, then there is a BRUTE, ABSOLUTELY INEXPLICABLE ***contingent existent or truth***.

CODA

Paraphrasing Demea in order to contradict his central claim, 'The question – Why all of this, that is, why *this* aggregate of contingencies? – is *not* still reasonable.' The question presupposes that there is *an aggregate of all contingencies*, for it would be about this aggregate. So it is 'improper', if there are no contingencies and thus no such aggregate, and thus in a manner is not still reasonable. And it has been recently established that if there are contingencies and thus this aggregate, then there cannot be a reason of any sort for *all* of it, so that, in this case, though the question is 'proper', it is demonstrably unaswerable and

thus in another manner is not still reasonable. To Sclauzero's declaration that *it is not conceivable that successions of causes and effects leading to now should have sprung from nowhere*, I say, on the contrary, that it is not only *conceivable* that there are not reasons and causes for all of this and that it has 'sprung from nowhere', but, if any of this is contingent, that it is *not conceivable* that there *are* reasons or causes of any sort for *all* of this from which it 'sprang'.

"Ah, but *is* any of this contingent?" What do you think?

POSTSCRIPT. FURTHER TO THE IRRELEVANCE OF NECESSITIES TO CONTINGENCIES

PS1 There is a deep divide. The program in which cosmological arguments are implicated is fatally flawed. It is inspired by the idea that contingencies, one and all, must be grounded in necessities. In fact, however, necessities can have nothing at all to do with contingencies. What, for example, can the fact that $5 + 7 = 12$ have to do with the rate of exchange today between Canadian and U.S. dollars, or with how much money I have in my pockets, or with anything contingent? My taking into account that fact might help me to calculate how much money I have in my two side pockets when I have \$5 in one and \$7 in the other. Citing the fact might be useful when presenting an explanation of the fact that I have \$12 in my side pockets. But, contrary to a suggestion that can be found in Adams (1987, p. 215), the mathematical necessity that $5 + 7 = 12$ cannot contribute to an explanation of the fact of \$12, which is fully explained by the facts of \$5 and \$7, there is a chasm between necessities and contingencies, and though relations of relevance and of reasons run on both sides of the divide, there are none that run across either way. Cosmological arguments would *per impossibile* bridge this gap.

PS2. Hume has Demea say:

> If there be no necessarily existent being, any supposition which can be formed is equally possible; nor is there any more absurdity in nothing's having existed from eternity than there is in that succession of causes which constitutes the universe. (See Section 1.2.1)

Demea's conditional sentence, 'If there be no necessarily existent being, any supposition which can be formed is equally possible', is worrisome. For one thing, it 'ifs' or hypothesizes what Demea must consider to be an impossibility. For he believes that there is a necessarily existent being, and if there is a necessarily existent being, then *that* there is is necessary. So there is the problem of what Demea could have *meant* with his sentence. The popular 'nearest antecedent world' line that works so well for counterfactuals is of no use for 'counterlogicals'. Setting aside as best we can this first considerable worry, we encounter the suggestion, intended by Demea, that necessarily

existent beings can make a difference, and that, given one, it is possible that not every supposition that can be formed regarding contingent reality is 'equally possible'. But what can this mean? It hardly needs saying that impossible suppositions are not equally possible with possible ones! And it is in any case false that not every possible supposition is *equally possible*, for there are no *degrees* of possibility. Furthermore, if what Demea meant to say was 'equally *probable*', then there is the problem that the existence of necessary beings, if any there be, can make no difference to probabilities. For necessary beings would exist no matter what was the complete character of the *cosmos*. They would exist in every possible *cosmos*. So their existence is necessarily neutral and irrelevant to the likely makeup of this *cosmos* of ours.

For a last swipe at Demea's conditional, it evidently depends on the assumption that a reason why there are any contingent *entities* at all, and why there is the particular succession of contingent entities that 'constitutes the universe', would have to run at least in part in terms of some *entity*. Against this unstated and undefended assumption, however, it is not obvious at all why explanations of these grand contingencies could not run *entirely* in terms of *nonexistential* hypotheses that explained them by making them probable. Stephen Hawking's 'wave function cosmology' posits just such a hypothesis – 'in the beginning there was the wave': See Section 8.3.3 of the next chapter. Leslie floats a 'no-creative-being' cosmology that would explain whatever an analogous 'creative-being' cosmology would explain: See Section 3.4.2 and Appendix B.[27]

PS3. As a matter of fact and logic, there *could have been* no contingent entities. Has anyone ever questioned this possibility? *Necessitarians*, who say everything is necessary, most certainly do not question the possibility that there should have been no *contingent* entities! They say only that this is not *merely* possible. And one does not expect *nonnecessitarians*, who say that there are contingent entities, to say that it was not possible that there should have been none. Leibniz, who would be a nonnecessitarian, *takes for granted* this possibility in his 'first question': "This principle [of sufficient reasons] having been stated, the first question which we have a right to ask will be, 'Why is there something rather than nothing?'" (Leibniz 1969, *Nature and Grace*, p. 638). The nonexistence of contingent entities would, of course, have made no difference to necessary entities, for they exist no matter what. Nothing that is contingent matters in the least to the existence and natures of necessary entities. How then *could* they matter to the existences and natures of things contingent? I pause for a reply.

PS4. Necessities cannot be essential parts of explanations of contingencies. But it has been written that "universals . . . figure essentially in natural laws governing the behavior and composition of all particulars that instantiate those universals. . . . [I]t appears that numbers play an important role in natural laws" (Lowe 1995, pp. 520, 523). Natural laws certainly figure in explanations of

contingents and contingencies. Does it not follow that these necessary entities matter to, that they contribute to and figure in, explanations of contingencies after all? No. The 'figuring in' is different. While natural laws, which are themselves contingent, can figure in, by contributing to, explanations of contingents and contingencies, the same cannot be said of the relations of universals and numbers, and necessary truths about them, to natural laws. Universals and numbers figure in natural laws by way of these laws being *framed in terms of them*, not by way of contributing to their 'contents.' Michael Friedman writes that

> the mathematical background of [say] Einstein's [general] theory [of relativity] functions . . . as a means of representation or a language, as it were. . . . Indeed, as the mathematician Elie Cartan showed in the 1920s, we can . . . formulate Newtonian gravitation theory using variably curved space-time, just as in general relativity. From this point of view it is . . . crystal clear that the mathematical machinery with which the concept of curved space-time is formulated is **part of the means of representation** or language of general relativity and **not part of its empirical content**. For . . . the theories . . . which differ widely . . . in empirical content – are now formulated within the very same mathematical language. (Friedman 1997, p. 12; emphasis added)

For the same reason, the mathematics of this language, the necessary truths that are analytic in it, its 'mathematical/grammatical' structure, cannot be an *explanation* of either theory why it is true or not true. Since various universals and natural numbers exist and have the natures that they have necessarily and, no matter what, that they exist and have the natures that they have cannot contribute at all to explanations of the particular shapes of the natural laws in which they figure or matter to which possible natural laws actually hold sway in this contingent *cosmos* of ours. Choices of conceptual and mathematical frameworks can *facilitate articulations* of laws and other propositions that may or may not hold sway and be true, which laws and propositions must look for their reasons and evidence not to these frameworks, but elsewhere to this *cosmos*. (Cf., Friedman 1997, in particular the concluding paragraph, pp. 18–19.)

PS5. It is part of the essential character of 105 that it is the anti-log of 2.0212. In contrast, that 105 is my favorite number is not part of its essential character. Since it is not necessary that 105 is my favorite number – since in some possible worlds it is not my favorite number – that essential anti-log feature of 105 cannot help, in other than a connecting way, to explain why 105 is my favorite number. As it happens, it is not my favorite number in part because it is the anti-log of 2.0212. I noticed that about it only after a high-school pal and I made it our favorite number, and it would still be my favorite number even if I never noticed that about it. Furthermore, even if that feature of 105 were one of the things about it that made me favor it, it would not contribute essentially to an explanation of my adoption of 105. It would merely detail

a connection between explaining contingencies of personal psychology and educational experience, and such, and that adoption.

PS6. There may well be *no* reason and absolutely no explanation why there is something rather than nothing contingent. What is in any case clear is that, if there are reasons of any sort why there is something contingent, that is, why there is some contingent entity rather than nothing of that sort, these reasons need to be *contingent* reasons. They need to be contingent truths that are somehow relevant to this grandly minimal contingent truth concerning contingent existents. What is important for our subject is that the existence and natures of various *necessary* entities are necessarily 'neither here nor there' as far as contingencies are concerned, this great one as well as all smaller ones. Necessities cannot be parts, they cannot be *essential* parts, of reasons for contingencies. When made explicit as parts of an explanatory story they serve only to bring into plain view logical relations between substantive, which is to say, contingent, parts of the story to make clear, when this is not immediately obvious, how they connect and how the story works to explain them.[28]

APPENDIX A. LEIBNIZ'S PROBLEM WITH NECESSITY

A1 What, according to Leibniz, is the reason for the existence of the World? How in outline does his cosmology go? He says that the reasons provided by finite, changing, contingent things – the series of which may be beginningless and eternal – "would consist in the very prevailing of inclinations" and would not necessitate (p. 85) or fully explain. The full reason for the world can be found in its essence, in its nature, but not in the way in which the full reason for a necessary being can be. It is the unique privilege, Leibniz maintained, of a necessary being that its essence, its nature, should entail its existence without regard to any other existents. In contrast, the essences of contingent existents harbor their reasons, but only by way of their recommending themselves to the Necessary Maker, who, verifying by an exercise in Divine Mathematics that they constitute the greatest and most perfect aggregate of possibilities, freely chooses to realize them.[29]

A2 Has he given a sufficient reason? That is an outline of Leibniz's story. It is so far consistent with the world's being contingent. But is it the outline of a *sufficient* story? Is it an outline of a story that when elaborated could give rise to no questions? Not yet. For it let's ask *why* God chose to create the best? This was, Leibniz would insist, a free choice. He could have chosen to create less than the best. He could have chosen to create nothing. So, for an outline of a sufficient reason, there is needed an indication of the reason for His free choice to create the best, a reason that could either finish the story or be clearly on the way to finishing the story. Prospects for a satisfactory development of Leibniz's sketch into a 'major finished cinematic' are, however, not bright as

soon as the problem of God's motivation is raised and are nearly extinguished by Leibniz's own effort to deal with this problem.

> If anyone asks me why God has decided to create Adam [and the rest], I say, because he has decided to do the most perfect thing. If you ask me now why he has decided to do the most perfect thing, or why he decided to do the most perfect thing, or why he wills the most perfect . . . I reply that he has willed it freely, i.e., because he willed to. So he willed because he willed to will, and so on to infinity . . . (Grua, 302)[30]. (Curley 1972, p. 96)

Leibniz must have known that that beginningless infinite series of acts of will, even if allowed to provide internal explanations for each act of will, left as still reasonable, to his way of thinking, the questions: "Why this beginningless series of acts of good will? Why not a beginningless series of acts of malevolent will? Why any exercises of the Divine Will at all in relation to possibilities for finite contingent existents?" The regress of willings that Leibniz proposes would be a paradigm of a 'reason' that would get one nowhere, that would not stop, and a reason with which one could not stop. It is a joke not intended. Leibniz's own case against the sufficiency as an explanation of a story of infinitely receding contingent moving movers, can be adapted to embarrass his story of infinitely receding contingent divine decisions and acts of will:

> Although the [last free decision] . . . arises from [a] preceding [free decision], and that in turn from [a free decision] which preceded it, we do not get further however far we may go, for the same question always remains. The sufficient reason, therefore, which needs not further reason, must be outside of this series of [free decisions] . . . otherwise we should not yet have a sufficient reason with which to stop. (*Nature and Grace*, pp. 638–9)

A3 Leibniz's 'trilemma.' He needed to say of three propositions, each of which he was very much inclined at least at times to view as necessary, that they are not *all* necessary. He needed to say that, on pain otherwise of accepting that everything is necessary, that nothing is contingent. Now come, with brief comments, the three propositions of his problem with necessity.

A3.1 Proposition One. The essence of the World – the aggregate of existent finite things – is the greatest and most perfect, the very best, of all essences of aggregates of compossible finite things.[31] This proposition about the merits of the World's essence – as every proposition about the merits of the essence of a possibility – is, to Leibniz's way of thinking, necessary, though it is demonstrable and knowable *a priori* only by an infinite intellect. Cf. G. E. Moore's view of general principles of intrinsic value – for example, the aesthetic enjoyment of a really beautiful object is good – as strictly necessary, though not analytic. Proposition One, Leibniz might say, borrowing words from Aquinas, is self-evident in itself, but not to us (ST I q2,a1 p. 19). He could add to this Thomistic point that "the claim that [this] possible world is the best involves a

comparison with infinitely many other possible worlds" (Sleigh 1995, p. 428a) is self-evident only to an infinite mind that can, as it were, 'run through infinitely many comparisons and infinitely complex alternatives'. But what would be knowable *a priori* – what is "conceptually true" – to a mind, to *any* mind, would, contrary to Leibniz's sometimes view, seem to be "necessarily true" (Sleigh 1995, p. 427b). Furthermore, even if some conceptual truths, because ferreting them out would involve infinite analyses, would not be known to any mind, since no mind can actually run through infinite comparisons, they would still be, because conceptually true 'in themselves', necessarily true. "[T]he *doctrine of infinite analysis*, which Leibniz applied to ward off . . . threat[s] to contingency" (Sleigh 1995, p. 427b; cf., p. 428a) seems inadequate to that essentially nonepistemological task.

Proposition One is not about the World, but about the essence of the World. The words 'the world' operate in Proposition One only as a device for identifying a certain 'world-essence' or 'possible world'. Were Proposition One about the World, it would presuppose that the world exists, and so could not figure in a sufficient reason for the World. Also, Proposition One could not figure in a sufficient reason for the world, if this proposition all by itself entailed the World, if the World is contingent. For then Proposition One would be contingent and not qualified for inclusion in a sufficient reason for anything. Lastly, Proposition One could not figure, as it is supposed to do, in the sufficient reason for the World that Leibniz outlines, unless it would be conceptually true at least to God's mind – unless it were actually conceptually true to that mind. And so again, if it is to do its job in Leibniz's cosmology, he cannot apply to this proposition his infinite-only-convergent-on-truth evasion and so must say that it, without a doubt, is a necessarily true proposition.

A3.2 Proposition Two. God exists. This premise is, according to Leibniz, demonstrated by Descartes's ontological argument, prefaced by a proof of the possibility of a perfect being. This proposition, he maintained, is 'necessary even for us'.

A3.3 Proposition Three. ***God chose the best.*** Leibniz sometimes says that this proposition is only contingently true, that while "God is metaphysically perfect of necessity . . . he is . . . morally perfect [not] of necessity, but rather [only] by choice" (Sleigh 1995, p. 428b). Besides being a desperate and arbitrary discrimination amongst what would be God's perfections – God would be essentially omniscient and omnipotent, but *not* essentially *good* in his choices?! – this line regarding Proposition Three is not consistent with its inclusion in the sufficient reason for the World that is the centrepiece of Leibniz's cosmology.

Insisting, as Leibniz sometimes does, on the contingency of God's goodness saves him from the collapse of all contingency into necessity. But in making room for contingency in this way, Leibniz is left with only an insufficient,

most certainly *not* final, reason for the existence of the World. He is left only with a reason, if we deign to call it such, in which "the same question always remains" (*Nature and Grace*, p. 639), with the question, "Why did he choose to be good?", leading to the question, "Why did he choose to choose to be good?", and so on to infinity.

A4. Leibniz had a problem, for he had two horrors. He had a horror of BRUTE FACT, and he had a horror of UNIVERSAL NECESSITY. He wanted to deny the first without falling into the second. And so he ran into difficulty, for he *wanted desperately* to 'square a circle'. He wished for *sufficient* reasons for all *contingencies*, whereas *sufficient* reasons, by their natures, are not possible for *any* contingencies. He wished to ground all contingencies in necessities. But contingencies can be grounded, if at all, only in other contingencies, so that it is *impossible* to ground them all.

. . . . .

A note on 'complete individual concepts.' "You say that Leibniz had a horror of Universal Necessity. But did he not maintain that every simple subject-predicate truth is, since analytic, necessary? And would he not have supposed that all other truths are compounded of simple ones in manners that make them necessary too? Are you not forgetting his 'predicate-in-subject' theory of truth, and his theory of 'complete individual concepts'?" No. I remember these doctrines of Leibniz. What I say is that *they* do *not* commit him to simple subject-predicate truths being analytic and necessary. Consider, for example, the truth that Adam sinned. According to these sometimes views of Leibniz that truth is (he *could* have said) only 'in a way' analytic and necessary, it is only 'conditionally' analytic and necessary. For this proposition presupposes, it entails, that Adam *exists*, and existence was in Leibniz's view included in only one individual concept, namely, the concept of God. Every other existent exists, he supposed, only contingently. This means that subject-predicate truths about all other existents have contingent existential presuppositions and so are themselves contingent, though one can add, if one likes, that they are analytic and necessary *conditional* on the satisfaction of their existential presuppositions. We should say that Leibniz's predicate-in-subject theory of simple subject-predicate truths was, *for subjects other than God*, intended by him to cover all and only *nonexistential* subject-predicate truths.

An 'individual concept' of such a one as Adam would determine a 'possible Leibnizian world,' and so the individual concept of an *existent* one such as Adam includes, Leibniz would say, the *reason* for this individual's existence in the excellence of its world, which is the World – every existent's world. But the concept of an existent finite individual does not, he would say, include existence itself, any more than the concept or essence of the World includes its existence.

It is, Leibniz maintained, "the privilege of divinity alone" that assuming, that "*it is possible, [entails that it] exists*" (Leibnitz 1949, p. 504). The Divine Essence alone, he held, amongst individual essences includes existence. That is what he thought entailed that, given that it is possible for the Divine Essence to be realized (which Leibniz believed he had demonstrated, thus remedying a deficiency in Descartes's proof), it is necessary that it be realized. Leibniz never supposed that the strategy of the ontological argument was available to demonstrate the existence of any other existent individual. The complete concept of every existent individual other than God contains, Leibniz averred, not existence itself, but grounds or reasons for it from which its existence 'follows.' Somewhat similarly, G. E. Moore held that goodness, while not a part of good natures, follows from them. Only *somewhat*, because while Moore says that goodness 'follows' in the sense of *follows with logical necessity*, Leibniz wanted to say, to protect their contingency and, in the case of beings such as us, freedom, that existence follows for existents other than God only *contingently*, thanks to God's choice of the best, which choice was contingent and free.

Leibniz's sometimes view was not that the truth that *Adam sinned* was analytic and necessary, but that the truth that *if Adam existed, then Adam sinned* was analytic and necessary. He never thought that everything that is true is analytic and necessary, and he was always concerned not to 'back himself into' Spinozistic necessitarianism. But he realized that that conditional necessity concerning Adam might make certain important matters as bad as that universal necessity. For if it is necessary that Adam, if he is to *be*, should sin, how could *Adam* have been free not to sin?! There are possible Adams, 'counterparts' of Adam in other only 'possible worlds' who are exactly like him, until his moment of sin, but though tempted as they are, do not sin. Adam *himself* when tempted to sin does so in every 'possible world' in which he exists, for he exists in only this actual world, and in this world he is tempted and sins. How then *could* he, *Adam himself*, have done otherwise? Leibniz is made by Arnauld to grapple with this problem in correspondence in 1686 and 1687 about the *Discourse*, and he grappled with it again in the appendix to his *Theodicy* of 1710 when, after writing approvingly of Lorenzo de Falla's handling of challenges to freedom that can seem to be posed by Divine Omniscience, he moves to what he considered more difficult challenges made by the Divine Will. Leibniz does not in that appendix take on the analog for his necessarily existent God, of this problem with Adam himself sinning.

To ponder is that very little is heard from Leibniz between 1686 and 1710 explicitly or implicitly about his 'complete individual concepts' that include in the case of an existent individual absolutely everything nonexistential that is true of him and the doctrine, in other words, that everything true of an individual is *essentially* true of him and that nothing true of him is 'accidentally' true of him. To ponder more deeply is the question of what reasons – logical, semantical, metaphysical, or whatever – could have led Leibniz in 1686 to

embrace this very unintuitive doctrine according to which everything non-existential that is true of an existent is essentially true of him.

Thanks to Ingvar Johansson for objections that occasioned these notes on problems Leibniz's complete individual concepts did and did not make for him. They did not make every truth about any existent individual analytic and necessary for Leibniz, as long as he could maintain that not every existent individual exists necessarily. But they did threaten the freedom to choose otherwise of existent individuals whom he wanted to have this freedom. Indeed, unremarked by Leibniz, is that God's complete concept threatened this freedom and more. Given the necessity of God's existence, it threatened the *possibility* of His choosing otherwise and so, horror of horrors, the possibility of *anything's being otherwise*!

APPENDIX B. CONTINGENCY IN JOHN LESLIE'S AXIARCHISM

According to Leslie's Axiarchism, "the world's existence and detailed character are products of a directly active ethical necessity . . . which is – **as a matter not of logic but of fact** – enough to guarantee its existence" (Leslie 1970, p. 286; emphasis added). And so it can seem that his cosmology is consistent with the existence of the world and with its detailed character being contingent. But it is not so. While accommodating inexplicable facts, it leaves no room for contingent facts.

Leslie 'says' in the words "as a matter not of logic, but of fact" that, though it is necessary that the existence of the world is guaranteed by its being "as good as can be" (p. 286), this is not *analytic* or demonstrable *a priori*: "We have two . . . separable concepts, of things ethically marked out for existence and of things effectively marked out . . . but the reality which these concepts mirror . . . [is] one of *marking out for existence and with creative effectiveness* . . . of *creative ethical requiredness*" (pp. 292–3). The creative and ethical aspects of this requiredness are said to be distinguishable in thought (they answer to "separable concepts") but not in reality. Leslie's "suggestion [which is plainer in Leslie (1979, Chapter IV) than in Leslie (1970)] is," Mackie writes, "that there [is] a synthetic but **necessary** connection between . . . ethical requirement and creative requirement" (Mackie 1981, p. 233; emphasis added). We are told that there is between them "*a* **necessary** *relation* . . . which is still not *a logically necessary relation* if this means one which Logic can demonstrate" (Leslie 1979, p. 76; emphasis added). Mackie writes 'synthetic' for Leslie's 'not logically necessary'. Moore would say that the best world would be of a *natural character* that *made* it *as good as it is*, where this was *necessary but not analytic* 'production'. Leslie was saying that that a world is *best* would *make* it *real*, where *this* was necessary but not analytic production.[32]

Leslie's suggestion – this is my first point concerning his theory – is that there is a necessary, albeit synthetic, connection of the strictest kind whereby

the ethical requiredness of the world secures its existence. He characterizes this as 'the *inexplicable* component' of his theory: the core brute fact from which flows, for all facts of existence, an element of 'brutishness,' *epistemological* brutishness. To this first point can be added the second that, according to his theory, the world, given its detailed nature, does not merely happen to be as good as worlds get: This "value . . . [follows] from [that detailed nature] with absolute necessity" (Leslie 1970, pp. 289–90), as Moore would say the intrinsic value of anything must do. So Leslie holds that the existence and detailed character of the world is explained by two necessities, namely: (i) that the best world is made by its excellence to exist; and (ii) that a world of 'this nature', I here advert to the detailed nature of the world that exists, is best among possible worlds. But then his theory implies that the existence of this world 'just so' is necessary. His theory of 1970 says not only that the real world is "the best of all possible worlds" (p. 287).[33] It says – though there is not clear and conclusive evidence that Leslie noticed this – that the real world is *the only really possible world*! While other worlds are *thinkable without contradiction*, it is a theorem of his cosmology that *they are not really possible worlds*, for it is a *theorem deducible* from what would be *necessary truths* of his cosmology that *precisely this world is the real world, that precisely it exists*. Given that there is just one really possible world, every truth and every existent is *necessary* – that is, true or existent *in every possible world* – albeit inexplicably so, since the creative power of ethical requiredness is inexplicable. Every truth and every existent is necessary whether or not there is a creative being. And *that* – *INEXPLICABLE*, *UNIVERSAL NECESSITY* – would have been Leibniz's nearly worst nightmare. For his worst one we may add to Leslie's 'mix' the godlessness of it.

APPENDIX C. ROBERT C. KOONS'S 'NEW LOOK' COSMOLOGICAL ARGUMENT

There are many 'good bits between.' Two are mentioned in note 18. But Koons's innovative cosmological argument neither starts nor ends well.

C1 A start-up problem. The problem here is with Koons's ontology, with what principles of his argument would be about.

C1.1 'Just the facts.'[34] It is not for convenience that I do not distinguish between facts and true propositions, but because they are the same things. For this identity, consider a fact, any fact – consider the fact that you are not dreaming; the fact that five plus seven equals twelve; the fact that while every swan is a bird, not every swan is white; the fact that there are many kinds of facts; the fact that there is something rather than nothing – it does not matter what fact you consider. Let this fact you have in mind be F. I have in mind for expository convenience an atemporal fact about the numbers 105 and 2.0212. It is a fact that 2.0212 is the log to four places of 105. What here is a fact? To repeat, *that 2.0212 is the log of 105*. It is that *proposition* which I said is a fact, and as it

happens, I am right, it is a fact, for it is as a matter of fact *true* that 2.0212 is the log of 105. That is, since it is true *that 2.0212 is the log of 105*, or in these words rearranged, since *that 2.0212 is the log of 105* is true, this, the proposition *that 2.0212 is the log of 105*, is a fact. This *that 2.0212 is the anti-log of 105* is *at once* a fact and a true proposition.[35] It, this fact, which could have been any fact, is a true proposition. There is not merely 'correspondence' between facts and true propositions; there is identity. Were there merely a correspondence of facts and true propositions, it could be that 'facts are what make true propositions true'. Given that facts are true propositions, they cannot be what make them true. I am not being dogmatic, nor am I propounding a theory. I am merely – while using words, especially the words 'fact', true', and proposition, in perfectly ordinary ways – promulgating some 'home truths,' some platitudes. The main ones, summed up, are that, for every proposition *p*, *p* is true if and only if *p* is a fact, and that, for every proposition *p*, if *p* is true, then *p* is identical with the fact *p*,[36] and if *p* is a fact, then *p* is identical with the true proposition *p*.

C1.2 Koons's 'facts'. A start-up problem with Koons's 'new-look' cosmological argument is with its 'facts'. He says that they are things that "in natural language . . . [we] pick out . . . in the standard way (i.e., by using a complement 'the fact that' clause)" (Koons 1997, p. 196a). From this one gathers that they are supposed to be facts as ordinarily conceived. And yet we are told that "[t]here is [not even a] one-to-one correspondence between true propositions and facts" (p. 194b)[37] and that "[i]t is very important not to conflate [i.e., identify] *facts* with *true propositions*" (p. 195a). But, as I have recalled, in 'natural language' the prefixes 'it is true that' and 'it is a fact that' are exchangeable; facts are true propositions, and true propositions are facts. In 'natural language' facts are, contrary to Koons, *not* "things in the world that make certain propositions true and others false" (Ibid.). Also, Koons writes that he does not take for granted that there are necessary facts and writes that, if there are any, they are very special. But in fact there are necessary facts, as there are necessary truths, and these facts are no more special in their 'factic-ity' than they are in their truth-value. Necessary facts and truths differ from contingent ones exactly in their modalities.

Perhaps we should allow Koons to retract the identification of his facts with things that are facts in 'natural language' and to say that he bases his cosmological argument on a theory of 'facts' as conceived by some philosophers. Then it would not be obvious that he gets the facts of which he speaks wrong at nearly every turn. But he would still be off to a bad start with them, for while there is no question but that facts as ordinarily conceived exist, there is a question concerning these 'facts' of which Koons would speak with 'that'-referentials and of which he does not say enough to allow questions concerning their being and natures to be decided. Supposing more were said that allowed the question of being to be favorably decided,[38] we might wish to look into what these 'facts' have to do with *facts* and to think about the wisdom of using this name,

'fact,' for them. With that ground clearing behind us, we could proceed to the business of Koon's argument and to the principles it employs for these 'facts' of his which would be what in the world made certain propositions true, as well as "special . . . objects that can serve as relata for causal relations" once "a robust sense of reality leads us to recognize causal connections as first-class citizens of our ontological inventory" (p. 194). Since his 'facts' are not facts, that some of his principles about these 'facts' are expressed by sentences that can express platitudes about facts does not argue for these principles's being true of 'facts.'[39]

C2 A terminal problem. Bracketing 'start-up' problems of Koons's 'new-look' cosmological argument, there is a serious problem with its conclusion. He explains how, in his system, if there is a contingent 'fact,' then there is a wholly contingent 'fact' C, the Cosmos, that is "the aggregate of all wholly contingent facts" (Koons 1997, p. 198b). He also maintains that there is "excellent empirical evidence for [now comes his Universality principle] the generalization that wholly contingent facts have causes . . . [and] that, at the very least, our experience warrants adopting the causal principle [i.e., Universality] as a default or defeasible rule." "This means," he says, "that in the absence of evidence to the contrary, we may infer, about any particular wholly contingent fact, that it has a cause" (p. 196b; cf., Section 3 of Koons 2001). Koons's position on the principle of Separate Existence – if x is a cause of y, then x and y have no part in common (p. 196) – is evidently similar. "I am not claiming that the axioms of causality I am appealing to are known by us prior to their application to the world of experience" (p. 202b).

The penultimate conclusion of the argument is that, "[i]f there are any contingent facts, then C has a cause that is a necessary fact" (p. 199a). Suppressed, but needed for this conclusion in his '*defeasible logic*', is the premise that *there is an absence of evidence contrary to his axioms of causality*. Attending, as he does, to Universality while taking Separate Existence for granted, *the essential premise suppressed is that*

> no positive reason can be given for thinking that . . . [C, 'the cosmos'] is an exception to the rule, for example, by showing that . . . [it] belongs to a category of things that typically does not have a cause. (p. 197a)

That, this suppressed premise, is the problem with Koons's 'windup.' The problem is that it is not true. Two points suffice to this assessment. First, the evidence, whatever it is, for Universality (wholly contingents have causes) and Separate Existence (causes do not 'overlap' effects), which entail that *C has a cause that is a necessary fact*, is evidence just as good *against* that and for *Contingent Causes for Wholly Contingents*, according to which *wholly contingent facts have only* CONTINGENT *causes*. It is said that "we have excellent empirical evidence for [Universality] the generalization that wholly contingent facts have **causes**" and that "this means that, in the absence of evidence to the contrary, we may

infer, about any particular wholly contingent fact, that it has a **cause**" (p. 196b; emphasis added). It is taken for granted that we have excellent empirical evidence for Separate Existence as it concerns wholly contingents. Not noticed, however, is that *all* of our evidence for Universality, *causes* for all wholly contingents, is at the same time evidence for Contingent Causes for Wholly Contingents, against which we have no evidence, as we are said to have no evidence against universality. That makes it a member of the club of *defeasible* principles of causality. But then, when we come to the *grand* wholly contingent C, the aggregate of all wholly contingent facts of which every contingent fact is a part, these axioms of causality, Koons's Universality and Separate Existence, and this Principle of Contingent Causes for Contingents, these '*defeasible* principles of causation' are at odds with one another and so are as a group '*defeased*'. More plainly put, the evidence regarding the causation of the 'fact' C – whether it has a cause and, if so, whether its cause is separate from it and a necessary 'fact' – is so far as the excellent evidence for these three principles is concerned at best 'a wash.'

Second, since C would be a wholly contingent 'fact', its cause, if it had one, would, by Separate Existence, be a necessary 'fact'. But then, since it is a necessary 'fact', it would obtain no matter what and so *seem* not to be of possible *matter* to any 'fact' that is contingent or a *possible cause* of C. To put words of Aquinas's to a purpose of which he would not approve, 'it seems not just that there is no case *known* to us in which something necessary is the cause of something contingent, but neither is it, indeed, *possible*.' Koons finds plausible "that all **effects** are contingent" (p. 199a; emphasis added). Certainly there is no case known to us in which something necessary is an 'effect' naturally and ordinarily so-termed. It is as plausible that all *causes* are contingent, and it is *very* plausible that all causes of contingents are contingent. Koons notices the question – "*Don't contingent facts have [to have] contingent causes?*" – and says that it begins what "is probably the most promising line of rebuttal to the cosmological argument" (p. 205a). I think he is right about that.

Koons's response to my first point would be that there is a reason for thinking that the grand contingency C is an exception to the Contingent Causes for Wholly Contingents and that *it* of the three principles is alone defeased. This reason would be based on the idea "that, in some precise sense, a cause is always *more necessary or less contingent* that its effect"(p. 205a; cf., Section 5 of Koons 2001) and an explanation of why no contingent fact can, in the relevant precise sense whatever it is, be more necessary or less contingent than C. Not knowing or understanding this sense, being in fact suspicious of the 'more necessary' and 'less contingent', to play out my hand in his game, I lay down my second point as a *decisive* reason (since it seems so to me) for thinking that C is an exception to Koons's Universality and Separate Existence combined for wholly contingents. It is, of course, his Universality that I think fails for wholly contingent C.

VII

Look 'Round!
Arguments from Design

Poems are made by fools like me,
But only God can make a tree.
Joyce Kilmer

What had that flower to do with being white,
The wayside blue and innocent heal-all?
What brought the kindred spider to that height,
Then steered the white moth thither in the night?
What but design of darkness to appall?–
If design govern in a thing so small.
Robert Frost

David Hume's great book, *Dialogues Concerning Natural Religion*, is a fictional report, by Pamphilus to Hermippus, of a conversation of his tutor, Cleanthes, with Demea and Philo on matters pertaining to religion. The main subject is a certain argument, a teleological argument, for the existence of God. Hume's masterfully orchestrated examination of this argument ends with a long rhetorical question and summing up by Philo, who holds that far less than Cleanthes sought can be concluded from facts of apparent design in nature, but that some sort of designer-deity, albeit not a humanly relevant one, can be reasonably inferred. Demea withdraws from the conversation after Philo has drawn it through facts of apparently unnecessary evil and before Cleanthes and Philo have given final statements of their positions, which agree on some points and disagree on others.

Section 1 to follow states the argument, considers its character, and compares it with cosmological arguments. Section 2 dispatches a certain radical objection to the argument and notes several factors relevant to its assessment. Precise Bayesian explications of these factors are developed in Sections 3 and 4. In Section 5, Hume's discussion and low assessment of Cleanthes' conclusion from the facts is reviewed in terms of these factors. Section 6 is about

what Hume was inclined to conclude himself from the facts, certainly not the Designer of Cleanthes', but nevertheless a designer. It is explained how Hume could have felt that that much was sufficiently evidenced to make belief reasonable, so that natural theology resolved itself into a theism of sorts. Sections 7 and 8 argue from the vantage point of present evidence and theories unavailable to Hume that the facts now fail to support any sort of designer and that today natural theology resolves itself into nothing. Richard Swinburne's argument from design is studied in an appendix.

1. THE ARGUMENT OF THE *DIALOGUES* – FIRST STATEMENT

Cleanthes: Look 'round the world: Contemplate the whole and every part of it: You will find it to be nothing but one great machine, subdivided into an infinite number of lesser machines, which again admit of subdivisions to a degree beyond what human senses and faculties can trace and explain. All these various machines, and even their most minute parts, are adjusted to each other with an accuracy which ravishes into admiration all men who have ever contemplated them. The curious adapting of means to ends, throughout all nature, resembles exactly, though it much exceeds, the productions of human contrivance; of human design, thought, wisdom, and intelligence. Since therefore the effects resemble each other, we are led to infer, by all the rules of analogy, that the causes also resemble, and that the Author of Nature is somewhat similar to the mind of man, though possessed of much larger faculties, proportioned to the grandeur of the work, which he has executed. By this argument *a posteriori*, and by this argument alone, do we prove at once existence of a Deity, and his similarity to human mind and intelligence. (Hume 1991, Part 2, p. 109. All page references, unless otherwise indicated, are to this edition of the *Dialogues*.)

Cleanthes is responding in the last sentence to Philo's suggestion that the question of the *being* of God is settled: "Nothing exists without a cause, and the original cause of this universe (**whatever it be**) we call God; and **piously** ascribe to him every species of perfection. . . . But let us beware, lest we think, that our ideas [of perfections] correspond to his perfections. . . . " (Hume 1991, p. 109; emphasis added). Cleanthes could have said that we cannot call the original cause God, *whatever* it be, but only if we can satisfy ourselves that it *merits* this name and is *worshipful*. The questions of the existence and nature of God cannot be separated completely. Nothing can count as a proof of the existence of *God* that does deliver reasons for the existence of a worshipful being.[1]

Philo explains the argument for Demea's benefit in Part 2, pages 111–12. Cleanthes elaborates in Part 3, pages 117–18. And Philo and Cleanthes cooperate in an expanded statement in Part 12, pages 172–5. The world, it is said there, evidently 'makes sense' to us, it makes a certain kind of very extensive sense that bespeaks in the clearest language an intelligent purposive maker.

Philo: A purpose, an intention, or design strikes everywhere the most careless, the most stupid thinker. . . . an anatomist, who had observed a new organ or canal, would never be satisfied till he had also discovered its use and intention. . . . Supposing there

were a God, who did not discover himself immediately to our senses; were it possible for him to give stronger proofs of his existence, than what appear on the whole face of nature? (Part 12, pp. 172, 173)

1.1. Cleanthes' argument enjoyed the authority of Isaac Newton

[T]hough these bodies [planets and comets] may . . . continue in their orbits by the mere laws of gravity, yet they could by no means have at first derived the regular position of the orbits themselves from those laws. . . . it is not to be conceived that mere mechanical causes could give birth to so many regular motions, since comets range over all parts of the heavens in very eccentric orbits. . . . This most beautiful system of the sun, planet and comets could only proceed from the counsel and dominion of an intelligent and powerful being. . . . We know him only by his most wise and excellent contrivances of things and final causes. . . . Blind metaphysical necessity, which is certainly the same always and everywhere, could produce no variety of things. (Newton 1953, pp. 41, 42, 44)

Can it be by accident that all birds, beasts, and men have their right side and left side alike shaped. . . . Whence arises this uniformity in . . . outward shapes but from the counsel and contrivance of an Author? Whence is it that the eyes of all sorts of living creatures are transparent to the very bottom, and the only transparent members of the body, having on the outside a hard transparent skin and within transparent humors, with a crystalline lens in the middle and a pupil before the lens, all of them so finely shaped and fitted for vision that no artist can mend them? Did blind chance know that there was light and what was its refraction, and fit the eyes of all creatures after the most curious manner to make use of it? These and suchlike considerations always have and ever will prevail with mankind to believe that there is a Being who made all things and has all things in his power. (Newton 1953, pp. 65–6)

1.2. Cleanthes argues from certain facts, to a theory – a hypothesis – that would explain them. The evidence that impresses is everywhere 'round. The hypothesis is 'creationist' for life forms and features God as elaborated by classical theologians. The argument simply stated is, as said, from evidence to an explanatory hypothesis.

Appearances of deliberate design in nature
Facts of inorganic order, mainly dynamic – for example, the solar system and the hydrogen atom. Facts of organic order, mainly functional – we find in nature purposive machines within machines adjusted to one another, a most "curious adapting of means ends" (Part 2, p. 109) – for example, the human eye (Part 3, p. 119), the correspondence of male and female parts and instincts of any species (Ibid.), and the whole human body (Part 12, p. 173).

Therefore, very probably, The Religious Hypothesis
According to this hypothesis the order and appearances of design in nature are the work of God, more or less as traditionally conceived. According to the hypothesis, this God went about the work of ordering nature and designing its

functional parts somewhat in the manner of a watch-maker or builder, 'hands-on' as it were (but of course not literally), making first progenitors for the species, arranging the stars and the planets, 'starting up' the latter for their perpetual revolutions about the sun, spinning the earth for days and nights, and so on.

1.3 It is not an 'analogical argument' but an argument for an analogical hypothesis. That Cleanthes' hypothesis is 'analogical', that it is recommended by a certain analogy in accordance with the grand principle that for like effects one expects like causes, is incidental to the argument. Cleanthes' hypothesis competes in the *Dialogues* not only with other analogical hypotheses, but also with hypotheses that do not trade on analogies. An example of the latter is the 'new hypothesis' that Philo elaborates in Part 8. The 'logic' of the argument is identified by Wesley Salmon.

> As Cleanthes presents the design argument, it is an . . . argument by analogy. . . . [Similarly] the use of . . . experiments on animals in order to ascertain the effects of various substances on humans constitutes [it is often said] an important application of analogical reasoning. . . . [But] these arguments, initially characterized as analogies, are more subtle and complex. They are arguments whose function is to evaluate *causal hypotheses*. . . . Hume was aware of this fact, I believe. If we look at the various facets of the discussion . . . , we shall find the main constituents of just such [subtle and complex] arguments. (Salmon 1978, p. 145)
>
> Although the argument of the *Dialogues* is not cast in formal terms, Hume showed a full appreciation of the . . . types of considerations which must be brought to bear in order to evaluate the theistic causal hypothesis. . . . he certainly recognize[d] it as something deeper and more subtle than a simple appeal to analogy. (p. 148)

Just so. Cleanthes at first appeals to "all the rules of analogy" (Part 2, p. 109) but soon sees that they do not serve well the argument he wishes to make. Rather than taking this as a reason for giving up on his argument, he quite sensibly takes it as a reason for giving up on their relevance to it: "[I]f [this] argument for theism be . . . contradictory to the principles of [that] logic: its universal, its irresistible influence proves clearly, that there may be **arguments of [an] . . . irregular nature**" (Part 3, p. 119; emphasis added). It is on these terms that the discussion proceeds. Good sense is displayed, and sensitivity 'to all the rules of causal speculation,' without any effort to make these rules explicit. This line to the irregular nature of Cleanthes's argument was an addition to the original manuscript of 1751, probably (this is according to Norman Kemp Smith) made before 1761, and thus before the publication of Thomas Bayes's two essays in 1763, and *long* before first statements by Laplace of forms of Bayes's Theorem relevant (as will be explained) to sundry causal speculations. (See note 4 to Chapter VIII.) Neither Hume nor anyone else of his day was in a *position* to articulate, as Laplace was to do, 'the logic of causal speculation' and of arguments from facts to explanatory hypotheses, though sensible discussants have always practiced this logic. For, in Laplace's words,

"the theory of probability [including centrally forms of Bayes's Theorem] is really only common sense reduced to calculus" (Laplace 1917, p. 196).

1.4 Teleological and cosmological arguments. They are both from facts to explanations, but different facts, to very different kinds of explanations, and reached by different kinds of inference.

Cosmological Arguments	Teleological Arguments
Facts cited	
Hardly remarkable facts such as that there is something rather than nothing.	*Striking facts of details* such as that there is 'this' order and 'these' appearances of deliberate design.
Explanations sought	
Extraordinary explanations, complete and final explanations. This at any rate is what Leibniz, 'The Cosmologist,' is after.	*Ordinary explanations* of the kind produced in courts of law, everyday life, and science. Always open-ended explanations. Never complete or final.

When Philo objects to the argument that "[w]e are still obliged to mount higher, in order to find the cause of this cause" (Part 4, p. 124), he asks rhetorically, "How can we satisfy ourselves without going on *in infinitum*?" (Part 4, p. 125). Cleanthes responds in rhetorical kind with the question: "Even in common life, if I assign a cause for any event; is it any objection, Philo, that I cannot assign the cause of that cause, and answer every new question, which may incessantly be started?" (Part 4, p. 126). Cleanthes declares that his purposes 'oblige him to mount higher': "You ask me, what is the cause of this cause? I know not; I care not; that concerns not me. I have found a Deity; and here I stop my enquiry" (Part 4, p. 127). Compare a prosecutor who is satisfied that a defendant's guilt has been established and leaves to others questions of motives. Compare a legislator who is satisfied that smoking tends to cause cancer and leaves to others questions of mechanisms.

Beings featured	
Necessary beings for reasons for the existent of contingents.	*Intelligent beings* for explanations of appearances of designed order and planned purposiveness.
Inferences	
Deductive – premises are said to entail necessary beings responsible for the world.	*Inductive* – facts are said to *make probable* that intelligence had imposed order and design.

2. ON ASSESSING ARGUMENTS FOR CAUSAL EXPLANATIONS

2.1 Dispatching a radical would-be conversation-stopper. As in science and in courts of law, when theories are put forward that would explain certain evidence, it is necessary to engage in a "particular discussion of the evidence" (Part 1, p. 103) in order that, 'assent may be proportioned to it' ("Of Miracles," Hume 1902, p. 110). Most of the *Dialogues* is such discussion designed to assess the Religious Hypothesis. Some space, however, is devoted to the possibility that we may not be qualified to engage in probable speculations regarding the origin of the world and that there is thus no occasion here for particular discussion of the evidence. Philo objects, as if to put a quick end to the debate, that "[e]xperience alone can point out . . . the true cause of any phenomenon" and that the origin of the world is "single, individual, without parallel, or specific resemblance" (Part 2, pp. 111, 115). He is, however, said to have seemed "between jest and earnest" and to stop short "when he observed some signs of impatience in Cleanthes" (p. 115). He "was a little embarrassed and confounded" (Part 3, p. 120) after Cleanthes had elaborated on the common sense that *appearances* of design *of course* bespeak a *designer*. No one would otherwise *describe* them as 'appearances of design'! Against Philo's brief radical opposition to empirical cosmogonies, it is certainly false that to reason responsibly about the origin of things, whether they be shoes, cars, watches, stars, or worlds, "it were requisite, that we had experience of the origin" (Part 2, p. 115) of these things. Experience of the origins of *other* things, indeed, experience of the ways of nature without particular regard to *origins* of any things of any kind, can be sufficient. Suppose I had never seen anybody *make* anything at all, but that I had seen people work with nuts and bolts and springs, and even done some of that myself, either without purpose or to repair and make again functional things. Then even though I had never seen anyone make anything, I could be well-placed to explain how a particular watch might have come to be. From experience I would know how such bits can be fitted together and how they work when fitted together. I would understand how some person could make a watch, and I might know why someone would want to make one. Similarly for stars. I could, from observing other stuff, have ideas about how they would make stars if brought together, and why someone who could would want to make them. Experience with mundane matter can lead to theories of matter in terms of which such events never witnessed can be explained. Indeed witnessing the birth of stars, over and over, might merely compound an observer's amazement. Similarly, though with greater stretches of creative imagination, for the evolution of the universe from a singular explosive point, and similarly for the establishment and evolution of life forms. We know that these things are subjects of possibly responsible reasoned investigation, because they have been and are the subjects of such investigations.[2]

2.2 Particular discussions of evidence

The order and arrangement of Nature, the curious adjustment of final causes, the plain use and intention of every part and organ; all these bespeak in the clearest language an intelligent cause or author. (Part 4, p. 127)

The Religious Hypothesis would explain facts of order and apparent design in nature. Cleanthes maintains that it does so very well. An issue is whether this is so. But this is not the main issue that his advocacy of the Religious Hypothesis raises, which is how it fares not merely as an explanation of the particular facts to which he adverts, but given all facts are relevant to it. To say, we want to compare it with other possible explanations, for which comparisons it can be helpful to consider separately what would have been its and their *powers as predictors* of facts they would explain, and its and their *intrinsic plausibilities*, these facts aside.

2.2.1 Total evidence. It is important, when assessing an explanation, that one consider not merely the facts to which it is first addressed, but all available facts that are relevant to its explanatory goodness, and so very possibly facts additional to those friendly to it that are marshalled by its advocates. The issue for Cleanthes's hypothesis is not merely how well it explains the facts he cites that can speak so clearly for it. In appealing as he does to facts in an entirely commonsensical way, he invites in other undisputed facts insofar as they are relevant. A prosecutor when arguing for a conviction on grounds that a defendant had the *means*, for example, a gun of the right caliber, and a *motive*, for example, prospects of an inheritance, must listen to defense counsel's argument that his client lacked the *opportunity*, for example, because he was at the time in another county. Philo does in the end concede that facts of order and apparent design tend to confirm Cleanthes' hypothesis, but Philo goes to considerable lengths in Parts 10 and 11 to bring out that by no means *all* the facts recommend it.

2.2.2 Alternative hypotheses. What Cleanthes needs to say is not merely that the Religious Hypothesis explains facts assembled, including facts that trouble it, but that it is the best explanation for them. Only then can the facts it explains recommend it for belief. For that it is not enough that they confirm it somewhat.

2.2.3 Imagined predictive powers and intrinsic plausibilities. When assessing the Religious Hypothesis, when determining how good an explanation it is of assembled facts, after lining up available competitors, it can help to *factor* its and their prowesses as explanations of these facts. How good a hypothesis is as an explanation of certain facts depends on its merits in two dimensions. First, an explanation is more or less good depending on its intrinsic plausibility, never mind the facts that it might be used to explain or needs to confront. That a

friend has a hangover would in this dimension be a rather bad explanation of his red eyes, if you know him to be a teetotaller. Second, a possible explanation of certain facts is more or less good depending on what would have been its predictive power for them, which is to say their likelihood supposing it, at a time when we were ignorant of them. Philo probes this dimension of the goodness of the Religious Hypothesis as an explanation of how the world came to be as we find it when he asks, "Is the world considered in general and as it appears to us in this life, different from what a man or such a limited being would, *beforehand*, expect from a very powerful, wise, and benevolent Deity?" (Part 11, p. 163). It is clear that the hangover explanation of your friend's condition, though bad in the first dimension, is very good in this second one.

Putting together these two dimensions of the goodness of an explanation for certain facts, it is plausible that the goodness of an explanation for certain facts – *in a sense that comprehends precisely what is relevant to the support provided for it by these facts* – is 'composed' of its strengths in the two dimensions of intrinsic plausibility when considered apart from these facts, and what would have been its predictive power for these facts. How much support, if any, certain facts provide for a particular possible explanation of them depends on how good an explanation in this factored sense *it* is of the facts, as compared with how good in this sense alternative explanations of these facts are. Regarding it and its competitors, everything will depend for one thing on its and their comparative intrinsic plausibilities these facts aside, and for a second thing on its and their comparative would-have-had predictive powers for these facts.

2.2.4 'Background information.' 'All the facts that they – some hypothesis and hypotheses alternative to it – would explain, or must confront' will in general *not* be all the facts that are *relevant* to the merits of members of a band of hypotheses, *that is*, to their would-have-been predictive powers and intrinsic plausibilities for the facts marshalled 'as evidence'. Other relevant facts will, however, typically not be marshalled in advance, but noticed and taken into account as these assessments in those two dimensions are conducted.

2.3 Confirming and disconfirming facts – that they need not *be striking or especially improbable.* Humdrum facts – a spot of gravy on a tie, a matchbook in a drawer – can take on the aspect of really potent forensic evidence for or against a theory of a crime. Such facts can, in the context of an investigation, be real clinchers for one theory of the crime over another. When they are, no one will say they are 'humdrum'. They are then perceived as 'significant', striking, or remarkable, which one may gather are not characteristics of facts in themselves, but only of facts in relation to explanatory theories of interest that invest them with significance. (Cf., van Inwagen 1994, p. 136.) What is important is, contrary to Swinburne, not that a fact "cries out for explanation . . . [or

that] it is . . . too striking to occur unexplained" (Swinburne 1994a, p. 50a), but that a possible explanation makes it 'striking'.[3]

Suppose that, as hands are being dealt in a bridge game, I am called to the telephone, and that on returning to the table I find for me a perfect hand. It would be no objection to my conviction that the hand was set up, that, on the supposition of a fair deal, it is no more improbable than any other specific hand. It is similarly not an objection to the argument from design that (Philo), "[w]ere a man to abstract from everything which he knows or has seen . . . every chimera of his fancy would be upon an equal footing" (Part 2, p. 111). What is important about the perfect bridge hand is that there are theories at hand of not inconsiderable initial plausibility that would with some force have predicted it. Supposing that my joking friends have made up a hand for me would make likely that I should find myself with a particularly good hand, perhaps a perfect hand. According to Peter van Inwagen,"if one can think of a possible explanation [for a certain fact – e.g., a perfect bridge hand] . . . , then it is wrong to say that that event **stands in no more need of an explanation** than an otherwise similar event [e.g., a hand of no particular value or disvalue] for which no such explanation is available" (van Inwagen 1994, p. 135; emphasis added). His point is that, when assessing competing theories, a fact that would be well explained by one of them is relative to its competitors in need of explanation. His point is the other side of mine, which is that what matters when assessing the significance of some fact for competing explanatory hypotheses – such as that a hand was made or that the deal was fair – is not whether or not this fact is in itself striking or improbable, but only whether or not there is a hypothesis in the pact of nonnegligible intrinsic plausibility, supposing which, this fact would have been rather likely.[4]

2.4. We could now proceed directly to Hume's 'particular discussion of the evidence' for and against Cleanthes's Religious Hypothesis. But I will digress to present formal explications of the ideas of explanatory goodness, inherent plausibilities, and would-have-had predictive powers, explications in terms of probabilities and conditional probabilities, the theory of which was taking shape in Hume's day, to reach its modern form in works of Laplace. A form of this theorem, a form that Laplace stated, says of things relevant to assessments of degrees of support that facts provide various possible explanations, *how* they are relevant. It makes plain precisely how intrinsic plausibilities and would-have-had predictive powers enter into the overall goodnesses of explanations for certain facts. The concepts of probability and conditional probability explained and related to ideas of evidence and confirmation in the next two section are important to coming discussions, but mathematical and logical details given regarding them, though elementary, are not. Thus these sections are blocked off as material that can be mined for main ideas without loss of continuity or substance.

3. PROBABILITIES, PLAIN AND CONDITIONAL

3.1 Probabilities. Here the ***probability*** of a proposition p – P(p) – for a person is a number that measures his *degree of confidence* for this proposition. If he is more confident of p than of q, then P(p) is greater than P(q). If he is precisely as confident of p as of q, then P(p) equals P(q). If he is certain of p, then P(p) is 1, and if he is sure that p is false, then P(p) is 0. In other cases P(p) is between 0 and 1.

3.2 Conditional probabilities. Of particular importance for us are certain relations amongst a person's probabilities for propositions, specifically relations between his probabilities for conjunctions and his probabilities for their conjuncts. These ratios are termed 'conditional probabilities.'

Suppose you are sure that initially there were marbles of different colors and textures distributed in barrels thus:

Barrel A	Barrel B	Barrel C
1 white 8 smooth yellow 1 rough yellow	4 white 4 smooth yellow 2 rough yellow	6 white 4 rough yellow

You know that I have drawn a marble from a barrel, and are sure that the draw, from whichever barrel it was made, was made in a random manner. You think that the draw *may* have been from barrel A. Assuming the abbreviations – Y: the marble drawn is yellow; A: the marble drawn was drawn from barrel A – your probabilities P(A & Y) and P(A) are related like this:

$$P(A \,\&\, Y) = 9/10 \cdot P(A)$$

How do we know? We have not said what is your probability for A, save that it is greater than zero. And, not knowing your probability for A, we do not know your probability for (A & Y). How is it that the relation for possible values of these probabilities is even so clear? We know that your confidence, whatever it is, in A consists in part of your confidence in (A & Y) and for the rest of your confidence in (A & W). *Indeed*, recalling that you are sure that the draw, from whichever barrel it was made, was made in a random manner, your confidence in A should be composed of nine-tenths of your confidence in (A & Y) and one-tenth of your confidence in (A & W). Which is to say that your probability for (A & Y) should be nine-tenths of your probability for A. If you think that I would not concentrate on barrel A, were it not the source barrel. If for that or some other reason you are sure that A, then for you $P(A) = 1$ and $P(A \,\&\, Y) = 9/10$. If you think that, before drawing randomly from a

barrel, I randomly selected a barrel from which to draw, then P(A) = 1/3 and P(A & Y) = (9/10)(1/3) = 3/10. But, whatever your views about, and consequent confidence in A, and thus in (A & Y), the *ratio* of the latter confidence to the former must for the clear thinker that you are be [dividing both sides of the displayed identity by P(A)]

$$P(A \,\&\, Y)/P(A) = \mathbf{9/10}.$$

That ratio is the ***conditional probability*** of Y given A, for which number we have the compact term '***P(Y/A)***'.

3.3 But why go on about this ratio? Why name it, and design a term for it? What is its significance? It has significance for the adjustment of your probability for Y upon learning A for sure. It has significance for the difference that this learning should make in your confidence for Y. It has significance for the *potential evidential bearing* of A on Y. Now comes substantiation, first in a discursive illustration and then in graphics.

3.3.1 Back to the marbles to illustrate the significance of ratios that are conditional probabilities. Suppose that for you it is as likely that the draw was made from one barrel as from another. Then your probability for the drawn marble's being yellow, P(Y), should be 19/30, for there are 30 marbles in all, and of these 19 are yellow. Now consider what would be your response – consider how you would adjust this probability – on learning that the draw was made from barrel A. Learning this without in the process changing your view of its relevance to Y, your new probability for Y, $P_n(Y)$, should be the same as the ratio of old probabilities $P_o(A \,\&\, Y)/P_o(A)$ – that is, your old conditional probability $P_o(Y/A)$ – which was 9/10 or 27/30, 'odd cases aside'. Think about it. Recall that *had* you been sure that A, your probability for Y would have been 9/10. When and if you *learn A for sure*, and nothing else of independent relevance to Y,[5] so that A's bearing on Y is not changed, odd cases aside, your probability for Y should be 9/10. The difference, $[P_o(Y/A) - P_o(Y)] = 8/30$, is the difference that learning A should make to your probability for Y in these circumstances. It is a 'measure', one wants to say, of the evidence that the possible news that the marble was drawn from barrel A harbors for you that the marble drawn was yellow. ['*Odd cases set aside are of two kinds*. First there are case in which, when learning A, you change your opinions regarding the setup, for example, your opinion regarding the randomness of the draw of the marble, or your opinions regarding the numbers and kinds of marbles in the barrels. *In these cases your opinion of the relevance of A to Y will have changed*. Second, there are cases in which, though your new-found certainty in A, that the marble was drawn from barrel A, is a *bona fide* case of learning, you suspect that it is not, and that it is, for the example, a product of

dyspepsia, drugs, hypnosis, subliminal influences, deception and misplaced trust, and so on. In these cases you may, reflecting on your credences, be prepared to allow them to be 'incoherent.']⁶

The significance of the ratio $P_o(A \& Y)/P_o(A)$ in this case is that of such ratios in all cases. It is because of this significance, this importance, that such ratios have a special name, 'conditional probability,' and a compact notation, '$P_o(Y/A)$'. This significance is – now comes my version of **the Principle of Conditionalization** – that if (odd cases aside) a person learns p for sure and in the process its possible evidential bearing on q does not change, so that his new probability for q conditional p, $P_n(q/p)$, equals his old one, $P_o(q/p)$, then his new probability for q, $P_n(q)$, should equal his old probability for q conditional on p, $P_o(q/p)$.⁷ This principle does not require that p be all that the person has learned for sure, that is, that it is "the strongest . . . proposition" he has learned for sure (Earman 1992, p. 34; cf., p. 239n11) or that it should 'fully capture the content of a learning experience'.⁸ It is enough, for this principle, that when p is learned for sure nothing else of independent relevance to q is learned, for then p's relevance, its evidential bearing on q, should (odd cases aside again) not change (cf., Sobel 1997a). *Given* this principle for updating probabilities on certain learning, a person's conditional probability P(q/p) at a time is a natural measure of *the evidential relevance of p to q* for him at this time. The difference [P(q/p) − P(q)] is a natural measure of *the potential evidential bearing of p on q* for him at this time. And P(q/p), again, is thus a natural measure of *the possible evidential bearing of p on q* for him at this time, since it is, for him at that time, the maximum possible potential bearing of p on q, given the evidential relevance, for him at that time of p to q. Given this relevance, P(q/p), the limit of the difference [P(q/p) − P(q)] as P(q) goes towards zero is, of course, P(q/p).

3.3.2 The significance of 'conditional probabilities', graphically demonstrated. Suppose that p is learned for sure and that nothing else of independent relevance to either q or r is learned. Then, while the relation between the probabilities of q and r may well change, relations between the probabilities of p, (p & r), (p & q), (p & q & r), and (p & ∼q & ∼r) should *not* change. For these relations between the probabilities to change, more of relevance to q and r independent of p's relevance would need to have been learned. If relations of those probabilities are *not* changed as that of p is elevated to 1, then simple probabilities for q and r are 'updated' to their initial conditional-on-p probabilities. Now comes a *graphic* demonstration of this conditional.

Let positively probable worlds correspond to points in the unshaded part of a rectangle, and let positively probable worlds in which a proposition is true correspond to points in the unshaded area assigned to this proposition. Suppose that the ratio of a proposition's unshaded area

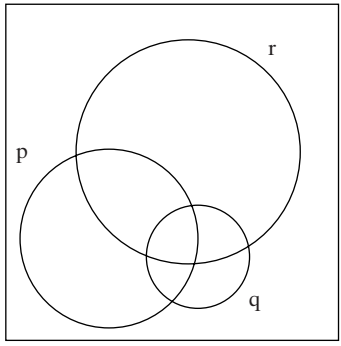


FIGURE 1

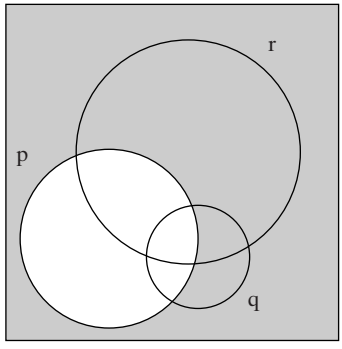


FIGURE 2

to the total unshaded area of the rectangle is the proposition's probability. Probabilities of p, q, and r are represented by the areas of their circles in Figure 1 in which there is no shading. Probabilities of their truth-functional compounds are represented by other areas, for example, that of ∼p is represented by the area outside of p's circle, and that of (p & q) by the area common to the circles of p and q. Consider areas for the probabilities of propositions p, (p & r), (p & q), (p & q & r), and (p & ∼q & ∼r). Note their relative sizes. Now suppose that p is learned for sure so that all positively probable worlds correspond to points in its area. To record this we shade the area outside the area assigned to p: That is Condition A. Suppose furthermore that nothing of relevance to r and q independent of the relevance of p is learned, so that in relation to r and q, the proposition p is all that is learned. To record this in the shaded diagram in Figure 2 produced to satisfy Condition A, we want the sizes of the areas of the five propositions to which we attended in the previous paragraph to be related one to another as they were in the unshaded diagram in Figure 1: That is Condition B. Observe that in satisfying Condition A, we have satisfied Condition B. "The thought is that after one

learns that ***p*** is true, credence in the shaded region . . . drops to 0, and the ***p*** circle becomes the new unit area. But nothing else changes; in particular, relative sizes of subregions of the ***p*** circle remain the same . . . as before" (Jeffrey 1981, p. 82). That is 'the thought' and the effect when, as we are assuming (and as Jeffrey was without saying so), p is all that is learned that is relevant to r and q, so that its possible evidential bearing on these propositions does not change.

Consider now the new unshaded areas of q and r, that is, consider ***the new probabilities P′(q) and P′(r)***. Note that they equal, respectively: (i) the new probabilities, P′(p & q) and P′(p & r); (ii) the ratios of the new probabilities P′(p & q)/ P′(p) and P′(p & r)/ P′(p), or, in other words, the new conditional probabilities, P′(q/p) and P′(r/p); *and* (iii) the ratios of the old probabilities P(p & q)/P(p) and P(p & r)/ P′(p), or, in other words, ***the old conditional probabilities, P(q/p) and P(r/p)***. Which is what was to be graphically demonstrated. As a dividend of this graphic demonstration, (ii) and (iii) substantiate that when a proposition p is learned for sure, and nothing else is learned that is of independent relevance to q, then, odd cases aside, the probability of q conditional on p does not change.[9]

3.4 Those barrels again. Left so far undetermined are your initial probabilities for the marble being drawn from one or another barrel. For these we suppose not that you think that I chose a barrel at random, but that you think I rolled a fair die, and if it came up either with one dot or two dots, drew from barrel A; if it came up with three dots, drew from barrel B; and otherwise drew from barrel C. So your initial probabilities for my drawing from one or another barrel are

$$P(A) = 2/6 \qquad P(B) = 1/6 \qquad P(C) = 3/6.$$

Suppose that I tell you that the marble drawn is yellow, and you believe me, so that you are now sure of that, that the marble draw was yellow: P(Y) = 1. The *question* is how confident are you *now* that I drew this marble from barrel A. Your new probability for A should be your initial conditional probability for A on Y,

$$\boldsymbol{P(A/Y)},$$

for you have learned that the marble I drew was yellow, and we assume nothing else that is of independent relevance to the issue from which barrel I drew it. But what *was* P(A/Y) before this learning? Not an easy question. We know what several other of your initial conditional probabilities were. In particular, since we know you were sure a marble would be drawn at random from a barrel, we can read off from the barrel diagram of Section 3.2.1 your initial probabilities **for** Y conditional on the barrel hypotheses. Counting yellow marbles in the barrels, we know that these conditional

probabilities were

$$P(Y/A) = 9/10 \qquad P(Y/B) = 6/10 \qquad P(Y/C) = 4/10.$$

But we cannot simply read off the conditional probability we want. It is a probability conditional **on** Y for the hypothesis that I drew from barrel A. For this probability we need help that a Bayesian theorem can provide.

4. BAYES'S THEOREMS

4.1. Forms of Bayesian theorems are implicit in the *Definition of Conditional Probability*, which is: For any positively probable proposition p, that is, for any proposition p such that $P(p) > 0$,

$$P(q/p) = P(p \,\&\, q)/P(p).$$

These theorems are little more than this definition, 'teased out.'

4.1.1 Bayesian theorem for a hypothesis and its negation. For $P(e) > 0$, $P(h) > 0$, and $P(\sim h) > 0$,

$$P(h/e) = \frac{P(h) \cdot P(e/h)}{P(h) \cdot P(e/h) + P(\sim h) \cdot P(e/\sim h)}.$$

This principle can be reached by a short argument from the ratio-definition of conditional probability together with three elementary principles for ideally consistent and well-thought-out degrees of confidence and probabilities. The first two principles have to do with degrees of confidence given logical relations between propositions.

Equivalence. Probabilities for logically equivalent propositions are equal: That is, for any propositions p and q,

$$\text{if } \Box(p \equiv q), \qquad \text{then } P(p) = P(q).$$

Incompatibility. If propositions are logically incompatible, then the probability of their conjunction is zero: That is, for any propositions p and q,

$$\text{if } \sim\Diamond(p \,\&\, q), \qquad \text{then } P(p \,\&\, q) = 0.$$

It will be evident that for the following demonstration weaker principles than these would suffice. Instead of principles for all logically equivalent and logically incompatible propositions, principles for propositions expressed by tautologically equivalent and truth-functionally inconsistent sentences would have sufficed. Equivalence and incompatibility are for ideally consistent and well-thought-out degrees of confidence. If, though p and q are logically incompatible, a person does not realize that they are, then he may well believe with some degree of confidence that they are both true.

The idealization relevant to the two principles stated includes 'logical omniscience', though uses to which we put these principles assume only modest powers of perceptions of truth-functional inconsistencies and tautologies. Our third principle is concerned exclusively with probabilities and a way in which a person's credences or degrees of confidence should be related.

Additivity. If a person is sure that not both of p and q are true, so that he places no credence at all in their conjunction, then his confidence in the disjunction of p and q sums his confidences in them considered singly: That is, for any p and q,

$$\text{if } P(p \,\&\, q) = 0, \qquad \text{then } P(p \vee q) = P(p) + P(q).^{10}$$

Now comes a derivation of Bayes's theorem for a hypothesis and its negation, for which we may assume the conditions

$$\text{(i)} \quad P(e) \leq 0, \qquad \text{(ii)} \quad P((h) \leq 0, \qquad \text{(iii)} \quad P(\sim h) \leq 0,$$

and derive

$$P(h/e) = \frac{P(h) \cdot P(e/h)}{P(h) \cdot P(e/h) + P(\sim h) \cdot P(e/\sim h))}.$$

Thus:

(iv) $P(h/e) = P(e \,\&\, h)/P(e)$	(i), Definition of Conditional Probability
(v) $P(h/e) = P(h \,\&\, e)/P(e)$	Equivalence
(vi) $P(h \,\&\, e) = P(h) \cdot P(e/h)$	(ii), Definition of Conditional Probability, algebra
(vii) $P(h/e) = [P(h) \cdot P(e/h)]/P(e)$	(v), (vi)
(viii) $P(h/e) = [P(h) \cdot P(e/h)]/P[(h \,\&\, e) \vee (\sim h \,\&\, e)]$	Equivalence
(ix) $P(h/e) = [P(h) \cdot P(e/h)]/[P(h \,\&\, e) + P(\sim h \,\&\, e)]$	Incompatibility, Additivity
(x) $P(\sim h \,\&\, e) = P(\sim h) \cdot P(e/\sim h)$	(iii), Definition of Conditional Probability, algebra
(xi) $P(h/e) = [P(h) \cdot P(e/h)]/[P(h) \cdot P(e/h) + P(\sim h) \cdot P(e/\sim h)]$	(vi), (ix), (x)

As said, this theorem of Bayes' is little more than the ratio-definition of conditional probability 'teased out'.

4.1.2 Bayes's theorem for a hypothesis in a partition. Now comes without proof, though a similar proof is possible, a theorem for a hypothesis in a finite partition, logical or probability. A *logical partition* is a set of propositions so related that it is necessary that exactly one of its members is true. A *probability partition* is like a logical partition except that, instead of its being a requirement that it be logically necessary that exactly one of

its member be true, it is sufficient that there be a probability of one that this is so. We have considered Bayes's theorems for the special case of two-member logical partitions of propositions and their negations. For any e such that $P(e) > 0$, and logical or probability partition $\{h_1, h_2, \ldots, h_n\}$ wherein for each h_i, $1 \leq i \leq n$, $P(h_i) > 0$, and h in this partition,

$$P(h/e) = \frac{P(h) \cdot P(e/h)}{\Sigma_n[P(h_n) \cdot P(e/h_n)]},$$

or, equivalently, assuming $h = h_1$,

$$P(h_1/e) = \frac{P(h_1) \cdot P(e/h_1)}{[P(h_1) \cdot P(e/h_1)] + \ldots + [P(h_n) \cdot P(e/h_n)]}.$$

The unconditional probabilities $P(h_1), \ldots, P(h_n)$ in these theorems are termed '**priors**' or 'prior or antecedent probabilities.' The conditional probabilities $P(e/h_1), \ldots, P(e/h_n)$ are sometimes termed '**likelihoods**.' The conditional probability P(h/e) is termed the '**posterior probability**.' In applications of these theorems, *available* explanatory hypotheses may not constitute a partition of either sort, to make up which one adds a 'null' or 'none-of-the-above' hypothesis. When it 'wins,' none of the available explanations is more probable than not on the evidence, and to 'proportion belief to the evidence' is to believe none of them, in other words, to suspend judgment regarding the how and why of it.

4.2 Interpretation. What does this general form of Bayes's theorem say when applied in contexts of confirmation? It says how the relative support that evidence e provides for a particular explanatory hypothesis h, of a partition of explanatory hypotheses, depends on how good an explanation it is of that evidence, as compared with how good its competitors are in that way, where the explanatory goodness involved is of a limited kind. The *goodness of an explanation* that is relevant to the support that evidence it explains affords is a function only of its *intrinsic plausibility* and its *would-have-had predictive power* for that evidence. *For 'evidence in hand'*, however, these factors of competing explanations are *from a certain perspective*. For this perspective a person imagines a time when the evidence e is not evidence, a time when its propositions are not known for sure when he has no evidence of particular relevance to them, though as much of his evidence as possible consistent with that is still evidence, and known for sure, and his credences for propositions are in general perturbed as little as possible consistent with all that. The credences that are *not* perturbed should include all 'background information' that is relevant to the plausibilities and predictive powers for e of hypotheses in the partition of the application. The imagined perspective consists of credences one would have at that time: It is represented by what would be your probability function for that time (we are pretending that there is such a time, and that you are

an ideal subject who would then have credences for all propositions). For 'cosmic speculation' concerning possible explanations of certain pervasive known features of nature, such as its order and appearances of design, this imagining can be difficult. Now come the formal explications promised in Section 2.4 of recently italicized terms. For every h in a partition:

The 'prior probability' P(h) measures the *intrinsic plausibility of h*.
The 'likelihood' P(e/h) measures the *would-have-had predictive power of h for e*.
The product [P(h) · P(e/h)] measures the *goodness of h as an explanation of e that is relevant to the support e affords h.*

This product is the way in which the intrinsic plausibility of h and its would-have-had predictive power for e determine its goodness as an explanation. They do so not, for example, 'by addition,' but exactly 'by multiplication or discounting'.

Bayes's theorem for a hypothesis in a partition says exactly how the support provided by evidence for a hypothesis depends not only on how good an explanation it is of this evidence, but on how good the explanations with which it is competing are. Indeed, this theorem does better than that by breaking down the goodness of explanations into intrinsic plausibilities and would-have-been predictive powers. It says exactly how the support provided by evidence for a hypothesis depends not only on its intrinsic plausibility and what would have been its predictive power for this evidence, but on the merits of competing explanations in these two dimensions. It makes the terms of competition between possible explanations for bodies of evidence precise in a way that can confirm intuitions and facilitate sensible adjudication.

4.3 The problem of old evidence. Here in Hume's words is a fundamental question when assessing the support provided by the facts of the world for the theory that it is the work of God, traditionally conceived. "Is the world considered in general and as it appears to us in this life, different from what a man or such a limited being would, *beforehand*, expect from a very powerful, wise, and benevolent Deity?" (Part 11, p. 163). I find in these words a response to 'the problem of old evidence' for Bayesian theories of evidence as this problem comes up when assessing theories of the origin and management of the world. It is a response to that problem of these theories made in advance of formulations of such theories, and notice of this major problem for them. It was the kind of response I made in the previous section without saying it was to the problem of old evidence of our subject. The *idea* is that, for assessing explanations for old evidence e, one works with what a subject's credences would be, not before all experience,

but were he to have had no experience specifically of e. This is not, as has been said, an easy exercise when the evidence is, as here, of order and apparent design in nature, but it is manageable, I think.

What is this problem for Bayesianisms? The measure proposed by such theories for "the confirmatory power of e for t" (Earman 1992, p. 119) is the difference, [P(t/e)− P(t)]. 'Old evidence e' is evidence already in hand in the sense that P(e) is 1. The *problem* with old evidence e is that if $P(e) = 1$, then P(t/e) = P(t) and $\mathbf{[P(t/e) - P(t)] = 0}$! Using a person's probability for a time to measure the bearing of evidence then in hand would say that it is not evidence for anything. Connectedly Bayes's theorem for a hypothesis in a partition would not say how the support of h by e depends on 'prior probabilities' and 'likelihoods': If P(e) = 1, then, for every probability h in a partition $\{h_1, h_2, \ldots, h_n\}$, $P(e/h) = 1$, $P(h) \cdot P(e/h) = P(h)$ and $\Sigma_n [P(h_n) \cdot P(e/h_n)] = 1$, so that

$$P(h/e) = \frac{P(h) \cdot P(e/h)}{\Sigma_n[P(h_n) \cdot P(e/h_n)]}$$

reduces to

$$P(h/e) = P(h),$$

which cannot serve "[o]ne of the great virtues of Bayesian confirmation theory . . . [namely] its ability to pinpoint and explain the strengths and weaknesses of rival accounts" (Ibid.). Of strategies that Earman identifies for responding to this problem, Hume's recently quoted words are closest to his second, counterfactual one. This strategy (i) "imagines what the agent's degree of belief in T would have been *ab initio* ['before learning E'?] if he were . . . a superhuman calculator [with credences for all propositions represented by a probability function][11] . . . and then [(ii)] . . . compares this number with the degree of belief this supercalculator assigns [to T] after learning E" (Earman 1988, pp. 134–5).

4.4 There is more to ordinary ideas of explanations and predications than what is relevant to Bayesian assessments. The explications given of 'explanatory goodness' and 'would-have-had predictive powers,' while suited to Bayesian assessments of the support provided by evidence for hypotheses, are imperfect as explications of ideas ordinarily expressed by these words. There is, for example, no requirement that, for the agent, h should be knowable independently of e, or that h should 'make sense' of e, or connect it with other things known. The absence of such conditions has dramatic consequences for paradigms of pseudo-explanations, for example, the would-be explanation of the fact that bread nourishes that cites the nutritive capacity bread! (Cf., Part IV, p. 126.) One letter can abbreviate both 'bread nourishes' and 'bread has a nutritive capacity,' since these sentences mean the same thing; let them be abbreviated by N. Since P(N/N) = 1,

the difference [P(N/N) − P(N)] is, for every possible *bona fide* explanation h of the fact that bread nourishes, at least as great as [P(N/h) − P(N)]. This notwithstanding that no one would say that the fact that bread nourishes has 'predictive power' for itself, or that it is an 'explanation' of itself.

The explications provided for 'explanatory goodness' and 'would-have-had predictive power' are imperfect for the ordinary ideas conveyed by these words. But they are right for Bayesian assessments of support for hypotheses by evidence. They work in applications of Bayes's theorem for a hypothesis in a partition, of which two will be made. The first, to our barrels and marbles, serves to fix ideas. The second, in Section 5, is to Hume's particular discussion of Cleanthes' argument from design, which will be reviewed in Bayesian terms.

4.5 Solving the marble problem. In real-life problems of explanation, usually only qualitative comparisons between relevant probabilities are possible. In our marble-case it is possible to fix the quantities of relevant probabilities. For you are sure (Section 3.4) that the barrel from which a marble was drawn was selected following the roll of a fair six-sided die in this way: The draw was from barrel A if the side with one dot or the side with two ended up, from barrel B if the side with three dots ended up, and was otherwise from barrel C. I have told you that the marble I have drawn from the barrel thus selected is yellow, and you believe me without a doubt. Using Bayes's theorem for a hypothesis in probability-partition and conditional probabilities that can be 'read off' from our diagram for the barrels (Section 3.2) given that you are sure that the draw was made randomly from the selected barrel, the support provided by the evidence Y, that the marble drawn was yellow, for hypothesis or theory A, that the marble drawn was drawn from barrel A, we have,

$$\mathbf{P(A/Y)} = [(2/6)\cdot(9/10)]/[(2/6)\cdot(9/10) + (1/6)\cdot(6/10) + (3/6)\cdot(4/10)] = 18/36 = \mathbf{\frac{1}{2}}.$$

P(A/Y) compares favorably with P(A), which is 2/6 = 1/3. The new evidence Y tends to *confirm* A: The measure of this confirmation is (18/36 − 12/36), or 1/6. And P(A/Y) compares favorably with P(B/Y) and P(C/Y), which are, respectively, 6/36 and 12/36 (verifications are left as exercises). The evidence provided by Y would detract from hypothesis C and would neither confirm nor detract from hypothesis B. Since barrel A is your favorite barrel, Y is for you *good news*.

But wait! I offer you the opportunity to inspect the marble drawn. You do this and find that it is yellow as I said. Your trust in my word was not misplaced. But, alas, you find also that it is rough! I hadn't told you that, knowing how partial you are to barrel A and not wanting to dampen your spirits. Possessed now of this additional information, what should you think? How

likely should you now consider the barrel A theory? For the answer we can calculate your old probability for A conditional on what you learned from me, Y, and with what you found out for yourself, R, combined, (Y & R):

$$\mathbf{P(A/Y \,\&\, R)} = [(2/6)\cdot(1/10)]/[(2/6)\cdot(1/10) + (1/6)\cdot(2/10) + (3/6)\cdot(4/10)] = 2/16 = \mathbf{1/8}$$

P(A/Y & R) compares *unfavorably* with P(A), which, recall, is 1/3. The combined new evidence tends to disconfirm A. P(A/Y & R) compares unfavorably with P(C/Y & R), which can be calculated similarly to be 12/16 = 3/4: Hypothesis A compares, on the evidence (Y & R), unfavorably with its competitor C. There is, as it happens, now 'nothing in' its competition with B: P(B/Y & R) is also 1/8. There is now a better hypothesis, and the combined new evidence *disconfirms* your favorite, barrel-hypothesis A. *Bad news*.[12] Not to be coy, our marble problem is a metaphor for the problem of the *Dialogues*. Barrel A plays the role of the Religious Hypothesis, Y is the role of order and apparent design, and R is the role of apparently unnecessary evil.

5. A 'PARTICULAR DISCUSSION OF THE EVIDENCE' – THE *DIALOGUES*, PARTS 5–8, 10, AND 11

Cleanthes: [R]efined and philosophical skeptics . . . are obliged . . . to consider each particular evidence apart, and proportion their assent to the precise degree of evidence which occurs. (Part 1, p. 103)

Having considered 'logical' challenges to arguments from experience to theories of the origin and management of the world (Part 2) and the *naturalness* of the theory of a designer and *immediacy* of the inference when, for example, the contrivance of the eye is surveyed (Part 3), Cleanthes and Philo enter in Part 4 a 'particular discussion of the evidence' that they agree is required for a proper assessment of the Religious Hypothesis. Now comes a summary of this discussion, organized under Bayes's theorem for a hypothesis in a partition, rather than under the simpler theorem for a single hypothesis and its negation that Salmon uses (Salmon 1978, p. 146). First, alternative hypotheses mentioned are recalled. Then facts are remarked, additional to those of apparently intended and deliberately imposed order and adaptations, that Philo stresses as relevant to the case. Next come comparative assessments implicit in the text of merits of competing hypotheses, their would-have-had-predictive powers, and their intrinsic plausibilities. Last, in this Section 5, comes the negative conclusion reached in the *Dialogues* concerning the Religious Hypothesis, which, as said, is found to be like our barrel A hypothesis.

5.1 Alternative hypotheses. Hundreds of theories (Part 7, p. 140) are suggested by analogies other than the one that inspires Cleanthes' theory, and

even that analogy suggests theories other than Cleanthes's. There are, furthermore (Part 8), significant nonanalogical alternative hypotheses.

5.1.1 Analogical alternatives

Other designer-theories. These differ from Cleanthes' with respect either to the character or methods of his designer. Featured, to mention only a few of the very many possible variations, can be

imperfect perhaps bungling designers, including designers who learn their world-making craft empirically, by trial and error (where our world might be one of the errors, one of the perhaps very many abandoned discards);
many designers working sometimes in harmony, and sometimes at odds;
mortal designers – included here are theories that would make the world the work of a young playful, an old failing, or a now dead designer;[13]
amoral designers who take no interest in conditions, good or bad, happy or miserable, of creatures;
corporeal designers, that is, designers that have 'bodies', if only in somewhat metaphorical senses, that they use in their work; and
instinctive designers whose activities might be likened to those of spiders spinning webs because they are made to do that, without choice, and without other than 'executive thought'.

Nondesigner analogical theories take off from the fact on which proponents of design-hypotheses insist, that it is not only *artifacts* and products of intelligence that display all the appearances of deliberately intended and imposed order, but, most prominently, also *animals and plants*.[14] Out of this reflection come,

the theory that our world grew from *a seed* cast off by another world (with worlds not only within but alongside worlds), for from seeds grow plants that display all the appearances of deliberately imposed design – and every world, or orderly part of the world, could have grown from seeds cast off by others, if there have always been worlds, or orderly parts of the world, *ad infinitum*; and
the theory that our world grew from *an egg* laid by another world.

5.1.2 Nonanalogical – order and appearances of design out of chance. This is the kind of theory that Philo, certainly speaking for Hume, 'says' would most please him: "Were I obliged to defend any particular system . . . I esteem none more plausible than that which ascribes an eternal, inherent principle of order to the world, though attended with great and continual revolutions and alterations" (Part 6, pp. 136–7). I gather from this passage only a preference on Philo's part for inherent-principles-of-order systems and the view that such a system, *if* there is one, *would* be most plausible. For I assume that Hume did not intend to portray Philo as changing his mind when he confesses, everything considered, "all the rules of good reasoning point to Mind and Thought as the

Supreme Cause of nature" (Part 12, pp. 174–5) and *not* to mindless inherent ordering principles.

5.1.2.1. "THE OLD EPICUREAN HYPOTHESIS" (PART 8, P. 143) SOMEWHAT REVISED. Would an *infinity of particles* that tried by random chance an *infinity of arrangements* throughout beginningless and endless *infinite time*, certainly hit about every arrangement, and indeed every sequence of arrangements, including this one of history as it is known to us, an infinite number of times? Philo implicitly asks, and answers in the negative, this difficult question. And so he suggests: "Instead of supposing matter infinite, as Epicurus did; let us suppose it finite. A finite number of particles is only susceptible of finite transpositions: And it must happen, in an eternal duration, that every possible order or position must be tried an infinite number of times" (Ibid.).

This theory makes certain that at infinitely many times there should be appearances of design, but it does not make appearances of design *especially likely*: It entails "that **every possible order** of position must be tried an infinite number of times" (Ibid.; emphasis added). So it does not make it especially likely that there should be order and appearances of design throughout the interval of time that happens to be *our* time.[15] The likelihood of appearances of design at a time on this hypothesis, including our time, is presumably (I am guessing at details of the hypothesis) 1/f, where f is the finite number of transpositions of which the finitely many particles are susceptible. The old Epicurean hypothesis is to facts of order and apparent design in the world known to us, somewhat as is the hypothesis *Fair Deals*, according to which deals always have been and always will be fair throughout the very long international history of bridge, is to a perfect bridge hand discovered by you on returning to the table after having been called away 'for a wrong number'. Fair Deals does not make likely that you should now hold a perfect hand, but it has an advantage over the simpler hypothesis, *Fair Deal*. For Fair Deals makes rather probable (perhaps, I have not done the history and the math) that there should be a perfect hand of thirteen spades dealt *to someone sometime*. Similarly, Philo's version of the old Epicurean hypothesis, though it does *not* make it likely that the world *of this interval of time that happens to be ours* should feature order and appearances of design, does make *certain* that order and appearances should take place for *some interval of time*. It makes certain that every pattern should "be tried an infinite number of times" (Ibid.).

5.1.2.2. HUME'S NEW HYPOTHESIS. The 'new hypothesis' of which Hume has Philo make much adds to the idea of transpositions of matter throughout 'eternal time' the thought that matter was *therefore* bound to fall into *forms that happened to be self-sustaining* and, having done that, to have continued for some time in these forms. According to this story, matter was bound in time (and there has been *infinite* time!) to fall into more and more such forms, so that eventually (and *long* before now) it took on the appearance of many machines within a great all-encompassing machine that we discover. Hume

writes of "an economy of things, by which matter can preserve that perpetual agitation, which seems essential to it, and yet maintain a constancy of forms" (Part 8, p. 144). Such an economy, with the self-sustaining form of the whole and the self-sustaining forms of a limitless variety of its parts down to the smallest, would *seem* to have been established by an intelligence to be through and through self-sustaining. The new hypothesis would explain this appearance of designed purposiveness in nature as an *inevitable* consequence of the restlessness of matter and the possibility of self-sustaining forms that 'channel' its perpetual agitation. (It is said that in seven years every atom of a person's body is replaced.)

5.1.2.3. Stanley Tweyman comments in his introduction to Hume (1991, pp. 33ff) on Philo's longing for "an eternal, inherent principle of order to the world; though attended with great and continual revolutions and alterations" (Part 6, p. 136). According to Tweyman, Philo has in mind not the kind of account of which the theories of Part 8 are his only examples, but the cosmological turn to eternal because logically necessary first principles conducted by Demea in Part 9. I think that *even if* Philo adverts in Part 6 *also* to such accounts, which I doubt, his primary reference is to an account that would work with *contingent* unchanging principles of order inherent in the world. Tweyman thinks that Philo says that he would "never willingly defend any system of this nature" (Part 6, pp. 136–7), because Philo has in mind 'cosmological systems' all of which are fundamentally ill-conceived, whereas I am persuaded that Philo speaks so because, of the kind of system he has in mind, he despairs of "success in any attempts of [their] nature" (Part 8, p. 146).

5.2 Evidence with which to conjure other than of design. Here we can be brief, even though we are concerned with a topic to which Hume devotes considerable space. In a word, there is the evidence of avoidable and unnecessary *evil*, that is, there are the facts of *apparently* avoidable and unnecessary evil. *Appearances* of *unnecessary* evil in nature are as pervasive as are *appearances* of design, where there are no observable designers. And in reasoning from the facts it is how things appear and are experienced that counts, and is the evidence. The evil insisted upon in the *Dialogues* is *apparently unnecessary* for the presumed ends of Cleanthes's at least "finitely perfect" (Part 11, p. 161) designer.

Hume attends in the *Dialogues* only to widely shared evidence, readily discovered when we 'look 'round', evidence upon substantial tendencies of which to incrementally confirm and disconfirm the Religious Hypothesis he expected *widely shared agreement*. It is presumably for these reasons, because he considered it is for most persons 'secondhand' at best, and weak evidence *for* God, that he does not bring into the *Dialogues* evidence of 'religious or mystical experiences' or 'theophanies' (Alston 1991; Fales 1996) or evidence of miracles (discussed in the next chapter). The omission of comment on the

evidence of 'common consent' (discussed in Meierding 1998), along with that of design and evil, is more difficult to explain.

5.3 The negative conclusion of the Dialogues. Both in terms of would-have-had predictive power and in terms of inherent plausibility, Hume was convinced that the Religious Hypothesis fared *worse than many* of its competitors, and *not better than any*.

5.3.1 Would-have-had predictive powers. The question here for the Religious Hypothesis: "Is the world considered in general and as it appears to us in this life, [as] a man . . . would, *beforehand*, expect from a very powerful, wise, and benevolent Deity?" (Part 11, p. 163). "Indeed not!" is the gist of Hume's answer. The world as we find it is most certainly not what one would expect beforehand on that hypothesis, not when one takes into account the extent of apparently unnecessary evil in the world. The unnecessary evil that seems to abound would be, if any of it really exists, logically compatible with the existence of the designer of the hypothesis if 'infinitely perfect'. The world is an 'an imperfect, unnecessarily rough, marble' or – with apologies to the oppressed, aggrieved, and miserable for my wimpish academic caution! – this is at least how it *seems*, which, as said, comes to the same thing for present inductive purposes. **Why** would a designer who in power and knowledge was up to the task of designing this world have let pass **'all of this'**? It is not *credible* that such a designer would have had no better choice in this matter. 'All of this' surely is the last thing one would have *expected beforehand* from Cleanthes's hypothesis, even when moderated to propose only a 'finitely perfect' designer. *Few* can, speaking hypothelically of themselves, deny *this* observation.

It is otherwise for alternatives to the Religion Hypothesis. Facts of evil that a 'finitely perfect' designer could have avoided for a net gain for the world, real as I think, or only apparent, which will do for present purposes, are quite *irrelevant* to amoral designer-hypotheses and also to all nondesigner hypotheses reviewed by Hume. They are understandable and even predictable on some designer-hypotheses considered. They are so on the hypothesis of a committee of concerned and well-intentioned, but somewhat at odds, designers as well as on the hypothesis of an old and failing designer, even if concerned and well-intentioned. The implicit finding of the *Dialogues* is that, because of facts of apparently unnecessary evil, *the would-have-had predictive power of Cleanthes's Religious Hypothesis for the facts it would, if not explain, at least accommodate, is less than that of many of its competitors and greater than that of none of them*.

5.3.2 Intrinsic plausibilities. The situation is, for different reasons, the same here.

5.3.2.1. Philo observes that "[a]n intelligent Being of such vast power and capacity, as is necessary to produce the universe . . . exceeds all analogy, and

even comprehension" (Part 5, p. 131). A *problem* with Cleanthes' Religious Hypothesis is that it insists on *vast* differences between its designer and any designing intelligence of which we have ordinary and direct experience. And even if sense can be made of its designer, that being's probability considered just in itself and without regard to things it might explain should, given its extraordinariness, be low. Furthermore, not only would this being be very unlike anything of which we have experience, but its proposed, and evidently required by what it would explain, vastness challenges comprehension and should thus be hard to credit and of low subjective probability. Additionally, Cleanthes' designer would, in its work, contradict rules that hold without exception in our experience. This One would be an *incorporeal* being, a mind only, and we have no uncontentious experience of such beings: "No man, Epicurus used to say, has ever seen reason but in human figure" (Part 5, p. 131); "that body and mind ought always to accompany each other . . . is founded on *vulgar experience*" (Part 6, p. 134). Connectedly, this great designing intelligence, since without a *body of its own*, would operate directly on matter in a manner, which, Philo reminds us, is contrary to all uncontentious experience: "In all instances which we have ever seen, thought has no influence upon matter, except where that matter is so conjoined with it, as to have an equal reciprocal influence upon it. No animal can move immediately any thing but the members of its own body. . . . [Y]our theory implies a contradiction of this experience" (Part 7, p. 147). That this designer would be "capable of . . . immediate fulfilments of intention . . . is a fundamentally mysterious element . . . which makes [the hypothesis] antecedently improbable" (Mackie 1982, p. 130). "Look, no hands!" this theory would allow its feature player to say. "**How** could He have *done* it?" one may ask. "Somehow," and "He just did it, that's all.", and "There is no *understanding* these great things." are not very satisfying answers. Better, it seems, to say that He didn't do it, that *no one* did it. Better to say even that order and apparent design *just happened*.

It is written that, *In the beginning God created the heaven and the earth*, and that then, in a language spoken and thought only by Him, for there was then no one but Him, *God said, Let there be light: and there was light*. He said it, and then, 'just like that,' what he said was done. But if so, then if He was of a mind anything like ours, He must have been *amazed*, supposing He learned of what had happened consequent to His creative speech. "'Did I do that?" a story in the style of one by Robert Nozick could continue. "Did I do that just by speaking my mind? Surely not!" Along with problems with His seeing the light (for it is written that *He saw the light*), and His private language, can be mentioned those of a "language a way beyond limitations of others" for His will "uniquely [to] determine a particular creation" (Nozick 1971, p. 26), assuming that this is what He did.

5.3.2.2. Cleanthes' designer hypothesis, indeed *every* designer hypothesis, not only departs in several ways from all of which we have experience, but contradicts exceptionless principles of experience and in several ways challenges

comprehension. "Allowing," Hume's Philo might say, "that such a designer is *possible*, does not advance Cleanthes' case" (cf., Part 10, pp. 159–60). For whether or not his designer is possible, its *inherent plausibility* even now should be negligible. In contrast, many of its competitors fare better, even if few do very well, in this dimension. Grounds for these conclusions concerning present inherent plausibilities work for like conclusions concerning inherent plausibilities that theories would have for us 'beforehand' when none of the evidence that "bespeak[s] in the clearest language an intelligent cause or author" (Part 4, p. 127) would be in hand. Then too we would have experience of only embodied minds that could not, 'just like that', let there be light, and then too an intellect of the vast powers of a world designer would exceed "all . . . comprehension" (Part 5, p. 131). And, for an important last point, *then* Cleanthes's designer hypothesis, indeed, every designer hypothesis, would suffer in another way. For, *absent evidence that clearly bespeaks a designer in nature*, the hypothesis of one could enjoy only evidence independent of that such as that of putative testimony by such a designer directly communicated. Left *for* designer hypotheses would be evidence to which few believers would confess and that no nonbelievers would not have. Hume does not make this point in the *Dialogues*. He missed it, I think, because he did not notice that, as the question of the predictive power of a hypothesis goes to what it would have been beforehand, so does the question of its inherent plausibility.

5.3.3 Taking the measure of the Religious Hypothesis. Initially, when one thinks of Cleanthes's designer hypothesis without regard to possible alternatives to it and thinks not of its would-have-been intrinsic plausibility, but only of its would-have-had predictive power, and this only for evidence of apparent design, without regard to that of apparently unnecessary evil, this hypothesis can look good and seem to be *confirmed*. However, after particular discussion of the evidence, Philo, speaking for Hume, considers Cleanthes's hypothesis to be decisively *disconfirmed*. This negative assessment followed for Hume largely from facts that should maximize its intrinsic *implausibility*, and from facts of apparently unnecessary evil that should minimize its would-have-had predictive power. The Religious Hypothesis was, in Hume's view, spelled out by Philo, damaged by how-questions, and devastated by why-questions.

6. PART 12 OF THE *DIALOGUES*: HUME'S 'PHILOSOPHICAL THEISM'

6.1. The evidence was found to be against Cleanthes' Religious Hypothesis. That is the negative conclusion of the *Dialogues* to Part 12. But it was not found to refute every hypothesis at all *like* it. Cleanthes and Philo agree in Part 12, and from that it is plain that Hume was convinced, that the facts did after all prove *a designer of sorts*, not Cleanthes' Deity, but an intelligence somewhat analogous to minds we know. Philo, but not Cleanthes, judged that

they proved nothing more, and so nothing relevant to our lives or sufficient for religion.

6.2 Last words regarding 'natural theology'

Philo: If the whole of natural theology, as some people seem to maintain, resolves itself into one simple, though somewhat ambiguous, at least undefined, proposition, *That the cause or causes of order in the universe probably bear some remote analogy to human intelligence*: If this proposition be not capable of extension, variation, or more particular explication: If it affords no inference that affects human life, or can be the source of any action or forbearance: And if the analogy, imperfect as it is, can be carried no further than to human intelligence, and cannot be transferred, with any appearance of probability to the other qualities of the mind: If this really be the case, what can the most inquisitive, contemplative, and religious man do more than give a plain, philosophical assent to the proposition, as often as it occurs, and believe that the arguments on which it is established exceed the objections which lie against it? (Part 12, pp. 184–5; part of an "addition made in the final revision, in 1776," N. K. Smith in Hume 1947, p. 227n1.)

Philo had spoken earlier in a similar vein,

So little . . . do I esteem . . . suspense of judgment in the present case possible, that I am apt to suspect there enters somewhat of a dispute of words into this controversy. . . . That the works of nature bear a great analogy to the productions of art is evident; and according to all the rules of good reasoning, we ought to infer, if we argue at all concerning them, that their causes have a proportional analogy. . . . and in particular ought to attribute a much higher degree of power and energy to the Supreme Cause than any we have ever observed in mankind. Here then the existence of a Deity is plainly ascertained by reason. . . . And if we are not contented with calling the first supreme cause a God or Deity, but desire to vary the expression, what can we call him but *mind* or *thought*. (Part 12, pp. 174–5; "addition . . . made in the final revision, in 1776," N. K. Smith in Hume 1947, p. 217n1.)

6.3. Having described the little that reason and natural theology can deliver, Philo goes on to suggest that it remains for a person "seasoned with the just sense of the imperfections of natural reason" only to "fly to revealed truth with the greatest avidity" (Part 12, p. 185). "To be a philosophical sceptic," he says, "is, in a man of letters, the first and most essential step towards being a sound, believing Christian. . . . (Ibid.). However, this step, we may observe, can for a philosophical sceptic be the last step he takes on that path. Witness Hume himself! It is, after all, a *problematic* step. It is problematic in one way in which Hume could not abide.

Qualified, measured assent to the hypothesis – let it be *intelligence* – that the cause of causes of order probably bears some remote analogy to human intelligence does 'raise the possibility' (in more or less Lorne Falkenstein's words) of communication between that mind or these minds and ours. It raises the possibility of revelation and "revealed truth" (Ibid.) and of receipts of devotion and

prayer, and so of nonquixotic worship. That is, assent to intelligence makes, for an assenter, grammatically proper the question of whether or not, between *it* or *them*, and us, there is discourse – it secures presumed referents for 'it or them'. It does not, of course, entail measured assent to the hypothesis – let this one be *communication* – that such discourse has taken place or, even if it has not taken place, that it is at any rate possible. While *communication* entails *intelligence*, *intelligence* does not entail *communication*. Furthermore, though we should say that Hume himself assented to *intelligence*, there is every reason to suppose that he placed no credence at all in *communication*, that he considered it improbable in the extreme. For one may gather from "Of Miracles" that he did not credit the testimony of others who claimed to have heard from nonhuman spirits, and it is a fair guess that he did not suppose that he himself had heard from any. Furthermore, as that intelligence (those intelligences) would, on the evidence, be without bodily organs to hear and see, and to speak and write, communication between it (them) and us would be contrary to all experience, for in all instances that we know, communication between minds is mediated by their bodies. While we may perhaps imagine direct, unmediated communication between minds, we should suppose that Hume would say that we have every reason to believe not only that it never happens, but that it never *can* happen, that it would be contrary to 'the laws of nature' of which Hume speaks in "Of Miracles."

Having made a plain assent to the somewhat ambiguous, at least undefined, hypothesis of philosophical theism and recorded the imperfections of natural reason, Hume himself stopped. Though he allows Philo to say that any person in the position Hume found himself "***will*** fly to revealed truth with the greatest avidity" (p. 185), it was Hume's opinion only that any such person ***may*** so fly, for Hume knew himself. He must have thought that though it is evident that persons *may* so fly to *revealed truth* for a god to their liking, those who do so engage in *contrary-to-the-evidence* irrational faith. I wonder why Hume had Philo say 'will' rather than 'may', and why he did not allow him further to characterize the faith to which he believed that sceptics can fly as irrational. My guess is that the explanation runs in terms of literary values such as dramatic balance.

In any case, we know that it is was not a flight that Hume took or, we should suppose, so much as contemplated as an adult. For though a *theist* of sorts, he was an antagonist of *religion*, settled in his opposition even when, in James Boswell's words, "just a dying" ("An Account of My Last Interview with David Hume, Esq.," Hume 1947, p. 76). Cf.:

> One Sunday forenoon the 7 of July 1776, being too late for Church, I went to see Mr. David Hume, who was returned from London and Bath, just a dying. . . . He was lean, ghastly. . . . He seemed to be placid and even cheerful. He said he was just approaching to his end. . . . I know not how I contrived to get the subject of Immortality introduced. He said he never had entertained any belief in Religion since he began to read Locke and Clarke. I asked him if he was not religious when he

was young. He said he was, and he used to read the *Whole Duty of Man*. . . . He then said flatly that the morality of every Religion was bad, and I really thought, was not jocular when he said "that when he heard a man was religious, he concluded he was a rascal, though he had known some instances of very good men being religious." . . . I asked him if it was not possible that there might be a future state. He answered it was possible that a piece of coal put upon the fire would not burn; and he added that it was a most unreasonable fancy that he should exist for ever. . . . I asked him if the thought of Annihilation never gave him any uneasiness. He said not the least; no more than the thought that he had not been. (Ibid.)

The book ends with the narrator's assurance that "Cleanthes and Philo pursued not this conversation much further . . . (Ibid.).

6.4 Who speaks for Hume. Hume at no point in the text of the book makes an appearance in his own acknowledged voice and person. More precisely, according to Smith, Hume speaks in his own person in modern editions only in two footnotes: The first, to Part 11 (p. 165n10), is an addition made no later than 1761 that is scored out in Hume's manuscript – Hume did not intend this to be part of the book when published; the second, of perhaps 1776, to Part 12 (Part 12, p. 177n18) is written on the last page of the original manuscript with an indication of its point of insertion, scored out, and then rewritten on a separate page (Smith in Hume 1947, pp. 94–5, 207, 219).[16] Given that Hume does not speak in his own person in the text, it is natural to wonder who in the *Dialogues* speaks for him. The answer, I think, is that usually Philo speaks for Hume; sometimes, perhaps often, Cleanthes too speaks for him; and sometimes Demea gives voice to Hume's views and sentiments. He was of several minds on subjects related to religion. He was conscious of propensities both to philosophic doubt and to commonsense belief regarding many things. Regarding some, he contrived, in alternation, both to doubt and to believe. For example, while impressed by the total and final lack of reasons for believing that the future will resemble the past, he was alive to the impossibility of not believing this most of the time, and of not reasoning and acting accordingly. So he believed in the uniformity of nature without pretending that he had reasons for doing so. Somewhat similarly, while impressed by the weakness of reasons for believing that the cause or causes of nature bore some analogy to human intelligence, Hume found in the end that he was inclined to believe this, and did believe this, although only in a tempered, severely curtailed, irreligious manner.

Philo's main job is to give expression to Hume's sceptical and critical tendencies. Cleanthes's principal function is to display Hume's natural disposition to down-to-earth belief in both "common life" and the "distant and high enquiries" of philosophy "which are nothing but the reflections of common life, methodized and corrected" (Hume 1902, p. 162). In the end, in Part 12, their voices are chorused for Hume, the sceptical believer, when his mind is settled and its tendencies to excess are in balance. Philo, when confessing to a certain

lack of caution "on the subject of Natural Religion," addresses Cleanthes as one "with whom I live in unreserved intimacy" (Part 12, p. 172) who will therefore never mistake his considered intentions. It may be that Hume is hinting at the oneness of Philo and Cleanthes, in that he is both of them.

6.5 In his own voice. According to Smith and J. C. Gaskin, *Dialogues Concerning Natural Religion* and *The Natural History of Religion* (1757) were composed simultaneously, though the former work was revised "in various points of detail about 1761 and again in 1776" (Gaskin in Hume 1998, p. xviii; cf., Smith in Hume 1947, p. 88). For Hume's views when he wrote the *Dialogues* we therefore have these lines of the *History*:

> As every enquiry, which regards religion, is of the utmost importance, there are two questions in particular, which challenge our attention, to wit, that concerning its foundation in reason, and that concerning its origin in human nature. Happily, the first question, which is the most important, admits of the most obvious, at least, the clearest solution. The whole frame of nature bespeaks an intelligent author; and no rational enquirer can, after serious reflection, suspend his belief a moment with regard to the primary principles of genuine Theism and Religion. (Hume 1993, *Introduction*, p. 134; cf., Part V, p. 150, Part VI, pp.153, 154).

To these may be added lines from the *concluding* part of the *History*:

> Though the stupidity of men, barbarous and uninstructed, be so great, that they may not see a sovereign author in the more obvious works of nature . . . ; yet it scarcely seems possible, that any one of good understanding should reject that idea. . . . A purpose, an intention, a design is evident in every thing; and when our comprehension is so far enlarged as to contemplate the first rise of this visible system, we must adopt, with the strongest conviction, the idea of some intelligent cause or author. The uniform maxims, too, which prevail throughout the whole frame of the universe, naturally, if not necessarily, lead us to conceive this intelligence as single and undivided. . . . Even the contrarieties of nature, by discovering themselves every where, become proofs of some consistent plan, and establish one single purpose or intention, however inexplicable and incomprehensible. (Part XV, p. 183)

I read the *Dialogues* as *detailing* and, in passages added after 1757, somewhat *curtailing* these opinions. Curtailment that I find goes mainly to the question of the *unity* of that intelligence, that is, to the perceived strength of reasons and arguments for philosophical monotheism over philosophical polytheism.

6.6. Hume evidently believed at times, and he believed *in the end*, that natural theology resolved itself into something positive, a theism of sorts. However (I thank David Brown for reminding me) Hume was *at times* of a mind to draw *no* conclusions from natural theology and not to translate "philosophical decisions" (Hume 1902, p. 162) into beliefs. He was at times of those "philosophical sceptics . . . who, from a natural diffidence of their own capacity, suspend, or

endeavour to suspend all judgments with regard to such sublime and such extraordinary subjects" (*Dialogues*, Part 12, p. 184). Hume wrote in the first *Enquiry* that:

> The *imagination* of man is naturally sublime, delighted with whatever is remote and extraordinary. . . . A correct *judgment* . . . avoiding all distant and high enquiries, confines itself to common life. . . . To bring us to so salutary a determination, nothing can be more serviceable, than to be once thoroughly convinced of the force of the Pyrrhonian doubt, and of the impossibility, that anything, but the strong power of natural instinct, could free us from it. . . . While we cannot give a satisfactory reason, why we believe, after a thousand experiments, that a stone will fall, or fire burn; can we ever satisfy ourselves concerning any determination, which we may form, with regard to the origin of worlds, and the situation of nature, from, and to eternity?
>
> This narrow limitation, indeed of our enquiries, is, in every respect, so reasonable, that it suffices to make the slightest examination into natural powers of the human mind and to compare them with their objects, in order to recommend it to us. We shall then find what are the proper subjects of science and inquiry. (Hume 1902, pp. 162–3)

Hume was at times inclined to exclude from the proper subjects of science and inquiry "the origin of worlds, and the situation of nature, from, and to eternity" because he suspected that all accounts in these areas must be, as he characterized his own 'new hypothesis', "incomplete and imperfect" (Part 8, p. 146). Having conceded its imperfection, Philo asked rhetorically, "But can we ever reasonably expect greater success in any attempts of this nature?" However, I suspect that even in his most sceptical moods he thought that though greater success in natural cosmology was very unlikely for him and his colleagues, it just *might* some day be realized by inquirers. And what is manifest is that Hume did *not* himself refrain from inquiring into nature from and to eternity, and that he was of "philosophical sceptics . . . who . . . [could only] **endeavour** to suspend all judgments with regard to such sublime and such extraordinary subjects" (Part 12, p. 184; emphasis added).

Suspension of judgment was not the option taken in the *Dialogues*. Hume *could* have had Philo say that the explanations he found to be available for appearances of intended design in nature were all so *bad* that the only reasonable response to this evidence was to say, "I cannot explain it. Nothing has been proposed that is believable and makes sense of it. If I *had* to choose, I would need to choose the hypothesis of designing intelligence or intelligences. It is the best of this miserable lot, and the most probable. But I do not have to choose. And the poor quality of these hypotheses as explanations of appearances of intended design, and conversely of these appearances as evidence for those hypotheses, makes reasonable that I not choose." He *could* have found that not one of the available hypotheses was on the evidence believable. But in the *Dialogues*, and *in the end*, that is not what he found.[17] Hume evidently did not in the end, when he finished his book and his life, consider the

hypothesis of designing intelligence or intelligences that bad.[18] He apparently agreed with Cleanthes that order and "the plain use and intention of every part and organ . . . bespeak in the clearest language an intelligent cause or author" (Part 4, p. 127). That presumably made up for the intrinsic implausibility of philosophical theism, which presumably was still of an incorporeal designing intelligence or intelligences.

6.7 Why not the New Hypothesis?

6.7.1. Hume believed that natural theology resolved itself in his day into philosophical theism, a doctrine of no human relevance that is continuous with traditional theism in this: It says that the cause or causes of order is or are somewhat analogous to a human mind.[19] This was partly because he was convinced that, as an explanation of appearances of design, philosophical theism, this vague, humanly irrelevant, designing-mind-or-minds theory, had no serious competitor. He would have preferred a theory that ascribed an "inherent principle of order to the world" (Part 6, p. 137) and has Philo sketch with some enthusiasm such a theory in Part 8. But though Hume would have liked such a theory, he could not in honesty find one "more plausible" (Ibid.) than theories of directing intelligence. In the end he has Philo confess that it is implausible that all the appearances of design should be the effects of blind, unguided by intelligence, natural forces, and that when resisting the natural aspects of Cleanthes' hypothesis he needed "all [his] sceptical and metaphysical subtlety" and employed disingenuously "objections [that] appear (what I believe* they really are) mere cavils and sophisms" on which no weight can be justly reposed (Part 10, p. 160). [*" 'perhaps' scored out; 'I believe' written in its place": note by Tweyman. "This alteration may have been made in 1776." (N. K. Smith in Hume 1947, p. 202.)]

6.7.2. Hume says of his theory that it improves on his revival of "the old Epicurean hypothesis" and is "not absolutely absurd and improbable" (Part 8, p. 144). But he realizes that it "is so far incomplete and imperfect" (Part 8, p. 146) and that it is not very probable given the *particular* evidence we have of appearances of design in nature. Nor did he have much hope that it could be completed and perfected and made formidable as an explanation of the evidence that most impressed him, which was the very complex and subtle arrangements of means to ends and appearances of intended design that are everywhere in live beings. Why not?

To understand, to appreciate the *reasonableness* of, Hume's dim view of the prospects of his 'new hypothesis,' we may begin with the fact that it was not an evolutionary theory, not even incipiently. Hume does suggest, while setting out his new theory, that an explanation for apparent design in living things is encapsulated in the question: "I would fain know how an animal could subsist unless its parts were so adjusted [that is, were adjusted in useful ways, in ways

apparently designed to secure its subsistence]?" (Part 8, p. 146). But there is *no* indication that Hume so much as considered the possibility of an *evolution* of apparently designed forms, from relatively simple ones to more complex ones. He says that, if "the universe goes on for many ages in continued succession of chaos and disorder . . . is it not possible that it may settle at last" into some self-sustaining order? (Part 8, p. 51). He adds that "we may not [only] hope for such a position . . . [but] be assured of it from the eternal revolutions of unguided matter" (Ibid.). But he *never* suggests that the universe might for some reason settle into some self-sustaining forms, and *then* into *other* (more secure? more encompassing?) self-sustaining forms, and so on eventually to very elaborate and complex self-sustaining forms, such as the ones we observe in ourselves. He never suggests that order and apparently intended design might have evolved, and so of course he never suggests possible principles, inherent in nature, that would have made antecedently probable the evolution and continuous development of forms apparently designed to survive and flourish. What is more, there is evidence that if he had contemplated such developments, he would have considered them unlikely in the extreme. Two things would in his view have told powerfully against evolutionary elaborations of his new hypothesis.[20]

Hume would not have believed that there had been enough *time* for evolution. Evolutionary processes take *time*, and Hume was not sure that the earth was very old. He was indeed, one assumes, at least somewhat impressed by those "proofs of the **youth** . . . of the world" that he had Cleanthes go out of his way to present (Part 6, p. 136; emphasis added).[21] Furthermore, not only would unguided evolutionary processes take time, but it seems that, in order to result in high orders of complex organic designs such as birds and bees, they would need long stretches of time in which these processes could work without violent disruptions. It is thus relevant that Cleanthes's 'proofs of the youth of the world' are opposed by the idea, put forward by Philo, that perhaps, though the earth is in fact ancient, its history has been marked by "great and continual revolutions and alterations" (Part 6, p. 137).[22]

The second obstacle for Hume to a development along evolutionary lines of his new hypothesis would have been that, though plausible evolutionary hypotheses make discards and eliminations of forms likely as forms better adapted to conditions evolve, Hume did not believe that many species that had once been were, by his century, *extinct*. He was not sure that even one species had been extinguished (Part 11, pp. 165, 167–8). That he was interested in the issue is, however, interesting. So is his interest in the age of the world, which he had Cleanthes and Philo go somewhat out of their ways to discuss. Cleanthes is allowed to say that Philo's hypothesis of the world as an animal the soul of which is the Deity "seems to imply the eternity of the world" (Part 6, p. 135). Instead of saying, "No, it implies rather the mortality of its Deity," Philo accepts Cleanthes' invitation to discuss the age of the world. Perhaps it was because of the apparent absence of extinctions and the youth of the earth that Hume

had Philo allow not merely that his new theory "[was] so far incomplete and imperfect," but to ask rhetorically whether "we can ever reasonably expect greater success in any attempts of this nature . . . [or] . . . ever hope to erect a system of cosmogony . . . [that] will contain no circumstances repugnant to our limited and imperfect experience" (Part 8, pp. 146–7).

7. NEW FACTS AND NEW THEORIES

Hume wrote Philo's last words in 1776. Given the facts and theories then known and available for consideration, an amoral intelligence or intelligences somehow responsible for appearances of design in nature seemed to him to be the best of a poor lot of possible explanations of these appearances, and believable. Hume was judicious, and reasonable, in his reluctant conclusion. Today, however, there are 'new facts' and new theories to be taken into account, and a very different conclusion seems to be in order.

7.1 'New facts'. Foremost among facts that are not now subjects of serious dispute of relevance to possible explanations of appearances of intended order and design is that the earth is *ancient*. It is approximately 5 billion years old. When to this is added the well-based estimate that, although there are many millions of living species, there are many more *extinct species*, including many *long* extinct species, even Hume's incomplete and imperfect new hypothesis, though it wants a mechanism, can take on some credibility relative to its competitors in his *Dialogues*, since its completion can be viewed as being, far from a lost cause, in principle doable. With only this new information, Hume could think, "There 'must' be a mechanism the hypothesis of which beforehand would have predicted the evolution of very complicated life forms, and the extinctions." And there are other facts that we now see as significant for the issues of natural theology, because of the character of explanatory options new, since Hume's day, to the debate. There are facts of certain *homologies* (e.g., the wings of birds and the forelegs of horses are homologous, or similar in form, though they differ in functions, as are and do the wings of bats and the hands of human beings); facts of *useless rudimentary organs* (e.g., the human appendix), and of *'make-do' adaptations* and "*maladjustments of structure to function*" ["birds and insects that use their wings for swimming" (Mackie 1982, p. 138), the Panda's 'thumb,' which is a rather inefficient tool for stripping bamboo, hour after hour]; and, William Seager has suggested, facts of now appreciated imperfections of even impressive 'natural designs', of the design of the human eye, for example. There are facts about *embryos* in what are termed 'advanced' species [e.g., human and chicken embryos both, at early stages, have 'gill slits' – These "are not gills as such" but structures that "serve as guides for the developing blood vessels. In fish, they turn into gills; in mammals, into glands" (Hitching 1982, p. 203).]. There is a slogan that, in a moderate form, summarizes and points toward an explanation of

such phenomena. According to it the study of embryos reveals that sometimes, in some ways, '*ontogeny recapitulates phylogeny*,' that is, the prenatal development of individuals recapitulates the evolution through eons of their species.

7.2 New theories. Prominent amongst new theories of importance to natural theology are evolutionary theories of forms of life, according to which over a period of about 3 billion years all forms of life, including long extinct forms, evolved from very simple ones that were little more than overgrown molecules with physical-chemical propensities to split into similarly incomplete halves that would then make themselves whole again by collecting needed materials from a primordial soup, each then to split again, and so on. The main mechanism giving direction to the grand process of 'biotic' evolution was and is natural selection.[23] Forms of life tend to multiply without limit. In ensuing competitions for space and sustenance, individuals that happen to vary from others in their times and places in useful ways tend to survive and to have offspring that by inheritance are similarly well-adapted to the conditions of the day. And so gradually, over time, populations change. They tend to change toward forms that are at the time better adapted, and thus sometimes toward more complicated forms. They tend to change in ways that, though the results of natural and unintelligent factors, display every appearance of deliberately imposed designs of successful means for intended ends. Other evolutionary theories play down the gradualism of Darwin and the importance of the tendency of species aggressively to propagate. Some say that the emergence of new species takes place not under conditions of intense competition, but following biological disasters (perhaps global) and during periods when competition is relaxed. But every scientific theory of the origin of species, and appearances of designs for life, is, with an eye to fossils, evolutionary. "Evolution of life over a very long period of time is a *fact*. . . . Despite the many believers in divine creation . . . the probability that evolution has occurred approaches certainty in scientific terms" (Hitching 1982, p. 12). The *issue* for biologists is not *whether* there is 'biotic' evolution, but only *exactly how*. There are differences among theorists, differences, for example, between modern synthetic Darwinists and 'really new synthesists' such as Stephen Jay Gould. But some of these differences seem more apparent and matters of emphasis and terminology than real and substantial, and what is plainer and more important for us is that no current scientific theory of order and apparent design in nature assigns *any* role to monitoring, and somehow manipulating, *intelligences*. As far as serious researchers are concerned, Hume's indeterminate designer hypothesis, his philosophical theism, is no longer in the running, not even as a *dimension* of a likely theory of appearances of the design of living organisms and their organs. Regarding the origins of life, while 'prebiotic' evolution (see the previous note) is not 'established fact', its confirmation seems, for what my opinion is worth, to be work in relentless progress.

7.3. Hume, realizing that his new hypothesis was unsatisfactory, asked with rhetorical pessimism, "Can we ever reasonably expect greater success in any attempts of this nature?" (Part 8, p. 146). He believed the answer was, "No."[24] But we, speaking into the past, can say to him, "Yes, though you will have to wait until 1859 for a Britisher to the south, Charles Darwin, to publish." The success of which Hume despaired was in large measure achieved in *The Origin of Species*. For Hume, natural theology resolved itself into a vague designer hypothesis. For many today it has resolved into nothing. Theisms have been robbed of what was their main evidential support, namely, the appearances of intended design in living things. Hearts look like pumps we make, larynxes like clarinets, eyes like cameras, and whole persons like many-purpose robots. Cleanthes, speaking to Philo, enjoins, "anatomize the eye: Survey its structure and contrivance; and tell me, from your own feeling, if the idea of a contriver does not immediately flow in upon you with a force like that of sensation" (Part 3, p. 119). Cleanthes is confident that "[t]he order and arrangement of Nature, the curious adjustment of final causes, the plain use and intention of every part and organ; all these **bespeak in the clearest language** an intelligent cause or author" (Part 4, p. 127; emphasis added), to which many can now say, "Not any longer! Not to me." "The phenomena to which Cleanthes refers now bespeak in the clearest language not an Author but an evolutionary process" (Mackie, p. 143). "These and suchlike considerations [the left/right symmetries of all animal, the eyes of living creatures]," Newton opined, "**always have and ever will prevail with mankind** to believe that there is a Being who made all things and has all things in his power . . . " (Newton 1953, pp. 65–6). He was right about recent history to his time, and for a while after. Such considerations had and would continue for a time to prevail. But he was wrong, we know, when he predicted that they *always* would do so.

7.4 Cannot evolutionary and designer theories be combined, *while retaining the best of both?* Well, some of them are *combinable* – some of them are logically compatible. The *question* for natural theology is whether, in combining evolutionary and designer theories, it is possible to 'preserve the best of both' for an explanation of order and appearances of design that is better than either.

7.4.1. There is some indication that van Inwagen thinks that two problems for Darwinian theories that rely exclusively on the process of natural selection might be reduced, and a better theory may be made by adding as another agency for change "an intelligent being [who sometimes guides] evolution by a series of actions that directly affect the genes of evolving organisms" (van Inwagen 1995, p. 151). The two problems concern: (1) macroevolution – the evolution of phyla – given the absence of fossil evidence for intermediate forms (pp. 145–50); and (2) the evolution of " 'cognitive capacities' . . . written on one's chromosomes" for breakthroughs in contemporary science (for example, Albert Einstein's and Kurt Gödel's chromosomes and breakthroughs),

which capacities seem not to be correlated with advantages for palaeolithic populations (pp. 152–8). Van Inwagen implies that adding a designer to evolutionary theory might help, and make for a better theory at these junctures. I think that that addition would, by most persons, be found to have just the opposite effect.

7.4.2. Let **E** be an evolutionary theory, Darwin's theory, if you like, that does not include a role for guiding intelligence. Let **D** be a designer hypothesis, Hume's philosophical theism or, if you like, Cleanthes's Religious Hypothesis (modified if necessary to be at least compatible with evolution). The question is whether an integration of these theories **(E + D)** could be a better explanation of the facts. This question divides usefully into two, one concerning the inherent plausibility of (E + D) and the other concerning its would-have-had predictive power for relevant evidence, including that which supports especially E, such as 'make-do' adaptations, imperfect organs, and the eventual extinctions of either or almost all species. [While the integration of E and D, (E + D), will entail, it will not be entailed by the conjunction (E & D).]

7.4.2.1 INHERENT PLAUSIBILITIES. Considered apart and in itself, (E + D) cannot be more probable than E. (E + D), this 'integration' of E and D, is a conjunction, though not merely of E and D: It is a conjunction of, along with these hypotheses, propositions that relate the designer or designers of D to the processes of E. A conjunction cannot be more probable than its least probable conjunct, and so it cannot be more probable than any of its conjuncts. Furthermore, depending on the character of the designer or designers posited in D and the roles assigned to it or them in relation to processes in E, (E + D) can be, when considered in itself, and without regard to the facts it might explain, harder to believe and intrinsically less plausible, perhaps much harder to believe and intrinsically much less plausible, than E alone. It should suffice to recall that (E + D) will invite questions concerning *how* a designing intelligence is supposed to influence material history, how it is supposed to 'lay hands' on natural material things, and how it is supposed to put into effect its will and plan for them. Any elaboration of (E + D) to respond to these questions could, I think, only detract further from its initial implausibility. It is better to say nothing at all regarding how the designer would have its way with nature. The reticence on this subject of all advocates of D can from this perspective be seen to have been well-advised, though its reason, I suspect, is that no one has thought of anything to say on the subject. It is certain, and hardly remarkable, that (E + D) cannot be more probable for a person than either E or D. But it could happen that a person's bringing together E and D led to an elevation of this person's probabilities for both E and D. The integration of E and D, the assembly of the theory (E + D), could result in 'old evidence suddenly falling into place' (Seager 1987, p. 312). And so it is remarkable that, for reasons just given, the integrated theory (E + D) figures to be inferior in initial plausibility not only to what is after the

integration and bringing together, but to what was before, the initial plausibility of E.

Furthermore, and more remarkably, the problem of *how* figures to make the intrinsic plausibility of (E + D) inferior to that of the contrary theory ***[E & ∼(E + D)]***, which theory affirms evolution E *explicitly without* the involvement of a designing intelligence that (E + D) affirms.[25] This point is more remarkable for two reasons. First, while [E & ∼(E + D)] is a proper rival of (E + D), E is not. Of [E & ∼(E + D)] and E, only the former is *logically incompatible* with (E + D). [E & ∼(E + D)], while maintaining E, denies the involvement in the processes of E of a designing intelligence that (E + D) affirms. In contrast, E alone merely does not affirm that involvement. Second, though P(E + D) need not be inferior to P(E), it cannot be superior to it: It is *necessary* that P(E + D) ≤ P(E). In contrast, it is not necessary that P(E + D) ≤ P[E & ∼(E + D)]. The composite theory (E + D) *could* be more plausible than this proper rival [E & ∼(E + D)]. It is thus remarkable that, regardless of its details, (E + D) figures, for essentially Humean reasons including most prominently *how-reasons*, not to be more plausible, and indeed to be decidedly less plausible than this rival [E & ∼(E + D)].[26]

7.4.3 Would-have-had predictive powers. It seems clear that the involvement of D in (E + D), whatever within broad limits its details, will detract from the would-have-had predictive power of E for relevant facts, whatever the details of E, and that (E + D) will compare unfavorably in this dimension with E, as well again as [E & ∼(E + D)]. The problem here is not with *how* the designer of D would work its will on the processes of E, but with *why* it would do that. One supposes that an intelligence of power and acumen suited to the work would have a choice amongst means to its ends in the world, and it is not easy to detail even fanciful reasons why it would employ *evolutionary* processes to establish designs in nature, rather than 'gentler', more direct, and more efficient means. Why, in order to make live organisms of present orders of complexity, choose a way that involves organisms eating and being eaten by other organisms, and a way in which the monumental struggles of most kinds, if not all kinds, are rewarded eventually with extinction? Why not, instead of cannibals and doomed species, make self-sufficient and mutually supportive organisms? Why not make kinds that never go out of style? That is presumably what benevolent and reasonable humans would do, were the task theirs and their powers adequate to it. Why not get things right from the start and let them be? Why settle for imperfect eyes and 'make-do' adaptations? The *difficulty* of these and other questions of *motivation* detracts from the would-have-had predictive power of (E + D) for evidence that is particularly *supportive* of E, and of [E & ∼(E + D)].

Humean reasons argue against the would-have-had predictive power of a theory that makes the processes of E the way of an intelligent designer. For facts bespeak not merely evolution, but *profligate* evolution that proceeds

without foresight, without the discipline of a plan, and without tinkering: [E & ~(E + D)] can in the tailoring of E predict this, while (E + D) cannot without difficulty and complication that can be expected to make its designer more trouble than it is worth 'explanatorily'. "Look 'round," Hume wrote, this time for Philo to speak; the first time (Section 1) had been for Cleanthes.

> What an immense profusion of beings, animated and organized, sensible and active! You admire this prodigious variety and fecundity. But inspect a little more narrowly these living existences, the only beings worth regarding. How hostile and destructive to each other! How insufficient all of them for their own happiness! How contemptible or odious to the spectator! The whole presents nothing but the idea of a blind nature, impregnated by a great vivifying principle, and pouring forth from her lap, without discernment or parental care, her maimed and abortive children! (Part 11, pp. 168–9)

Darwin, *sans* 'intelligent principles', *predicts* this profusion, this prodigious variety, hostility, destructiveness, and, remember, extinctions the extent of which Hume did not dream. Everything that Hume brings here of "the idea of a **blind nature**, impregnated by a great vivifying principle, and pouring forth from her lap, without discernment or parental care, her maimed and abortive children" (emphasis added) tells the story of Darwin *without a guiding intelligence*. That story makes all this exactly what, beforehand, one would have expected.

8. THE ARGUMENT FROM DESIGN – MILLENNIAL EDITIONS

8.1 From fine-tuning to a Fine Tuner

> Theism, you say, has been robbed of its main evidence, appearances of intended design. But surely there is what can now be appreciated to be much grander evidence than it, namely, that the *cosmos* gives every appearance of having been *fine-tuned for life*. The whole may present nothing but the idea of blind nature *impregnated by a great vivifying principle*, but that is already a something *extraordinary*. Does not the presence of a vivifying principle of some kind, does not the bare possibility of life, does not the fact that the world is as if fine-tuned for life, testify to the hand of a *fine-tuner* of staggering intelligence to know what to do, and of staggering power to do it? How else can the fine-tuning on which the very possibility of life depends be explained? Given that the world is a place in which things can live, Darwinisms, extended to the 'prebiotic', may explain the emergence of life, and of more and more complex forms of life. But, given the exquisite adjustments and forces, quantities, and constants required for that, what can be said other than that we have to thank for it a Fine Tuner, an awesome intelligence joined to a power to 'tune' the fundamental laws of nature?

There is a remarkable difference between the 1776 version and the millennial versions of the argument from design. The evidence for design in 1776 was

for all to see: "Look 'round," Cleanthes could enjoin. The *new* evidence for design is for all but a very few to take on faith and authority.

> Our present picture of the cosmos . . . supplied by physics . . . [and] cosmology . . . involves a lot of numbers . . . constants . . . that . . . cannot be predicted theoretically. They are numbers that, as the physicists say "have to be filled in by hand". . . . The interesting thing . . . is that it appears that if the cosmos were much different at all, there would be no life . . . Small changes in various of these numbers would result in a cosmos that lasted only a few seconds or in which there were no atoms or in which there were only hydrogen and helium atoms[27] . . . . It seems to be the lesson of modern physics and cosmology . . . that the cosmos is [as if] "fine-tuned . . . to enable . . . life. . . . [that o]nly a vanishingly small proportion of the totality of possible cosmoi [that answer to the present picture of the cosmos supplied by theoretical physicists, that is, to "the standard model of quantum particle physics" (Smolin 1997, p. 204), except for the constants that they fill in by hand] are suitable abodes for life. (van Inwagen 1994, pp. 129–30)

There you have a general indication of the new evidence, about fourth hand, from me, who got it as you can see from van Inwagen, who probably got it from teachers and reporters of science, who got it from some front-line researchers' reflections on the significance of some features of contemporary physical theory. Lee Smolin speaks with some authority when, to put a number to it, he writes of "how probable [it is] that a universe created by randomly choosing the parameters [of the standard model] will contain stars" that it "comes to about one . . . in 10^{229} . . . [a] truly ridiculous . . . number" that may be compared with the number "10^{80} protons and neutrons" in "the universe we can see from earth," which gigantic number is "infinitesimal compared to 10^{229} " (Smolin 1997, p. 45, calculations on p. 325). Smolin says that life as we know it is possible only in a universe of *light* and of *stars* (pp. 1ff). Job One in *Genesis*, recall, was to light up the world.

8.2 Many cosmoi hypotheses. "What *else* can we say to explain the fine-tuning?" The main response has been to say, 'for openers,' that this world, the cosmos, of ours is one amongst many. The common thread of these hypotheses is that this cosmos in which we live and breathe – this closed spatial-temporal region of causally interacting entities, let us say – is only one amongst *many*. Such theories are of two kinds.

8.2.1 Simple many cosmoi theories. The cosmoi of these theories are, in the values of the fundamental constants, *independent*. In these theories the many cosmoi do not differ in the forms of their basic laws and the positions in them for constants: It is a seldom stressed feature of these theories that, even if the many cosmoi are all possible 'standard-model' cosmoi, they are not all the cosmoi that are 'epistemically possible' and so may well be not all that are logically possible. This feature of many cosmoi theories of the current debate is made much of in Fulmer (2001). Some simple many cosmoi theories array

the cosmoi 'side by side'. Others have them coming one after the another as in the "Phoenix model . . . in which the collapse of a whole universe leads to a bounce that gives rise to a . . . new universe" (Smolin 1997, p. 95). The *essential* point about these simple theories is that each cosmos is 'a whole new ballgame' as far as its basic parameters are concerned. "The crucial feature of the various Multiple Universe theories, is that those physical parameters on which *inhabitability* depends, are understood to be assigned **randomly for each universe** [from the 'space of initial conditions and fundamental constants of all possible outcomes of big bangs']" (White 2000, p. 261[260, 261]; bold emphasis added). White identifies 'multiple universe' theories with my 'simple many cosmoi' theories. That 'simple many cosmoi' theories include this 'crucial' feature (which, note, implies the omni-applicability of 'the standard model') means that they are not all *that* simple. Let MU be a 'simple many cosmoi,' where M is the hypothesis that there are many cosmoi and R is this 'crucial feature.' Then MU is (M & R).[28]

Additional variety amongst simple many cosmoi theories concerns the 'placement' of the random processes by which the fundamental constants of the 'standard model' are determined. Some, for example, Wheeler's Phoenix model, locate these processes *in* the cosmoi and say that they take place shortly after the 'big bangs' of these cosmoi. These theories enjoy the evidence that this is the way of our cosmos. They can speak easily of "possible outcomes of a big bang" (White 2000, p. 260 – he should have written of 'physical parameters being assigned randomly not *for*, but *in*, each universe', p. 261). Other simple many cosmoi theories advert to 'supercosmic' random processes that spew out cosmoi the parameters of which are *predetermined*. For example:

> [T]hink of our cosmos-designing machine as containing a . . . device . . . [that] sets the dials on the machine [for the parameters of a cosmos] at random. The [supercosmic] machine turns out a cosmos. Then the randomizing device resets the dials and the machine turns out another cosmos, and so on through a very large number of resettings. (van Inwagen 1994, p. 142)[29]

These theories do not have 'big bangs followed shortly by random determinations of the basic physical constants' (Smith 1990, p. 34).

8.2.2 Evolutionary many cosmoi theories. I know of one example of such a theory, Lee Smolin's. A theory of Quentin Smith's is (inadvertently, perhaps) 'on the way' to an evolutionary theory. These theories have cosmoi *coming from* cosmoi. In this respect they are like the Phoenix model mentioned. A difference is that the determinations of basic parameters for a cosmos are said to be somewhat restricted in the space of all possible outcomes of big bangs. That opens the possibility of restrictions that favor the establishment of cosmoi with parameters fine-tuned for life, given some selection mechanism and something like the inheritability of parameters. Smolin's theory includes a promising selection mechanism. It is 'the whole package'.

8.3 Limitations of simple many cosmoi theories. The many cosmoi in these theories are generated by processes that make very probable an enormous variety in their basic parameters and make "statistically unsurprising that there are a few that actually contain life" (van Inwagen 1994, p. 143). These theories will include explanations of 'observational selection effects,' explanations of why "[w]e cannot observe . . . cosmoi . . . unsuitable for life . . . [explanations that say how] something restricts the scope of our observations" (van Inwagen 1994, p. 144), for example, that there cannot be causal interactions of any kind between events in different cosmoi. Protected by these explanations, simple many cosmoi theories have considerable advantages in **intrinsic plausibilities** over fine-tuner theories. There is a great difference between, on the one hand, (i) intelligent designers fashioning from materials found, plants and animals, or, backing up, DNA molecules, these to go forth and multiply – that is almost *imaginable* (I am imagining myself willing nuts and bolts together and having them come together consequent to my willing), and on the other hand, (ii) 'intelligent designers tweaking the physical constants for life' – that, to my mind, is *just talk*.[30] There is not only a problem with the *how* of this 'tweaking' of the constants, but, before getting to that, a problem with the *what* of this, 'on the ground.' There is the problem, in the dynamic realities of the elementary particles described by the 'standard model', of what *happens* when the constants are set this way and that? In terms of Smith's exposition of a big bang, what *happens* when "superparticles interacting by means of the superforce" are *differentiated* into particles and forces and "the basic physical constants" are determined (Smith 1990, p. 34)?

Simple many cosmoi theories are, however, in terms of **what would have been their predictive power** for *life in this cosmos*, at a considerable disadvantage, for reasons that have been found against this power of Philo's revision of 'the old Epicurean hypothesis' (Section 5.1.2.1). By construction they make statistically unsurprising that there should be cosmoi that accommodate life as we know it. But these simple many cosmoi theories make no more likely that there are, thanks to their basic parameters, cosmoi that support this life than that there are cosmoi of any other variety. The likelihood of any given particular cosmos, including the one in which we find ourselves, randomly generating for itself the life-supporting parameter assignments (to simplify I am assuming that there is exactly one such assignment), *or any other particular parameter assignments*, assuming finitely many possible assignments, f, is 1/f, given that, to recall a 'crucial feature' of simple many cosmoi theory, *assignments for each cosmos are determined by some random chance process operating on the 'space' of all possible assignments*. That, furthermore, is the likelihood of a particular cosmos receiving a particular parameter-assignment, *whether or not there are many cosmoi*, given that 'crucial feature'.[31] Their advantages of inherent plausibility and disadvantages in would-have-been predictive power to my mind balance to place simple many cosmoi and rational tuner theories about on a par, and equally probable on the evidence of fine-tuning. That makes for me

a case to van Inwagen's general claim that, "[a]s far as our present knowledge goes (aside from any divine revelation that certain individuals or groups may be privy to) we have to regard the following two hypotheses as equally probable:

- This is the only cosmos, and some rational being has (or rational beings have) fine-tuned it in such a way that it is a suitable abode for life.
- "[MU] This is only one among a vast number of cosmoi, some few of which are suitable abodes for life" (van Inwagen 1994, p. 145).

These hypotheses seem to me to be to be equally probable, but to this I add that they are both in my view very improbable. Van Inwagen would not agree with this addition. There are lines that seem to say that our situation vis-à-vis those hypotheses is like one in which two hypotheses make a 'probability partition.'[32] The two hypotheses of a pro-life tuner and MU do not make a 'probability partition' for me, since my probabilities for them do not sum to one. Far from it.

8.4 Smith's many cosmoi theory. A less simple many cosmoi theory of a *branching* sequential kind that it is claimed would in a manner explain the existence of a cosmos with the physical constants of ours, though again without making cosmoi that are fine-tuned for life especially likely, is developed in Smith (1990). The object of this theory is not to begin an explanation of why our cosmos is as fine-tuned for life, but only to take explanations of its laws and constants 'deeper'. According to Smith's theory, every cosmos c is related to exactly one cosmos c′ by a 'singularity' that is a 'black hole' of c, and the 'big bang' of c′, and this relation of *cosmoi* is noncyclical. This relation is 'timelike' but not temporal since a cosmos is a space-time, every position in which is spatiotemporally related to every other position in it, and to no position in any other cosmos (p. 25), and 'black holes' and 'big bangs' of cosmoi are not positions in them but past and future 'boundaries' of positions in them (pp. 32–3). 'Big bangs' are followed by intervals occupied "by superparticles interacting by means of the superforce. . . . Following this interval . . . the superparticles become differentiated into . . . forces. This differentiation . . . occurs in chance or random ways . . . [which] determine the basic physical constants" (p. 34). Differentiations are governed by 'metastatistical laws' for differentiations of initial 'early-on' conditions of cosmoi. "In the scenario we are envisaging there is **no universe** that exists **unexplained** and **no set of basic laws** whose obtaining is **unexplained**. **Our ultimate 'brute facts'** are not the existence of a universe or the obtaining of a set of basic laws of a universe but **the existence of an infinite series and the obtaining of the metalaws** . . . " (p. 35; emphasis added). The 'explanations' envisaged are for all possible cosmoi. As said, nothing in the theory explains why our cosmos is as if tuned for life.

In Smith's story "universes that have laws or . . . conditions that do not permit [the] formation [of 'black holes'] . . . are 'dead ends'" (p. 35). The theory

is not an evolutionary theory, but it has an extension that is.[33] Suppose speculation is added regarding metastatistical laws relating laws and conditions of cosmos c for which singularity S is a 'black hole,' to 'early-on' conditions in cosmos c′ for which singularity S is a 'big bang.' Suppose in particular speculation to the effect that the likelihood of 'black holes' in a cosmos is likely to be approximately equal to their likelihood in the cosmos from which it is descended. The result could be an evolutionary theory 'of sorts' that favored the 'propagation' of cosmoi in which the formations of 'black holes' was more and more probable. Such a selective, evolutionary theory could make *especially likely* the appearance in the infinite branching series of cosmoi, cosmoi such as ours that are rather prone to '*black holes*' and *therefore*, perhaps, *as if* tuned by a great Tuner *for life*, not 'black holes', if conditions that favor 'black holes' also happen to favor life. The rough unfinished speculation of this paragraph was, I have learned, in the direction of Smolin's overtly evolutionary many cosmoi theory.

8.5 Smolin's evolutionary many cosmoi theory. Here are excerpts from Abner Shimony's outline of this theory.

> The standard model of elementary particles requires nineteen fundamental dimensionless parameters. . . . This proliferation of apparent arbitrariness . . . is generally regarded as a serious blemish of the standard model. . . . What Smolin proposes is to keep the standard model but to supplement it with an evolutionary explanation of the values of the nineteen parameters. He makes two postulates: "The first of these is . . . time does not end in the centers of black holes, but continues [through a big bang] into some new region of space-time . . . (Smolin 1997, p. 93). Smolin's second postulate is that "the basic forms of the laws don't change during [such a] bounce, so that the standard model of particle physics describes the world both before and after the bounce. However, I will assume that the parameters . . . do change. . . . I . . . postulate . . . that these changes are small and random."[34] (Smolin 1997, p. 94) . . . Universes with different values of the nineteen parameters differ greatly in their propensity to generate other universes. . . . [T]he fitness of a universe [meaning roughly 'the expected number of descendants'] is determined by its propensity for producing black holes. The centre of Smolin's argument . . . is the sketch of a demonstration that the range of the nineteen parameters which determine universes with high fitness is extremely narrow . . . the distribution of values of the parameters is highly peaked. Most actual universes [therefore] have parameter values in the narrow range which generates many descendants. If one then assumes . . . that our universe is with overwhelming probability 'typical', then an explanation is provided for the values of the parameters . . . found in *this* universe. . . . [Smolin] argues further . . . that the values of the parameters that are conducive to the production of black holes are also conducive to life. (Shimony 1999, pp. 216–18)

Now come Smolin's words for salient features of his theory, and my reports of some of these features.

> The idea that the parameters . . . might change at a bounce . . . [is present already in the Phoenix model] championed . . . by John Archibald Wheeler. He called it the

"reprocessing" of the universe. What I am adding . . . is only . . . that change at each bounce is **small** (p. 95; emphasis added).[35] Added also, I report, are (1) that the standard model of quantum particle physics prevails before and after every one, and that *only* the *parameters* change at a bounce; (2) that there is a 'bounce' into a new universe at each collapse, in or of, a universe, that is, at each local collapse into a black hole, as well as at the global collapses of whole universes; and (3) that "*our universe is a typical member of the collection*" (p. 101; italics original).[36] From these postulates it is only a few steps to the conclusion that "*the parameters of the standard model of elementary particles physics have the values we find them to have because these make the production of black holes much more likely than most other choices*." (Smolin 1997, p; 96, italics original.) "[*I*]*t is probable that a universe chosen at random from the collection [of universes] has parameters that are near a peak of the production of black holes*." (p. 101; italics original)[37] The preponderance of universes in the collection are at or near such a peak. And, though we have not chosen our universe at random, since it is typical it was probable when it began that it would be at or near a peak. "[A]t least one way for a universe to make a lot of black holes requires there be carbon and other organic elements, as well as stars that produce these elements in large quantities. The theory then predicts that our universe has these ingredients for life, not because life is special, but because they are typical of universes found in the collection [of universes]. (p. 204)[38]

Voilà!! Tuning for black holes that is *incidentally* sufficient for stars, and so for life. What a nice theory! It at least matches *tuner theories* in terms of predictive/explanatory powers for as if fine-tuned-for-life parameters, and it bests them without contest in terms of intrinsic plausibilities, free as it is of why's and of the especially difficult how's, that plague them. Contrary to Smolin "there is [*not*] little [we] can say against" intelligent tuner theories, and for his theory, other than, pejoratively, that they are "mysticism . . . dependent on a faith about something outside the domain of rationality" (p. 45).

This evolutionary many cosmoi theory has an advantage in predictive/explanatory power over simple many cosmoi theories in that *it* makes especially likely that a cosmos, this one of ours, for instance, should have parameters as if tuned for life. Detracting from that advantage *may* be inferior intrinsic plausibility, given the greater structure established by its postulate that changes in parameters at bounces are random in a narrow range. But simple many cosmoi are themselves 'baggaged' by the assumption, essential to them, that parameters of cosmoi are established in a manner that makes very probable that "cosmoi exhibit a vast number of 'cosmos designs'" (van Inwagen 1994, p. 142).

8.6 John Earman keeps an open mind. John Earman suspects advocates of many cosmoi theories, which are for him all *simple*, nonevolutionary many cosmoi theories, of bad logic: "I don't see that an improbable outcome for a chance experiment gives warrant to think that this particular experiment has been run many times [or in many places]" (Earman 1987, p. 316n5). He observes that Ian Hacking has labeled that inference 'the Inverse Gambler's Fallacy'. There is *something* in this supposedly quite fallacious inference: For if

an improbable result of a chance experiment obtains today, then it obtains on some day, and while the hypothesis of this experiment's being run many times does not make the first fact especially likely, it does do that for the second fact. As has been said, many cosmoi theories are not quite without would-have-had predictive power for facts in evidence concerning fine-tuning, though they are without such power for some that include those they would, if they could, make very probable.[39] Earman may think less of them on this score than I do. But his main disappointment is with their intrinsic implausibilities. They have no "independent scientific justification" (p. 315a). They are at best only coherent "speculation" (p. 315b). Even so, they can be intrinsically more plausible than fine-tuner hypotheses, which are at best mysterious as regards their means.

Largely discounting many cosmoi competitors to design, Earman is left with the hope that there will some day be a good 'deep theory' of the appearances of fine-tuning: "Perhaps the answer lies in a deeper scientific theory which allows the [as if fine-tuned for life] values of fundamental constants to be computed from first principles" (pp. 314b–315b). But what are we to believe, assuming no deep theories are on offer and simple many cosmoi theories are not to be believed? Earman had not heard of evolutionary many cosmoi theories. He all but ignored designer hypotheses (p. 314a), though he seems to have been, vis-à-vis apparent fine-tuning for life, in much the position that Hume was in vis-à-vis apparent design of living things when he opted for a designer hypothesis as the best of the bad lot available. Earman, surveying the lot available to him, chooses instead to rest with 'none of the above'.

8.7. Some say that the appearance of fine-tuning that makes life possible testifies to a pro-life Tuner: FT. Others say that it testifies to this cosmos being just one amongst many, each of which is 'tuned' by a chance mechanism that makes every possible tuning equally probable: MU. Given a choice between just those possibilities, with Earman I would accept neither. I place hardly any credence in either FT or MU. My probabilities for them sum to very little. Given especially the how-problems of FT that *severely* curtail its inherent plausibility, and given the easy consistency of MU with 'the tumbling spectacle' that would make a mystery of a pro-life Tuner's motivation, despite what would have been the considerable predictive advantage of FT, I think that I place no more credence in one hypothesis than in the other. But I am not sure, and it does not matter whether I am right about that. Redirecting words of Philo's, "it is [to my mind] a thousand, a million to one, if either . . . be the true system" (Part 8, p. 143). Were I persuaded that the apparent fine-tuning of which I hear talk cannot be grounded in a 'deeper theory', I would put down the appearances of fine-tuning to our dumb luck. Otherwise, supposing a *worked-out* deep theory of fine-tuning of this cosmos, say a theory along the lines of Smolin's evolutionary many cosmoi theory, were not only more plausible than FT and MU (as in my view his present largely speculative theory already is *by far*) but

actually *acceptable*, that is, more probable than not (as in my view his theory presently is *not*), I would accept it as an explanation of the facts of fine-tuning of this place and put *that* theory's fundamental assumptions down to that luck. After all, not everything can be explained, not everything *can* have a reason (if, it is hardly necessary to qualify, anything is contingent), and a good 'deep theory' of 'parameters for life' could very well be a place at which reasons and explanations for them run out.

8.8 'Perhaps in the beginning there was a wave of chances for cosmoi, with which a deity had nothing to do'. Quentin Smith has favored over both FT and MU a certain deep theory, namely, Stephen Hawking's "'wave function of the universe' . . . quantum cosmology" (Smith 1994a, pp. 236–7). "It is not part of my argument to defend [***Hawk***], although I believe [Hawk] is confirmed by observational evidence . . ." (p. 236). According to Smith, Hawking 'says' that in the beginning, when there was nothing – when there was no matter distributed in space and time – there was the wave, that is, there were in the beginning unconditional objective chances for "'the Universe to appear from nothing' ([J. Hartle and S. W. Hawking, "Wave Function of the Universe," *Physical Review* 1983, 2960–75)" this way or that, with so much matter distributed thus, in space of such and such curvature, *and with basic parameters just so*, all governed by the wave function (p. 237). Calculations are envisioned that would show that "our universe is not just one of the possible histories [which presumably goes without saying] but one of the most probable histories" (Hawking 1996, p. 177).

The theory may be felt to have an explanatory advantage over

> the standard hot big bang model [according to which] the universe began to exist . . . from a physical singularity. . . . At this singularity, all physical laws break down . . . it is in principle impossible to predict what will emerge . . . the existence of the singularity itself is an unexplained given. Hawking's quantum cosmology omits the initial singularity and implies that it is probable (to a degree less than one) that the universe begins to exist with a nonsingular state in accordance with the wave function law.[40] (Smith 1994a, p. 237)

According to Smith, "[i]t is precisely this implication [which can seem to give Hawking's cosmology an advantage] that precludes the existence of God" (p. 237). How so? Like this, according to Smith: "If there is a theism consistent with [Hawk] . . . it must be an acausal theism [***AcausTh***.]" (p. 242). "God's classical role as the cause of the universe [that condensed out of the cloud] is inconsistent with . . . divine attributes . . . [including] rationality" (p. 241). Why? Because there could be no reason for God to ordain a law that gives this world a 95% chance supposing no divine interference, "if God intends to bring this universe into existence by his will [if need be] and thereby vitiate [this] condition laid down in the law" (p. 241). Classical causal theism, **ClssCausTh** is out: Only composites of AcausTh and Hawk are

consistent theories.[41] According to these theories, "God does not create the universe. . . . God wills that the wave function law obtains but does not will that the universe [that condenses out of it] exist. Rather, God leaves to chance . . . that the universe [this fine-tuned one for which we would thank its Maker] . . . will begin to exist uncaused" (p. 239). Against any theory that cobbled an AcausTh and Hawk, Smith writes that

> there seems to be no empirical evidence that would confirm the conjunction of acausal theism and the wave function to an equal or greater degree than it would confirm the wave function alone. The anthropic coincidences do not count as separate evidence for theism, ***since these are predicted by the wave function law***. As Hawking writes, "[Calculations show] that, using the sum over histories, our universe is not just one of the possible histories but one of the most probable ones." ([*A Brief History of Time*], p. 137). (Smith 1994a, p. 242; bold emphasis added)

An (AcausTh + Hawk) theory is of course of less intrinsic plausibility than Hawk (as well as [Hawk & ∼(AcausTh + Hawk]), on account of the extra baggage of AcausTh and, even more so, Hume can add, because of the how-questions that AcausTh invites which one cannot imagine being answered.[42] And it should be inferior for anyone, in would-have-had predictive power to Hawk, and to [Hawk & ∼(AcausTh + Hawk)]. God, according to the theory, "wills that the wave function obtains" (p. 239), and as a consequence it does obtain. Marvellous! But if He was capable of that, why, rather than risking disappointment, did He not simply will and thereby execute the universe He wanted? Could He establish, just by willing it, the wave function that makes this universe highly probably, but not just by willing make this universe or any other universe for sure? If not, why not? Executing a *definite* intention sounds easier. But if he could do that, why didn't he? Did he not care which obtained of the several universes to which he allowed some chance?

8.9 A digression from fine-tuning to big bangs. According to some big bang theories there was a "beginning-point to space-time" – there was a first instant of time when "the radius [of the world] was zero and the density of matter, temperature and curvature of the universe were all infinite" (Smith 1991, p. 50). These theories posit "an instantaneous state of lawlessness . . . during [which] no physical law obtains that could connect [this first state] to later instants. . . . At any instant arbitrarily close to the [first] instant . . . physical laws do obtain . . . connecting [the configuration C at this instant] to configurations occupying later instants [and also to configurations at all post-big-bang earlier instants] but there obtains no law connecting C to the [first unarticulated state that banged]. C adopts a lawful evolution but has its ultimate origin in primordial lawlessness" (p. 52).

A theologian who would meld such a big bang with a God-hypothesis is faced with vexing how-questions and why-questions. Why, supposing

God had some ends in view – for example, moderately happy animate creatures in profusion, though any ends will do – would He have chosen to have His creation begin with an instantaneous inherently lawless state out of which just anything at all could have naturally emerged? That choice would seem to have called for planned intervention to control the 'explosion' into space-time. Why would He have had everything start in that way, given that He could instead have started with an initial state that led naturally and lawfully for sure to His ends, or 'started' with a beginningless series of states each of which led naturally and lawfully for sure to His objectives for later states? It is no answer to say, "Why not?" or "Only God knows." These big bangs, Smith says, would be inconsistent with God's rationality (p. 58). He might better have said *prima facie* inconsistent as long as that 'Why?' goes unanswered.[43] "It is [*prima facie*] a mark of inefficiency, incompetent planning, and poor design to create as the first natural state something that [by all objective chances] needs supernatural intervention 'right off the bat' to ensure that it leads to the desired outcome" (Craig and Smith 1993, p. 239; Smith's words.)[44]

9. IT IS BEST TO LEAVE GOD-LIKE BEINGS OUT OF OTHERWISE NATURAL EXPLANATIONS

Adding D, a designer theory, to E, an evolutionary theory, or to (SmolinEvMC + E) or to (Hawk + E) would be like combining T, a teetotaller dehydration theory, to H, a hangover theory when the facts to be explained are bloodshot eyes, parched throat, headache, and so on. While H could be a good explanation and T a not completely negligible explanation, putting them together would make a bad explanation inferior to both and to its explicit contraries [H & ∼(H + T)] and [T & ∼(H + T)]. The composite (E + D) is, I think, related similarly to E and D alone and, more remarkably, to its contrary [E & ∼(E + D)]. The same holds, if not more so, for the composite theories [(SmolinEvMC + E) + D] and [(Hawk + E) + D] in relation to their nondesigner parts as well as to their explicit nondesigner contraries. It is a matter of the how and why problems that D brings with it. It is because of them that these cosmologies are better off without intelligent principles. "[T]o suppose . . . in these cases, an unknown voluntary agent is mere hypothesis; and hypothesis with no advantages" Hume wrote some time ago (Part 8, p. 143). It is, he could have added, hypotheses with considerable disadvantages.

A CONCLUDING SCIENTIFIC POSTSCRIPT

Theistic explanations have always drawn their evidential support from facts for which no natural explanations were known. And so as the bounds of science expand, and more and more of nature's puzzles are solved in natural terms, the evidential support for theistic hypotheses contracts. At the limit it vanishes.

When this kinematic becomes evident to persons who are in initial sympathy with the methods of natural theology, something remarkable can happen to their arguments. The claim that facts, in order to be made intelligible, need to be understood in theistic terms, can change. It tends to change from the claim that this is required for purposes of a good, open-ended, ordinary explanation of facts concerning mainly living things to the claim that only by recourse to supernatural terms and necessary beings can one reach a really complete and finished understanding of any thing at all. This tendency to change was known to Hume, who had Demea give voice to it: "if so many difficulties attend the argument *a posteriori*, said Demea; had we not better adhere to that simple and sublime argument *a priori*. . . . Whatever exists must have a cause. . . . In mounting up . . . from effects to causes, we must either go on in tracing an infinite succession, without any ultimate cause at all [which can be proved absurd], or must at last have recourse to some ultimate cause, that is *necessarily* existent . . ." (Part 9, p. 148).

What begin as teleological arguments for designers are apt to end as cosmological arguments for necessary beings as self-explaining ultimate causes or reasons for contingencies. What are initially entirely reasonable quests for ordinary explanations of certain aspects of nature have a way of degenerating into what are quite unreasonable demands for utterly impossible kinds of explanations of, or grounds for, contingencies. They have a way of doing this as the suspicion grows that in the end science and ordinary natural explanations are bound to appropriate to themselves all would-be evidence for supernatural explanations and all explanatory roles that theisms might perform. A better response for the religious to the relentless advance on all fronts of secular science, and a more secure response, is to give over the field of explaining nature to science and to base belief and religion not on arguments and reason, but on one's personal experience of God, if persuaded they have some, or maintain it in faith, if they have, or can acquire it. Another response to the relentless progress of science is, pending undeniable personal encounters, to give up on God and religion and live alone in the world, save for the rest of humanity and creatures great and small.[45]

APPENDIX. SWINBURNE'S TELEOLOGICAL ARGUMENT, AND HIS CUMULATIVE ARGUMENT, FOR THE EXISTENCE OF GOD

A1 Swinburne's teleological argument. It is an argument from fathomable uniformities. Swinburne maintains that a certain grand fact **E** tends to confirm an intelligent agency hypothesis G (I use 'E' and 'G' where Swinburne uses e and h). The fact, E, is not that there is widespread adaptation of means to ends in the biological realm, nor that, for example, the solar system and hydrogen atoms 'run like clockwork,' but rather that the universe always has been, is, and ever will be orderly according to "simple, formulable, scientific laws . . . which men can recognize and describe" (Swinburne 1994a,

p. 45b; Swinburne 1994a comes from his *The Existence of God* of 1979). Swinburne "takes the common-sense view that [belief in E is] justified" (Ibid.). He does not explicitly represent E as 'certain evidence' of probability 1. Leaving us out of it, E is the presumed fact of "the vast **uniformity** in the powers and liabilities of bodies through endless time and space, and also the **paucity** of kinds of components of bodies" (p. 47b; emphasis added). It is not that the world accommodates living beings, but, roughly, that it accommodates beings such as us who can learn from experience not only enough to get around with, but deep and great truths of nature, the ways of which not just where we live, but everywhere, are accessible to our limited minds. Swinburne claims that this great fact tends to confirm G, that there is "an agent of great power and knowledge who brings about through his continuous action that bodies have the same very general powers and liabilities [always and everywhere] . . . [specifically, since this is] the simplest to postulate . . . [that there is] one [such agent] of infinite power, knowledge, and freedom, i.e., God" (Ibid.). It may be observed that G, as spelled out here, *entails* E. Swinburne in his argument takes as "the alternatives– . . . first, that the temporal order of the world is where explanation stops [and that there is no cause of E], and second, [G]" (pp. 48a–b [49b]). Let this first hypothesis be hypothesis N. He "ignore[s] the less probable possibilities that the order is to be explained as due to the agency of an agent or agents of finite power" (p. 48a). He also ignores the possibilities that the order is to be explained as due to an agent of infinite power who is not God, G, and that it is to be explained by a many cosmoi theory S+ along the lines of Smolin's, according to which the temporal order of the world is *not* where explanation stops. ("But S+ will have it stop with the 'supertemporal order' of cascading cosmoi." True, but not distinguishing. Swinburne's explanation stops with God. It is the way of explanation contingencies to stop somewhere, with some contingency.) Swinburne proceeds as if the disjunction of possibilities other than G and N is so improbable, that N and ∼G – though they are not logically equivalent, or even exactly equivalent probabilistically – can be treated in conditional probabilities as if they were equivalent probabilistically.

A2 The 'logic' of this argument. Swinburne's *thesis* – what he would demonstrate – is that E *tends* to confirm G, $\mathbf{P(G/E) > P(G)}$. His thesis is *merely* that. He offers what he characterizes as a good **C-inductive** argument. He does not claim that E serves a good **P-inductive** argument for G, $\mathbf{P(G/E) > \frac{1}{2}}$. He offers, in other words, to demonstrate only that E *incrementally confirms* G, and not also that it *absolutely confirms* G. His incremental confirmation thesis does *not* entail that G is more probable on this evidence E than is ∼G, P(G/E) > P(∼G/E). It does, however, given that P(∼G) > 0, entail that E tends to disconfirm ∼G, P(∼G/E) < P(∼G); for P(G/E) > P(G) is equivalent to P(∼G/E) < P(∼G).[46] To see this consider that P(∼G/E) = 1 − P(G/E) and

$P(\sim G) = 1- P(G)$. That $P(G/E) > P(G)$ does not, however, entail that $P(\mathcal{G}/E) < P(\mathcal{G})$. It is indeed compatible with $P(G/\ /E) > P(E)$. It is similarly compatible with $P(S+/E) > P(S+)$.

Swinburne reaches the conclusion that $P(G/E) > P(G)$, by way of the inequality $P(E/G) > P(E/\sim G)$, which, given that G and ∼G are positively probable, is equivalent to it.[47] Swinburne's argument for $P(E/G) > P(E/\sim G)$ rests on the to-his-mind 'near enough to probabilistic equivalence' of ∼G and N that makes them interchangeable in conditional probabilities terms. It is plausible that, from a perspective in which one knew nothing of E, learning that N, that there is no cause or reason for the orderly or chaotic way of nature, would not enhance one's probability for: (0) $P(E/N) \not> P(E)$. Given that 'near-enough' equivalence, Swinburne can help himself to (i) $P(E/\sim G) \not> P(E)$. From that perspective, (ii) $P(E/G) > P(E)$: This is because, from that perspective, $P(E) \neq 1$ and $P(E/G) = 1$, since, as observed, G entails E. Swinburne maintains this identity (p. 51b), though (see pp. 50b–51b) not for this simple reason. It follows from (i) and (ii) that, from that perspective, (iii) $P(E/G) > P(E/\sim G)$, which, as said, is equivalent to his conclusion, (iv) $P(G/E) > P(G)$. A problem for this reasoning is its reliance on the idea of a 'near-enough' equivalence between N and ∼G to reach (i) from (0) by putting '∼G' in place of 'N'. Let '≈' here stand for 'almost equal to.' It is not clear what, if anything, is in the idea that if $P(p \equiv q) \approx 1$, then $P(p/r) \approx P(q/r)$ and $P(r/p) \approx P(r/q)$. Another problem with this reasoning is that it is unnecessarily complicated, since (ii) is equivalent to (iv): One can go back and forth between (ii) and (iv) by way of the 'intermediaries' $P(G \& E)/P(G) > P(E)$, $P(E \& G)/P(G) > P(E)$, and $P(E \& G)/P(E) > P(G)$.

A3 The argument compared with Cleanthes'. There are important differences here other than Swinburne's down-playing, because it is taken care of by Darwin (p. 44b) of evidence of adaptations of means to ends in living beings. For one, Cleanthes's argument aspires to be a good *P-inductive* argument. This makes 'prior probabilities' or intrinsic plausibilities of hypotheses relevant to assessments. Swinburne's argument, given its modest *C-inductive* aspirations, can ignore prior probabilities, though they do figure quietly in his undeveloped (and I think inadequate) reasons for letting ∼G and N 'come to the same thing' in his argument. For a second contrast, several alternatives to the Religious Hypothesis are given respectful hearings. There is no suggestion in the *Dialogues* that it is either Cleanthes's detailed and religiously loaded explanation of the facts of apparent design or no explanation at all. For a third, Cleanthes's argument, again because it is offered as a good P-inductive argument and not merely a good C-inductive argument, opens the door to other facts. Swinburne's C-inductive argument can confine himself to the evidence E that it targets. The question for Swinburne is merely whether this particular evidence, taken by itself, tends to confirm G, on which *very* narrow question

Philo for one might be happy to allow that E does tend to confirm G *a little*, while wondering why Swinburne goes on about this so.[48] After all, that E tends to confirm G is consistent with its tending to confirm each of its alternatives $\mathcal{G}$ and S+.

A4 'Cumulative confirmation' – 'Don't try this at home!'

A4.1. In defense of the modest aspirations of his argument from design, Swinburne would say that it is part of a stage of a two-stage cumulative argument to justify belief in God. Section A5 is about his cumulative argument. Now I consider something like the first stage of it. In this argument various kinds of evidence are considered one after the other in relation to theism: for example, first the sheer existence of a complex world, next that the world is in many ways as if purposefully ordered, eventually that it can seem that one would expect to find more and better in a world that was subject to a perfect being, and so on. Some evidence tends to confirm, some disconfirm. But the 'yeas' have it: *Taken all together*, the evidence 'on balance' tends to confirm, that is, theism is incrementally confirmed by the combined evidence. Cf.:

> The *consensus gentium* argument argues for the truth of the hypothesis that God exists based on the evidence of the common belief of men and women that he exists. . . . Cicero may have thought that this evidence by *itself* was sufficient for believing in God's existence, [but] of greater interest is whether the argument provides any support for God's existence. . . . With the exception of the ontological argument, arguments for God's existence are essentially inductive arguments claiming that belief in God is justified based on various kinds of available evidence. . . . [The *consensus gentium* argument] may, when combined with other evidence . . . provide sufficient support for rational belief in God's existence. With the addition of the evidence of common consent to other available evidence, the scale may be tipped in the favour of theism. (Meierding 1998, pp. 272–3)
>
> Mere incremental confirmation may not be what theists want for their doctrines, but it is a start. And once the start is made, there does not seem to be any principled road block to achieving a substantial degree of confirmation. For example, testimonies to a number of New Testament miracles can each give bits of incremental confirmation to [some tenet . . . of Christianity] that together add up to substantial confirmation. Or the evidence of miracles can combine with the evidence of prophecy and design to provide grounds for the credibility, or even moral certainty of religious doctrines. (Earman 2000, pp. 66–7)

A4.2. Cumulative confirmation cannot rest on the principle – let it be Cumulative Confirmation – that, for a body of evidence, E, where $\Box$ [E = (e_1 & e_2 . . .e_n)], and a hypothesis h, the incremental confirmation of disconfirmation provided by E, [P(h/E) - P(h)], equals the 'algebraic average' of

the incremental confirmations and disconfirmations provided by e_1, $e_2, \ldots, e_n$,

$$\frac{\sum_{1 \leq i \leq n}[P(h/e_i) - P(h)]}{n}.$$

The method cannot rest on this principle because, even if each of several lots of evidence e 'tends to confirm' h, in the sense that for each, $P(h/e) > P(h)$, they can when combined into one body of evidence E *disconfirm* it in that sense, it can be that $P(h/E) < P(h)$. The principle, for just two lots of confirming evidence, e and e',

$$([P(h/e) > P(h)] \,\&\, [P(h/e') > P(h)]) \supset (P[h/(e \,\&\, e')] > P(h))$$

is not valid, and similarly, therefore for n lots of confirming evidence. "With the addition of . . . [supporting] evidence [e for a hypothesis h] . . . to other available evidence [e′] the scale may be tipped" (Meierding 1998, p. 273) *not only for* the hypothesis *but against* it, even when the other evidence e′ is also supporting in the sense that $P(h/e') > P(h)$![49] Wesley Salmon tells a story of radioactive decay to make the point that "[e]ven if each set of measurements [of different dimensions of an experimental result] confirms [a] hypothesis. . . . [that] the conjunction of the findings . . . confirm[s] the hypothesis . . . does not follow automatically. . . . [Whether this conjunction confirms it] depends on more circumstances, including . . . that the conjunction itself is one of the predictions of the theory. . . . there are broad and basic questions about the legitimacy of . . . accumulation of many confirming test results" (Salmon 1973, p. 80).

Another problem for the method of cumulative confirmation as presently conceived is that a body of evidence E will not have a unique 'division' into pieces of evidence of which it is the conjunction. For example, if $\Box[E \equiv (e \,\&\, e')]$, then $\Box\,[E \equiv ((e \vee f) \,\&\, (e \vee \sim f) \,\&\, e')]$. The problem for the method as presently conceived is that

$$([P(h/e) - P(h)] + [P(h/e' - P(h)])/2$$

need not equal

$$([P[h/(e \vee f)] - p(h)] + [P(h/(e \vee \sim f)] - P(h)] + [P(h/e') - P(h)])/3.^{50}$$

A4.3. Perhaps, however, as Salmon suggests, these challenges to Cumulative Confirmation can be met by *answering* those "broad and deep questions about the legitimacy of . . . accumulation of many confirming test results" (Ibid.) Perhaps, for example, there are conditions **K** that are common in cases in which we seem to practice 'adding up the evidence to find where its balance lies', which conditions are such that the principle is *valid* that,

> *Cumulative Confirmation**. For a body of evidence $\Box[E = (e_1 \,\&\, e_2 \ldots e_n)]$ and hypothesis h, such that the division E into $e_1, e_2, \ldots, e_n$ satisfies conditions **K** for hypothesis h,

$$[P(h/E) - P(h)] = \frac{\sum_{1 \leq i \leq n}[P(h/e_i) - P(h)]}{n.}$$

Perhaps, for part of another possibility, confirming evidence as ordinarily conceived when we seem to practice 'adding up the evidence' is not well-explicated by the difference [P(h/e) > P(h)]. Perhaps, when well-explicated, confirming evidence can be seen necessarily to cumulate. For example, suppose E is the 'total evidence' for h and that e is a piece of E. Rather than say that *e supports h* if and only if P(h/e) > P(h), *perhaps* we should say that e supports h if and only if *e makes a positive contribution to h* in the sense that P(h/E) > P[h/(E ∨ ∼e)]. That would be a response of sorts to those 'broad and deep questions' raised by the invalidity of Cumulative Confirmation, *if* the following principle is valid:

Cumulative Positive Contributing Evidence. [(P(h/E) > P[h/(E ∨ ∼e)]) & (P(h/E) > P[h/(E ∨ ∼e′)])] ⊃ (P(h/E) > P[h/(E ∨ ∼(e e′))])

A4.4. I leave the difficult question of the validity of that principle, since even with a solution in hand to the problem of accumulation of many tests confirming and disconfirming test results, 'we would only be getting started here' on deep questions concerning the idea of cumulative confirmation *in what would be Loren Meierding's sense*, and I think Earman's. For we would have managed only questions raised by cumulating *incremental* confirmations and disconfirmations, provided by pieces of evidence, to reach *incremental* confirmations or disconfirmations by the *bodies* of evidence composed of these pieces. Meierding's idea, and it seems Earman's, is of a more wonderful process. It is of a process wherein from *incremental* confirmations and disconfirmations of several kinds of evidence bearing on God's existence, or some religion, one may reach an *absolute* confirmation or disconfirmation: "With the addition of the evidence of common consent to other available evidence, **the scales may be tipped**" (Meierding 1998, p. 273). Evidence "can combine . . . to provide grounds for . . . even moral certainty of religious doctrines" (Earman 2000, p. 67). Envisioned is an *argument* by accumulation of incrementally confirming and disconfirming evidence to *absolute* confirmation or disconfirmation. But it is *impossible*, from only incremental confirmations and disconfirmations, to reach absolute confirmations and disconfirmations.

Incremental confirmations and disconfirmations that evidence can provide for a hypothesis are, as Meierding sees, "independent of the *prior probabilities*" of this hypothesis and its competitors (Meierding 1998, p. 278). The method of 'P-argument' by accumulation of evidence would solve 'the problem of prior probabilities' when measuring the support provided by evidence for a hypothesis, and saying whether it 'tip the scales' for it, by leaving these probabilities out of its calculations. It is, however, a lesson of Bayes's theorems that, in cases in which neither the evidence when combined into a conjunction

nor the hypothesis, is antecedently absolutely improbable, 'prior probabilities' *cannot be left out*. In these cases, the support provided by evidence e for a hypothesis h, P(h/e), and whether this evidence 'tips the scales' and is 'sufficient for rational belief', P(h/e) $\frac{1}{2}$, is *always* in important part *dependent* on the 'prior probability' of this hypothesis.[51] That is so even when the total evidence can be 'broken up' into parts, each of which can be seen incrementally to confirm, whereas, according to the idea of argument by acccumulation of evidence, at least in that case the scale should be definitely tipped. The problem of *cumulating incremental confirmations* that Salmon leaves *may* have a happy solution. The problem with what would be *'P-arguments' by accumulation of evidence for and against* is *not* solvable: Getting absolute confirmations out of incremental confirmations is impossible.

The only presently developed theory of what to do in well-conducted assessments of the bearing on balance of several pieces of evidence to be explained or at least accommodated on a hypothesis recommends *combining them into one body of evidence* and, surveying alternatives to this hypothesis in a partition (logical or probabilistic),[52] proceeding as best we can within the framework provided by Bayes's theorem for a hypothesis in a partition to assess 'prior' intrinsic plausibilities and 'likelihoods' (would-have-had predictive powers for the body of evidence). This exercise is worth getting into only when the evidence being processed is *all* the evidence available that the competing hypotheses should explain or at least accommodate. The meager result of Swinburne's argument from design, according to which a certain *limited* body of evidence is 'incrementally positive' for what he wants to believe is not a step, it is not *part* of a step in this Bayesian process, and similarly for the evidence of common consent that Meierding promotes. *Their discussions provide only things to think about*, of which there are many, when assessing the 'likelihoods' of hypotheses of theisms and alternatives to them of *all* evidence to be explained, or at least accommodated, *combined*. And even with these assessments in hand to plug into Bayes's theorems, there can be left to assess (if this has not already been done) the 'priors' of hypotheses in the partition, which Meierding (perhaps inadvertently) and Earman (certainly inadvertently) imply can be irrelevant to the establishment of sufficient support in evidence for rational belief.[53]

A5 On Swinburne's cumulative argument for the existence of God

One unfortunate feature of recent philosophy of religion has been a tendency to treat arguments [e.g, the cosmological and teleological] for the existence of God ['a person without body (i.e., a spirit) who is eternal, is perfectly free, omnipotent, omniscient, perfectly good, and the creator of all things' – Swinburne 1979, p. 8] in isolation from each other. There can of course be no objection to considering each argument initially, for the sake of simplicity of exposition, in isolation from others. But clearly the arguments may back each other up or alternatively weaken each other. . . . That arguments may support and weaken each other is . . . more evident, when we are

dealing with inductive arguments. . . . In order to consider the cumulative effect of arguments, I shall consider them one by one, starting with the cosmological argument and including the argument from evil . . . , and ask how much the premisses of each argument add to or subtract from the force of the previous arguments. . . . I shall argue that most of the arguments (taken separately and together) for the existence of God are good C-inductive arguments – that is to say, their premises make it more likely that God exists than it would otherwise be . . . [and] that the argument from evil [against] . . . is not a good C-inductive. . . . If that is right, then the net effect of taking together the premises of the arguments which I consider is that we will have a good C-inductive argument to the existence of God. . . . In the last chapter I shall reach a conclusion on whether or not the balance of all the relevant evidence favours theism [for a good P-inductive argument]. . . . I shall devote most of my time to assessing the inductive strength of [the] arguments . . . [and of each] whether it is a good C-inductive argument. . . . [I]t is a lot easier to see when we have a good C-inductive argument than when we have a good P-inductive argument. (Swinburne 1979, pp. 13–14)

A5.1. Swinburne argues that each of six arguments for the existence of God is a good C-inductive argument. He reaches these conclusions by comparing the likelihood of the evidence of an argument's premises, e, on the hypothesis that God exists, G, with its likelihood on the hypothesis that God does not exist. Finding in each case that $P(e/G) > P(e/\sim G)$, he concludes (validly) that $P(G/e) > P(\sim G/ e)$, and that $P(G/e) > P(G)$ (op. cit., Chapters 7 through 10, 12, and 13). He considers one argument against the existence of God, the argument from evil (Chapter 11), and concludes that here $P(G/e) = P(G)$. Given these assessments of "the main evidence for and [possibly] against the existence of God" (p. 277), Swinburne *takes for granted* that, if E is the evidence of the premises of these arguments *combined*, then $P(G/E) > P(G)$. "The crucial question remaining . . . is just how probable all of the evidence which I have considered makes the hypothesis. . . . the important issue [remaining] is whether" $P(G/E) \frac{1}{2}$ (p. 278). That is, *though*, from what his chapters on the arguments claim, specifically, that

$$\text{for each conjunct of e of E but one, } P(G/e) > P(G),$$

and that

$$\text{for no conjunct e of E, } P(G) > P(G/E),$$

it does not necessarily follow that

$$P(G/E) > P(G);$$

Swinburne does not provide reasons for this penultimate conclusion of his argument for the existence of God. He must have thought that this grand result for the evidence of the arguments combined *did* follow necessarily from the particular results for the evidence parcelled out for which he does argue. He

wrote that "[t]here can of course be no objection to considering each argument initially, for the sake of simplicity of exposition, in isolation from others" (p. 13) to see whether their evidence tends to confirm or disconfirm the existence of God. I agree that there can be no objection to that 'in principle,' which is consistent with its not being a good idea. *Prima facie* it is a *complete waste of time*, if the object is to see whether their evidence *combined* tends to confirm or disconfirm the existence of God.

A5.2. Swinburne uses, to reach the ultimate conclusion of his argument, that $P(G/E) > \frac{1}{2}$, Bayes's theorem for a hypothesis and its negation,

$$P(G/E) = \frac{P(G) \cdot P(E/G)}{P(G) \cdot P(E/G) + P(\sim G) \cdot P(E/\sim G)}.$$

That makes *relevant* his penultimate conclusion, that $P(E/G) > \underline{P}(E/\sim G)$, to which he has labored (but not well, as recently observed),[54] though seeing this reminds that every bit as relevant is the relation between the 'priors', P(G) and P(∼G). It can be gathered from the displayed application of Bayes that $P(G/E) > \frac{1}{2}$ if and only if

$$\frac{P(E/G)}{P(E/\sim G)} > \frac{P(\sim G)}{P(G)}.$$

That makes unfortunate his decision to devote almost all of his book to the issue of the first ratio, regarding which the 'priors' of the second ratio are irrelevant and can be ignored. *Why* did he devote most of the book to an argument for the 'C-confirmation' (incremental confirmation) of his theism on all the evidence, when the *issue* for him is whether there is a good argument for its 'P-confirmation' (absolute confirmation)? He says this is because "[i]t is a lot easier to see when we have a good C-inductive argument than when we have a good P-inductive argument" (p. 14). He does not add, but might have, that this is precisely because when investigating issues of 'C-induction' those troublesome 'priors' can be ignored. Cf.: "Why are you looking for your keys only here, under this lamp, when you know you may well have dropped them there amongst the bushes?" "Because there is more light here." Not a good reason.

Swinburne does not, as I say that Hume implictly does so well, use Bayes's theorem for a hypothesis in a partition. This, though his summing up is in its terms:

> The phenomena which we have been considering are puzzling and strange. Theism does not make their occurrence very probable;[55] **but nothing else makes their occurrence in the least probable**, and they cry out for explanation. *A priori*, theism [Swinburne means his particular version of perfect-being theism] is perhaps very unlikely, **but it is far more likely than any rival supposition**. Hence our phe-

nomena are substantial evidence for the truth of theism. (p. 290; bold emphasis added)

Nor does Swinburne execute the shift that Hume does at least for the assessments of relevant likelihoods to 'priors' of theism and its rivals, and likelihoods of the phenomena on them, *beforehand*. Executing the necessary shift has particular relevance to the supposed great superiority of the 'prior' of his theism to that of every alternative to it. Many, reflecting on the internal tension of Swinburne's idea of an *eternal* perfect-being *creator* and continuous manager of the world, will not agree that his theism would have been 'beforehand' even the most probable *perfect-being* hypothesis.

VIII

Clouds of Witnesses*
"*Of Miracles*"

> Therefore let us also, seeing we are compassed about with so great a cloud of witnesses . . . run with patience the race that is set before us. . . .
>
> *Hebrews* ***12***.1

> There surely never was a greater number of miracles ascribed to one person, than those, which are lately said to have been wrought in France upon the tomb of Abbé Paris. . . . Where shall we find such a number of circumstances, agreeing to the corroboration of one fact? And what have we to oppose to such a cloud of witnesses, but the absolute impossibility or miraculous nature of the events, which they relate?[1]
>
> David Hume

INTRODUCTION AND PREFATORY REMARKS

The evidence of testimony depends not only on the reliability of those testifying, but also on the credibility of what they attest apart from their testimony for it.

> [When] the fact which the testimony endeavours to establish partakes of the extraordinary and the marvellous – in that case the evidence resulting from the testimony admits of a diminution, greater or less in proportion as the fact is more or less unusual . . . "I should not believe such a story were it told to me by Cato" was a proverbial saying in Rome. . . . The incredibility of the fact, it was allowed, might invalidate so great an authority. (p. 113)

How great the diminution, when the fact partakes of not merely the extraordinary and marvelous, but the miraculous?[2]

0.1. The object of this chapter is to make the best of Hume's "Of Miracles." There are points, though not many, at which I think that better can be done than the best that can be found in "Of Miracles": One of major significance is

elaborated in a passage at the beginning of Part Two. To come, eventually, is a formal probability articulation of Hume's principal thesis, which is, as Laplace wrote, "simple common sense" of a kind, and "the theory of probability is really only [that kind of] common sense reduced to calculus" (Laplace 1917, p. 196).[3] In defense of my "imposition of a twentieth century framework, with its language of conditional probabilities, upon an eighteenth century debate" (Gower 1989, p. 4), this twentieth-century framework is not only common sense reduced to calculus, but most of the essentials of this framework were reached in developments in the eighteenth century that culminated in the work of Laplace. The framework was not fully spelled in time for Hume's mid-century work on miracles, but its general shape was certainly not alien to the intellectual environment of his day. It turns out that Hume's central thesis, regarding when testimony is sufficient to make reasonable belief in a miracle, admits of a formulation that is a theorem of elementary modern probability theory, a valid principle of Bayesian confirmation theory.

0.2. Barry Gower objects not only to the use of modern principles for conditional probabilities, but to the use of simple unconditional probabilities as now understood. In his view every mathematical theory that represents probabilities of repeatable events as ratios is foreign to Hume's thought. Certain passages on the formation of probable belief in which Hume uses mathematical terms such as 'subtracting' and 'deducting' for mental procedures are taken by Gower to imply that probable beliefs of repeatable events would be represented not by ratios, but differences:

> Suppose, for example, that the die has four sides marked with a circle and two marked with a cross. Then, said Hume, "the impulses of the former are . . . superior to those of the latter . . . and the inferior destroys the superior, as far as its strength goes." Evidently Hume's idea was that it is the *difference between*, rather than the *ratio of*, the number of chances favourable to an outcome and the number of chances unfavourable to it, that measures the strength of our belief for or against its occurrence. (Gower 1989; cf., 1990 and 1991.)

To respond, Hume's primary concern when theorizing about "the probability of chances" and "the probability of causes" (especially in the *Treatise* Book I, Part III, Sections XI and XII, and the *First Enquiry*, Section VI) was with the psychology, not arithmetic, of probable beliefs. Inferences from his discussions to implicit mathematics should be tempered by the tenability of views inferred. In particular, notwithstanding mathematical imagery and metaphor employed, I think that in so far as Hume had in mind a measure for probable beliefs in chancy events, it was a ratio-measure. An argument against Gower's contrary suggestion is the untenability on the least 'experimentation' of the difference-measure he would have Hume endorse. Gower, without elaborating, makes the understated concession that "the cogency of these [lost in history, difference-measure] ideas is . . . questionable" (Gower 1989). Consider against these ideas

that the difference between chances for a circle, and those against, on the toss of a balanced equilateral tetrahedron (triangular pyramid) with a circle on three sides and a cross on the fourth is the same as the difference between circle-sides and cross-sides on Hume's cubic die that has circles on four sides and crosses on two. No one would expect circles on tosses of that tetrahedron with the same degree of intensity that they would expect circles on that die. No one would say that their probabilities were the same. It is not plausible "that Hume's idea was that it is the difference between . . . the number of chances favourable . . . and the number . . . unfavourable . . . that measures the strength of our belief" (Gower 1989). It is not plausible that this was an idea that Hume, without thinking long enough about, intended in his writings. The inadequacy of the difference-measure for degrees of probable belief is just too obvious, too easily exposed, to presume, without unequivocal textual evidence, that it is intended by an author.

Alberto Mura suggests another reason against Gower's proposal that the difference between favorable and unfavorable chances, or experiments, measures the strength of belief. This formula implies negative measures (Mura 1998, p. 318). Thus the measure of belief in a cross on a toss of my tetrahedron would be $(1 - 3) = -2$. "[I]t is not clear . . . what the idea of a negative probability (or degree of belief) stands for" (Ibid.). Indeed. Might Gower, to stay away from 'negative degrees of belief,' intend for Hume a theory of absolute, not algebraic, difference-measures of degrees of belief? Perhaps, but still not with any plausibility. A theory of *absolute* differences would – since $|1 - 3| = |3 - 1|$ – equate the degree of an attentive person's belief in a cross on a toss of our tetrahedron that has a cross on one side and circles on three with the degree of his belief in a circle!

There is ample positive textual evidence that Hume considered psychologically relevant to probabilities of chances not only the difference between the numbers of favorable and unfavorable chances, but these numbers themselves. Both factors are of course mathematically relevant to ratios of favorable cases to all cases and to proportions and frequencies of favorable cases, which supports the usual reading of Hume's implicit ideas concerning probabilities for kinds of events. Turning to a passage concerned with probabilities of causes of not merely repeatable but actually repeating events, we find that, according to Hume, "where different effects have been found to follow from causes which are to *appearance* exactly similar . . . [t]hough we give preference to that which has been found most usual, and believe that this effect will exist . . . we must . . . assign to each of [the other effects] a particular weight and authority, in ***proportion*** as we have found it to be more or less ***frequent***. . . . [W]hen we transfer the past to the future . . . we transfer all the different events, in the same ***proportion*** as they have appeared in the past. . . ." (p. 58; bold emphasis added). Hume here all but says that the weights we assign – that our degrees of beliefs – are, mathematically speaking, proportions or ratios.

Notwithstanding problems with accommodating some mathematical imagery and metaphor of Hume's discussions of the psychology of probable beliefs, it is appropriate to employ modern ideas of probability in interpretations of passages in which he is not discussing, but using, ideas of probability. Grounds for this appropriateness are first that these ideas, the ones I will use, were 'in the air' when he wrote, and second that they merely make precise and tractable common good sense and speech concerned with matters of evidence. In the end, of course, 'the proof is in the pudding.' The proof is in the extent to which the not-so-modern notions I use contribute to a reading that explicates the text, a reading that illuminates its arguments and theses.

0.3. *Part One*, below, is on 'miracles' and 'laws of nature' in "Of Miracles." *Part Two* studies Hume's main conclusions for the evidence of testimony for miracles. It does so without worrying problems posed by his attribution of *extreme* improbabilities to all miracles "the proof against [which he says would be] as entire as any argument from experience can possibly be imagined" (p. 114). (Section B3 of Appendix B addresses these problems.) Featured is an interpretation in probability terms of Hume's *general maxim* concerning the evidence of testimony, the first half of which, that in order for testimony to be sufficient to establish a miracle, it is necessary "that its falsehood would be more miraculous, than the fact which it endeavours to establish" (p. 116), is 'found' to be a theorem of probability theory. *Part Three* relates Hume's ideas to Bayes's theorem for a hypothesis and its negation. Though it is certain that Hume did not have anything like Bayes's theorems in mind when he wrote "Of Miracles," and that he indeed never had in mind anything very much like them,[4] he can be "seen as applying a proto-Bayesian argument" (Owens 1984). *Part Four* takes up objections to the regard that Hume says should be paid when assessing the evidence of testimony for facts to the improbabilities of the facts testified. The objections, made by Richard Price in Hume's day, are still made today. *Part Five* is about recent experiments in which people are found systematically, and without signs of confusion or patent irrationality, to discount prior improbabilities of reported aspects of events, such as the color of a taxi involved in an accident, when 'updating' on the evidence of witnesses to these events. These experiments suggest that an adequate theory of updating by actual subjects may need to be more complicated than the simple Bayesianism that I say was Hume's position. It is maintained that, even if complications are needed for applications to ordinary, less than ideal intellects, Hume's main conclusions regarding the evidence of testimony for miracles for ordinary intellects would be left intact. There are two appendices. The first details a proof of that theorem in Part Two. The second goes into Part I of "Of Miracles" that culminates in Hume's 'general maxim' regarding testimony sufficient to establish a miracle. One objective is to make connections with an idea started by Condorcet; a second is to study problems posed by Hume's implicit references in places to possibly *unequal proofs* for laws of nature and against miracles,

and in other places to proofs against miracles being as *full and as entire* as can possibly be imagined, so that, for a wise man, they gives rise to 'the last degree of assurance.'

PART ONE. CENTRAL TERMS

1 'Miracles'

1.1. "A miracle may be accurately defined, *a transgression of a law of nature by a particular volition of the Deity, or by the interposition of some invisible agent*" (p. 115n). I assume that when Hume writes of miracles in "Of Miracles" he means miracles in this accurate sense, unless there are explicit indications otherwise, as in "there may possibly be miracles, **or violations of the usual course of nature**" (p. 127; emphasis added). His 'accurate definition' melds definitions in *Webster's Third International Dictionary*: "**1 a:** an extraordinary event taken to manifest the supernatural power of God fulfilling his purposes . . . **b:** an event or effect in the physical world deviating from the laws of nature" (p. 1441). It is more or less the idea that informs most 'learned' discussions of miracles. "In the literature . . . miracles have generally been understood to be temporary suspensions of one or more laws of nature accomplished by divine power" (Keller 1995, p. 54). The main difference is that Hume's idea countenances miracles done by invisible supernatural agents other than God.[5]

1.2. John Earman has suggested that the explicit definition that Hume placed in a footnote was "an organizational aberration" (Earman 1993, p. 296), meaning (*perhaps*) that it deserved to be prominent and explicit in the main text as *the* definition of a miracle for its purposes. Earman thinks that Hume, when he wrote "[a] miracle is a violation of the laws of nature" (p. 144), was defining a miracle differently as merely a violation of the laws of nature.[6] In my view Hume was in these words not defining the term, but stressing the aspect of its idea that was important for his argument. In his use of the word 'violation' it is indicated that there is more to the idea, for to speak of violations implies the violators. When Hume writes that "[a] miracle is a violation of the laws of nature" (p. 144), he is, without saying so, giving a serviceable for purposes at hand *elliptical* definition in which the 'by'-clause of his 'accurate definition' is suppressed. Hume does not say that a miracle can be an inexplicable exception to the laws of nature, a mere 'glitch' in the natural order. He may well have thought that the 'idea' of the laws of nature (more of which below) leaves no room for mere glitches, that is, for exceptions for no 'cause' or 'reason', known or unknown. He may well have thought that exceptions to 'laws of nature' would *have* to be transgressions by agents of potent wills. In any case, Hume was in no doubt that a miracle, properly and accurately speaking, would be something more than an exception to the natural

order. He ruled that a *miracle*, even if not every departure from the laws of nature, would have to be by the potent will of God or some other invisible agent.

1.3. We may gather that Hume, given his definition of a miracle, could have viewed every putative miracle, 'accurately' so-termed, as antecedently improbable for a reason other than that it would be a violation of a law of nature. He considered a significant negative against Cleanthes' Religious Hypothesis that, "[I]n all instances which we have ever seen, thought has no influence upon matter, except where that matter is so conjoined with it, as to have an equal reciprocal influence upon it" (*Dialogues*, Part 7, p. 147). All miracles would contradict this uniform experience, for Hume's 'invisible' agents would be incorporeal agents. This contradiction in which miracles, 'accurately' so-termed, would partake is in addition to their sundry particular contradictions of the natural order. It is remarkable that Hume does not make this point in "Of Miracles." He argues for the antecedent improbability of miracles solely on the ground of their being violations of various laws of nature. He nowhere argues against miracles that they would all be immediate realizations in material things of the wills and intentions of 'invisible' and so bodiless agents, contrary our exceptionless experience of agents' wills and intentions operating only by way of their bodies.

Nor does Hume in "Of Miracles" mention motivational puzzles, which would detract from the probability of some marvelous events being *bona fide* miracles by particular volitions of the Deity. William Peterson et. al. observe that all heretofore purported 'healing-miracles' run into this trouble.

> [Suppose that] John believes that God can and will occasionally intervene and, thus, when his daughter becomes gravely ill, he beseeches God for assistance. When she recovers, he attributes the recovery in part to God's direct activity and thanks God for this demonstration of divine compassion. . . . [Now suppose that] Tom, a friend of John . . . encounters a number of starving children during a business trip to India [and] . . . beseeches God for help. After all, he reasons, if God can heal John's daughter, surely God can also help some of these children. Yet in spite of his fervent prayer, all the children die slow, painful deaths. . . . The *prima facie* moral tension here is obvious. Why did God intervene in one case but not in the other? [Why would anyone who could intervene in the one case not intervene in the other?]. . . . Of course, many responses are available to believers. . . . But. . . ." (Peterson et. al. 1991, pp. 170–1)

The question detracts from what would have been the predictive power of Divine healing-miracles hypotheses. Many miracles attributed to Jesus in the Gospels are acts of healing "those that were lame, blind, dumb, maimed" (*Matthew* 15:30), leprous, and in other ways diseased. Cf., I *Kings* 17–24, for the 'revival' of a boy whose 'breath has left him' in response to Elijah's prayer. The mother accepts this as proof that Elijah is a man of God and a true prophet who says the word of God as it is. Keller (1995) argues that not only proposals

of healing God-miracles, but of all irregularly distributed God-miracles, give rise to serious why-questions.

1.4. Miracles of the Bible

FRESH WATER INTO STINKING BLOOD. "YHWH said to Moshe: Say to Aharon: Take your staff and stretch out your hand over the waters of Egypt and stretch out your hand over the waters of Egypt, over their tributaries, over their Nile-canals, over their ponds and over all their bodies of water, and let them become blood! There will be blood throughout all the land of Egypt – in the wooden containers, in the stoneware. Moshe and Aharon did thus, as YHWH had commanded them. He raised the staff and struck the water in the Nile, before the eyes of Pharaoh and before the eyes of his servants, and all the water that was in the Nile changed into blood. The fish that were in the Nile died, and the Nile reeked, and the Egyptians could not drink water from the Nile. . . . "[7] (*Exodus* 7:7–23; Fox 1995, p. 295)

A PARTING OF SEAS. "Moshe stretched out his hand over the sea, and YHWH caused the sea to go back with a fierce east wind all night, and made the sea into firm-ground thus the waters split. The Children of Israel came through the midst of the sea upon the dry-land, the waters a wall for them on their right and on their left." (*Exodus* 14:21–2; Fox 1995, pp. 332–3)

LOAVES AND FISHES. In a desert place: ". . . he . . . took the five loaves, and the two fishes, and looking up to heaven, he blessed, and brake. . . . And they did all eat, and were filled. . . . And they that had eaten were about five thousand men, beside women and children." (*Matthew* 14:19–21) On a mountain: "And he took the seven loaves and the fishes, and gave thanks. . . . And they did all eat . . . four thousand men, beside women and children." (*Matthew* 16:36–8)

WINE OUT OF WATER – LET THE MIRACLES BEGIN. " . . . the mother of Jesus saith unto him, They have no wine. . . . His mother saith unto the servants, Whatsoever he saith unto you, do it. . . . Jesus saith unto them, Fill the waterpots with water. . . . And he saith unto them, Draw out now, and bear unto the governor of the feast. And they bare it . . . the ruler of the feast tasted the water that was made wine. . . . This beginning of miracles did Jesus in Cana of Galilee, and manifested forth his glory; and his disciples believed on him." (*John* 2:1–11)

WATER-WALKS. " . . . Jesus constrained his disciples to get into a ship, and to go before him . . . when the evening was come . . . the ship was in the midst of the sea, tossed with waves: for the wind was contrary. And in the fourth watch of the night Jesus went unto them, walking on the sea. . . . And Peter . . . said, Lord, if it be thou, bid me come unto thee on the water. And he said, Come. And Peter . . . walked on the water, to go to Jesus. But when he saw the wind boisterous, he was afraid; and beginning to sink, he cried, saying, Lord, save me. And immediately Jesus stretched forth his hand . . . and said unto him, O thou of little faith, wherefore didst thou doubt?" (*Matthew* 14:22–31)

RESURRECTIONS. " . . . there came a certain ruler, and worshipped him, saying, My daughter is even now dead: but come and lay thy hand upon her, and she shall live. . . . And when Jesus came into the ruler's house. . . . He said unto them, Give place: for the maid is not dead, but sleepeth. And they laughed him to scorn. But when the people were put forth, he went in, and took her by the hand, and the maid arose. And the fame hereof went abroad into all that land." (*Matthew* 9:18–26) "Then said Jesus unto them plainly, Lazarus is dead . . . he had lain in the grave four days already. . . . When Jesus . . . saw . . . weeping . . . he groaned in the spirit. . . . Jesus said, Take ye away the stone. . . . And Jesus lifted up his eyes, and said, Father, I thank thee that thou, has heard me. And I knew that thou hearest me always: but because of the people which stand by I said it, that they may believe that thou has sent me. And when he thus had spoken, he cried with a loud voice, Lazarus come forth. And he that was dead came forth. . . . " (*John* 11:14–44). "Then Jesus . . . came to Bethany, where Lazarus was which he had been dead, whom he raised from the dead" (*John* 12:1).

THE RESURRECTION. " . . . Jesus . . . bowed his head, and gave up the ghost . . . the soldiers . . . when they came to Jesus, and saw that he was dead already, they brake not his legs: But one of the soldiers with a spear pierced his side, and forthwith came there out blood and water. . . . Joseph . . . Nicomedus . . . they took . . . the body of Jesus . . . [to] a new sepulchre. . . . There laid they Jesus . . . " (*John* 19:30–42; cf., *Matthew* 27:50–60, *Mark* 15:37–46, *Luke* 23:46–53) " . . . the [next] day . . . at evening . . . came Jesus and stood in the midst, and . . . shewed unto them his hands and his side. Then were the disciples glad, when they saw the Lord. . . . But Thomas . . . was not with them when Jesus came . . . he said, Except I shall see in his hands the print of the nails, and put my finger into the print of the nails, and thrust my hand into his side, I will not believe. And after eight days again his disciples were within, and Thomas with them. . . . Then saith he to Thomas, Reach hither thy finger, and behold my hands; and reach hither thy hand, and thrust it into my side: and be not faithless, but believing. And Thomas answered and said unto him, My Lord and my God." (*John* 20:19–27; cf., *Matthew* 16, *Mark* 16, *Luke* 24)

2 *'Laws of nature'*

2.1. It is an easy negative point that Hume does not use the phrase 'the laws of nature' in "Of Miracles" for true exceptionless generalizations. This negative, spelled out, is not that laws of nature, though not *mere* exceptionless generalizations, are for Hume *necessary* exceptionless generalizations, wherein the necessity is not logical but natural. For if he intended a sense that involved the laws of nature being exceptionless, whether or not necessarily so, then 'a violation of the laws of nature' would be a contradiction in terms, since whatever else violations would be, they would be contrary to, they would be departures from, they would be *exceptions* to, laws of nature. Were any exceptionless-generalization sense intended by Hume for 'laws of nature,'

then, to his way of speaking, miracles would be not merely *difficult*, but *impossible* to prove by testimony, for they would be themselves impossible: It could be known *a priori* that they never happen, as it never happens that a newborn infant is already an adult. When reading "Of Miracles" we should assume that Hume's use of 'laws of nature' was not so disingenuous as secretly to make his elaborate arguments at best unnecessary.[8] Earman writes: "whatever else a law of nature is, it is an exceptionless regularity" (Earman 2000, p. 8). I say that on the contrary, whatever a law of nature is in *"Of Miracles,"* it is *not* 'in part' (it does not entail) a statement of an exceptionless regularity.[9] 'Laws of nature' in "*Of Miracles*" are *not* "[n]atural laws on a [so-called] Humean view" (Keller 1997, p. 96). They are *not* a kind of "exception-less regularities" (Ibid.).

What then *are* 'laws of nature' in "*Of Miracles*"? I think that Hume would mean* by 'laws of nature' patterns of 'necessary connections' between events in nature, patterns revelatory of "inherent tendencies and dispositions of things in the world to act and react in certain ways" (Peterson et. al. 1991, p. 157). We should assume that when Hume writes of "the absolute impossibility or miraculous nature of . . . events" (p. 125), he would mean* not logical but 'natural' impossibility, and (awkwardly) that he would intend that the 'absolutely impossible' can happen, that it is logically possible, and that it is even possible that there should be good evidence for its happening that makes belief reasonable. Swinburne aligns himself with Hume on part of this point: "[O]ur account of laws of nature suggests that it makes sense to suppose that on occasion the physically impossible occurs" (Swinburne 1989, p. 80). Hume would add that the 'physically impossible' can happen *repeatedly* (cf., Clarke 1997, p. 100). (*Why '*would* mean'? "Because," he might begin to explain, "I only would, if I could, if I had the requisite ideas." Section 2.3 below elaborates.)

2.2. The *evidence* for a law of nature, Hume writes, is always a 'uniform experience', an experience of an exceptionless pattern of events. It is only such experience that in his view gives rise to 'perceptions' of objective principles controlling nature and of 'necessary connections' between events in nature. Which is not to say that we cannot without incoherence so much as *contemplate* departures from heretofore uniform experiences that have led to such 'perceptions', departures that would be due to interference by the Deity or other invisible agents with the necessary order of nature. Nor is it out of the question that one should have an experience that one reasonably interprets as of such interference, evidence that *inter alia* convinces you that something that is naturally impossible has happened. So, to amend, we should understand Hume to mean that it is only the *initial* evidence for a law of nature that is always an experience of an exceptionless pattern in nature. Once established in our view, a law of nature can be sustained in our view by a uniform experience of its pattern when we do not 'see' potent wills interfering. We should understand that Hume intends that there is for us a 'proof' – a uniform, or

unalteratble, or infallible experience – against an event when and only when it is of a kind that, in our experience, has never happened *without what we judge to have been interference in nature of a potent will*, so that there can be a 'proof' for us against an event, even if we think that it would not be the first of its kind.

While Hume would agree that "to show that a law of nature obtains just is to show that the occurrence of exceptions is **naturally** impossible," he would *deny* that *therefore* "to show that even one 'exception' has occurred would be to show that that law . . . did not obtain" (Flew 1985, p. 8; emphasis added). He thought that to show that an exception to what one has hitherto taken to be the laws of nature has obtained, while of course demonstrating the possibility of exceptions to what one has taken to be the natural order, does not demonstrate the 'natural' – *without the intervention by agents of potent wills* – possibility of these exceptional events. Nor does Hume say that "it [must be] always **more likely**, i.e., conformable to experience, that those claiming the event to be a miracle are mistaken rather than that the event is a genuine violation of a law of nature" (Coleman 1989, p. 338; emphasis added – I have replaced 'is' by 'must be'). He could have agreed that "past experience show[ed] that what are at one time considered violations of natural laws are frequently found at some later time not to be so" (p. 339). But he *denied* by an unmistakable implication of his 'general maxim' that "proportioning belief to evidence . . . it is [always, of necessity] more reasonable to believe that the claim that an event is a miracle is mistaken than it is that the event is a violation of natural law" (p. 339).

2.3 That in Hume's view, 'accurately speaking,' there are no 'laws of nature'! Saying that it is *a law of nature* that A's and B's are constantly conjoined would I believe for Hume be '*popular speech*' for the 'popular speech' that A's and B's are *necessarily connected*. I think that Hume, in "Of Miracles," is speaking with and to the *sophisticated vulgar*, and that he would confess that since, no nondebunking sense can be made of their 'laws of nature,' necessarily he intends none. If I am right, then Hume, without saying so, must have thought that would-be miracles, since they would breach 'causal connections', are plagued not only by problems of *proof*, but by deeper problems of *sense*. These deeper problems would, I think, be compounded by the 'idea' that miracles would be breaches of natural necessary connections *by volitions of God or some invisible agent*. For that idea would involve the profoundly challenging notion of 'powers over powers', of 'powers' of certain agents by mere volitions and bents of mind to interfere with 'powers' of things in nature. Given what would be their 'powers over natural powers', the Deity and those other invisible agents could be termed 'supernatural agents'. The transgressions of laws of necessary connections that they would effect by potencies of their volitions are termed "supernatural events" (p. 118). As he wrote in a note to Section IV of his *First Inquiry* of "the word, Power," so might he have written of the phrase

'violation of a law of nature' in Section X, that "[t]he more accurate explication of it would give additional evidence to this argument" (p. 33).

Miracles, I am saying for Hume, since they would breach 'necessary connections' and be exercises of 'powers' of God or other invisible agents, are in an even worse case than intelligible but logically impossible would-be things such as unmarried husbands and adult infants. Reasons for this radical judgment are implicit in "Of the Idea of Necessary Connexion," *First Enquiry*, Section VII, in which Hume deconstructs that idea, and all ordinary ideas of cause, power, and natural necessities, and maintains that thoughts of these are all nothing more than illicit projections onto events, of inner expectations of repetitions of conjunctions engendered by uniform experiences of these events. That said, I will from now on in this chapter follow what I take to be Hume's lead, who, to speak to the sophisticated vulgar who believe in miracles, speaks with them of 'laws of nature' and 'miracles.' Mackie also speaks with them of these things, but he thinks (as Hume must not have) that the notions involved are "coherent and by no means obscure" (Mackie 1982, p. 20).[10]

2.4. Hume's laws of nature in the 'loose and popular sense' are statements of *necessary connections between causes and effects* and of *exceptionless conjunctions* of causes and effects, *in the absence of interventions by potent wills*. A law in this sense is not of the form

$$\Box([C \,\&\, \textit{lawful}(C \supset E)] \supset E),$$

where *lawful* abbreviates 'it is entailed by the laws of nature that', but of the form

$$\Box([C \,\&\, \textit{lawful}(C \supset E) \,\&\, \sim \textit{vol}(\sim E)] \supset E),$$

where *vol* abbreviates 'it is, by the particular volition of an agent, the case that'. Miracles, that is, interventions by supernatural agents, would *violate* natural laws by preventing their natural operations. They would violate, *without changing*, the laws of nature, that is, without changing the powers and dispositions that are popularly supposed to be inherent in the natures of things and responsible for the natural course of events. According to this popular idea of a law of nature, there is a possible world in which laws of nature are sometimes violated by particular volitions of agents.[11]

2.5. Hume rarely writes of 'laws of nature' elsewhere in the *Enquiries*, except to refer to purported moral laws governing the conduct of persons. Briefly, regarding 'laws of the natural order' elsewhere in the *First Enquiry* (for some references to which I thank James Dye), Hume writes in Section I of a philosopher (Newton) who has "determined the **laws** and forces, by which the revolutions of the planets are governed and direct" (p. 14; emphasis added). In Section IV he writes of "a **law** of motion . . . [concerning] the moment or force of any body, in motion" and implies that such a law concerns an "inseparable and

inviolable connexion" of causes and effects (p. 31; emphasis added) and again of "natural powers and **principles**" (p. 33; emphasis added). In Section VIII, "Of Liberty and Necessity," Hume writes that "matter, in all operations, is actuated by a necessary force. . . . The degree and direction of every motion is, by **the laws of nature**, prescribed with . . . exactness" (p. 82; emphasis added). Here he cautions that we must look into this idea of necessity and explains that there is nothing to it beyond constant conjunction and inference consequent to experience thereof. He intends in Section VIII the 'accurate explication' of 'necessity or connexion' of the previous Section VII. That serves his purpose in Section VIII to 'dissolve' the problem of liberty and necessity, with his philosophy of 'ideas' to 'make it go away'. It serves his purpose in Section X to use these suspect words in their loose and popular sense as they are used by those to whom his argument is addressed, who have not heard of, or been affected by, his deconstruction of 'necessary connections' and are not suspicious of the *idea* of miracles as accurately defined.[12]

3 Evidence for miracles, and for God. A miracle would be "a transgression of a law of nature by a particular volition of the Deity, **or by the interposition of some invisible agent**" (p. 155n; emphasis added). And so evidence sufficient for belief in a miracle is *not necessarily* evidence for a transgression of a law of nature by *a particular volition of the Deity*. It will be, if attributing it to the Deity rather than to other invisible agents makes better sense. It will be evidence against that, if attributing it to invisible agents other than the Deity makes better sense.[13] Christopher Hitchens wonders, "If one leper can be cured, the flock may inquire, then why not all lepers? Allow of a . . . miracle and it becomes harder to answer questions about infant leukemia or mass poverty and injustice. . . ." (Hitchens 1996, pp. 13–4). It is harder to answer the question on the assumption that *God* made the miracle than on the assumption that it was made by some other invisible agent who is not as powerful, or not as caring, or not as fair as the Deity would be. However, though attributing sporadic occurrences of miraculous healings and such to other invisible agents does not occasion "unsatisfying formulae about the Lord's preference for moving in mysterious ways" (Ibid.), it does, assuming the Lord exists, leave mysterious what would be the Lord's quiescence and tolerance of the parsimony of other miracle-workers. A loving God would presumably want more good miracles. A fair God would presumably want miracles not to be 'sporadic' and distributed arbitrarily to only some of otherwise similar potential beneficiaries.[14] In this somewhat attenuated way, evidence for miracles of sorts can be evidence not only that God did not do them, but that God does not exist. The existence of God is not, however, a subject of this chapter. Rather, we are concerned with aspects of the logic of testimonial evidence, especially of miracles. It is therefore sufficient to record that though evidence for some miracles can be for God's

existence, evidence of other miracles can in one way or another be against that existence.[15]

PART TWO. HUME'S PRIMARY ARGUMENT

A miracle is a violation of the laws of nature; and as a firm and unalterable experience has established these laws, the proof against a miracle, from the very nature of the fact, is as entire as any argument from experience can possibly be imagined. . . . There must . . . be uniform experience against every miraculous event, otherwise the event would not merit that appellation. And as a uniform experience amounts to a proof, there is here a direct and full *proof*, from the nature of the fact, against the existence of any miracle; nor can such a proof be destroyed, or the miracle rendered credible, but by an opposite proof, which is superior. The plain consequence is (and it is a general maxim worthy of our attention), "That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind that its falsehood would be more miraculous, than the fact which it endeavours to establish; and even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior." (pp. 114–16)

'Doing "Of Miracles" one better'. There is here the point, anticipated in Section 0.1, at which I think we can do significantly better for Hume than the best that can be found in "Of Miracles." He writes that "[a] miracle is a violation of the laws of nature . . . a firm and unalterable experience. . . . There must . . . be uniform experience against every miraculous event, otherwise the event would not merit that appellation" (p.114.). There is for this reason, he might have said, "a very strong **presumption** against [an event that would merit that appellation] having happened" (Mackie 1982, p. 26; emphasis added).[16] I agree that there is for everyone a strong presumption against any event that has happened meriting that appellation, but I do not agree with what would be Hume's grounds for it. It is, I think, not because any candidate for this appellation must have going against its occurrence extensive uniform experience: After all, there was the weight of such experience against the first reports of transatlantic solo slights (cf., van Inwagen 2002, p. 407). The main reasons for the very strong presumption against a miracle's happening are *entirely independent* of the evidence of that experience against it.

First, *there is the natural impossibility* of '*violations of a natural law' properly so-termed*. This makes for a person a presumption against any event's meriting that appellation *before he knows anything about it* other than that someone says it was a miracle: " 'And he quieted our doubts by making a miracle,' they tell me, but though they are all honourable men, I do not believe them." There is in the imagery of 'breaches of necessary natural connections' a presumption-against for the 'sophisticated vulgar', a Humean sceptic of 'laws of nature' and such

might say, perhaps adding that everyone, when not on his philosophic guard, thinks with the vulgar of these things. There is this presumption-against for 'scientific realists', a more neutral commentator might say, reserving judgment on the objective reality of causal connections. (Cf., Clarke 1997, p. 96.)

Second, *there is a standing presumption against violations of natural laws*, since each would be a violation of "the law which includes all others . . . [that] phenomena . . . depend on some law" (Mill 1874, p. 222), or, in other words, the principle that events in nature have causes in nature.[17] We operate with a great assurance that events are lawful, that they have causes, though this is *not* as John Stuart Mill would have it, because we have for this principle extensive confirming experience. It is not because "in the progress of science, all phenomena have been shown, by indisputable evidence, to be amenable to law" (Mill 1874, p. 223)!! Rather, a person's *first confidence* that events have natural causes – this presumption in its first appearance in a person's experience – is *natural*: It is not a conviction that ***comes from*** experience, but a conviction we are designed to ***bring to*** experience. Hume, in words somewhat like those he uses in Section V for 'the determination by custom alone, after the constant conjunction of two objects, to expect the one from the appearance of the other,' *First Inquiry*, p. 43, could have said in Section X, of the strong convictions in all cases that there are causes for events in them, "that, as this operation of the mind [to presume causes] . . . is [also] so essential to the subsistence of all human creatures, it is not probable that it could be trusted to the fallacious deductions of our reason, which . . . appears not, in any degree, during the first years of infancy." He could have said that "[i]t is more conformable to the ordinary wisdom of nature to secure [this disposition] . . . by some instinct . . . which . . . may discover itself [if not] at the [very] first appearance of life and thought [then as soon thereafter as experience has, in the manner explained in Section VIII, provided an inkling of the 'idea' of cause] . . . independent of all the laboured deductions of the understanding" (*First Inquiry*, p. 55). This elaboration of his position – 'in which the projection or objectification of expectations of repetitions of experienced regularities can be viewed as not merely a trick of human psychology, but as something useful for the disposition to presume causes' (cf., Mackie 1980, p. 72) – would have provided him with a better theory of the principles of the understanding whereby we learn from experience and better foundations for a critique of the evidence of testimony for miracles not very different from the critique run in "Of Miracles."

"But this elaboration of Hume's position would, by *explaining* them, argue only for the *reality* of presumptions against violations of laws of nature. It would not harbour arguments that would in any manner also *justify* them." This is true. *At least* here and now I choose not to 'go there,' to Kant, for arguments that would aspire, at least, not merely to explain, but also to justify, these presumptions.

My claim is not, as Mill would have it be, that there is for everyone always a single collectively general presumption against violations of laws of nature: That is, my claim is not *that for everyone presumably every event has a cause* and does not violate a natural law. My claim is rather distributively general and to the effect *that every event for everyone presumably has a cause* and does not violate a natural law. Mackie says "it is most unlikely that any testimony will be able to outweigh" (Mackie 1982, p. 26) the presumption against a particular event being a violation of a law of nature. I agree at least in part. In agreement, I think that reasons sketched to explain the presumptions we experience against violations predict that for everyone they are strong and seldom overcome by that of testimony.[18] But, contrary to a perhaps unintended suggestion of Mackie's words, I think that the strength for a person of the presumption against a violation in a particular case, and how that compares with the strength for him of the evidence of testimony for a violation in this case, can be different than they are for a similarly informed and equally reasonable person, and that, as a consequence, their conclusions for the case can be different. These can certainly differ when one party believes, *independently of this evidence*, that there exists an agent capable of making what would be the miracle of this case, and the other does not. On the evidence of their testimony (see notes 7, 15, and 18), the presumption is, for Johnson, much more easily overcome than it is for Earman, and for me. (Cf.: "From a naturalist perspective . . . the arguments for evolution are much more powerful than from a theistic perspective" (Plantinga 2002, p. 350.)

4 Of the first part of Hume's general maxim. "That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind that its falsehood would be more miraculous, than the fact which it endeavours to establish." This first part of Hume's maxim can be read as a *theorem* of probability theory, and so as certainly true notwithstanding the controversy of it.[19]

4.1. Hume claimed that "[u]pon the whole . . . it appears, that no testimony for any kind of miracle has ever amounted to a probability, much less than a proof" (p. 127) and "that a miracle can never be proved so as to be a foundation of a system of religion" (p. 127). The words 'have ever amounted' first appeared in the edition of 1768: Earlier editions had instead 'can ever possibly amount'. The words 'can never be proved' are I think a hyperbole for 'it is extremely unlikely that a miracle that would found a religion should ever be proved'. Hume begged that the limitations of his statement should be remarked. "For [he owned] that otherwise, there may possibly be miracles, or violations of the usual course of nature, of such a kind as to admit of proof from human testimony; though, perhaps, it will be impossible to find [even] any **such** in all the records of history" (Ibid.).[20]

4.2. My main concern here is not with Hume's treatment of the substantive historical matter of whether there ever has been credible testimony of a miracle, but with the necessary condition for testimony that Hume provides in the general maxim with which he concludes Part I of "Of Miracles." Here, however, before the main business, are several comments on Hume's treatment of the historical record and on the difference he makes between prospects for credible testimony for religious and nonreligious miracles.

To illustrate the difficulty of credible testimony even for miracles without significance for a religion, Hume observes that if all authors in all languages agreed that there was from the first of January 1600 eight days of total darkness, though it would be an "extraordinary event," we ought accept it as a fact" and *"to search for the causes whence it might be derived"* (pp. 127–8).[21] We ought to accept it as a DEPARTURE from THE USUAL COURSE OF NATURE, but not as a *VIOLATION* of *A LAW OF NATURE*, and so not as a 'miracle' *accurately* speaking. But what if Hume were convinced not only that there was that period of darkness, but that there were no natural "causes whence it might be derived" (p. 128)? Would he *then* conclude that it was a miracle as accurately defined, that is, not merely a 'transgression' of the laws of nature, but one done by a particular volition of an invisible agent? I doubt it, for there would be the question of the *point* of, the *motivation*, for intervention to make those eight days of darkness. "But surely he would not, surely he could not, conclude even given such a plausible point that the event had been so brought about, for he did not believe in the existence of such agents." I disagree with his beliefs. On the evidence of the *Dialogues*, Hume *did* believe in such an agent or agents, if Philo spoke for him when he said, "*That the cause or causes of order in the universe probably bear some remote analogy to human intelligence*" (Hume 1991, Part 12, p. 184). Less ambiguous evidence in *The Natural History of Religion* of such belief on Hume's part was quoted in Section 6.6 of the previous chapter.[22]

Credible testimony for miracles is, for Hume, unlikely, for miracles that would establish systems of religion it is especially unlikely. Why, in his view, would persons of sense on sober reflection *always* reject testimony for what would be 'religious miracles' (pp. 128–9), for example, miracles that would have us believe in Jesus as it is written that his disciples did (*John* 2:11), and as it is written that he wished people to do (*John* 11:42). Hume allows, without giving an example, that persons of sense *may sometimes* on reflection accept testimony for violations of laws of nature by the Deity or some invisible agent. And he gives an example of testimonial evidence that would amount to proof of a 'miracle' in the thin 'inaccurate' sense of an extraordinary departure from the usual course of nature (pp. 127–8). But in Hume's view, testimony for *bona fide religious* miracles 'never can' prove them, this for two reasons. There is first our experience of "many instances of forged miracles, and prophecies" (p. 119) and of "ridiculous stories of that kind" – "the violations of truth are more common in testimony concerning religious miracles, than in that concerning any other matter of fact" (p. 129). And there is second the consideration

that any testimony for a 'religious miracle', or a miracle that would be the foundation of a system of religion, is opposed by all testimony for miracles on which would be founded other systems of religion. This, Hume says, is because "in matters of religion, whatever is different is contrary" so that testimony that would establish miracles "wrought in any [religion] . . . as its direct scope is to establish [that religion]; so has it the same force[23] . . . to overthrow every other [and with them] the credit of [testimony that would establish their miracles]" (p. 121).[24] But he exaggerates. Witness "miracle[s] . . . pretended to have been wrought in" (p. 121) the Old Testament, testimony for many of which does not tend to discredit testimony for miracles pretended to have been wrought in the New Testament, and vice versa. Of course vice versa, since Christianity can embrace the miracles of the Old Testament and the evidence for them. Judaism and Christianity are different religions, but they are not 'contrary' in whatever sense of that term Hume intended in this passage. Their differences do not have the consequences for the evidence of testimony for their miracles that Hume describes.

'Important' miracles in Swinburne's sense would be religious miracles in Hume's sense. Evidence for these miracles would not be of certain events merely as demonstrations of "the power of god or gods and their concern for the needs of men, and little else" (Swinburne 1994b, p. 315b). Evidence for these miracles would thereby be evidence of God's *name*, or of the identity of God's prophets or divinely ordained scripture. "We need to know that it is more probable that God became incarnate in Christ than that Muhammad was his chief prophet. . . . [We need] information about the . . . details of the way to worship" (Swinburne 1981, p. 184). Evidence for 'important' miracles would be proofs of the revelations of particular religions. So "evidence for the miracles of one religion" is "evidence against the miracles of any other" when the miracles involved are *important* ones. From which Swinburne has said it follows that "evidence for [important] miracles in each [is] poor" (1968, p. 315a), exactly as Hume claimed. But he exaggerates as Hume did, since religions can share prophets and scripture, as Judaism and Christianity share Isaiah and the Tōrāh (Pentateuch).

4.3 'Hume's Theorem' – a necessary condition for 'testimony sufficient to establish a miracle.' I take Hume's maxim to be addressed primarily to cases in which a subject knows of some testimony for an event that, in his view, would, if it happened, have been a miracle. However, to put off a problem (that of 'old evidence'), I proceed in the present section with cases in mind in which a subject is thinking about some *possible* testimony of that sort, the occurrence of which he considers somewhat probable. For a formal interpretation, I let M assert what in a subject's view would be a miraculous occurrence and *t(M)* assert the occurrence of testimony for M, the occurrence of which testimony is positively probable. M might say that the stars rearranged themselves last night to spell 'GO CANADA!', and then, after a minute returned to their

original positions. *t(M)* might say that Helen, a sincere and sober woman, says that she witnessed all that, or *t(M)* might say that all that is what a 'cloud of witnesses' say. My casting of the first part of Hume's maxim,

> That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind that its falsehood would be more miraculous, than the fact which it endeavours to establish. . . .

in probability terms *begins* with the proposal that "testimony is sufficient to establish a miracle" only if that miracle's having actually occurred is more likely than not, *conditional upon* the occurrence of that testimony: This hardly remarkable proposal is formally that testimony *t(M)* is sufficient to establish M only if

$$\mathrm{P}[\mathrm{M}/t(M)] > \tfrac{1}{2}.$$

"But," Johnson might say to challenge this symbolization of 'the evidence of *t(M)* is sufficient to establish the miracle M,' "what if in learning of the testimony *t(M)* for M, you learn also of M? In *that* case even if just before learning of it *the credibility of t(M)*, as measured by P[M/*t(M)*], was less than $\frac{1}{2}$, *immediately after* this credibility is 1" (cf., Johnson 1999, pp. 65, 67). That is true. However, while the *credibility* of *t(M)* should in this case change as indicated, *the amount of evidence for M that precisely t(M)* provided, and so whether or not it was sufficient to establish M, should not. (These are not distinguished in Sobel (1987), the subject of Johnson's comments.) *And* the evidence it PROVIDES immediately after it is learned should be the same evidence it PROVIDED just before, *unless my view of its bearing on M has changed*,[25] as it does in some 'out of the way' cases, in which cases 'all bets are off' regarding relations of the evidence provided by *t(m)* when it is learned and just after is learned. There is no sign that Johnson has in mind 'out of the way cases' when he by implication criticizes the displayed condition as being proper 'probability-speech' for 'evidence of testimony sufficient to establish a miracle.'

Regarding the kind of case Johnson has in mind, he would say, "It is nothing special. Consider, for example (for which I assume that you have not read *my* book): 'When [you] learn[ed as you read the entry in References (Owen 1984) – I assume you did that] of the existence of the . . . remark by Sobel that he commented on David Owen's paper in 1984, [did you] not at the same time learn that Sobel commented on David Owen's paper in 1984?' (Johnson 1999, p. 67.)" Before answering, I suggest you consider whether you *have* learned that. You might think about the possibility that I have Johnson here asking a 'trick question' for which I set you up when I wrote that entry. It is an issue whether, if ever, in learning that testimony to the effect that p has taken place one *learns* that p. The problem is not with learning that p *when* one learns that *t(p)*, but with learning that *in* learning that *t(p)*, though just before learning of the testimony, your conditional probability P[p/*t(p)*] was less than one. For the former unproblematic possibility, I have in mind a case (a somewhat out

of the way case) in which you open your front door to someone who tells you that it is raining when, as you can see over his shoulder, it is raining. Regarding the problematic case, it can seem that things that are strictly speaking *learned* in an experience in which one learns of the existence of certain testimony that p, for example, by being on the receiving end of it, should be not only certain but '***ensconced***' in one's background information (cf., p. 57) and that with more resistance to dislodgement than the proposition testified can be merely on the strength of the testimony for it.[26]

To continue my 'casting' of the first part of Hume's maxim, I put '*it is not the case that*' for '*that no*' and take the pattern 'it is not the case that X, *unless* Y' here to mean '*either* it is not the case that X, *or* Y' in the exact sense of '($\sim$X $\vee$ Y)' or equivalently '(X $\supset$ Y)': That is, I take 'unless' here to express a *weak inclusive disjunction*. I assume that for Hume the falsehood of testimony for a possible miracle would be 'more miraculous' than the fact to which it testifies, if and only if this possible miraculous fact, though very improbable, would not be as improbable as that falsehood, or equivalently, that it would be more probable than that falsehood. In that case, I assume that Hume would say that this false testimony would, in the subject's view, be itself a 'miracle,' a *more improbable* miracle than would be the testified fact.[27] By these assumptions I reach, as an articulation in probability terms of the displayed maxim,

$$(\mathrm{P}[t(M)] > 0) \supset \left[\sim\left(\mathrm{P}[\mathrm{M}/t(M)] > \tfrac{1}{2}\right) \vee (\mathrm{P}(\mathrm{M}) > \mathrm{P}[t(M) \,\&\, \sim\mathrm{M}])\right],$$

or equivalently,

$$\left[(\mathrm{P}[t(M)] > 0) \,\&\, \left(\mathrm{P}[\mathrm{M}/t(M)] > \tfrac{1}{2}\right)\right] \supset (\mathrm{P}(\mathrm{M}) > \mathrm{P}[t(M) \,\&\, \sim\mathrm{M}])],$$

which I term 'Hume's theorem'.[28] It is demonstrated in Appendix A to be a consequence of simple intuitive principles of probability, which principles are consequences of Kolmogorov axioms for probabilities, together with the standard ratio-definition for conditional probabilities.

4.4 The old evidence of known testimony. There is a problem with cases of 'known testimony,' since regarding it we have that $\mathrm{P}[t(M)] = 1$. In these cases Hume's theorem reduces to a near tautology, for if $\mathrm{P}[t(M)]) = 1$, then $\mathrm{P}[\mathrm{M}/t(M)] = \mathrm{P}(\mathrm{M})$ and $\mathrm{P}[t(M) \,\&\, \sim\mathrm{M}] = \mathrm{P}(\sim\mathrm{M})$. So in these cases *of course* (I now repeat Hume's theorem), $[(\mathrm{P}[t(M)]) > 0] \,\&\, (\mathrm{P}[\mathrm{M}/t(M)]) > \frac{1}{2})] \supset (\mathrm{P}(\mathrm{M}) > \mathrm{P}[t(M) \,\&\, \sim\mathrm{M}])$. When $\mathrm{P}[t(M)] = 1$, this conditional is equivalent to the nearly tautologous conditional $[\mathrm{P}(\mathrm{M}) > \frac{1}{2}] \supset [\mathrm{P}(\mathrm{M}) > \mathrm{P}(\sim\mathrm{M})]$, which I note includes no mention of testimony $t(M)$. Specifically for known testimony, which is the kind that Hume perhaps always has in mind in "Of Miracles," I assume that, for a significant nontautological principle, he would want either something like this,

That no testimony that one knows has been given is sufficient to establish a miracle, unless this testimony *would have been*, just before one learned that it had been

given, of such a kind that its falsehood was more miraculous than the fact which it was momentarily to endeavor to establish.

or something like this,

That no testimony that one knows has been given is sufficient to establish a miracle, unless this testimony is of such a kind that, *were one not to know that it has been given, its falsehood would be* more miraculous than the fact which it would endeavor to establish.

The formal interpretation that I am promoting can, for 'known testimony', be addressed to one or the other of these counterfactaul perspectives. So addressed, it does not reduce to a near tautology, though it does, since it is a theorem of probability theory, state something that is necessarily true. It happens to state something that is deducible without difficulty from intuitively compelling consequences of familiar axioms.

Earman writes, with reference to my implicit notice of the problem of old evidence in Sobel (1991), that he thinks that it "poses one of the most difficult challenges facing Bayesian confirmation theory" (Earman, op. cit., p. 307n7). The first solution offered above to Hume is like a solution Earman describes for a problem of new theories and old evidence (Earman 1992, p. 122). The second solution follows what David Christensen thinks is "[t]he most immediately appealing . . . approach to . . . the . . . problem [of old evidence]" (Christensen 1999, p. 445). In Section 4.2 of the previous chapter I handled, in a manner somewhat similar to the first solution, the particular problem of old evidence that comes up when it is a matter of assessing theories to explain appearances of design in the world. (There is no notice of the problem of old evidence in Earman (2000).)

4.5. Hume's theorem states a necessary condition for "testimony [being] sufficient to establish a miracle," namely, that "the testimony be of such a kind that its falsehood would be more miraculous, than the fact which it endeavours to establish," which condition I have rendered thus, $P(M) > P[t(M) \,\&\, \sim M]$. This condition is *not* also sufficient, *even if testimony on which a miracle is more probable than not is sufficient to 'establish' it* (which would be a much lower standard of proof than the Roman Church applies) for it could be, it is a logical possibility, that $P(M) > P[t(M) \,\&\, \sim M]$ even though $\sim(P[t(M) \,\&\, M] > P[t(M) \,\&\, \sim M])$, or equivalently, $P[t(M) \,\&\, \sim M] \geq P[t(M) \,\&\, M]$. And when this condition obtains, $P[M/t(M)] = P[t(M) \,\&\, M]/(P[t(M) \,\&\, M] + P[t(M) + \sim M]) \not> \frac{1}{2}$. The probability of the proposition that $[t(M) \,\&\, \sim M]$ could be very small; it *could* be even smaller than that of M, because the occurrence of this testimony $t(M)$ is very improbable for reasons that have nothing to do with the probable falsity of it: Consider, for example, "*They* would *never* say such a thing, whether or not it was true. Whether or not they were sure it was true." In the case of *such* testimony, $[t(M) \,\&\, M]$ would be 'doubly improbable', and

so it *could well* be even less probable than the conjunction [*t(M)* & ∼M] whose improbability would, by hypothesis, be due only to *t(M)*'s being more improbable than M. For the possibility of such testimony, I assume the probability distribution *P[t(M)* & M] = .25, P([*t(M)* & ∼M] = .3, P[∼*t(M)* & M] = .25, and P(∼*t(M)* & ∼M) = .2. For this distribution, P(M) = .25 + .25 > *P[t(M)* & ∼M] = .3, while *P[t(M)* & ∼M] = .3 > *P[t(M)* & M] = .25. I owe these numbers to Sandy Zabell, who used them in correspondence *circa* 1990 to make a closely related point.

5 A condition that is not only necessary, but also sufficient, for testimony sufficient to establish a miracle

Testimony for Miracles. (*P[t(M)*] > 0) ⊃ (P[M/*t(M)*] > $\frac{1}{2}$)] ≡ (***P[t(M)* & M]** > ***P[t(M)* & ∼M]**), in words, positively probable testimony is sufficient to establish a miracle if and only if it is more probable that it occur and be true than that it occur and be false.[29] A theorem 'close' to the Testimony for Miracles, suggested in passing by Zabell in correspondence, is

$$P\,[t(M)] > 0 \supset (\mathrm{P}[\mathrm{M}/t(M)] > \tfrac{1}{2} \equiv \mathrm{P}[\mathrm{M}/t(M)] > \mathrm{P}[\sim \mathrm{M}/\mathrm{t}(\mathrm{M})]).$$

Let this be *Hume's maxim**. This principle has a very easy proof. Given that P[t(M)] > 0, P[M/*t(M)*], and P[∼M/*t(M)*] exist and sum to 1, if the first exceeds $\frac{1}{2}$, it exceeds the second. For the 'proximity' of this theorem to Testimony for Miracles, consider that, given *P*[*t(m)*] > 0, it is rather obvious that (P[M/*t(M)*] > P[∼M/*t(M)*]) ≡ (*P*[*t(M)* & M] > *P*[*t(M)* & ∼M]. Hume's maxim* is Earman's proposal for a probabilistic reading of Hume's maxim (Earman 2000, p. 41).[30] Whereas I take Hume's 'unless' to express a weak inclusive disjunction, he takes it to express a strong, exclusive disjunction.[31] There are indications cited by Earman that that was Hume's intention, which is to say that he intended in his maxim a condition that is not only necessary but also sufficient for testimony that would establish a miracle. These indications go against my proposal. Against Earman's proposal is that P[M/*t(M)*] is not a plausible take on the 'miraculousness of the fact,' for which I have P(M). We agree that Hume had at least primarily in mind for his maxim the situation in which "it is known that" there is testimony "to the occurrence of a miraculous event" (Ibid.), and in this situation P[M/*t(M)*] equals P(M). "But," to adapt (and edit) a remark of Earman's, "the fact that they are [equal in this situation] does not make them equally good readings [in this situation]" (Ibid.). Also, for 'known testimony' we are, according to the previous section, by one counterfactual move or another, in a situation in which it is not known.

In my view, (1) Hume did intend to give in his maxim a necessary and sufficient condition for testimony that would establish a miracle and thought that the words 'the testimony be of such a kind that its falsehood would be more miraculous, than the fact which it endeavours to establish' (p. 116), understood naturally of course, expressed such a condition; (2) he was wrong about

that – not a great surprise, since he did not have formal logic and probability theory with which to 'check his words'; and (3) his words, naturally understood, can express only the necessary condition that I say that they do. He deserves credit for this, though he intended more. The disagreement that Earman and I are in over the first half of Hume's maxim is subtle and not important.

6 On the second part of Hume's maxim. Having said "[t]hat no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish," Hume adds that, "**even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior**" (p. 116; emphasis added). A problem with these words of Hume's, on my reading of the first part of his maxim, is that they would rest on the falsehood that the inequality $[\mathrm{P(M)} > \mathrm{P}[t(M) \ \& \sim \mathrm{M}]$ is not only a necessary condition but also a sufficient condition for $\mathrm{P}[\mathrm{M}/t(M)] > \frac{1}{2}$. Another problem is that Hume *seems* to be saying that whenever $\mathrm{P}[t(M) \ \& \sim \mathrm{M}] < \mathrm{P(M)}$, then $\mathrm{P(M)} = \mathrm{P(M)} - \mathrm{P}[t(M) \ \& \sim \mathrm{M}]$. That would mean that whenever $\mathrm{P}[t(M) \ \& \sim \mathrm{M}] < \mathrm{P(M)}$, then $\mathrm{P}[t(M) \ \& \sim \mathrm{M}] = 0$, which Hume could not have thought is so. I despair of a 'saving' symbolization of Hume's 'second-part' words. Cf., "[T]he second half of the Maxim appears to be nonsensical" (Earman 2000, p. 43). For something to put in place of it, we have that the "assurance" provided by testimony, as measured by $\mathrm{P}[\mathrm{M}/t(M)]$, is the following simple function of $\mathrm{P}[t(M) \ \& \ \mathrm{M}]$ and $\mathrm{P}[t(M) \ \& \sim \mathrm{M}]$:

$$\mathrm{P}[\mathrm{M}/t(M)]) = \frac{\mathrm{P}[t(M) \ \& \ \mathrm{M}]}{\mathrm{P}[t(M) \ \& \ \mathrm{M}] + \mathrm{P}[t(M) \ \& \ \sim \mathrm{M}]}.$$

This says that a suitable assurance results when $\mathrm{P}[t(M) \ \& \sim \mathrm{M}]$, the improbability of false testimony for M, is, *in a certain manner* 'deducted' from $\mathrm{P}[t(M) \ \& \ \mathrm{M}]$, the improbability of true testimony for M (*not* the improbability of 'the fact it endeavours to establish'). The manner of 'deduction' is that of the ratio displayed above, the value of which 'varies oppositely' to the value of $\mathrm{P}[t(M) \ \& \sim \mathrm{M}]$. It is not a matter of simple deduction, that is, of subtraction, of the inferior from the superior. A *somewhat* different reconstruction, based on what I call 'The Hume-Condorcet Rule of the Evidence of Testimony', is given in Section B2 of Appendix B.

PART THREE. ENTER BAYES

7 Bayes's theorem for the evidence of testimony. According to Hume, "the evidence resulting from . . . testimony admits of diminution, greater or less in proportion as the fact is more or less unusual" (p. 113). Confidence in a

reporter, and the improbability of facts he reports, are held to pull in opposite directions. My first and main proposal has been that we take as a measure of 'the evidence that would result from a piece of testimony', the conditional probability, **P[S/*t(S)*]**, wherein *S is a proposition affirming some possible state of affairs* (for example, that the taxicab involved in some accident was blue) and *t(S) is a proposition affirming the existence of a piece of testimony affirming S* (for example, that a witness said that the taxicab involved in that accident was blue). My proposal is that the evidence of this testimony is measured by the conditional probability, P[S/*t(s)*], that is, by the probability of testimony's *truth* given this *testimony's occurrence*. P[S/*t(S)*] can, of course, differ in value from the reverse conditional probability, **P[*t(S)*/S]**, which is the probability of *testimony's occurrence* given what would be its *truth*. It is essential that these two conditional probabilities not be confused, and it is important to an appreciation of much controversy that it is, given various expressions of them in ordinary English, easy to confuse them. Consider for example the words 'the probability that Jones would speak truly about the possible fact that the taxicab involved in the accident was blue'. "Would, given what?" one may well ask. Would, given that the taxicab involved in the accident was blue, *or* would, given that Jones has said the taxicab involved in the accident was blue? Depending on which, those words express something of the form P[*t(S)*/S] *or* something of the form P[S/*t(S)*]!

Taking P[S/*t(S)*] as a measure of capacity of testimony *t(S)* to establish S – it can be seen how this depends not only on 'the testifier's reliability' – a measure for which is proposed in Appendix B – but also on the antecedent probability of what would be the fact attested, which is measured by P(S). It can be seen how it depends on these things in precisely the way Hume would have it do. And it is possible to see, in a useful way, how we may assume Hume considered the antecedent improbabilities of events to militate the likelihood of testimony for them being true. We can see how according to another theorem of probability – and so according to "common sense reduced to calculation" (Laplace 1917, p. 196) – this discounting takes place.

Bayes's theorem for a hypothesis and its negation – Section 4.1 of Chapter VII – is, if P(e), P(h), and P(∼h) > 0, then

$$P(h/e) = [P(h) \cdot P(e/h)]/[P(h) \cdot P(e/h) + P(\sim h) \cdot P(e/\sim h)].$$

Replacing 'h' by 'S', to suggest not specifically 'explanatory hypotheses' but any state of affairs, and 'e' by '*t(S)*', to suggest not evidence in general for S but the specific evidence of some testimony for it, leads to

Bayes's theorem for the evidence of testimony:

$$P[S/t(S)] = \frac{P(S) \cdot P[t(S)}{P(S) \cdot P[t(S)/S] + P(\sim S) \cdot P[t(S)/\sim S],}$$

wherein *P[t(S)]*, P(S), and P(∼S) > 0. This principle provides structure for defence and possible elaboration of Hume's position in Parts Four and Five later.

8 Thomas Bayes and Bayes's theorems

8.1. Bayes does not in his essay state Bayes's theorem for a hypothesis and its negation. Nor does he state a many-hypotheses generalization of it. In his essay he deals with a special case in which prior probabilities are equal. Now when they are equal, prior probabilities 'cancel out' in Bayes's theorem: If P(h) = P(∼h), then P(h/e) = P(e/h)/[P(e/h) + P(e/∼h)]. When equal, prior probabilities *can be ignored*. "[Hume] would not have found Bayes' theorem [in Bayes' essay], for the theorem . . . was the invention of later writers" (Earman 2000, p. 27). True. "What Hume would have found was the recognition of the importance of prior probabilities for inductive reasoning." (Ibid.) False, or misleading, depending on the intended extension of 'inductive reasoning.' "[T]here is no evidence in [his] essay that Bayes visualized the general problem: so far as the evidence goes he attacked his problem *ab initio* and not as a particular species of a broader genus" (Molina 1963, p. ix). And in the problem that Bayes deals with, *prior probabilities*, once established as equal, *are*, and they can be when equal, *ignored*. They do not have places in his formulas or calculations. Richard Price – who added an introduction, notes, and appendix to Bayes's essay, and transmitted the whole to *Philosophical Transactions* in 1763 – seems also not to have an idea of theorems inspired by Bayes's work that have places for prior probabilities. It is salutary to read Price's criticisms in Part Four of Hume's views with this in mind. We have in his criticisms objections to what can be read as Hume's nonmathematical Bayesian *attention* to prior probabilities made by a mathematician who may have had only a limited understanding of *the role* of prior probabilities, of their relevance *always*, in 'Bayesian calculations'! Formulations of what we know as Bayes's theorems complete with places for possibly unequal 'priors' were I think first made by Laplace (cf., note 4). He puts the general rule in words on pages 15–16 of Laplace (1917). Perhaps he had the idea in 1785 when Condorcet, after citing Bayes and Price regarding methods for finding probabilities of future events, given past events, writes that "M. de la Place est le premier qui ait traité cette question d'une manière analytique" (*Essai sur l'application de l'analyse à la pluralité des voix*, Paris 1785, "Discours préliminaire," lxxxiii).

8.2. Earman provides a nice statement of the problem of Bayes's essay.

> Consider a repeatable process (such as coin flipping) that is [you are sure] characterized by a fixed objective chance p . . . of yielding an outcome with property B on each trial. . . . The problem Bayes set himself then is this: Given that in n trials m of the outcomes are B . . . what is the rational degree of belief that p lies between

given limits? The answer is fixed if, and only if, the prior (degree of belief) probability distribution over the objective chance parameter p is given. Bayes supplied an ingenious argument for the conclusion that in the absence of any further background information . . . **the prior distribution over p should be uniform** [that is, for intervals i and i′ in the interval 0 through 1, if i and i′ are equal in length, then it should be that P_o(p lies in i) = P_o(p lies in i′)]" (Earman 2000, pp. 28–9; emphasis added)

As said, the problem that Bayes takes up assumes equal 'priors'. Bayes's work is relevant to the 'logic' of learning from experience, and in particular to Hume's idea that a firm and unalterable experience of a pattern gives rise to a *proof* that it obtains always that is as entire as any argument from experience can possibly be imagined (p. 114).[32] But Bayes's essay has *no easily discernible application* to the question of the evidence of testimony for more or less unlikely events. The relevance of Bayes's essay to the assessment of testimonial evidence for events, regarding which one may have *all sorts of background information* that lead to *unequal* 'priors', was, I think, not fully understood before Laplace's work.

PART FOUR. "[T]HE MAN WHO . . . STOOD UP TO DAVID HUME"[33]

9 Richard Price

[T]he turning point in Mr. Hume's argument is . . . the principle, that no testimony should engage our belief, except the improbability in the falsehood of it is greater than that in the event which it attests. (Price 2000, p. 163)

9.1. We have seen that this principle has a natural interpretation in modern probability terms that is *demonstrable*. That makes remarkable that Price, an able contributor during the formative period of the theory, *objects* to it. He maintains "that improbabilities *as such* do not lessen the capacity of testimony to report truth" (p. 165). Allowable – and this can seem to give Price an argument – is that, speaking somewhat loosely, there is a *sense* in which improbabilities do not necessarily lessen 'capacities of testimony to report truth': Capacities of *testifier* to report truth can be independent of, and even be enhanced by, improbabilities of truths they report – the testifier may be more careful when reporting the improbable; also, *the likelihoods of testimony for truths* can be enhanced by improbabilities – the improbable can be more remarkable. So for two reasons **P[*t(S)*/S]** can be for a reporter positively responsive to P(∼S) for some S. However, as Hume claims, and as has been displayed – Section 6, Bayes's theorem for the evidence of testimony – P(∼S) is necessarily, in every case, negatively relevant to **P[S/*t(S)*]**, and so to whether or not "testimony [*t(S)*] should engage our belief" (p. 163).

9.2. There is what has been termed a 'persistent cognitive illusion' in which credibilities of pieces of testimony, P[S/*t(S)*], are confused with likelihoods of

truthful testimony, P[*t(S)*/S] (Diaconis and Freedman 1981, pp. 331, 332n1: see Section 7).[34] Price seems, in his main criticisms of Hume, to suffer it. For the rest, his case consists of allusions to calculations – calculations to the results of which he supposed Hume was committed – that he thought would dramatize the supposed error of allowing prior improbabilities to affect credibilities of testimony. He supposes that since so many facts are antecedently improbable, the taking of prior improbabilities into account would render testimony and indeed *sense*, vision, hearing, and the rest all but useless. Price writes:

> TESTIMONY is truly no more than SENSE at second-hand: and improbabilities . . . can have no more effect on the evidence of the one, than on the evidence of the other. (Price 1811, p. 166)

"And no less," Hume could say, sticking to his guns.

9.3. Thinking that he was scoring against Hume, Price might suggest that we suppose that *The Toronto Star* reports truth two times in three 'across the board' on all subjects, and without regard to antecedent improbabilities of facts reported: Suppose that this is given to a person's satisfaction, so that, for any event E, for him P(E/ *The Toronto Star* reports that E) = 2/3, without regard to P(E). Then, Price could observe, this is of course so when this person learns that *The Toronto Star* reports a very improbable event E, as it is after all in the *business* of doing, and is usually doing (p. 164). The probable is hardly news. For this person the report "communicates the probability [that is, odds] of 2 to 1 to the event" (p. 166): After learning of the report, it is for him P(E) = 2/3, even if, before learning of it, his P(E) had been close to 0. "Evidence [of a kind that is] *generally* right ought to be received as being so, notwithstanding improbabilities by which we have found it not to be affected" (p. 165). "A *given probability* of testimony communicates itself always entire to an event" (p. 167), and does so quite regardless of the event's antecedent improbability.

These observations are to embarrass Hume. In fact they are consistent with his position. If, as often, the credibility of a testimony is given to a person's satisfaction *without regard to the improbability of the fact attested*, then certainly this person need not attend to this improbability in order to satisfy himself regarding the testimony's credibility. Though improbabilities of facts attested *are* relevant to the credibilities of testimony, a person is *of course* not doomed to take into account the improbabilities of facts attested *again and again*. Going back to *The Toronto Star*, it is to the person's satisfaction *given* that its testimonies are all 2/3 credible without regard to their subjects or the improbabilities of the facts reported. That *settles* P(E/*t(s)*] = 2/3 for him.

10 Lotteries – Price thought they made his case

10.1. "The improbability of drawing a lottery in any particular manner . . . is such as exceeds all conception" (p. 164). Suppose that a trustworthy reporter has said that the number drawn from some lottery was 79. Price implies that if one were to take into account the great antecedent improbability of this number's being drawn, *as Hume would have one do*, then, notwithstanding the reporter's veracity, one would not believe him. But Price is wrong about this, if, as we should assume, Hume would have one reason soundly in a Bayesian manner. For sound Bayesian reasoning that takes into account the improbability of the draw says that in common circumstances, notwithstanding this improbability, we should *believe* the report according to how truthful we consider the reporter to be. Let us, by spelling out the case in a natural way, see how taking prior probabilities and improbabilities into *proper* Bayesian account, far from leading us to doubt a trustworthy reporter, has us 'communicating his truthfulness undiminished to what he reports', exactly as Price would have us do.

Suppose possible draws are from numbers 1 through 1000 (not 1 through 50,000 as Price supposed; p. 175n16) and that for each number n in this range the probability, P(n), that it will be drawn, is .001. Suppose it is certain that the reporter has said of a particular number, perhaps 79, that it was drawn. *Let the 'veracity' of the reporter in this case be such that, for each n in this range*, P[*t(n)*/n] = .9, *and by implication* P[∼*t(n)*/n] = .1. (I have picked .9 for computational convenience. A good reporter would score much higher given the importance of getting it right in this case.) *Suppose additionally that, whatever number had been in fact drawn, it is as likely that the reporter would have erred in favor of* one other number, *as that he would have erred in favor of* another number. (This is a special case, chosen to keep things relatively simple. Earman may imply that it is "[t]he type of case Price had in mind" – Earman 2000, p. 50.) For example, had 78 been drawn (perhaps it was), it is as likely that he would have misreported that 79 had been drawn as it is that he would have misreported that 998 had been drawn, and similarly for every pair of possible misreports. *I am assuming that the reporter is not particularly prone to misreport in favor of 79*, as he would be if he had "some interest in choosing number 79" (Laplace 1917, p. 111). Following Laplace I am assuming that, for any two distinct numbers n and n′ in the lottery's range, P[t(n′)/n] = .1/999: The .1- probability of not reporting a number drawn correctly is, I am supposing, distributed completely and evenly among the 999 possible misreports of it.

What credit would be due to the report that 79 had been drawn, according the following application to the case of Bayes's theorem for a hypothesis and its negation?

$$\mathrm{P}[79/t(79)] = \frac{\mathrm{P}(79) \cdot \mathrm{P}\,[t(79)/79]}{\mathrm{P}(79) \cdot \mathrm{P}[t(79)/79] + \mathrm{P}(\sim 79)\mathrm{P}[t(79)/\sim 79]]}.$$

We may begin by substituting numbers, where this is easy:

(i) $P[79/t(79)] = [.001(.9)]/[.001(.9) + P(\sim 79) \cdot P[t(79)/\sim 79]]$.

The mathematical *problem* is exactly the product $P(\sim 79) \cdot P[t(79)/\sim 79]$. It turns out to be equal to .001(.1).

Here is how. Given that $P(\sim 79)$ is positive, we have by the definition of conditional probability that

(ii) $P(\sim 79) \cdot P[t(79)/\sim 79)] = P(\sim 79) \cdot \{[P[\sim 79 \,\&\, t(79)]]/P(\sim 79)\}$

(iii) $P(\sim 79) \cdot P[t(79)/\sim 79)] = [P[\sim 79 \& t(79)]$ from (ii).

(iv) $P(\sim 79) \cdot P[t(79)/\sim 79)] = \sum_{n:\ 0<n\leq 1000\ \&\ n\neq 79} P[n \& t(79)]$ from (iii).

For this inference, consider that $P[\sim 79 \equiv 1 \vee \cdots \vee 78 \vee 80 \vee \cdots \vee 1000] = 1$ and that, for any distinct possible draws n and n′, $P(n \,\&\, n') = 0$. Then by Equivalence and 'Generalized Additivity*' (see Section 4.1 for Additivity*),

$$P[\sim 79 \,\&\, t(79)] = P[1 \,\&\, t(79)] + \cdots + P[78 \,\&\, t(79)] + P[80 \,\&\, t(79)] + \cdots + P[1000 \,\&\, t(79)].$$

The sum on the right is the Σ-sum on (iv). Proceeding, substituting in (iv), $P(n)P[t(79)/n]$ for $P[n \,\&\, t(79)]$, with which it is identical, given that for each n, $P(n) > 0$ yields

(v) $P(\sim 79) \cdot P[t(79)/\sim 79)] = \sum_{n:\ 0<n\leq 1000\ \&\ n\neq 79} P(n)P[t(79)/n.]$

(vi) $P(\sim 79) \cdot P[t(79)/\sim 79)] = 999[.001(.1/999)]$ from (v).

For this last, somewhat tricky, inference we have that, for each n greater than 0 and not greater than 1000 *including 79*, $P(n) = .001$, and that in our case, for each of the 999 numbers in this range *other than* 79, $P[t(79)/n] = .1/999$. Proceeding,

(vii) $P(\sim 79) \cdot P[t(79)/\sim 79)] = .001(.1)$ from (vi)

(viii) $P[79)/t(79)] = \dfrac{.001(.9)}{.001(.9) + .001(.1)}$ from (i) and (vii)

(ix) $P[79/t(79)] = .9$ from (viii) – Q.E.D.

Laplace (1917, pp. 109–10) gets the same value by a somewhat different calculation. Diaconis and Freedman (1981, p. 334n10) present a general treatment of reports of lottery draws.

10.2. Price does not explain why he thinks that Hume is committed to rejecting as incredible *all* reports of lottery draws. He does not spell out the calculations that he thinks Hume's position on the relevance of prior probabilities validates or explain why he thinks these calculations – they would be bad calculations – are validated by that position. Recall that the reporter's veracity relative to this lottery has been assumed to be .9: It has been assumed

that for each possible draw n, P[*t(n)*/n] = .9. And so, as Price would have it, the "given probability of testimony [for 79, which is P[*t(79)*/79] communicates itself . . . entire to [this] event" (p. 166) or, in this case, *as if* the improbability of the event were *irrelevant* and as if it were *necessary* for testimony for any event S that P[*t(S)*/S] should equal P[S/*t(S)*] without regard to P(S). But this is *not* necessary. We have reached this identity in the *present* case by taking prior improbabilities into account, by taking them *properly* into account, as, in the absence of compelling contrary reasons, we should assume that Hume would have us do.

10.3. The important thing about reports of lotteries in common circumstances that lends credibility to these reports and makes credibility in a case identical with the reporter's veracity is not that there are many ways in which draws can be misreported – that is true of all facts – but that *there are no reasons favoring one misreport of a draw over another*, so that it is equally unlikely that a draw should be misreported in the various ways in which it can be misreported. In our case, for example, though a misreport of 78 is not extraordinarily unlikely, P[∼*t(78)*/78] = .1, the *particular* misreport that 'says' 79 rather than 78 *is* extraordinarily unlikely, P[*t(79)*/78] = .1/999. In this, reports of lotteries are like reports of the ordinary run of unremarkable possible facts. And in *precisely this* they are *unlike* reports of what would be marvelous and wondrous things (with all of this Price agrees – pp. 167–8) and very unlike what would be miraculous happenings of religious significance. For a nonreligious example, if T is the statement that a bridge hand just dealt but not turned up marvelously and wondrously contains 13 spades and U is a statement to the effect that the hand is of some particular mixed and unremarkable character, then even if, for some person P(T) = P(U), it could be that for this person P[*t(T)*/∼T] is *much* greater than *P*[*t(U)*/∼U]. There are *reasons* why a person might misreport hands of all sorts as T-hands, to win a bet or to have some fun, because for fun he is deluded (cf., "*surprise* and *wonder* . . . being agreeable . . . gives a . . . tendency towards . . . belief of. . . . events from which it is derived" "Of Miracles," p. 117), and so on. But this is not so for a U-hand: It is by hypothesis unremarkable and in no way special. The case is similar for what would be wonders of religious significance. "A religionist may be an enthusiast, and imagine he sees what is no reality; he may know his narrative to be false, and yet persevere . . . for the sake of promoting so holy a cause. . . ." (pp. 117–18).

Calculating credibilities of reports in a manner that takes into account, as Price would have not done, the prior improbability of the draw reported results, in common lottery cases, in credibilities that equal reporter's 'veracity', as Price would have them do.[35] Similar considerations explain the credibility of the ordinary run of reports despite the antecedent improbabilities of facts that are in the ordinary run reported, which are often great. *One* thing that, on my reading of Hume, distinguishes what, in a person's view, would be *miracles* from the general run of improbable happenings is that their improbabilities

are to be not merely great, but *extremely* great. *Another* thing – the thing I *now* stress – that he says distinguishes reports of would-be miracles, as well as the wider range of the extraordinary and marvelous that fill the pages of checkout-counter tabloids, is that *these reports are relatively likely, even when they are false*. This is because of motives to lie and, more importantly perhaps, because of the appeal of the marvelous and miraculous, and the desire to believe, which not only provide opportunities for manipulators but can persuade gullible persons to think they have witnessed, and to report sincerely, though mistakenly, what they would like to believe that they have witnessed. Compare your readiness to accept a report from a "serious, capable, honorable, sober, sincere fellow" that a certain Helen O'Donnell has beaten odds of a trillion to one to win a stupendous lottery with your readiness to accept from this same fellow a report that he observed, while "attentively watching"(Johnson 1999, p. 54), an instantaneous rearrangement of one minute's duration of the stars in the sky of South America that spelled out 'Hi y'all!'. Your antecedent improbability for the temporary rearrangement of the stars could not be *much* greater than that for Ms. O'Donnell's great luck, though I suspect you would be far more reluctant to accept the second report than the first. One difference between the reports is that you may (when thinking with the sophisticated vulgar) think that stellar rearrangement would be a "salient and gross empirical event" (Ibid.) that *went against all the laws nature*, an event that was not merely wildly improbable but *naturally impossible*. Another difference between the reports is that the content of the second report makes not unlikely that a fellow should be duped to make it, especially a sober, serious, sincere fellow like the reporter. It sounds so much like a joke someone somehow played on him. Johnson writes, "I am to be told by the Humean why *even these* [sobriety, attentiveness, and the rest] – are unavailing" (Ibid.). I am trying to remind him of how even these conditions of the witness and circumstances *can* be unavailing, and of why for *most* people they would be unavailing, in many cases of would-be miracles in which he believes on the basis of testimony. There is for all, going against belief, *the presumption against miracles* (see the black-boxed text at the beginning of Part Two).

11 Hume, 'I must weigh this'. Hume, in a letter to Price concerning his dissertation on miracles, wrote: "I own to you, that the Light, in which you have put this Controversy, is new and plausible and ingenious, and perhaps solid. But I must have some more time to weigh it, before I can pronounce this Judgment with Satisfaction to my self" (Raynor 1980, p. 105). I know of no evidence that says that Hume communicated with Price again on this subject, and consider it likely that on reflection Hume found that he was still satisfied in the main with his own argument in "Of Miracles," even though he had nothing useful to say to Price that would clarify their differences. He had good reasons to be satisfied with

his argument in the face of Price's criticisms, and this even if he was not able to state these reasons in clear and persuasive controverting terms. He could have felt that the simple common sense of the part of his general maxim that stresses the importance of prior probabilities of attested facts ensured that it did not imply the contradictions and absurdities that Price brought specifically to it. We can understand that conviction, and approve of his persistence in it. For we know that this part of his general maxim has a formal interpretation that is a theorem of probability theory. Despite Price's superior mathematics and intimate acquaintance with Bayes's essay, Hume was, I think, the better intuitive Bayesian. He 'knew probable reasoning', he knew how to do it, and he was very good at it

PART FIVE. TVERSKY AND KAHNEMAN'S TAXICABS

12 Two experiments. Subjects in experiments reported in Tversky and Kahneman (1977) ignored or discounted initial probabilities when coming to conclusions on the basis of testimony. They in some cases accorded to witness's 'reliabilities' (in a sense made exact in Appendix B) what, from a simple Bayesian perspective, would be inordinate weights. One possible reaction to these experiments is to say that persons tend to make mistakes in them; another is to think that simple Bayesianism is not an adequate theory of rational updating – that simple Bayesianism is insensitive to dimensions of credence-states that are important to rational assessments in some cases. After reporting results of two experiments, I consider what Hume might make make of them.

12.1. Subjects in one set of experiments are told that 85 taxi cabs in a town are green and that the rest, of which there are 15, are blue. Subjects are also told that one of these 100 taxi cabs was involved in an accident at night. If, without having any further information to go on, a subject were asked for the probability, expressed as a decimal, that the taxi cab involved in the accident was blue, he would of course have to say, ".15." Subjects, however, were told more. They were told that a witness to the accident has reported that the taxi cab involved in it was blue, that a court has tested this witness for his ability to identify green and blue taxi cabs by presenting him with equal numbers of the two kinds, and that in these tests the witness got each color right 80% of the time. The subjects were then asked for the probability that the taxi cab involved in the accident was blue.

One assumes that, before being told what the witness said, the subjects would have revealed the following probabilities and conditional probabilities: $P(B) = .15$, $P[t(B)/B] = .8$, and $P[t(B)/\sim B] = .2$. (Regarding this last probability, explanations of the case will ensure that, before being told what the witness reported, the subjects are sure that the taxi cab was either blue or green, and

that the witness had reported either that it was blue or that it was green, so that P[*t(B)*/∼B] = (1 − P[*t(G)*/G]).) One may thus calculate that before being told what the witness said, P[B/*t(B)*] = [.15(.8)]/[.15(.8) + .85(.2)] = .41 A deliberate calculating Bayesian, on receiving the witness's report that the taxi cab was blue, would it seems update his probability for its being blue from .15 to .41. But, and this is what makes these first experiments interesting, the median assessment of subjects in the experiment was .8, and there was little variation from this value. It is as if, when updating their initial probabilities for the taxi cab's being blue, the subjects in fact *ignored and set aside* these probabilities, as Price, but not Bayesians, would have them do.

There was another set of experiments, the same as those just described except that the subjects, *instead* of being told what was *the ratio of green taxi cabs to blue ones in the town*, were informed only of the ratio of green taxi cabs to blue ones *involved in accidents* in the town. They were told that this ratio was 85 to 15. Assessments of the accident cab's being blue varied widely. That is one contrast with the first experiments; another is that the median assessment was .55, and so much closer to the Bayesian calculation of .41 than the median assessment of the first experiments. Comparing results of these experiments with those of the first ones thickens the plot.

13 Responses to these results

13.1 A hard line. What might Hume make of these two sets of results, supposing he has in hand not only these results, but modern forms of Bayes's theorem? He might of course take a simple hard line and say, "I do not claim that people are always rational and wise, or that they always proportion their beliefs as they should to the evidence. People are prone to confusions, and to irrational prejudices concerning kinds of evidence. Witness the assessments of subjects in those experiments." He could, however, take a more conciliatory line, concede that assessments of subjects in these experiments were *not* necessarily unreasonable, and elaborate his position on testimony to make room for this conciliation. He might allow that simple Bayesian assessments are not always mandatory, and even so continue to insist that, when testimony is for a *miracle*, proper assessments will need to take into account their improbabilities, and do so in much the way that has been described. Let me indicate a way in which such a measured response to the base-rate experiments might go.

13.2 Possible lines of a more measured response. Of course (one might say) persons are not always perfectly rational and wise. People are often confused and are sometimes inconsistent in their beliefs and incoherent in their degrees of belief. *Furthermore*, people are often *inexact* in their degrees of belief. However, unlike inconsistency and incoherence, inexactness is not a mode of irrationality, but rather a natural and indeed rational condition for persons who are variously limited in their capacities to store and process data, and

who are possessed of variously imperfect and limited data. Consider a person who, while limited in data and capacities, is otherwise perfectly rational. As a simplification, pretend that 'logical omniscience' is a part of perfect rationality, and so consider a person who is quite certain of every logical necessity. How might the total credence-state of this person be represented? The main thing to say is that it might be better represented by a many-membered *set* of probability functions than by any *single* probability function.[36] Suppose it would be. Even so, we could speak of the person's probabilities for propositions – his 'singular probabilities' for propositions – meaning by his 'singular probability' for a proposition an average, perhaps a somehow weighted-average, of his probabilities for it. A person's 'singular probabilities' would thus have both *quantities* and *qualities*, the latter being functions of the distributions of, as well perhaps as the 'significance' or relative 'weights' of, the probabilities averaged. Qualities of 'singular probabilities' would correspond to what some might term the 'weights' or 'degrees of ambiguity' of evidential bases for propositions[37] and to the confidence a person had in his various 'singular probabilities', displayed perhaps in his readiness to accept bets based on them.

Turning to the special topic before us – the evidence of testimony – certain elaborations and qualifications are now in order. When qualities of relevant 'singular probabilities' are equal, assessments of credibilities should proceed as has been maintained. When unequal, 'singular priors' should be discounted only if their qualities are inferior to the qualities of 'singular probabilities' that determine witnesses' reliabilities.[38] They should be discounted dramatically to the point of being ignored only when they are markedly inferior. One consequence of these points is that always when testimony is for a miracle against which there is a 'proof' that is not opposed by a 'counterproof', 'singular priors' are fully relevant, since a person's 'singular priors' for and against such miracles will, it seems plausible to maintain, be not only of extreme quantities but of highest qualities – probabilities averaged of such miracles, given their unambiguous inconsistency with what one takes to be the natural and necessary order of nature, will be concentrated closely around the average value. Indeed it could be that it is this that marks the most important difference in a person's credences between what he would view as merely marvelous and extraordinary and what he would view as miraculous. It could be that it is not that probabilities for what would be miracles are of smallest quantities (infinitesimal quantities, I suggest in Appendix B, in contrast with the merely small finite quantities of the marvelous), but that they are exceptionally concentrated and focussed, and of highest qualities.[39]

What more can be said of cases in which relevant 'singular probabilities' are not more or less equal in quality? One possibility is that no more can be said than that in these cases it may not be unreasonable to discount 'singular priors' (cf., Cohen 1981, p. 366) or *other* relevant 'singular probabilities,' depending on which 'singular probabilities' are of lower quality.[40] Another possibility, however, is that more can be said, and that when qualities are not equal a rational

updater will calculate using not only quantities but also qualities of relevant 'singular probabilities' in accordance with suitably ramified Bayesian principles, discounting probabilities of inferior qualities more or less depending on the details and degrees of their inferiorities.

13.3. I like this measured response that I have indicated Hume might make to our base-rate experiments. I wish I could say more, in particular, more about the averages that are to yield 'singular probabilities,' more about assessments and measures of their qualities, and more about calculations that would involve both quantities and qualities of 'singular probabilities', supposing there *is* more that can be said along these several lines. In fact, I can only express without proof or argument the sense that, notwithstanding the complications generated by recognition of a 'qualitative dimension', simple Bayesianism provides the central structure for the best theory of rational updating, and that even if the best theory is considerably more complicated, it will yield for the assessment of testimony for miracles essentially the simple Bayesian analysis provided in Part Two.

LAST WORDS

It was I suppose clear from the start why, if anyone were to tell you that a man had died and come back to life, you had best not believe him without substantial corroboration. "The statement that a man has been raised from the dead would," Leslie Stephen wrote, "prove that its author was a liar" (Stephen 1949, p. 341) or, at any rate, to temper Stephen's words, it would prove, in the absence of substantial corroboration, that its author was the speaker of an untruth, that "this person either [would] deceive or [was] deceived" (Hume 1902, p. 116). More fully and precisely, it would prove this to anyone who, after the news of the statement, still thought the thing would be a *miracle* and a *natural impossibility* and did not think that the existence of this testimony, if it is false, would be not only also a miracle and a natural impossibility, but an even *greater* miracle and impossibility. It would not prove this to me – it would take more than the word of some serious and sober fellow to convince me of the occurrence of a resurrection miraculous in the accurate sense of this word, much more.

APPENDIX A. A PROOF OF HUME'S THEOREM – $[(\mathrm{P}[t(M)] > 0) \;\&\; (\mathrm{P}[\mathrm{M}/t(M)]) > \frac{1}{2})] \supset (\mathrm{P}(\mathrm{M}) > \mathrm{P}[t(M) \;\&\; \sim\mathrm{M}])$

Principles used are, for any propositions p and q:

Definition of Conditional Probability (DefCP). $[P(p) > 0] \supset [P(q/p) = P(p \;\&\; q)/P(p)]$.
Equivalence. if $\Box(p \equiv q)$, then, $P(p) = P(q)$.
Incompatibility. if $\sim\Diamond(p \;\&\; q)$, then $P(p \;\&\; q) = 0$.
Additivity. if $P(p \;\&\; q) = 0$, then $P(p \vee q) = P(p) + P(q)$.

all stated in Section 4 of Chapter VII, and

Consequence. $\Box(p \supset q) \supset P(q) \geq P(p)$.

and

Bounds. $0 \leq P(q/p) \leq 1$.

With the exception of Definition of Conditional Probability, these principles are consequences of the Kolmogorov axioms, for any propositions p and q: (i) $P(p) > 0$; (ii) if $\Box p$, then $P(p) = 1$; and (iii) if $\sim\Diamond(p \,\&\, q)$, then $P(p \vee q) = P(p) + P(q)$.

1.	$(P[t(M)] > 0) \,\&\, (P[M/t(M)] > \frac{1}{2})$	assumption for a conditional proof
2.	$P[M/t(M)] = P[t(M) \,\&\, M]/P[t(M)]$ |	1 (first conjunct), DefCP
3.	$P[M/t(M)] = P[t(M) \,\&\, M]/P([t(M) \,\&\, M] \vee [t(M) \,\&\, \sim M])$	2, *Equivalence*
4.	$P[M/t(M)] = P[t(M) \,\&\, M]/P([t(M) \,\&\, M] + [P[t(M) \,\&\, \sim M])$	3, *Incompatibility*, *Additivity*
5.	$P[t(M) \,\&\, M] > 0$	1 (second conjunct), 1 (first conjunct), DefCP, algebra
6.	$P[M/t(M)] = \dfrac{1}{1 + (P[t(M) \,\&\, \sim M]/P[t(M) \,\&\, M])}$	4, 5, algebra
7.	$P[(M) > 0$	5, *Consequence*
8.	$P[M \,\&\, t(M)] = (P(M) \cdot P[t(M)/M])$	7, DefCP, algebra
9.	$P[t(M) \,\&\, M] = (P(M) \cdot P[t(M)/M])$	8, *Equivalence*
10.	$P[M/t(M)] = \dfrac{1}{1 + (P[t(M) \,\&\, \sim M]/(P(M) \cdot P[t(M)/M])}$	6, 9
11.	$\dfrac{1}{1 + (P[t(M) \,\&\, \sim M]/(P(M) \cdot P[t(M)/M])} > \frac{1}{2}$	1 (second conjunct), 10
12.	$1 > (\frac{1}{2}) \cdot [1 + (P[t(M) \,\&\, \sim M]/(P(M) \cdot P[t(M)/M])]$	11, algebra
13.	$2 > 1 + (P[t(M) \,\&\, \sim M]/(P(M) \cdot P[t(M)/M])$	12, algebra
14.	$2 \cdot (P(M) \cdot P[t(M)/M]) > (P(M) \cdot (P[t(M)/M + P[t(M) \,\&\, \sim M]$	13, algebra
15.	$P(M) \cdot P[t(M)/M] > P[t(M) \,\&\, \sim M]$	14, algebra
16.	$P(M) > P[t(M) \,\&\, \sim M]$	15, *Bounds*

17. $[(\mathrm{P}[t(M)] > 0)\&(\mathrm{P}[M/t(M)] > \frac{1}{2})] \supset (\mathrm{P}(\mathrm{M}) > P[t(M)\& \sim M])$ 1–16 make a 'conditional proof' for this conditional.

For a proof of

$$(\mathrm{P}[t(M)] > 0)\ \&\ \left(\mathrm{P}[\mathrm{M}/t(M)] > \tfrac{1}{2}\right) \supset (\mathrm{P}[t(M)\ \&\ \mathrm{M}] > \mathrm{P}[t(M)\ \&\ \sim \mathrm{M}]),$$

the 'only if' half of Testimony of Miracles, replace (16) and (17) by

16′. $\mathrm{P}(\mathrm{M}) \cdot \dfrac{\mathrm{P}[\mathrm{M}\ \&\ t(M)]}{\mathrm{P}(\mathrm{M})} > \mathrm{P}[t(M)\& \sim \mathrm{M}$ 15, 7, DefCP

17′. $\mathrm{P}[t(M)]\ \&\ \mathrm{M} > \mathrm{P}[t(M)\ \&\ \sim \mathrm{M}]$ 16′, algebra, *Equivalence*

18′. $(\mathrm{P}[t(M)] > 0)\ \&\ \left(\mathrm{P}[\mathrm{M}/t(M)]\tfrac{1}{2}\right) \supset (\mathrm{P}[t(M)\ \&\ \mathrm{M}] > \mathrm{P}[t(M)\& \sim \mathrm{M}])$ 1-17′

APPENDIX B. *CONDORCET'S RULE, WITNESS RELIABILITY, AND 'LAST DEGREES OF ASSURANCE'*

B1 Bayesing Condorcet's rule

B1.1. There can be found in the work of Condorcet the suggestion that, when it is known and certain that some witness has testified regarding some event, the probability that he has told the truth is given by the ratio

$$\frac{\mathrm{vv}'}{\mathrm{vv}' + \mathrm{ee}'}.$$

Karl Pearson informs us that "v and e. . . . represent the probabilities of the truth and falsity of the event and v'. . . . the probability that the witness confirms the truth and e' the probability that he does not" (Pearson 1978, pp. 459–60). Cf.: "Supposons maintenant que u & e représentent les probabilités de la vérité d'un évènement extraordinaire & de la fausseté du même évènement, & qu'en même-temps u' & e' expriment la probabilité qu'un témoignage fera ou non conforme à la vérité, & qu'un témoin ait asuree'de la vérité de cet évènement" (Condorcet 1783, pp. 554–55). Pearson uses 'v' where Condorcet uses 'u'. I note that $\mathrm{e} = (1 - \mathrm{v})$ and, since it is taken to be certain that the witness has testified to the event, that $\mathrm{e}' = (1 - \mathrm{v}')$. Isaac Todhunter, taking some liberty, writes that Condorcet gave the formula,

$$\frac{\mathrm{pt}}{\mathrm{pt} + (1 - \mathrm{p})(1 - \mathrm{t})},$$

where "p is the probability of the event itself. . . . t. . . . the probability of the truth of a certain witness" (Todhunter 1965, p. 400).

B1.2. Condorcet gives his formula "with very little explanation" (loc. cit.). How did he 'find' this formula? There is a 'derivation', which could have been Condorcet's way to the formula, in Edgeworth (1911, p. 383, paragraph 48). Francis Edgeworth begins with a rule for *the probability that concurring independent witnesses* have spoken the truth:

$$\frac{\mathrm{tt}'}{\mathrm{tt}' + (1 - \mathrm{t})(1 - \mathrm{t}')},$$

wherein t and t' are *truthfulness measures for these witnesses*. This rule has some initial plausibility. Its final assessment, not undertaken here, would depend on the exact interpretations of italicized phrases. For the evidence of testimony given by a *single* witness, Edgeworth treats 'nature' as a concurring independent witness and allows all evidence, other than the evidence of this testimony, that bears on the attested fact to determine a truthfulness measure for nature *qua* concurring witness! Regarding this 'derivation,' it is significant that the rule for the probability that concurring witnesses have spoken the truth is plausible only for concurring *independent* witnesses. "The application of probabilities to testimony proceeds upon two assumptions," the second of which is "that the statements of witnesses are *independent* in the sense proper to probabilities" (Ibid.). Presumably the intended independence is 'relative to S′, and testimonies *t(S)* and *t(S)′* are to be independent in this sense if and only if $\mathrm{P}(t(S)'/[t(S)\ \&\ \mathrm{S}]) = \mathrm{P}[t(S)'/\mathrm{S}]$. But testimony to some fact on the one hand, and what one takes to be other evidence relevant to this fact on the other, are of course often *not* independent 'relative to this fact.' Edgeworth's 'derivation' of Condorcet's formula is unBayesian, and unsound.

B1.3. Condorcet was not entirely happy with his formula for the probability of the truth of given testimony. He felt it led "to results too far removed from those given by common reasoning" (Pearson 1978, p. 461). He was particularly concerned with problems for reports of numbers drawn in lotteries. Notwithstanding, however, Condorcet's misgivings and the rather chequered history of commentary on his formula,[41] his formula admits of an interpretation – albeit one different from Condorcet's and I think from that of others who have used and discussed it – under which it is a theorem of probability theory, a theorem that can be related usefully to "Of Miracles." For this relation, the Bayes-Laplace Rule for the Evidence of Testimony (see Section 6) may be recast. It is equivalent to the following rule, which has the form of Todhunter's version of Condorcet's rule.

The Hume-Condorcet Rule for the Evidence of Testimony: If $P[t(S)]$, $\mathrm{P(S)}, \mathrm{P}(\sim\mathrm{S}) > 0$, $\mathrm{p} = \mathrm{P(S)}$, *and*

$$\mathrm{r} = \frac{\mathrm{P}[t(S)/\mathrm{S}]}{\mathrm{P}[t(S)/\mathrm{S}] + \mathrm{P}[t(S)/\sim\mathrm{S}]},$$

then

$$\mathbf{P[S/}\boldsymbol{t(S)}\mathbf{]} = \frac{\mathbf{pr}}{\mathbf{pr} + (\mathbf{1} - \boldsymbol{P})(\mathbf{1} - \boldsymbol{r})}.$$

The only principle of probability on which the equivalence depends is that P(∼S) = [1 − P(S)].

B2 Witness reliability. In the Hume-Condorcet rule, *r* is defined in a way that makes it a measure that can be naturally termed 'the *reliability* of the testifier to S': *r* varies as such a measure should according to what are taken to be the testifier's propensities to error and deception in connection with S. Two things matter to that reliability – one is how likely it is that the testifier would testify that S supposing that S did obtain, P[*t(S)*/S]; the other thing is how likely it is that he would misrepresent in *this* way, namely, by testifying to S rather than to some other nonevent, supposing that S did *not* obtain, P[*t(S)*/∼S]. These things are involved in the factor *r* in the way required given its role as a measure of 'the reliability of the testifier to S.'[42]

There are other words for what *r*, as defined, measures. It recommends itself as a measure of *the evidence for the truth of the testimony (and so of the fact testified) 'considered apart from other evidence relevant to the fact testified in it'* (the evidence of "the testimony considered apart and in itself" – p. 114), the factor that Hume saw as in competition with *the evidence against the fact 'considered apart from this testimony for it'* which is measured by (1 − p) in the Hume-Condorcet Rule (the *improbability* of a fact being the *probability* of its *negation*). 'The evidence of possible of testimony to S, '*considered apart and in itself*' in Hume's words, should depend 'positively' as *r* does on the likelihood that the possible testifier would testify to S supposing that S obtained, *and* 'negatively' as *r* does on the likelihood that he would testify to S even though it did *not* obtain. For a possible 'derivation' of *r* as a measure of 'the evidence of testimony *considered apart and in itself*,' one could begin with the observation that this evidence should be what the evidence of the existence of this testimony would be if the fact, S, were as probable as not, so that this fact's antecedent probability made no difference to 'the evidence of this testimony for it': In that case, 'the evidence of testimony considered apart and in itself' should equal 'the evidence of this testimony.' The derivation could then conclude with the observation that, when P(S) = .5, prior probabilities P(S) and P(∼S) are equal and cancel out in the Bayes-Laplace Rule, so that by that rule P[S/*t(S)*] = r.

According to Hume, the evidence of testimony to a fact depends on two things: the evidence of this testimony considered apart for this fact; and the probability of this fact considered apart from this testimony. In the Hume-Condorcet Rule for the Evidence of Testimony, *r* can be taken to measure the first of these things, and *p* the second. The rule then states how, to

arrive at the evidence of testimony, the inferior is to be 'deducted' from the superior.

B3 On last degrees of assurance

B3.1. It has been written that "Hume . . . continually employs the term 'miracle' . . . to signify anything that is highly *improbable* and *extraordinary*" (Whately 1853, p. 57n, a putative *reductio ad absurdum* of Hume's critique of the evidence of testimony for miracles that "first appeared in the year 1819", p. 8). In fact, Hume equates the extraordinary with the marvelous (p. 113) and *contrasts* the miraculous with the marvelous: "Let us suppose that the fact . . . instead of being only marvellous, is really miraculous" (p. 114.). "It's a *miracle*!" said Father Dominic, a medieval scribe in a TV commercial, though the photocopier by Xerox that so vastly reduces his labor is, we know, merely marvelous. A miracle would be a 'violation of the laws of nature,' against which there are always 'full proofs'[43] that, if unopposed by counterproofs, give rise to 'more than probabilities' for denials of miracles and make miracles themselves not only highly improbable but improbable '*in the extreme*' and 'less than probable' as laws of nature are 'more than probable' (p. 114). There is a 'proof' for every law of nature that, unopposed, gives rise in a wise man to "the last degree of assurance" for conforming events (p. 110) and presumably 'the least degree of assurance' or 'the last degree of doubt' regarding 'violating' events. In standard probability theory last and least degrees of assurance must be probabilities of 1 and 0, and a representation in standard probability theory of these ideas of Hume's must include that if, in a person's view, M would be miracle, then for this person $P(M) = 0$. But then the Bayes-Laplace and Hume-Condorcet rules are not applicable to evidence of testimony for miracles.

B3.2. A simple solution to *this* problem could consist in the following rule, which *is* applicable even to absolutely improbable testified propositions.

The General Rule for Evidence of Testimony: If $P[t(S)] > 0$ and whether or not $P(S) > 0$),

$$P[S/t(S)] = P[S \,\&\, t(S)]/(P[S \,\&\, t(S)] + P[\sim S \,\&\, t(S)]).$$

There is, however, a more serious problem with representations in standard probability theory of Hume's ideas. They leave no room for a possibility that Hume stressed, namely, that the falsehood of testimony for what in a person's view would be a miracle might *itself* be something that in this person's view would be a miracle. On the standard analysis suggested above, if M asserts what would be in the subject's view a miracle, then $P(M) = 0$ and $P(\sim M) = 1$. But then, if it is positively probable that certain testimony to M should exist,

$P[t(M)] > 0$, then $P[\sim M \,\&\, t(M)] > 0$, and it *is not* a miracle that this testimony should exist though false. Furthermore, on the standard analysis, no testimony for a miracle could afford evidence for it: If $P(M) = 0$, then $P[M/t(M)] = 0$ for any testimony $t(M)$. According to Hume, however, testimony *can* afford evidence sufficient to *establish* a miracle, if its falsehood would be an even greater miracle, which he implies it might just be.

Hume writes once of infallible experience as leading a wise man to expect "the event with **the** last degree of assurance" (p. 110), but he does not say that such experience always leads a wise man to expect the event with *certainty*, and the oddity of the construction '*the* last *degree* of assurance' may not be without significance. He writes of "a firm and unalterable experience" as being "as **entire** as any argument from experience can possibly be imagined" (p. 114; emphasis added), but he does not say of such experience that it is always as *strong* as any argument from experience can be. Nor did he hold these and related uncompromising views. He believed in the possibility of weaker and strong 'full proofs,' of inferior and superior 'arguments as entire as arguments from experience can be', of 'last *degrees* of assurance', and of the possibility of *greater and lesser* miracles. Such opinions, which are plain enough in "Of Miracles," are explicit in comments, solicited by Hugh Blair, on George Campbell's soon-to-be published critique (it was published in 1762): "The proof against a miracle . . . is of that *species* or *kind* of proof, which is full and certain when taken alone . . . but there are degrees of this species, and when a weaker proof is opposed to a stronger, it is overcome" (Mossner 1980, p. 293).

'Proofs,' according to Hume, give rise to last *degrees* of assurance *in the absence of 'counterproofs,'* but not always. In the presence of 'counterproofs' net, *nonextreme*, not-last-degree assurances are produced that can go either way.[44] *Ruled out*, as a numerical interpretation of Hume's idea of a proof, given this doctrine of weaker and stronger 'full and certain' proofs, is therefore "that when experience provides a proof, the conditional probability of the conclusion, given the evidence of experience, is 1" (Earman 2000, p. 25).[45] One way to accommodate Hume's idea of extreme degrees of assurance in an interpretation is to employ a 'nonstandard' theory in which probabilities are 'hyperreal' numbers,[46]including nearly-zero 'infinitesimals' and nearly-one numbers that are 'within infinitesimals' of one. That seems just the environment for a reconstruction of Hume's ideas in terms of mathematical probabilities. (I beg the limitations here made be remarked. What follows is a *reconstruction of Hume's ideas* regarding full and certain proofs of various strengths. I do not *propose* that when a person 'perceives' events to be necessarily connected, and their separation naturally impossible, his credences should be represented by a probability function that assigns a positive infinitesimal to their separation. It would I think make for a better theory to excise 'proofs' and 'extreme probabilities' from "Of Miracles" than to deal with them as best as one can.)

B3.3. For Hume, M asserts what would in a person's view would be a miracle, only if M is logically possible and there is what Hume would term a 'proof' for this person against M that has moved him to view it as *naturally impossible*. For a quantitative gloss on Hume's idea of a miracle, I say that there *is* a 'proof' for a person against M *if*, for this person, $\boldsymbol{P(M) < i}$, for some positive ***infinitesimal*** *i*,[47] and that such an inequality holds for a person *if and only if* there is, for this person, such a 'proof' against M, *and no such 'proof' for M*. A 'firm and unalterable contrary experience' provides a person with such a proof against if and only if *it has in fact given rise (causal, not justificational, notion) in this person to a credence for M that is represented by such an* EXTRAORDINARILY *small number*.

A positive infinitesimal is a positive hyperreal number that is less than every 'real (hyperreal) number'. There is an isomorphism between the real numbers and a subset of the set of hyperreals. Hyperreals in that subsets are real (hyperreal) numbers. Positive infinitesimals are 'just the numbers' for contemplated transgressions of 'laws of nature' that, without being absolutely improbable, are 'less than probable' as these laws are themselves 'more than probable': Transgressions of laws can be *less than n-probable for every standard real n greater than 0, though not 0-probable*; the laws themselves can be *more than n-probable for every standard positive real less than 1, without being 1-probable*.

B3.4. According to Hume, a person views a logical possibility as a miracle only if he views it as a violation of a law of nature, and so views it as a *natural impossibility*. We have such views. Hume considers them to be philosophically suspect and incapable of fully face-saving analyses in terms of ideas derived from experience, but he thinks that they are natural and indeed irrepressible 'views' for *everyone*, including sceptics such as himself when they are not 'engaged in their scepticism' (in which they are usually not engaged). There is, he might say, a sense in which 'we cannot do without them': He might say that though we do not for any theoretical or practical purposes *need* them, we cannot, psychologically, *avoid* them in our ordinary thinking. The proposal I am making for reading "Of Miracles" is that such 'views' (scare-quotes in deference to Hume's philosophic suspicions) be accorded distinctive treatment in a probabilistic representation of a credence-state, with all and only 'views' of natural impossibilities having *infinitesimal* probabilities in the representation. Similarly, all and only things 'viewed' as *natural necessities* will have probabilities that are, though less than, 'infinitely close' to, 1. (For distinct hyperreals n and m, n is 'infinitely close' to m if and only if the absolute difference $|n - m|$ is less than some infinitesimal.)

This treatment accords to 'views' of natural impossibilities and necessities a kind of resilience that seems appropriate. For any positively probable E that does not in a person's view express a natural impossibility, if M does in this person's 'view' express a natural impossibility *against* which there is thus a

proof, and *for* which there is *not* a 'proof', then P(M/E) ≈ P(M). 'Proofs' when unopposed by proofs give rise, according to my proposal for "Of Miracles," to extraordinary probabilities from which no amount of ordinary conflicting evidence can significantly detract, though they are not absolutely fixed and immune to significant diminution, which *can* be called for by opposing 'proofs.'[48] My suggestion is that the presumption established by an unopposed 'proof' against what in a person's view would be a miracle and a violation of a law of nature is to differ not merely in degree from the presumption that lies antecedently against ever so many marvelous facts that 'would be too good to be true'. The presumption generated by an unopposed 'proof' is to be of another *order*:[49] *It* is overcome by testimony, *only if* the falsehood of the testimony is 'infinitesimally improbable', and similarly for *bodies of independent testimony*, so that, *contrary* to Charles Babbage,

> [I]f independent witnesses can be found, who speak the truth more frequently than falsehood, *it is* [**not**] ALWAYS *possible to assign a number of independent witnesses, the improbability of the falsehood of whose concurring testimonies shall be greater than that of the improbability of the miracle itself.* (Quoted in Earman 2000, p. 54)[50]

That is possible only when there is included amongst the witnesses at least one the falsehood of the testimony of whom it itself 'infinitesimally improbable'. This consequence of the present infinitesimal treatment of the probability of any would-be miracle coheres with Hume's not unreasonable intransigence before extraordinary testimony to what Hume viewed as would-be miracles "upon the tomb of Abbé Paris" (p. 124):

> The curing of the sick, giving hearing to the deaf, and sight to the blind, were every where talked of. . . . But what is more extraordinary; many of the miracles were immediately proved upon the spot, before judges of unquestioned integrity, attested by witnesses of credit and distinction. . . . Where shall we find such a number of circumstances, agreeing to the corroboration of one fact? And what have we to oppose to such a cloud of witnesses, but the absolute impossibility or miraculous nature of the events, which they relate? And this surely, in the eyes of all reasonable people, will alone be regarded as a sufficient refutation. (Ibid.)

Hume, at any rate, could believe in the events attested in this historical episode, only if he at the same time was persuaded that they were *not* miraculous but only inexplicable and such that 'instead of doubting, we should look for causes' (cf., p. 128). The claim regarding 'all reasonable people' is at best contentious.

B3.5. From now on let 'M' express what in a person's view would be a miracle *for* which there is *not* a 'proof' – that is, M is to be an event against which there is a 'proof' that is *not* opposed by a 'counterproof', at least not yet. Let '*t(M)*' express the existence of possible testimony for M such that for the person in question $1 > P[t(M)] > 0$. By an application of the Rule for

the Evidence of Testimony for Miracles (Section B3.2), it follows that, for a positive infinitesimal i that is less than or equal to P(M),

$$\mathrm{P}[\mathrm{M}/t(M)] = \frac{i}{i + \mathrm{P}[\sim \mathrm{M}\ \&\ t(M)]}.$$

There are two possibilities for this ratio. P[~M & *t(M)*] either is not, or is, an infinitesimal. *If it is not*, then

$$(\mathrm{P}[\mathrm{M}/t(M)] > 0)\ \&\ (\mathrm{P}[\mathrm{M}/t(M)] \approx 0),$$

and everything considered the testimony would lack not all, but 'nearly all' credibility: The 'near-equality' says that the difference (P[M/*t(M)*] − 0) is a positive infinitesimal. To prove this, assume that P[~M & *t(M)*] ***is not*** an infinitesimal, and suppose, for an indirect derivation, ~(P[M/*t(M)*] > 0) & (P[M/*t(M)*] ≈ 0), or equivalently,

$$\sim(\mathrm{P}[\mathrm{M}/t(M)] > 0) \vee \sim(\mathrm{P}[\mathrm{M}/t(M)] \approx 0).$$

FIRST CASE: ~(P[M/*t(M)*] > 0). Probabilities are nonnegative, so (P[M/*t(M)*] = 0). But $i \neq 0$, so ~P[M/*t(M)*] = 0), for a contradiction. End of first case. SECOND CASE: ~(P[M/*t(M)*] ≈ 0). Therefore, for an infinitesimal i,

$$i/(i + \mathrm{P}[\sim \mathrm{M}\ \&\ t(M)]) \not\approx 0.$$

Then, for some noninfinitesimal n that is greater than 0 and less than 1,

$$i/(i + \mathrm{P}[\sim \mathrm{M}\ \&\ t(M)]) = \mathrm{n},$$

and so

$$\mathrm{P}[\sim \mathrm{M}\ \&\ /t(M)] = i[(1 - \mathrm{n})/\mathrm{n}].$$

But then P[~M & *t(M)*] ***is*** an infinitesimal,* for a contradiction. End of second case. End of proof. (*If n is a 'real hyperreal' r, then [(1 − r)/r] is a 'real hyperreal' and the product i[(1 − r)/r] is an infinitesimal by Theorem 4.1 of Henle and Kleinberg, op. cit., p. 34. If n is a 'nonreal hyperreal' h, then, since n is not an infinitesimal, for some 'real hyperreal' r: h > r; so [(1 − h)/h] < [(1 − r)/r]; and so i[(1 - h)/h] is smaller than the infinitesimal i[(1 − r)/r], and is thus itself an infinitesimal.)

If P[~M & *t(M)*] *is* an infinitesimal, then that the testimony should exist though false, [~M & *t(M)*], would *itself* be a miracle, and, supposing that P[~M & *t(M)*] = i',

$$\mathrm{P}[\mathrm{M}/t(M)] = \frac{i}{i + i'}.$$

Everything in *this* case would thus depend on which would be the greater miracle, that the testimony should exist and be *true* so that i' = P[~M & *t(M)*] > P[M & *t(M)*] = i, *or* that the testimony should exist and be *false*, so

that $i = \mathrm{P}[\mathrm{M} \,\&\, t(M)] > \mathrm{P}[\sim\mathrm{M} \,\&\, t(M)] = i'$. If that the testimony should exist and be *true*, then

$$\mathrm{P}[\mathrm{M}/t(M)] < \tfrac{1}{2}.$$

If that the testimony should exist and be *false*, then

$$\mathrm{P}[\mathrm{M}/t(M)] > \tfrac{1}{2}.$$

In this last case, in which "the falsehood of testimony would be more miraculous" than would be its truth, and *only* in this case, can testimony for a miracle "pretend to command . . . belief" (p. 116). Hume does not give an example of such testimony. He was, on the present mathematical treatment of his 'proofs' and extreme probabilities, *bound* to be nearly certain that there has never been testimony to a would-be miracle whose falsity would be a miracle, let alone a greater miracle. If, in someone's view, *t(M)* asserts the existence of *such* testimony, then both $\mathrm{P}[\mathrm{M} \,\&\, t(M)]$ and $\mathrm{P}[\sim\mathrm{M} \,\&\, t(M)]$ are infinitesimals. But then $\mathrm{P}[t(M)]$ is an infinitesimal, for the sum of any infinitesimals is itself an infinitesimal.

ON TWO PARTS OF THE COMMON CONCEPTION

IX

Romancing the Stone

"When I use a word," Humpty Dumpty said, in rather a scornful tone, "it means just what I choose it to mean – neither more nor less."
Lewis Carroll

I call a fig a fig, a spade a spade.
Meander[1]

1. ON THE 'COMMON NAMES' OF GOD

[W]hen most of us . . . think of God, the being we think of is in many important respects like the God of the traditional theologians. It will be helpful, therefore, in clarifying our own thoughts about God to explore . . . the conception of God that emerged in the thinking of the great theologians. (Rowe 1993, p. 5)

'Exploring,' we find the big three of omnipotence, omniscience, and perfect goodness, and also such as everlastingness (sometimes eternality or atemporality) and being the creator. These are frequently enhanced properties that God would have essentially. This chapter is about omnipotence, bare and essential. Other attributes and aspects of the common conception come in for comment, but only in connection with omnipotence. The next chapter is about omniscience considered quite alone. An appendix to Chapter XII studies omniscience in combination with freedom, mainly human. Not pursued in this book are problems with what would be perfect goodness alone, if it would include justice and mercy. Reconciling virtues of justice and benevolence is a problem not only for what would be their divine realizations, but there should be special difficulties to the project in that case. David Hume's subtle ways of positioning the virtues of justice and benevolence in an ideally good human nature are of doubtful relevance to what would be the divine case, and other discussions of the human issue can be expected to be at best suggestive of approaches to that case.[2] While possible problems raised by perfect goodness alone are not taken up, problems made for it by the world, when it is

combined with omniscience and omnipotence, familiar problems, are subjects of Chapters XI and XII.

2. OMNIPOTENCE

2.1 A provisional definition. An 'omni-potent' being would be all powerful, all mighty, capable of doing anything, of unlimited power and might. Such, in other words, is what one would guess from the word's evident roots and what a fully versed speaker of English, at least one not acquainted with learned discussions, would say about its meaning. Peter Geach agrees: "[T]he English word 'omnipotent' would ordinarily be taken to imply ability to *do* everything" (Geach 1973a, p. 7). Of course, no well speaker, innocent of philosophy, who said that an omnipotent could do anything would mean that an omnipotent would be capable of changing the past, or making three less than two, or anything else that is impossible and the doing of which would make true a proposition that is necessarily false. Even Descartes, when ordinary speech served his purposes, drew this line on God's power: "I have never judged that God was incapable of something, except when it was incompatible with being perceived by me distinctly" (Descartes 1979, p. 45 – first paragraph of Meditation Six). The idea, the ordinary idea, of the word is that an 'omnipotent' would be able to do 'anything that can be done'. As a first approximation to an articulation of the idea, I offer:

> A being *b* is **omnipotent** if and only if, for any action *a*, *b* can do *a* (i.e., it is logically possible that *b* should do *a*) if and only if someone can do *a* (i.e., it is logically possible that someone should do *a*).

To persuade that an omnipotent would be capable of some action, this definition would have one persuade that some being could do it, that it is logically possible that some being should do it. For a *proof* of a capacity of an omnipotent we can thus point to the fact that some being has performed some action, for example, driving a nail, proving Fermat's last theorem, or running a mile in under four minutes. For proofs sufficient to decide intellects that actions that have never to our knowledge been performed are even so logically possible, we have no other way than (i) to conceive or imagine them, where that involves satisfying ourselves that our ideas of these actions do not harbor *a priori* contradictions; and (ii) to satisfy ourselves that the *prima facie* 'evidence'[3] for possibilities provided by our conjurings would stand up under ideal critical scrutiny. And this, though not without its problems, is I think a good way. By this means we can, according to the present definition, conclude that an omnipotent would be able to leap tall buildings at a single bound, run faster than speeding bullets, and stop locomotives. No problems with conceiving, no problems with imagining, these things.

There is in the displayed formula implicit quantification over potential tasks such as leaping tall buildings (a possible task) and making 2 plus 2 equal 5 (an impossible task). Some discussants of omnipotence would prefer quantification that ranged over states of affairs or propositions, and a formula such as

> A being X is omnipotent if and only if, for any state of affairs S, X would be capable of bringing about that S if and only if it is logically possible that there is a being y such that y brings about S.

or

> A being X is omnipotent if and only if, for any proposition P, X would be capable of bringing about that P is true if and only if it is logically possible that there is a being y such that y brings about that P is true.

There is little to be gained by such circumlocutions. Rather than make one of them official over simpler forms that run in terms of tasks or powers, it is possible to have recourse to their ideas of bringing about states of affairs and propositions when they are wanted to make clearly unproblematic applications of the simpler forms.

2.2 Powers and capacities. Thomas V. Morris sometimes promotes a distinction between capacities and powers (Morris 1987, pp. 72ff; 1991, pp. 69–73, 78–80); takes as primitive an idea of a power (Morris 1991, p. 69) for which idea, he warns, that the "surface grammar of power locutions can be misleading in numerous ways" (Morris 1991, p. 79; cf., 1987, pp. 73–4); holds that 'A can do x' is *not* logically equivalent with 'A has the power to do x' (wherein 'power' is understood in his way); and says that "there is no discrete power to sin" (Morris 1987, p. 73; cf., 1991, p. 78). Lacking a confident grasp of the distinctions that Morris sees, and not being clear about when and how ordinary language is supposed to mislead regarding them, I move freely in this chapter between 'can'-, 'able'-, and 'power'-locutions, though my definitions of omnipotence run always in terms of what an omnipotent could do, of what it would be capable of doing. Morris might say, "Right. And there are problems with what would be 'omni-capableness', problems that do not arise for what would be 'omni-powerfulness'." My proposal (Section 8.2 later) for what would be God's power (not to say 'omnipotence') or capacity, for I do not make Morris's distinction, agrees with Morris's proposal for that power.

2.3 A problem for that provisional definition. Suppose that Lois Lane is an omnipotent. We want to say that L can do anything that can be done. It is of course consistent with that that L should not actually do everything that can be done. Indeed, it is necessary that L should not actually do everything that

can be done. Consider that though it is possible for someone at some time 'to marry for the first and last time', and that it is possible for someone at some time 'to divorce again', it is not possible for anyone at some time to marry for the first and last time and also at some time to divorce again. Divorcing *again* is possible only for one who has twice-married.

There must be things that can be done that L does not do. To be an omnipotent is not to be a 'do-it-all,' but only to be a 'can-do-it-all.' No one can do everything that can be done. No one can do everything at once. And no one can do everything at some time or other, or during some or other periods, for amongst things that can be done are, for example, marrying for the first and last time, and marrying again, and marrying forever, and marrying twice. Let us consider one thing that L does not do. For concreteness, let it be not lifting a certain small stone. It follows from this that there is a task that L cannot perform, which task is one that even I can perform, namely, *lifting this stone that L does not lift*. It is logically impossible that L should lift this stone that L does not lift, for, were L to do that, L *though not lifting it would lift it*. Certainly, however, this 'incapacity' that L 'suffers' does not compromise her omnipotence. Why not? It is because L's doing that would be a case of a logically inescapable general incapacity: *Lifting this stone that L does not lift* would be a case of *lifting a stone that one does not lift*. *No one* can do anything that he does not do: Though it is not necessary that if x does not do t, then x cannot do t, $\sim\Box(\sim xDt \supset \sim\Diamond xDt)$, necessarily x cannot, not doing t, do it, $\Box \sim\Diamond(\sim xDt \,\&\, xDt)$.

As we do not require that an omnipotent O should be capable of performing logically impossible tasks such as making 2 plus 2 equal 5, so we should not require the capacity to perform would-be instances of tasks that are logically impossible in the manner in which lifting a stone that one does not lift would be: We should not require that O be capable of tasks that it is logically impossible that any agent should do. That a being can never stand up, not because it is essentially incorporeal, but because it is always standing, should not disqualify it from the title 'omnipotent' as naturally deployed. The provisional definition wants to be revised to make this plain.

2.4. We start with the *preliminary revision* adequate for most cases.

> *A being b is* ***omnipotent*** *if and only if, for any universal action a, b can do a* (*i.e., it is logically possible that b should do a*) *if and only if someone can do a* (*i.e., it is logically possible that someone should do a*).

A **universal action** is a performance of which does not essentially involve any particular person as its agent, its subject, or in any other way. Lifting a stone that has not been lifted is a universal action, and so is lifting a stone that *one* has not lifted, but lifting a stone that I have not lifted is not. It is *not* a problem for a being's omnipotence as here defined that, though many people can, he cannot,

lift a stone that *he* does not lift. On the other hand, a being for whom *rendering oneself sightless* was impossible because he was *essentially sighted* (if that is a possibility) would not be omnipotent according to this definition, for that is a universal action of which some beings are capable. Similarly, a *necessarily everlasting active being* (if *that* is a possiblity) would not be omnipotent, for it would be incapable of *suicide*, the universal action of ending one's own life. To deal with a subtle case that comes up in Section 4.1.2, I add a clause for beings without 'distinctive essential natures' that do not exist necessarily for our ***Final Definition***.

> *A being b is* ***omnipotent*** *if and only if, for any universal action a, b can do a if and only if (i) someone can do a (i.e., it is logically possible that someone should do a) and (ii) b's doing a would not be a case of b's doing a′, where a′ is a universal action such that, for every being c, even if c does not have a distinctive essential nature* and is not a necessary existent, c cannot do a (i.e., it is not logically possible that c should do a′).*

[*A being has a distinctive essential nature if and only if there is a universal property such that it has this property essentially, and not every being has this property essentially. A being that has only properties such as *being self-identical* essentially does not have a distinctive essential nature.] Clause (ii) is meant to be equivalent to

> *b's* doing *a* would not be a case of *b's* doing *a′*, where *a′* is a universal action such that, for every being *c*, *regardless of its essential nature or necessary existential status*,* c cannot do *a* (i.e., it is not logically possible that *c* should do *a′*).

[*A being *c*, regardless of its essential nature or necessary existential status, cannot do a universal action *a′* if and only if there is a being *c′* that differs from *c* only in its *essential* nature or in its existential status not being necessary; such *c′* cannot do *a* (i.e., it is not logically possible that *c′* should do *a′*). I say that if a universal action cannot be done by some agent regardless of its essential nature or necessary existential status, then this action a is a ***'general impossibility'***.] By this final definition an omnipotent being would be capable of doing *anything*, that is, any 'generally possible universal action', which is not to say that an omnipotent would be capable of doing *everything*, that is, every 'generally possible universal action'. I have avoided the locution 'capable of doing everything', in order to stay away from its collective/distributive ambiguity, and from any suggestion that an omnipotent should be capable of doing the *conjunction* of things that meet the conditions of our definition. Suppose there is a one-time appeal for money for some cause to be given at one time, and that Betty is both capable of giving

to this cause and capable of not giving to it. Certainly Betty is not capable of giving and also not giving. She is capable of doing each of these things, but neither she nor anyone else is capable of doing both of them. The definition is framed so it does not demand that omnipotents should be capable of doing both.

2.5 Richard Swinburne's way. Swinburne identifies difficulties with an analysis of 'omnipotence-at-a-time,' difficulties similar to those with which I have been dealing. Here is his troubled analysis, (C): "A person P is omnipotent at a time t if and only if he is able to bring about any (logically possible) state of affairs after t" (Swinburne 1993, p. 155). One reason given against this analysis is that "it requires that for P to be omnipotent before 1976, P must be able to bring about . . . a state of affairs in 1977 not brought about by P" (Swinburne 1993, p. 156). Swinburne proposes as a solution an analysis (D), according to which: "A person P is omnipotent at a time t if and only if he is able to bring about any logically contingent state of affairs after t, the description of which does not entail that P did not bring it about at t" (Swinburne 1993, p. 156). I am not happy with analysis D's management of cases of 'essentially natured' agents. For example, essentially sighted B's incapacity to render itself unsighted does not entail that he is not '(D)-omnipotent'. However, an essentially sighted being's incapacity to render itself unsighted *should* – as it does under my definition – tell against its omnipotence, as should what would be the incapacity of an essentially honest person to tell a lie. Inabilities that a being could not, because of its essential nature, escape are still inabilities. Perhaps this is more obvious when the essential characteristics generating inabilities are not desirable. As a corrective one can think, for example, of essential deafness and essential meanness and the inabilities that these would entail.

3. 'ESSENTIAL PROPERTIES'?

Essentialism . . . is the doctrine that among the attributes of a thing some are essential, others merely accidental. Its essential properties are those it has necessarily, those it could not have lacked. . . . Some properties are essential to everything whatever – the attribute of being self-identical, for example. . . . Others – for example, the attribute of being greater than 7 – are essential to all things that have them. Still others are essential only to some of the things that have them. Thus truth is essential to the proposition that 9 is greater than 7 but not to the proposition that the number of the planets is greater than 7. Advocates of the doctrine can be expected to disagree over particular cases. What are the essential attributes of, say, Dancer's Image? No doubt it will be counted essential that he is a horse, and accidental that he was disqualified in this year's Kentucky Derby. [Cartwright 1987, p. 150] I see no reason . . . for thinking essentialism unintelligible. At the same time, I do not mean to suggest that it is without its perplexities. Chief among these is the obscurity of

the grounds on which ratings of attributes as essential or accidental are to be made. (op. cit., p. 158)

3.1. It is time for confessions that might have been entered straight out when 'essential properties' entered in Chapter I. I have been writing, and will continue to write, of them in order to stay with philosophical/theological discussions that often veer to essential properties, even though I am not sure that *any* essential properties of possible interest are logical possibilities. Certainly they would be extraordinary conditions of which we have no unproblematic experience. While, as can be observed, some people are really blond, if any are essentially blond, that they are would need to be told by means special powers of observation. I want to say that no one is essentially blond. I want to say that no one *could* be essentially blond. Similarly, I find, for my intuitions concerning the essentially sighted, honest, and, to move to our subject, omnipotent. "I understand in theoretical terms these characters, but. . . ." The problem is with what would be 'the grounds' for such essential attributes, their 'real grounds' in the natures of their possessors. How, for example, could Margaret[4] be *essentially* unilingual? What about her could make her so? Something about the 'shape' of her brain? That, however, could make her essentially unilingual only if her brain is *essentially* so shaped, and how could that be? It seems that if she is essentially unilingual, then either she just is so 'basically' or there is some other attribute that is 'basically essential' to her and that 'makes' her also essentially, this way, unilingual. There is a sense in which neither alternative makes sense to me that I express in the wonder, "How *could* Margaret be *essentially* unilingual?" My problem is not with the *essentiality* of it. Margaret is, I am pretty confident, an essentially *animated* being. She could not have been a begonia, nor could she become one. Buckingham Palace is essentially *a building*. While it could have been a public housing complex, it could not have been a fence, not to mention a doctor. It is not the essentiality but *the entire essential uninlingualness* that puzzles (doesn't it?).

It is much the same with the ideas of beings who would be essentially perfectly good, of beings who would be essentially omniscient, and of beings who would be essentially omnipotent. Bracketing 'nice' difficulties with the *core* ideas of perfect goodness, omniscience, and omnipotences, I am not sure I 'get at all' the ideas of what would be their *essential enhancements*. I am inclined to think that these ideas can have no instantiations, that the beings they would depict are not so much as possible, and that in *this* sense no one of us that goes on about them 'gets them at all.' Cf.:

Some thinkers hold that such a concept – even if we are inclined to use it – is incoherent. . . . I do not believe that it is incoherent: I can find no actual contradiction implicit within it. Nevertheless it cannot be denied that it is . . . a very strange concept. . . . We should hesitate to postulate that this strange concept has any real instantiations. . . . (Mackie 1982, pp. 238–9)

John Mackie is writing of objective value. But his lines work for me when addressed to the ideas of essential sightedness, honesty, omnipotence, and so on of persons and beings. While I do not think that the words 'essentially sighted' are nonsense, while I do not think that their concept is incoherent and harbors an *a priori* contradiction, I suspect that they are 'unintelligible' in another sense, and that in any case their concept lacks 'any real instantiations' in any world. I suspect that essential sightedness, honesty, omnipotence, and so on are impossible conditions, not *a priori* but *really*, that is, *logically*. However, though there is a sense in which I do not 'understand' these properties, I understand well enough I hope what would be their 'logic', and how to talk about them. I will do that without always reminding of the extraordinariness and strangeness of the ideas of discussed essential attributes, and without *constantly* repeating that there is a sense in which I do not know what I am talking about when I speak of these attributes, and without *always* repeating that I doubt the relevance of these concoctions to longings of "the religious frame of mind" (Findlay 1955[1948], p. 52 [180]).

3.2 On our 'evidence for' (i.e., reasons for believing in) their possibilities. There is, I have said, a problem with what would be the 'real grounds' of these essential attributes. There is a somewhat related problem with what would be the 'epistemic grounds' for their possibilities. Stephen Yablo writes of conceiving as involving the appearance of possibility (Yablo 1993, p. 5). *Conceiving* a winged horse can include an 'appearance of the possibility of a winged horse' somewhat as *perceiving* includes an 'appearance of a winged horse.' There can be no such appearance of the possibility of an essentially winged horse, or an essentially unilingual being. There can be nothing remotely like still or moving pictures of instantiations of these essential attributes. For these essential attributes we have, it seems, only their words to go by. But that we have words for them, and that there is no contradiction lurking *a priori* in, for example, 'Margaret is essentially unilingual', is 'on the face of it' no more relevant to their possibilities than to what would be their realities. *Indeed*, for a formal point that may have more damaging implications, given the definition of 'x is essentially ϕ,' that is, 'Ess(x,ϕ)',

$$(x)(\phi)\Box[\mathrm{Ess}(x,\phi) \equiv \Box(\mathrm{E!}x) \supset \phi x],$$

it can be seen that

$$(x)(\phi)\Box([\Diamond\mathrm{Ess}(x,\phi) \;\&\; \mathrm{E!}x] \supset \phi x);$$

from *which* it follows for Margaret, who without a doubt exists, that she is *possibly* essentially unilingual, *only if she is unilingual*, and therefore* that only 'evidence for' her *being* unilingual can be 'evidence for' the possibility of her being essentially unilingual. [5] (*I am assuming that 'evidence for', i.e., 'reasons for believing', is 'closed for this entailment'. It is not 'closed in general for entailment'. That Margaret's favorite color is brown can be 'evidence for'

here wearing a brown dress tomorrow, without its being 'evidence for' her wearing a dress tomorrow.) That there is no contradiction lurking *a priori* in the words 'Margaret is essentially unilingual' is, however, *not* 'evidence for' her *being unilingual*; if it were, I would have reason to think that she is unilingual and not, as is the case, *have no idea* whether or not she is unilingual. Similarly, it seems, for *whatever* might be suggested as 'evidence for' the possibility that she is essentially unilingual.[6]

4. ON WHETHER OMNIPOTENCE IS POSSIBLE

4.1 Throwing stones. Can there be an omnipotent being? There is an old argument that says no.

'THE PARADOX OF THE STONE'[7]

4.1.1. Making a stone that one cannot lift is a possible universal task. It is something I can do; so it is something that an omnipotent being would be capable of doing. But then there is something that an omnipotent being cannot do: "[L]ift the stone he can create" (Rowe 1993, p. 7). So an omnipotent being would not be omnipotent, which is a contradiction.[8] Right? No. Section 4.1.2 explains why.

Making a stone that one cannot lift is a 'self-referential task'. Thinking that there may be problems somehow inherent to these, one might shift the argument from them to their generalizations, from, for example, making a stone that *one* cannot lift to making a stone that *no one* can lift. I could not with a quantity of cement demonstrate that this task is possible. I could not say, "See this big stone that I cannot lift. I made it." That argumentative *advantage* in the present context of the self-referential stone-task would have been lost. But, it can seem, the general stone-task would remain possible, and discussion adapted to it with a net gain. That is so, however, only if the dialectical difficulty is manageable that an omnipotent could lift any stone, so that, if there is an omnipotent, then making a stone that *no one* can lift is not a possible task. There may after all be no getting away from self-referential tasks for present argumentative purposes. But then there is, I think, nothing wrong with such tasks for present argumentative purposes, and I will proceed in terms of overt ones rather than generalizations that conceal related involution.[9]

4.1.2 Catching stones. There is, under the words 'but then', a *non sequitur* in the little argument of the previous section. Suppose a being is omnipotent. Then it can make a stone it cannot lift, for making a stone that one cannot lift is a universal action that many, even we, can do. Of course, though it can make a stone that it cannot not lift, it has not done so. If it had, there would be something that it could not do: lift this stone that it had made. So it would

not be omnipotent. There is not, confronting this omnipotent, a particular stone that it cannot lift, and there is not a *kind* of stone, a stone of such and such size, shape, and substance (and whatever), that it cannot lift. For, by hypothesis, it is omnipotent and can lift every stone there is, and every kind of stone there could be. Suppose, however, that an omnipotent being were to do this thing that it can do, namely, make a stone that it cannot lift. It can do that, for even my neighbor can do that, namely, make a stone that he cannot lift. "But what if it were to do that, what then?" Well then it would not be omnipotent, for then there would be a stone, a stone that it had made, that it could not lift. There would then be a particular stone of a kind that it could not lift. How could this come to be? It could come to be by this omnipotent's at once making a stone of a certain kind, it need not be a very large or heavy stone!, *and diminishing its lifting power* with respect to stones of the kind that it had made. It is a general point that a being's not being able to lift some stone is a *relation* between, on the one hand, this being's stone-lifting capacity and, on the other hand, this stone's character, usually only its size, shape, and substance, though other stone-attributes are of conceivable relevance. When people make stones they cannot lift their lifting-capacities remain constant, but when an *omnipotent* being made a stone that it could not lift, it would need to adjust that term of its relation to the stone that it made. An omnipotent's making a stone that it could not lift would necessarily involve this being's doing something to itself. It would need somehow to diminish its lifting-power with respect to the kind of stone that it made. But that should not be a problem for an omnipotent being! Though I am not inclined to, especially not as a permanent thing, I am certainly capable of reducing my lifting-powers with respect to all kinds of stones. Since I can do that, so can an omnipotent.

No being that is omnipotent has made a stone that it cannot lift, for, if it had, there would be something that it could not do, namely, lift that stone that it had made, and it would not be omnipotent. *But* an omnipotent being *can make a stone that it cannot lift*. No problem! To do that it would simply make a stone, and somehow or other diminish its lifting powers, and *relinquish its omnipotence*. There is no contradiction in an omnipotent being's making a stone that it cannot lift. There is no contradiction in the idea that the condition of omnipotence is a relinquishable condition. Indeed, it is *part* of the idea of omnipotence that it is relinquishable.[10] Cf.: "the omnipotence of a person (in the ordinary sense of 'person') at a certain time **includes** the ability to make himself no longer omnipotent, an ability which he may or may not choose to exercise" (Swinburne 1993, p. 161; emphasis added). I have not put my point in exactly Swinburne's way because I am not offering an account of 'omnipotence at a time', but of omnipotence *simpliciter*, and I understand that as an inalienable 'career-condition' in a sense to be explained. Also, I would adapt Swinburne's words, by making them about beings in general without the suggestion that they are right only for 'ordinary' beings.

There is in the story a universal action that an *omnipotent* cannot perform, though *lesser* agents can, namely, *making a stone that one cannot lift without diminishing one's stone-lifting power*. But an omnipotent's *per impossibile* doing that would be a case of, *without diminishing one's stone-lifting power, making a stone that one cannot lift, which stone one can lift*, and *no one* can do that. This 'incapacity' does not, therefore, detract from omnipotence as defined in Section 2.3, thanks to clause (ii) of the *final definition*.

4.2 Recasting the stone. Omnipotence is not a problem. It is possible. But there are many ways in which it can seem to be impossible. To bring out an important one, I will 'articulate' our stone.

4.2.1 An articulation of the putative paradox

PREMISES

(1) Necessarily, if a being is omnipotent, then it *can* make a stone that it cannot lift.

That is a consequence of the final definition of Section 2.3.

(2) Necessarily, if a being is omnipotent, then it *cannot* make a stone that it cannot lift.

It is, as the argument (for what is worth) 'goes', no more possible for an omnipotent to make a stone that it cannot lift than it is possible for a thoroughly and always honest man to tell a lie. In each case, were the thing done (the stone made, the lie told), the agent would not be as the agent is (omnipotent, honest).

CONCLUSION

(3) Necessarily, there is not an omnipotent being.

To demonstrate the validity of this argument it is sufficient to derive (3), *sans* 'necessarily', from (1) and (2), both *sans* 'necessarily'. And it can seem that that should be easy in the system of Appendix B in Chapter III. And so it is, *if* its second premise is interpreted in one of the ways it can be.

4.2.2 Trouble-shooting. The trouble with the argument, when 'necessarily' is stripped from its premises and conclusion, is that either it is invalid or it employs a premise that, if modal intuitions count for anything, is false. The trouble is with,

(2′) If a being is omnipotent, then it cannot make a stone that it cannot lift.[11]

This sentence is ambiguous. It has a 'wide-modal-scope' interpretation,

(2′w) It is logically impossible that a being that is omnipotent makes a stone that it cannot lift.

or equivalently,

> It is logically necessary that a being that is omnipotent does not make a stone that it cannot lift.

Let 'x *is* omnipotent' and 'x *makes* a stone it *cannot lift*' be for the same time, and understand that when x makes a stone of some sort there is then a stone of this sort. Symbolizations for the two expressions of wide-scope (2′) are – using the abbreviations, Ox: x is omnipotent; Mx: x makes a stone that it cannot lift – respectively,

$$\sim\Diamond(\exists x)(Ox \,\&\, Mx)$$

and

$$\Box(x)(Ox \supset \sim Mx).$$

These symbolizations begin and end the following sequence of obvious logical equivalents: $\sim\Diamond(\exists x)(Ox \,\&\, Mx)$, $\Box\sim(\exists x)(Ox \,\&\, MX)$ (modal negation), $\Box(x)\ \sim(Ox \,\&\, Mx)$ (quantifier negation), $\Box(x)\ (\sim Ox \vee \sim Mx)$ (DeMorgan), $\Box(x)(Ox \supset \sim Mx)$ (def of '$\supset$'). Sentence (2′) also has the 'narrow-modal-scope' interpretation,

(2′n) For an omnipotent being, it is logically impossible that it makes a stone that it cannot lift.

$$(x)(Ox \ \supset \sim\Diamond Mx)$$

or equivalently,

> For an omnipotent being, it is logically necessary that it does not make a stone it cannot lift.

$$(x)(Ox \ \supset \ \Box\sim Mx)$$

Of the two interpretations of (2′), the wide-scope interpretation is a trivial consequence of the definition of 'omnipotent' (for 'x is omnipotent' and 'x makes a stone it cannot lift' are about the same times, and when a stone of some sort is made, there it is). In contrast, nothing recommends the narrow-scope interpretation, and if modal intuitions count for anything, it is simply false. To see this suppose that Clark Kent is and will ever be omnipotent, and consider the instance of the last displayed formula, $(Oc \supset \Box\sim Mc)$, where 'c' stands for 'Clark Kent'. By supposition we have the antecedent that Oc is true: We have supposed that Clark Kent is omnipotent. But certainly the consequent that $\Box\sim Mc$ is false. Though it is a consequence of his assumed now-and-ever omnipotence that Clark Kent *does not* make a stone that he cannot lift, it is not logically necessary that he not do that. One can imagine his at once making a stone and diminishing his powers so that he cannot lift it. We are supposing he is a *permanently* omnipotent being, not that he is an *essentially* omnipotent.

We are supposing only that he is a super man, and not also that he is a super *strange* man, as even an essentially *middling strong* man would be.

4.2.3 To study the play of (2), depending on its interpretation, in the argument, we consider the arguments:

A	(1′)	(x)(Ox ⊃ ◊Mx)	**B**	(1′)	(x)(Ox ⊃ ◊Mx
	(2′n)	(x)(Ox ⊃ ∼◊Mx)		(2′w)	∼◊(∃x)(Ox & Mx)
	(3′)	∼(∃x)Ox		(3′)	∼(∃x)Ox

Of these arguments, A is valid. To see this, *suppose for purposes of argument* that the conclusion is false, so that it is true that,

(i) (∃x)(Ox). [There is an omnipotent being.]

Let c name one of these omnipotent beings so that,

(ii) Oc.

It follows from the universal premises (1′) and (2n) that,

(iii) Oc ⊃ ◊Mc

and

(iv) Oc ⊃ ∼◊Mc.

And it follows from (ii) and (iii), and then from (ii) and (iii), that

(v) ◊Mc

and

(vi) ∼◊Mc.

Contradiction! This means that our supposition that the conclusion (3′) is false is untenable given the premises (1′) and (2′n). It cannot be false, if they are true, which is to say that argument A is valid. However, it has a premise, namely, (2′n), which, if our modal intuitions are to be trusted, is false. The premises of argument B, on the other hand, are impeccable. In particular, premise (2′w) is a trivial consequence of definitions, for 'is omnipotent' and 'makes a stone that it cannot lift' relate to the same time. However, argument B is not valid. For the invalidity of argument B, it suffices that there should be a 'world' in which its premises are true and its conclusion false. If there is such a world, then it is possible that its premises should be true and its conclusion false whereas, were it valid, this would be impossible. We 'construct' such a world, by first making false B's conclusion and then making true its premises. Let there exactly the being s (for Superman, not Sobel) in α be omnipotent: Os. Then it is true in α that (∃x)Ox and *false* that **(3′)** ∼(∃x)Ox. Now 'construct' a world β in which it is true that Ms. This makes true *in* α that ◊(∃x)Mx; *and* **(1′)**, that (x)

($Ox \supset \Diamond Mx$) – for this, we have that s is in α the unique omnipotent. Recall that **(2′w)**, as a consequence of definitions, is true in every world, so it is *true in* α;. And there we have it, a world α this world α that we have 'constructed' – in which the premises, (1′) and (2′w), are true, and its conclusion, (3′), is false.

4.3 But not all at the same time. Now comes a challenge to the possibility of omnipotence that runs in terms of learning and forgetting. "Some beings can learn the log of 105, and some beings can forget it. So, according to its definition, omnipotence would involve being able both to learn and to forget the log of 105. But no one can both learn and forget anything, since one can learn only what one does not know and can forget only what one knows. And no one can both know and not know anything." Right?

This time I say, Yes and No. Certainly no one can at the same time know and not know anything, but there is no problem with a person's knowing and not knowing something at different times. Similarly for marrying for the first time and committing adultery. Though no one can do both of these things at the same time, since in order to commit adultery one must be married, many people manage to bring them off at different times, and that would be no problem for an omnipotent being. Similarly also for marrying for the first time and marrying for the second time, for sitting down and standing up, and for countless other pairs of tasks. [Several of these pairs are suggested by remarks in Swinburne (1993, p. 154).] So that it should not succumb to these slight challenges, my definition of Section 2.3 should be understood as short for the following formula that pays explicit intention to temporal matters.

> **An omnipotent being** *b* would at one time or other (or in one interval or another) be capable of doing any *universal* action *a* such that (i) it is logically possible that *a* should be done by someone, and (ii) *b's* doing *a* would not need to be a case of *b's* doing a universal action *a′* where this is a 'general impossibility' (as defined in Section 2.3).

Swinburne considers it to be a "reasonable assumption" that "it is logically impossible for an agent . . . [after its time] to bring about a past state" (Swinburne 1993, p. 155). Cf.: "Perhaps changing the past is just a special case of doing something impossible" (Wierenga 1989, p. 16). A consequence of Swinburne's reasonable assumption is that my definition does not require that an omnipotent should ever be able to do something that would be a case of 'changing the past'. On Swinburne's reasonable assumption, my definition of omnipotence satisfies the condition that, according to it, "an omnipotent being need not be able to do something that is incompatible with what has already happened" (Wierenga 1989, p. 17).

4.4 The past. "But how do you understand 'the past' and 'what has happened' when you say that it is not required of an omnipotent that it should be capable of changing the past, or of doing something incompatible with what has already happened?" I understand 'the past' for present purposes in a way adapted from Zemach and Widerker (1989, p. 113). Let a proposition be about a time t if and only if it entails (i) that the world exists, that is, that some contingent thing exists, at t; and (ii) either that a certain thing happens or obtains at t, or that a certain thing does not happen or obtain at t. (Cf., Fischer 1989, p. 36.) That Jones mows his lawn today, that he does not mow his lawn today, that he mows his lawn for the first time today, and that he mows his lawn for the last time today, are four propositions about today. My first proposal is that **the past through t** is the strongest true proposition about a time not later than t that is compatible with (i) the proposition that "the world ceases to exist at t" (Zemach and Widerker, p. 113), which is to say that at t, but at no later time, something contingent exists, and (ii) every proposition about a time or times later than t and about only times later than t. **The past to t** is the weakest proposition that entails, for every $t' < t$, the past through t'. I assume for these definitions that a truth is incompatible with the world's ceasing to exist at t if and only if it entails a proposition the nontruth of which is entailed by the proposition that the world ceases to exist at t. Suppose that Jones will mow his lawn next Thursday – J, and that Jones knows this now a week in advance – K. Since neither J nor K are compatible with the proposition that the world will cease to exist at noon next Wednesday, neither J nor K are part of (i.e., entailed by) the past to then. Suppose that Jones will mow his lawn for the last time today – L. L is about today, and it is compatible with the world's ceasing to exist next Thursday. But L is not compatible with the proposition that Jones will mow his lawn next Thursday, so L will not next Thursday be a part of the past. To say that 'the world ceases to exist at t' is to imply that nothing contingent exists after t, and that nothing contingent happens after t. Arguably, to say this is to imply that "there are no times after t" (Zemach and Widerker 1989, p. 113) and that t is the "last moment of time" (Wierenga 1989, p. 102). It is not necessary, however, for present purposes to insist on this point, which raises issues concerning the relation of time and times to contingent happenings. (Section A3.3 of Appendix A to Chapter XII returns briefly to the past.)

4.5. Swinburne and Edward Wierenga respond differently than I do to temporal conceptual challenges. To deal with them they define in the first place not omnipotence *simpliciter* – a 'career-condition' – but 'momentary conditions' that they cast as omnipotence at times (Swinburne 1993, pp. 155–6; Wierenga 1989, p. 25). Presumably they would define omnipotence *simpliciter* so that a being would be omnipotent *simpliciter* either if and only if it is omnipotent at all times, or if and only if it is omnipotent at all times that it exists. Either way there would be substantive differences between their accounts of

omnipotence *simpliciter* and mine. It would be consistent with their accounts that an omnipotent being should never be capable of adultery or divorce, of remembering or forgetting anything, or even of rising from a sitting position. Think of a being who never marries, never knows anything, and never sits. Not so for omnipotents on my account, who must be capable of each of these actions – of adultery, divorce, remembering, forgetting, and rising from a sitting position – at some time or other. I would define omnipotence at a time somewhat as Wierenga does. It would be a consequence of my definitions that a being could be omnipotent at a time, at every time to some time, or even at every time, though it was not omnipotent *simpliciter*.

4.6 On the doctrine of Humpty Dumpty. "But who wants God to be able sometimes at least to commit adultery, and who cares whether or not, like Theatetus, he can sometimes sit?!" I answer with a question: "Who's been talking about God? Not me. I've been talking about omnipotence and omnipotent beings. Left open for me is whether a being worthy of that name, and of worship, needs to be, or can be, omnipotent." Many philosophers and theologians seek to make something nice of omnipotence and suited to a God of worship. Cf.:

> Theists typically hold that God is almighty or all-powerful. . . . But theists are usually quick to add that there are many things God *cannot* do. . . . Moreover, holding that various limitations on ability are compatible with being omnipotent is not restricted to the Christian tradition. . . . My strategy is to [categorize] . . . limitations on ability that are compatible with being omnipotent. I then . . . formulate a definition of omnipotence. Finally, I show that this definition accords with my initial list of conditions on omnipotence. . . . (Wierenga 1989, pp. 12–4)

Swinburne mentions an alternative to practice that is 'different in name only':

> [T]o knock theism into . . . shape, we are faced with the choice of providing a narrower [than natural] definition of 'omnipotence . . . , or choosing another word to describe the extent of divine power. . . . Traditional theological use favours the former . . . modern secular understanding of the natural meaning of 'omnipotent' . . . suggests that we ought to describe the power of [God] by some other word than "omnipotent". The latter alternative has been taken . . . by Professor P. T. Geach [Geach 1973a] who prefers the word "almighty" to the word "omnipotent" to describe how powerful God is. (Swinburne 1993, p. 165)

Rather than practice the doctrine of Humpty Dumpty on 'omnipotence' and ordinary synonyms of it such as 'almightiness,' I have tried to call a spade a spade, and having done that to defend the possibility of 'omnipotence' naturally understood. Questions concerning omnipotence – what it comes to and whether it is possible – are properly *prior* to questions concerning God and omnipotence. Peter Geach runs together these questions and critiques not proposed accounts of omnipotence *simpliciter*, but proposed accounts of 'divine

omnipotence', or of 'God's capacity to do everything' (Geach 1973a, pp. 9ff). Plantinga suggests that we may, without answers to the first general question of omnipotence, shift in reasonable hope of doing better to the second particular questions of 'God's omnipotence'. After finding difficulties with two definitions of omnipotence, he says: "But perhaps . . . even if we cannot give a *general* explanation of omnipotence, we may be able to say what *God is omnipotent* comes to" (Plantinga 1967, p. 170). This methodology complicates matters, and is strange. For complications, having found a reason for rejecting as necessary for *a being x's omnipotence*, that x is capable of performing any action A such that the proposition *x performs* A is logically possible, Plantinga must (and does), only for *another* reason, reject as a necessary and sufficient condition for *God's omnipotence*, that God is capable of performing any action A such that the proposition *God performs* A is logically possible. Additional to its complications, Plantinga's proposal is strange for the reason he has recourse to it: It would have us make sense of 'God is omnipotent' wherein it seems that something generally predicable would be predicated of God, before we make general sense of that predicate, and while we allow that very possibly it has no general sense.

5. ON ESSENTIAL OMNIPOTENCE

"Perhaps there is no problem with the possibility of omnipotence unqualified, but there is a problem with the possibility of *essential* omnipotence." Rowe could say this, though these are not his words. He might add that in his own little argument he is concerned not with unqualified omnipotence, but only with omnipotence qualified in a certain way, that he is concerned only with *essential* omnipotence. And this means, he might say, that there is a problem for what would be God's omnipotence, since omnipotence would be a *part of God's nature or essence*, a "basic attribute" (Rowe 1993, p. 7) that God *could not* be without: "Being perfectly good is as much a part of God's nature as having three angles is part of the nature of a triangle. God could no more cease to be perfectly good than a triangle could cease to have three angles" (Rowe 1993, p. 7). Similarly, according to Rowe, for omnipotence, and *all* elements of the common conception. God, according to John Findlay and *many* others, if as appropriate an object of religious attitudes as is possible, cannot merely *happen* to be omnipotent, omniscient, and perfectly good. Many philosopher/theologians are what might be termed *essentially perfect being* theorists, or theists if they believe (Rowe does not). For *these* theists it is at best small comfort that bare omnipotence is possible, *if* essential omnipotence is *not* possible. I am sorry to disappoint.

5.1 It is not possible. An essentially omnipotent being could not exist without being omnipotent. Its omnipotence would be essential or necessary to its existence. What then of the possibility of essentially omnipotent beings? How do

they fare under the weight of stones? Is it possible that there is a being who is omnipotent in *this* manner? Now comes a proof that this is not possible.

ANOTHER STONE

Making a stone that one cannot lift is a possible task. That is something I can do. So an essentially omnipotent being would be capable of doing that. But then there is that an *essentially* omnipotent being could not do. I do not say it is to "lift [a] stone he can create" (Rowe 1993, p. 7), but that it is to make a stone that he cannot lift. So an essentially omnipotent being would not be omnipotent, which is as near as needs be to a contradiction, since an essentially omnipotent being would of course be omnipotent. Why could not an essentially omnipotent being make a stone that it could not lift? Because if it did, it would no longer be omnipotent, which cannot happen, since this being is not merely omnipotent but *essentially* omnipotent. It *cannot* be without being omnipotent: *It* cannot *be* without being omnipotent. The problem comes out if one considers *when* it could not lift this stone, or when *it* could not lift this stone. To make a stone that it cannot not lift, it is necessary that it bring about that at some time, at which both it itself and a stone it has made exist, it cannot lift that stone. But, to repeat, there cannot be a stone that it cannot lift. It cannot *co-exist* with a stone that it cannot lift. It is essentially omnipotent, and when it exists it can lift every stone.

An essentially omnipotent being would *be* omnipotent, but also it would be incapable of diminishing its power, and so, because of this incapacity, it would *not be* omnipotent. Diminishing one's power is something that many beings can do, it is something than an ordinary omnipotent could do, and it is something that an essential omnipotent could not do. It is furthermore not an instance of a 'generally impossible' kind of performance. This incapacity of an essential omnipotent would be due to its essential nature, it would be due to its being *essentially* omnipotent. It is not an incapacity that every being would 'regardless of its essential nature' suffer. A consequence, by the final definition of 'omnipotence,' is that this essential omnipotent would not be omnipotent: Clause (ii) was designed for this consequence. Our second stone dramatizes what would be the general incapacity of an '*essential* omnipotent' to relinquish its omnipotence – it particularizes this condition that spells the impossibility of essential omnipotence. (Appendix A 'articulates' the argument of this section.)

5.2 On approaching the impossible. Suppose that an essential omnipotent were to attempt the impossible! Suppose that it were to try to create a stone that it could not lift. The 'best' it could do, the closest that it could come to that impossible performance, would be to create a stone while arranging that there should be in place of itself when the stone is made someone else 'just like' it, but for the fact that this being was incapable of lifting that stone. The 'best' it could do would be to 'move everything *other than itself* to another possible

world' that in the way described would be very similar to the actual world. An essentially omnipotent being who was not also a necessary existent could not diminish its powers, but it could **commit 'ontological suicide.'** It would be capable of a performance such that, *were* it to execute it, things would be as detailed in the previous paragraph. It *could*, in the manner of the previous paragraph, 'move all other things' to a possible world in which *it* itself did not *exist*, a world that featured in its place a hardly discernible only *nearly* omnipotent being. An essential omnipotent would have the capacity of coming that close to creating a stone that it could not lift. It could do this only by committing ontological suicide. It, this essential omnipotent of which I would write,* would not be anyone or anything at all in the world of its stone-making near-success. (*'Would write' because my point is that essential omnipotence *is not possible*.)

5.3 An Aristotelian interlude. Ontological suicide would be a rather desperate expedient that proved nothing. Aristotle, it seems, would say that no agent could reasonably engage in it, not even to make a philosophic point.

> Everyone <not a morally good man alone> wishes good things for himself [everyone seeks the good – this is everyone's 'life-work']; <but he wishes only for what is good for himself as a man:> no one would choose to become another kind of being and to have that other being possess everything good. (Aristotle 1962: 1166a17–20, tr. M. Ostwald.)

That is egoism, and we should not believe it. A person can, in his wishes, find good reasons for letting another 'have it all'. Aristotle tells us that we should "try to become immortal [and a god] as far as that is possible" (1177b34), taking comfort, perhaps, in the fact that it is only so far possible – that is, in the fact that no matter how close we get to this condition, we will not make it all the way to divinity, and so cease to be ourselves. I should do all I can to become a god, taking comfort that I will not make the final movement, and lose everything to the perfect one that, through my efforts, would have come to be in my place to enjoy the fruits of my good labor. That would not be fair. Were it possible, Aristotle implies, there would be no answer to the question, Why should I do all the work, and this other guy get all the good?

Strange questions, strained reflections, and fittingly so, for the doctrine out of which they are spun, That it would not be me, for I am a man, and so essentially imperfect and not purely contemplative, whereas those are just the conditions and states of a god, for whom they are essential. The strangeness is not in this idea of Aristotle's of pure uninterrupted contemplation, or in the idea of a perfect being, but in his notion of the *modal* attachments of states and conditions, and their negations, to individuals. It is his *essentialism*. What does it *mean*, That it would not, that it could not, be *me*?

What, to return to our main subject, would it *mean* that this *essentially* omnipotent being could do nothing short of suicide to rid itself of the burden

of its omnipotence, were it of a mind to do that? I know what it means in *formulaic modal terms*; I know well enough to play with the idea. But . . . Cf.:

"[W]e are led . . . to hold that an adequate object of our worship must possess its various qualities *in some necessary manner*. . . . we are led to a queer and barely intelligible scholastic doctrine, that God isn't [for example] merely good, but is in some manner indistinguishable from His own . . . goodness. (Findlay 1955[1948], p. 53[181])

It *is* barely intelligible, this doctrine to which so many are now led, according to which God would be essentially good and would possess, 'not merely as it happened' (pp. 52–3 [180–1]), the other qualities that would make it worshipful, but essentially and 'in some necessary manner'.

6. ON NECESSARILY EVERLASTING EXISTENCE[12] CONJOINED WITH ESSENTIAL OMNIPOTENCE

Essential omnipotence is impossible, so all conjunctions of essential omnipotence with other attributes and conditions are impossible. Sometimes, however, its impossibility in conjunction with another attribute or condition is more obvious than its solitary impossibility. For example, it is easy to see that the conjunction of essential omnipotence and necessary existence is impossible, which is another attribute that Findlay maintains would need to belong to God: "[T]he worthy object of our worship can never be a thing that merely *happens* to exist . . . his . . . non-existence must be wholly unthinkable in any circumstances" (Findlay 1955[1948], p. 52 [182]).[13] An essential omnipotent who was also a necessary everlasting existent could not make a stone that it could not once it was made lift. Suppose, for *reductio* purposes, it were to make a stone that it could lift. Then it, *this* essential omnipotent, *would still be*, since by the present hypothesis it is a necessary everlasting existent. But it would not be omnipotent, since there would be this stone that it could not lift. That, however, is impossible, since it is by hypothesis essentially omnipotent, and so, since it would be, it would still be omnipotent.

7. ON OMNIPOTENCE CONJOINED WITH OTHER CONDITIONS AND ATTRIBUTES

Omnipotence, in contrast with essential omnipotence, is not impossible. That makes more interesting that its conjunction with necessary everlasting existence is impossible, for an omnipotent being could commit suicide, and suicide is not possible for necessarily everlasting existents. And omnipotence cannot be combined with certain essential attributes.[14] Consider, for example, essential honesty. Suppose, for *reductio* purposes, that an omnipotent being is essentially honest. Then this being cannot lie. Since it is essentially honest, there is no possible world in which it itself, this very being, lies. Morris agrees:

An essentially good being "cannot lie" (1987, p. 30); there is no possible world in which he lies (1987, p. 28). So there is something that an essentially good omnipotent cannot do, it cannot lie, though even I can do that. (Here Morris might say, "Right. An 'omni-capable' being cannot be essentially honest, for if he were, then, contradiction, this capable-of-everything being would not be capable of lying. But an 'omni-powerful' being can be essentially honest, for there is no discrete power to lie for him to lack." See the cautionary note in Section 2.1.) Again, there cannot be a being who is both omnipotent and essentially omniscient. Being essentially omniscient, such a being would be *incapable* of deceiving itself. There would be this that it could not do, and so, impossibly, this omnipotent being would not be omnipotent. The general point is that omnipotence cannot be combined with the essential mode of any attribute such that it is conceivable that a being who had that attribute nonessentially should rid itself of it.

8. WHAT IS LEFT FOR GOD OF OMNIPOTENCE?

8.1. No being can be essentially omnipotent. And no being who is a necessarily everlasting existent, and so not a possible suicide, or who is, for example, essentially good, and so not a possible liar, or essentially omniscient, and so not a possible self-deceiver, can be omnipotent. What is possible for any being x is that it should be capable of each task t that it is logically possible that some being should do, which is such that (i) for each attribute, if any, that x has essentially, x's performing t is consistent with its having this attribute (so lying is not something x can do if x is essentially perfectly good, and deceiving itself is not something it can do if it is essentially omniscient); and (ii) if x has necessary everlasting existence, then performing t is consistent with its continuing to exist (so suicide, ordinary and 'ontological', are not possible for x if it has necessary everlasting existence). Let any being of such extensive power be an **only necessarily self-limited power**, or an **ONSLIP**.[15] I am not going back on my word of Section 4.3 and offering here, under other words, a whittled-down revision of my definition in Section 2.3 of 'omnipotence'. Whether an ONSLIP would be an omnipotent would depend on whether it had any features essentially and on whether it existed always necessarily. It would be omnipotent if it has no features essentially and does not exist always necessarily. That is a sufficient condition for an ONSLIP's being omnipotent. For a necessary condition I propose *not* existing always necessarily, and *not* having any attribute essentially of which a being who had it only nonessentially could rid itself.

An ONSLIP can be far from omnipotent. Consider a being who was essentially incapable of creating something from nothing. This being would be incapable of doing something that it is (assume!) possible that some being should do. And its performance of this task would not be an instance of a 'generally impossible' performance. Consider a being who was essentially a

bodiless spirit. This being could not sit or stand, or scratch its chin – it can be capable of doing anything that it is logically possible that it should do – even though, if the limit to its lifting power is very low, it cannot lift a feather. The possible challenge – "But is it really possible that there be such beings?" (cf., Wierenga 1989, p. 29) – lacks punch against these beings. Certainly there can be beings who are incapable of creating from nothing, and who are of limited lifting power, for you and I are such beings. Traditional theologians who say that it is possible for a being to be *essentially capable* of creating from nothing and to be *essentially unlimited* in lifting power are in no position to deny similar privileges to the restricted and limited essential *opposites* of those positions. They are certainly in no position to challenge the possibility of essentially bodiless spirits.

The particular challenge – "But is it really possible that there be a being whose abilities are *essentially* limited in ***this*** way?" (Wierenga 1989, p. 29) – lacks punch against the would-be possibility of a certain Mr. McEar. McEar would be essentially limited to powers minimally sufficient to ear-scratching.[16] What reason is there for thinking that McEar does not represent a possibility'? Wierenga's only suggestion is that such an essentially limited being would be incompatible with the existence of beings of the power God would have: "[I]t would seem to be at least possible that God confer on [McEar] greater powers that include the ability to tie a shoe. In that case, it would be possible for any such limited being to do more than it is able to do" (Ibid.). But this thought has no force, and it does not advance discussion of McEar's possibility. For one sufficient point against Wierenga's suggestion, God *could* confer greater powers on *McEar* if and only if McEar is *not* in his powers *essentially* limited. Not even God could change Buckingham Palace into a road, if Buckingham Palace is essentially some kind of house. God could have the palace ground up and a road made of its gravel, but in the process God would have had Buckingham Palace destroyed, probably collapsed in a demolishing explosion. After that, it would be gone, and its stuff made in a road that could not be it, because it would not be. Similarly for McEar, who is by hypothesis essentially limited in his powers. Not God could help him. All God could do would be to *replace* McEar by a more gifted creature, for which he would get no thanks from poor McEar.

8.2. God, it can be said by those concerned that He should be possible, would be an ONSLIP.[17] God would not, God could not, if God is to exist, be essentially omnipotent, for that is not a possible condition. Whether, if God were an ONSLIP, God would be omnipotent will depend, for example, on whether He would have certain basic attributes essentially, and whether He would exist always necessarily.[18] If he would not, then His power would be in no way limited, and He would be omnipotent. If God would, then His power would not extend to doing things inconsistent with either His essential nature or His necessary everlasting existence (cf., Rowe 1993, p. 7) and thus might well be

limited, and He, therefore, not omnipotent. But even if restricted, it seems that the power of God could extend to everything of which the religious would have Him capable. This conception of God's power as limited only by His essential nature and necessary everlasting existence – supposing God would have an essential nature or exist always necessarily – assigns, I think, to God all the power that anyone who would not only be in awe of Him, but who would worship and adore Him, could want him to have.

An ONSLIP need not be awesome. Whether one would be depends on what if any essential attributes it would have, and on whether it would have necessary existence. As a crown to all traditional views of God's nature and existence, even if being an ONSLIP would not make God omnipotent, it would in a manner sufficient for religious spirits make God great. It would make Him "'almighty' . . . [in the sense of] having power *over* all things . . . [and being] the source of all power" (Geach 1973a, pp. 7–8). It would I think make That One "a maximally powerful perfect being" (Morris 1987, p. 71). That would be a being whose power was subject to "no independent, externally determined constraints" and to no "internally determined constraints . . . such that he lacks any possible . . . power it is intrinsically better to have than to lack," and who was "the sole source and continuous support of all the power there is or could be" (Ibid., and cf., 1991, p. 78). Being a *maximally powerful* perfect being is, according to Morris, "what the theist means to ascribe to God when characterizing him as 'omnipotent'" (1987, p. 71). It is only putting this, which I believe, in other words to say that what a perfect-being theist means to ascribe to the deity with the word 'omnipotent', what he means to add to being otherwise perfect, is **being an ONSLIP**, which is **not** to say being perfectly powerful, being all-powerful, or **being omnipotent**, if we would call spades spades, as I wish we would.

APPENDIX. A FORMAL ARTICULATION OF THE ARGUMENT OF SECTION 5

Here, in the form of Section 4.2.1, is the argument for the impossibility of essential omnipotence in Section 5.1.

(1e) Necessarily, if a being is essentially omnipotent, then it can make a stone that it cannot lift.

(2e) Necessarily, if a being is essentially omnipotent, then it cannot make a stone that it cannot lift.

(3e) Necessarily, there is not an essentially omnipotent being.

Premise (1e) is entailed by premise (1) of Section 4.2.1 and inherits its secure status: This entailment is proved below. Premise (1e) entails (by 'dropping' the word 'necessarily') that if a being x is essentially omnipotent, $\Box(E!x \supset Ox)$,

then it is logically possible that it should make a stone that it cannot lift, ◊Mx:

$$(x)[\Box(E!x \supset Ox) \supset \Diamond Mx]. \qquad (1e')^*$$

Grounds have been explained in Section 5.1.1 for premise (2e), which is, *sans* 'necessarily':

$$(x)[\Box(E!x \supset Ox) \supset \sim\Diamond Mx]. \qquad (2e')^*$$

It follows from (1e′) and (2e′) that (3e), which is *sans* 'necessarily':

$$\sim(\exists x)\Box(E!x \supset Ox) \qquad (3e')^*$$

Conclusion (3e′)* follows from (1e′)* and (2e′)*, as – see Section 4.2.3 – (3′) follows from (1′) and (2n′). The main difference between these inferences concerns the values of their second premises. Premise (2n′) is false. In contrast, (2e′) is, by the argument of Section 5.1.1, true. Now comes the promised proof that (1) entails (1e): It is in the system of Section B3 of Appendix B of Chapter III.

1.	~~*SHOW*~~ □(x)[□(E!x ⊃ Ox) ⊃ Mx]	ND
2.	~~*SHOW*~~ (x)[□(E!x ⊃ Ox) ⊃ Mx]	FUD
3.	~~*SHOW*~~ E!x ⊃ [□(E!x ⊃ Ox) ⊃ Mx]	CD
4.	E!x	assumption for conditional argument
5.	~~*SHOW*~~ □(E!x ⊃ Ox) ⊃ Mx	CD
6.	□(E!x ⊃ Ox)	assumption for conditional argument
7.	□(x) (Ox ⊃ Mx)	premise(1)
8.	E!x ⊃ Ox	6, N
9.	(x)(Ox ⊃ Mx)	7, N
10.	Ox	4, 8, MP
11.	Ox ⊃ Mx	4, 9, FUI
12.	Mx	10, 11, MP

X

'God Knows (Go Figure)'*

1. Introduction

1.1. There is a problem with what would be the knowledge of an omniscient being, a problem that derives from what would be its knowledge of itself. An omniscient being who knew everything would know that it knew everything. And of each thing that it knew, it would know that it knew this thing. And it would know more. It would know of each pair of things that it knew, that it knew each thing in this pair, of each trio of things that it knew each thing in this trio, and so on for every set of things it knew. But does not this mean that, *per impossibile*, an omniscient being would need to know more than it knew?

1.2 Plan. Section 2.1 of Part I presents an argument for the impossibility of omniscience from the impossibility of a set that would include everything a know-it-all would know. This argument harbors one for the stronger conclusion that complete self-knowledge is impossible. Notice is taken in Section 2.2 of arguments that Patrick Grim has made against omniscience, from the impossibility of a set of concepts an omniscient would need to have and from the impossibility of a set of all truths. In Part II, an idea of 'totalities' is explained that demands less of them than standard axioms demand of sets, so that the primary argument may be trimmed to its essentials. Part III elaborates the Cantorian perspective of this study, according to which there can be 'Manys' – things of a kind of which we can speak generally – that are not collected in sets, classes, 'totalities', or any 'Ones'. Section 7 of Part IV reviews Grim's critique of 'nonsets', collector options, that might contain an omniscient's knowledge: I think he is right, and that arguments such as those of Sections 2 and 4 cannot satisfactorily be met with "*a nonset something else*" (Grim 1991, p. 98). Section 8, responding to those arguments, casts omniscience as knowledge of all truths while allowing that this may be an 'inconsistent multitude' (Cantor's term) not collected in any One, and Section 9 floats a conception of divine knowledge as partly actual and virtually complete.

According to this conception the actual part would be nicely circumscribed in a One, while the presumably uncircumscribed rest would be only virtual, albeit immediately available, knowledge. Sections 10 through 12 of Part IV cue, present, and resist an argument due to Grim against the very idea of knowing 'all truths'. This argument maintains, without recourse to principles for sets or any kind of collecting Ones, that there cannot be a *proposition* about all truths, so that we cannot so much as think or speak of them or a being who would know them. I say that there is, and that we can (as I think I just have), that this possibility's 'costs' would be 'diagonal propositions' crucial to this last argument, and that it is possible that all truths should be known as they would be, in one manner or another, by an omniscient.

1.3 Knowledge and propositions. This chapter is about the possibility of omniscients whose knowledge would include "'propositional' [knowledge] . . . *knowledge that p*" (Alston 1986, p. 288) of every truth. So some might say that it is not relevant to the possibility of *divine* omniscients, for "[s]ome thinkers . . . have maintained that God's knowledge is not broken up into proposition-sized bits . . . , but rather constitutes a seamless whole, an undifferentiated intuition of all there is" (Ibid.).[1] However, they say that God knows *in His way* all there is to know in our way, that He *somehow* knows *all things that are, that they are, and how they are*, and arguments to come can be adapted to challenge the possibility of such a knower, on the ground that a being who knew everything would need to know, never mind how, more things than it knew, or, on the more radical ground, that we have no idea of an omni-knower, that we cannot so much as speak coherently about everything knowable.

PART I. FROM THE IMPOSSIBILITY OF SETS OF KNOWN TRUTHS

2.1 The primary argument from the impossibility of a set of the reflective parts of an omniscient's knowledge

Premises

(1) There is for every knower a set that contains precisely the propositions that this knower knows.
(2) An omniscient knower would know every true proposition.
(3) (The Power Set Axiom) There is, for every set S, a **power set** *Pow*(S) that is the set of all subsets of S.
(4) (True Propositions About Subsets of Sets of Known Propositions) For every knower and every set K composed of propositions known to him, for each member K′ of the power set *Pow(K)* of K there is a true proposition kn(K′) that of him and of K′ says precisely that he knows each of

its members;[2] and, for any distinct members K′ and K″ of *Pow(K)*, any proposition, kn(K′), that of him and of K′ says precisely that he knows each of its members, is distinct from any proposition kn(K″) that of him and of K″ says precisely that he knows each of its members.

Now comes a lemma that presupposes (3).

(5) (**Cantor's Theorem**) The power set *Pow(S)* of any set S contains more things than does that set *S*: That is, *Pow(S)* is of **greater cardinality** than is S.

Conclusion

(6) There is not an omniscient being.

For a deduction of (6), *suppose* for purposes of an indirect proof that, contrary to (6), there *is* an omniscient being O. Then there is a set CK (for 'complete knowledge') that contains precisely the propositions that O knows: (1). The set CK has a power set Pow(CK): (3). CK includes for each member K of Pow(CK) a proposition to the effect that O knows each proposition in K, where each such proposition is distinct from every other: By (4) there is for each K such a true proposition, and they are distinct, and by (2), O knows each since each is true. So *CK contains at least as many propositions as there are sets of propositions in its power set Pow(CK)*. But *CK does* not *contain at least as many propositions as there are sets of propositions in its power set Pow(CK)*, since, if it did, its cardinality would be greater than it is, by Cantor's Theorem, and no set is of greater cardinality than it is. The emphasized contradiction completes an indirect proof for (6). (An appendix expands on items in boldface and gives a diagonal-argument for Cantor's Theorem.) This argument elaborates one that would demonstrate the impossibility of "even a nonomniscient being of a certain sort" (Grim 1991, p. 95). If sound, my argument shows *inter alia* that no being can have complete knowledge of its knowledge. If sound, it shows not only that complete knowledge, but that complete introspective knowledge, is impossible.

2.2 Two set-centered arguments by Grim. These arguments also use Cantor's Theorem against certain sets said to be required for the possibility of omniscience.

2.2.1 An argument from the impossibility of a set of what would be the concepts of an omniscient

Any omniscient mind would . . . be self-reflective . . . : among its objects of knowledge, among those things it knows something about, would be its own conceptions of properties. . . . But by Cantor's power set theorem, there will be more actual properties of its objects of knowledge than objects themselves [since there is for

every subset of these the property of being a member of this set]. Actual properties will outnumber its conceptions of properties. Thus some genuine property of its objects of knowledge, and therefore some truth, will fall outside even its range of *conception* [and thus outside its knowledge]. (Grim 1991, p. 83)

Where my argument of Section 2.1 has that propositions known to an omniscient would comprise a set, this argument spelled out has the premise that an omniscient's conceptions of properties would comprise a set. As the primary argument harbors an argument against the possibility of a being whose knowledge is 'introspectively complete,' so the present argument harbors an argument against the possibility of a being whose conceptual resources for self-reflection (not to mention their exhaustive employments) are complete.

2.2.2 An argument from the impossibility of a set of all truths. This argument comes from that of Section 2.1 by replacing premise (4) – True Propositions about Sets of Known Propositions – by

(4″) (True Propositions about Subsets of Sets of True Propositions) For every set T composed of true propositions, there exists a set tr[*Pow*(T)] of true propositions about members of *Pow*(T), the power set of T, such that, for each member T′ of *Pow*(T), tr[*Pow*(T)] includes exactly one true proposition tr(T′) about T′, and for any distinct members T′ and T″ of *Pow*(T), propositions tr(T′) and tr(T″) of tr[*Pow*(T)] are distinct. [Numeral '(4′)' is being saved for Section 4 later.].

Let t be a true proposition in T. Then, Grim suggests, tr[*Pow*(T)] can be the set that, for each T′ in *Pow(T)*, includes, if t is a member of T′, the proposition that t is a member T′, and includes otherwise, if t is not a member of T′, the proposition that t is not a member of T′. These will be necessary truths. If one wants contingent truths corresponding 1–1 with sets in *Pow*(T), one can conjoin to each of these necessary truths some one contingent truth c (Cf, Grim 1991, p. 92). I use 'some *one*' so that there is not a problem with there being sufficiently many. Proceeding indirectly as in Section 2.1, to deduce the conclusion from premises, suppose that there is an omniscient being O. Then, from (1) and (2), there is a set of all truths. CK, the set of truths that O knows, is the set of all truths. But, for the anticipated contradiction, there is not a set of all truths. This follows 'by indirection' from The Power Set Axiom (3), (4″), and Cantor's Theorem (5) thus: Suppose there is a set of all truths CT (for 'complete truth'). CT has a power set *Pow*(CT): (3), The Power Set Axiom. There is a set tr[*Pow*(T)] of true propositions that is of the same cardinality as *Pow*(T): (4″). Each member of this set tr[*Pow*(T)], since it is a true proposition, is a member of CT, by definition of CT. So CT contains as many propositions as there are sets of propositions in *Pow(CT)*. But CT does not contain as many propositions as there are sets of propositions in *Pow*(CT), Cantor's Theorem,

and that no set is of greater cardinality than itself. There are problems for this argument for (4″) which, Beall (2000) observes, is unsound, *if* – as on theories that 'correlate' propositions with functions from worlds to truth values – (i) there is exactly one necessary truth, and (ii) logically equivalent truths are identical. To maintain it, one needs to reject such theories. Arguments for (4) are not similarly burdened: Truths that it correlates with elements of *Pow(K)* are clearly distinct.

2.2.3 Simmons on Grim. Grim's argument in Grim (1991, Chapter 4, Section 1) reworks arguments in Grim (1984) and (1988, Section VI) to show that there is no set of all truths. That result gives an argument against omniscience, assuming that for any knower "there is a . . . set of things it knows" (p. 95). Keith Simmons criticizes these early arguments. Half of his criticisms are predicated on the false claim that "Grim's argument relies on [a certain] (A)" (Simmons 1993, p. 23). He says that Grim assumes that, "given any member of the power set of I [the would-be set of all truths], there is a truth *about* that member," and that "[b]ehind the assumption lies a more general one:

(A) To each set, there corresponds a truth.

justified as a consequence of

(A′) Given any set, there is a truth about that set" (Simmons 1993, p. 23).

Grim does not assume or rely on (A), and he is not responsible for things that can 'lie behind' principles he does rely on, unless they are entailed by these. For his argument to show that there is not a set of all truths, Grim does not assume, but maintains by a little subsidiary argument limned above under (4″), that the power set of any set of truths can be put into 1–1 correspondence with a set of truths (see Grim 1984, p. 207; 1988, p. 256; 1991, p. 92). That little subsidiary argument can be adapted to show that (X), the members of any set of sets of truths can be put into 1–1 correspondence with a set of truths. It *can* seem that (X) commits Grim to (A) restricted to sets of truth, which is the proposition that, to every set of truths, there 'corresponds' a truth. But it does not. (X), *together with* the proposition that ***there is a set of all sets of truths***, do entail (A) restricted to sets of truths. But Grim is not by that argument committed to this latter proposition. Which is good, since he of course does not *believe* it and is sometimes arguing against it. Simmons observes that some "regard as a paradox" what Grim offers as a "*reductio* proof that there is no set of all truths" (Simmons 1991, p. 26). I take this to mean that there are 'perplexed' people who, while seeing that *something* is wrong when they assume that there is a set of all truths, are not prepared to point a finger at this assumption, or at anything in particular that contributes to the contradiction Grim develops. *If* so, I can only encourage them to think again about Grim's proof, particularly the subsidiary argument under (4″).[3]

PART II. THE ARGUMENT TRIMMED TO ITS ESSENTIALS

An argument from weaker premises can proceed in terms of objects for which only a few of the properties of sets are stipulated. The argument comes in Section 4 after the introduction in Section 3 of these objects. Section 5 elaborates aspects of the argument of Section 4 that differ somewhat from their counterparts in Section 2.1.

3 'Totalities.' To trim the primary argument against omniscience to its essentials, I adapt it to 'totalities', objects that, for one difference from sets, do not necessarily have 'power totalities'. The trimmed argument does not require, and so I do not assume, that elements of a totality T that satisfy a stateable condition K and of which we can speak specifically should themselves make a totality. Nor do I insist that for any two things there is a totality that comprises precisely these things. To explicate totalities, instead of analogs for totalities of The Power Set Axiom, and such principles as Separation, Pairing, and Union, I assume a single principle that contains everything about totalities on which the coming trimmed argument depends. For more structure than is presently required, one might stipulate for totalities analogs of other principles of Zermelo set theory, without stipulating or implying an analog of The Power Set Axiom. Even given such principles there would be the question of whether the conceptual room provided for 'totalities' that are not sets is occupied. I know of no likely candidates for it, and concede that there may be no totalities that are not also sets. The end of this part is not to include introducing a novel nonempty kind of 'collector', but merely to trim the primary argument to essentials.

4 The argument trimmed. Terms in boldface are for things commented upon or explained in Section 5 later.

Premises

(1′) There is for every knower a totality that contains precisely the propositions that this knower knows.

(2) An omniscient knower would know every true proposition.

(3′) (**Subtotalities of Totalities**) For every totality T: (i) For each member x of T, there exists the singleton totality {x}; and (ii) for every **mapping** M of T onto a totality *Sub*(T) composed only of subtotalities of T, there exists a totality T* that is a subtotality of T such that T* includes x of T if and only if x is not a member of M(x), the totality in *Sub*(T) with which x is paired by M.

(4′) (True Propositions About Subtotalities of Totalities of Known Propositions) For every knower and for every totality K composed of propositions known to him, for each subtotality K' of K there is a true proposition *kn*(K') that of K' and of him says precisely that he knows each of its member propositions; and, for any distinct subtotalities K' and K'' of K, any proposition *kn*(K') that of him and of K' says precisely that he knows each of its members is distinct from any proposition kn(K'') that of him and of K'' says precisely that he knows each of its members.

Now comes a lemnma that follows from (3′) and a **More-Than** rule to be explained.

(5′) **(Cantor for Totalities)** Every totality has more subtotalities than members.

Conclusion

(6) There is not an omniscient being.

Suppose that there is an omniscient being O. There is a totality CK that contains precisely the propositions that O knows: (1'). CK includes for each subtotality K of CK a proposition that 'says' that O knows each proposition in K, with the propositions for distinct subtotalities of CK being distinct: (2), (3′), and (4′). *So* there are more propositions in CK than propositions in CK: (5′). *Impossible*! For no things are there more than there are.

5 Subtotalities, mappings, more than, and Cantor for totalities

5.1 Subtotalities of totalities. This says everything about subtotalities on which our trimmed-down argument depends. It affirms the existence of certain subtotalities of totalities *without* affirming the existence of a totality of them. [Let the analog of Subtotalities of Totalities for sets be *Subsets of Sets*. This is a consequence of intuitive principles for sets *other than the Power Set Axiom* that ensure that every subset of set S is given by Subsets of Sets: If S' is a subset of S, such principles ensure that there is a 1–1 correspondence C between S and a set SS of subsets of S such that, in C, precisely the members of S' are not members of the subsets in SS with which they are paired; S' is then identical with $\{x: x \in S \ \& \ x \notin (x)\}$, the existence of which set is entailed by Subset of Sets. For such a 1–1 correspondence C: (a) consider the union (*Sing*(S) U *Comp*(S)) wherein *Sing*(S) includes precisely, for each x in S,{x}, and *Comp*(S) includes precisely, for each x in S, the complement in S of {x}; (b) separate out from (*Sing*(S) U *Comp*(S)) the set *Sub*(S) that includes {x} if x is not in S' and the complement in S of {x} if x is in S'; (c) form the 1–1 correspondence C between

S and this *Sub*(S) in which, for each x in S, C(x) is {x} if x is not in S, and C(x) is the complement in S of {x} if x is in S'.]

5.2 Mappings. Mappings, mentioned in Subtotalities of Totalities, figure in the rule stated below for comparing quantities of things that may not make sets, as is done in Cantor for Totalities. Mapping relations do the work that is done by 1–1 correspondences, certain sets of ordered pairs (Cf. Grim 1991, p. 118), for sets.

Mappings into and *mappings onto*: For these relations, consider F's and G's of which we can speak generally, for example, frogs, green things, sets, propositions, things that are not members of themselves, things of which we can speak generally, and so on. A relation M maps F's *into* G's – $F \succeq G$ – if and only if every F stands in relation M to exactly one G, and at most one F stands in relation M to any given G:

$$(x)[Fx \supset (\exists y)(z)([Gz \,\&\, M(xz)] \equiv z = y)] \,\&$$
$$(x)(y)(z)[Gx \supset ([(Fy \,\&\, Fz \,\&\, M(yx) \,\&\, M(zx)] \supset y = z)].$$

A relation M maps F's *onto* G's – $F \approx G$ – if and only if every F stands in relation M to exactly one G, at most one F stands in relation M to any given G, and for every G there is an F that stands in relation M to it:

$$(x)[Fx \supset (\exists y)(z)([Gz \,\&\, M(xz)] \equiv z = y)] \,\&$$
$$(x)(y)(z)[Gx \supset ([(Fy \,\&\, Fz \,\&\, M(yx) \,\&\, M(zx)] \supset y = z)] \,\&$$
$$(x)(Gx \supset (\exists y)[Fy \,\&\, M(yx)])$$

For a mapping relation M of X's to Y's, let precisely the ordered pairs (x,y), where x is an X and y is a Y, such that M(x,y), be *the map of M*. Maps are certain Many's. Some but not all maps are sets. For an example of one that is not, consider the identity relation that maps precisely the objects that are not members of themselves onto themselves. (There *are* objects that are not members of themselves, even if there are no other objects.) The ordered pairs determined by this relation do not make a set, for "there is **nothing** the members of which are precisely the things that are not members of themselves" (Cartwright 1994, p. 7; emphasis added). Why not? Because if there were, then, *per impossibile*, it would both be and not be a member of itself.

5.3 More than. We can say, for X's and Y's of which we can speak, that:

> There are more X's than Y's if and only if (i) there is a mapping of Y's into X's and (ii) there is not a mapping of Y's onto X's: $X \succ Y$ if and only if $X \succeq Y$ and $X \not\approx Y$.

More-than relations between things that may not make sets or even totalities are defined here "in terms of whether there is a [mapping] relation that satisfies

the first condition and there is not a relation that satisfies the second" (Grim 1991, p. 118). It is an easy consequence of this rule that there are not more X's than X's, for there is a mapping, namely, the identity mapping, of X's onto X's, and of X's onto X's!!! It is thus an easy consequence that Cantor for totalities has as a corollary that there is not a totality of all totalities. (It is *not* an easy consequence, if it is a consequence at all, of this rule that if $X \succ Y$, then $Y \not\succeq X$. This is a consequence *if* the following analog for mappings of the Schröder-Bernstein Theorem for '1–1 correspondences into' is valid: For X's and Y's of which we can speak, if $X \succeq Y$ and $Y \succeq X$, then $X \approx Y$. Patrick Suppes writes of the Schröder-Bernstein Theorem that its "proof is the most difficult of any theorem yet stated in these first four chapters" (Suppes 1960, p. 94).) This More-Than rule makes possible more-than relations (and not-more-than relations) between X's and Y's the many X's and the many Y's make sets or even totalities, or 'collecting Ones' of any sort, more of which we shall see shortly. All that is required now is that we should be able to speak generally, quantificationally, of all X's and all Y's. Grim observes that, though "the only formal semantics for quantification we have is in terms of *sets"* (Grim 1991, p. 115), it is not clear that our understanding of quantification is limited to applications in which the range of quantifiers are sets, or again, any kind of 'collecting Ones.'

5.4 Cantor for totalities. Cantor for Totalities follows from Subtotalities for Totalities and our More-Than rule by an adaptation of the argument in Section A3 of the appendix at the end of this chapter. To be proved: For every totality there are more subtotalities than members. Consider any totality T. (i) There *is* a mapping of members of T *into* subtotalities of T. [Each member x of T can be paired with the subtotality $\{x\}$. The existence of these subtotalities is secured by the Subtotalities of Totalities. The mapping relation is simply that of a member x of T to the subtotality of which x is the sole member.] (ii) There *is no* mapping of members of T *onto* subtotalities of T. [Suppose, for argument, that there is a mapping M of members of T onto subtotalities of T. Consider the subtotality T* of T that includes x if and only if x is not a member of the subtotality of T with which M pairs $x - T^* = \{x: x \in T \ \& \ x \notin M(x)\}$ – the existence of subtotality T* is secured by the Subtotalities of Totalities. The subtotality T* of T ***is*** paired by M with a member of T: Every subtotality of T is paired by M with member of T. But T* ***is not*** paired by M with a member of T. (For this last 'line' there is the following subsidiary proof. Suppose that T* is paired by M with a member x of T: $T^* = C(x)$. This x *is* a member of T*: For if x is not a member of T*, then, by the definition above for T*, x is a member of T*; so x is a member of T*. And, a contradiction, x *is not* a member of T*: For if x is a member of T*, then, by that definition, x is not a member of T*; so x is not a member of T*.)] *Therefore*, from (i) and (ii), by the More-Than rule stated above, there are more subtotalities of T than there are members of T.

5.5 ZF-Power. "Christopher Menzel has proposed saving the notion of a set of all truths: [S]tandard ZF set theory minus [only] the power set axiom, he suggests, offer[s] such a possibility" (Grim 1991, pp. 111–2; see also Menzel 1986). Let a 'set' of ZF-Power be a set$^{ZF\text{-}P}$. Totalities are not by definition necessarily sets$^{ZF\text{-}P}$. ZF-Power includes analogs of all axioms of ZF except The Power Set Axiom. It includes, in particular, a Separation principle. No such principle is a part of Subtotalities of Totalities, which is my sole stipulation for totalities. But sets$^{ZF\text{-}P}$ are totalities. An analog of the Subtotalities of Totalities is valid in ZF-Power.

5.6. The argument of Section 2.2.2 establishes that there cannot be a *set* of all truths. Subtotalities of Totalities, and maps for more-than relations, afford an argument that there cannot be a *totality* of all truths. Grim indicates how one can show that there cannot be a *set*$^{ZF\text{-}P}$ of all truths. His argument employs Separation for sets$^{ZF\text{-}P}$ to establish that, if there were a set$^{ZF\text{-}P}$ T of all truths, there would be a subset T′ of T that contained those truths that satisfied the condition of being about a subset of truths. T′ would be a set of all truths about sets of truths. The argument then proceeds by a Cantorian *reductio* to show that there is not a "mapping [of] truths onto truths concerning sets of truths" (p. 112), so that *per impossibile* this subset T′ of T would be larger than T. An argument to show that there is no totality of all truths would not rely on an analog for totalities of Separation.

PART III. BEYOND COLLECTIONS

6 On kinds of multiplicities

6.1 Manys that are not collecting Ones. Every set is a totality, and, because less is demanded of totalities than standard axioms demand of sets, it is not necessary by definition that every totality is a set. Also, as not all things of kinds of which we can speak make sets, so do they not all make totalities. The many totalities themselves do not compose a totality; that is a consequence of the Subtotalities of Totalities and the 'more-than' rule for comparing not only totalities, but of all things of kinds of which we can speak generally, some such things 'collected' in anything. For example, there are the many things that are not members of themselves, that do not belong to themselves, and so on. We can speak generally of things that are not members or elements of themselves, that do not belong to themselves, and so on; but such things, we have learned, cannot be members or elements, and so on, of anything. These 'uncollectible' things make, Cantor would say, '*inconsistent multitudes*'.

6.2 Cantor on sets and other Manys. Grim tells us that the great man once defined a set as "'a Many which allows itself to be thought of as a One'"

(Grim 1991, p. 124): "Unter einer 'Mannigfatigkeit' oder 'Menge' verstehe ich nämlich allgemein jedes Viele, welches sich als Eines denken läßt" (Cantor 1932, p. 204: 1895). Rudy Rucker reports that "[i]n 1895, Cantor restated this definition as follows: 'By a "set" we mean any gathering into a whole *M* of distinct perceptual or mental objects *m* (which are called the "elements" of *M*)' [Cantor 1932, p. 282]" (Rucker 1984, p. 323). Cantor held that objects *are* 'gathered together into a whole' if and only if the Many of them '*can be thought of as a One*', that is, can be so thought of *without contradiction*. A 'set' is a One that *comprises* a Many, that *collects* its many things. I take Cantor's view to have been that there is a collecting One for several objects whenever the supposition that there is such a collecting One is *consistent*.

Cantor's view was, in part, that not every Many *is* collected in a One, that some kinds of things are not collected because they are *uncollectible*. Cf.:

> Cantor found it necessary to distinguish 'two kinds of multiplicities': "For a multiplicity can be such that the assumption that *all* of its elements 'are together' leads to a contradiction, so that it is impossible to conceive of the multiplicity as a unity, as 'one finished thing' [or to think of it as a One]. Such multiplicities I call *absolutely infinite* or *inconsistent multiplicities*. As we can readily see, the 'totality of everything thinkable,' for example, is such a multiplicity. . . . If on the other hand the totality of elements of a multiplicity can be thought of without contradiction as 'being together', so that they can be gathered together into *'one* thing', I call it a *consistent multiplicity* or a 'set'." [Cantor 1932, p. 443; 1967, p. 114] (Simmons 1993, p. 28)

It is evident that in calling such multiplicities 'inconsistent,' Cantor does *not* mean that they are *impossible* or "not quite real" (Rucker 1984, p. 9). He does mean that there are no such multiplicities, that there is not, for example, the 'inconsistent multiplicity' of thinkable things. He *speaks* of these multiplicities and of what he takes to be this one of them. He says, "such multiplicities I call absolutely infinite" (Ibid.), which would be strange, if he believed that there are no such multiplicities, that 'they' are merely 'figments of inconsistent thought'. He thought that there *are* such multiplicities, perhaps many of them. He may have thought – I am guessing – that the multiplicity of them is itself such a multiplicity. His negative point is only that they are not 'single finished things', which words want, I think, to be understood as short for 'single finished things, *with all that this entails*.' One can think about them consistently, as long as one does not think about them in that way, as single finished things 'with all that this entails'.

What is entailed for Cantor's 'single-finished-thing multiplicities'? It is a fair guess that a 'power-multiplicity axiom' is entailed along with 'diagonal submultiplicities' to embarrass various interesting would-be single-finished-thing multiplicities that, given these resources, could be demonstrated not to 'measure up'. 'Consistent multiplicities', whose elements can be thought of without contradiction as being together in a collecting thing, are, Cantor says, 'sets'. Cf.: "Cantor suggested explicitly that consistency be taken as a criterion

of set existence, thus presaging a doctrine central to Hilbert's work," (Hallett 1995, p. 71).

6.3 'Talk about things'.[4] "It is one thing for there to *be* certain objects; it is another thing for there to be a *set*, or set-like object, of which those objects are members" (Cartwright 1994, p. 8). Richard Cartwright persuades that the "All-in-One Principle" (Cartwright 1994, p. 7) is not *true*; that it is not true that, wherever there are many things of which one can speak generally, there is a class, collection, multiplicity, or One of some sort that can properly be said to 'comprise' these things, or to have them as members or elements. One might guess that one can speak only distributively and not also collectively of Manys that do not make collecting Ones. But this is evidently not so: Consider that it is possible to speak of things that are not members of themselves, that it is possible to speak of all of them, and to say, for example, that they do not make a collecting One.

PART IV. OMNISCIENCE AND DIVINE KNOWLEDGE

7 Taking the measure of these challenges to omniscience

7.1 Problematic premises. Premise (2) of the arguments of Sections 2 and 4 – that an omniscient would know every true proposition – seems secure. This premise says nothing about the manner of an omniscient's knowledge. It does not say that it would be composed of discrete knowledge-bits of true propositions (see Section 1.3). It does not say that an omniscient would always in some manner have every true proposition 'equally in or before its mind'. Nor does it say that an omniscient's knowledge would be exhausted by its knowledge of true propositions. It does not say that the things an omniscient would in some manner know would be of some collecting One. "But why say only that premise (2) *seems* secure?" Because there is a recondite challenge coming in Part V to the idea of 'every true proposition'. Rather than anticipating the response that satisfies me, I am waiting until the end to say that premise (2) *is* secure.

Premises (3) and (3′) are unproblematic, constituting as they do mere explications of 'set' and 'totality.' The remaining premises are problematic. Premises (4), (4″), and (4′) say that certain *propositions* about subsets or subtotalities of sets or totalities *exist*. These premises are in my view plausible enough for the arguments they serve in Sections 2 and 4, notwithstanding the unavailability of a general theory of propositions that gives conditions for their existence, uniqueness, and 'aboutness'. 'Lines' (5) and (5′) are not premises but 'lemmas' that are provable from stipulations concerning 'sets' and 'totalities'. As such they are unproblematic.

Which brings us, no surprise, to premises (1) and (1′), which say that *sets or totalities* that comprise precisely all and only the propositions known by

knowers *exist*. Initially, and on reflection, it is in my view credible that omniscient knowers are at least 'conceptually possible' (that is, not conceptually impossible) and that the knowledge of an omniscient would be absolutely unbounded, and so perhaps the holding capacity of any collecting One. The arguments of Sections 2 and 4, rather than recommending the abandonment or curtailment of the idea of omniscience, to my mind tell against their first premises and are strides toward converting from conjecture to conclusion the thought that the truths known by an omniscient are *not* collected in any kind of One. Persuaded in part by Grim's discussion, which will be reviewed in the next section, I think that there is no kind of collecting One of all truths or all propositions that would be known to an omniscient.

7.2 Grim against several set-like options. Having said that "[t]his much seems to be a solid result, there can be no set of all truths"(Grim 1991, p. 98), Grim considers and finds wanting responses that would avoid sets by packaging the propositions known to an omniscient in collecting One's afforded by one or another alternative-to-Zermelo-Fraenkel set theories. He considers five alternative set theories and finds that "none . . . offers any hope, except at quite unacceptable cost, for even a *class* of all truths [let alone a class of all propositions known to an omniscient]" (p. 100). None, he argues, affords a kind of collecting One appropriate to all and only truths:

> An unrestricted principle of comprehension for classes – such as that in KM [Kelsey-Morse set theory], ML [Quine's system of *Mathematical Logic*], or A [Ackerman's set theory] – if extended to give us classes of truths, allows too much of the basic mechanism of a Cantorian argument. . . . Comprehension restricted to stratified or predicative conditions, on the other hand, such as in NF [Quine's 'New Foundations'] and VNB [the von Neumann-Bernays system] . . . cripples the basic mechanism of the Cantorian [diagonal] argument. But . . . only at the cost of seriously crippling mathematical induction as well. (p. 109)

Grim reports that Cantor's theorem is not provable for all sets in Quine's NF: "It is not provable, in particular, for *non-Cantorian* sets within NF, which have the anomalous feature of containing more elements than [does the set of] their singleton subsets" (p. 100). Nor is Cantor's Theorem provable for classes in VNB. Within VNB "the 'diagonal' element crucial for such a proof . . . is simply not guaranteed to exist as a class" (p. 104). The 'cost-horn' of the argument, on which horn Grim hangs NF and VNB, is of somewhat uncertain strength in the present context. It is thus welcome that Grim provides implicit and explicit additional criticism of options *for all truths* afforded by 'proper class' theories such as NF and VNB. The 'non-Cantorian sets' of NF are, as already noted, very strange. For one thing, they have elements that are not also elements of singleton classes. But surely if all truths are collected in a class of some sort, then each truth occupies all by itself a class of that sort. Also, while some

Cantorian arguments against a VNB class T of all truths are blocked, others, Grim maintains, are not:

> Consider, for example, not subclasses of T . . . but merely *conditions* [*on elements of T*] . . . that may or may not define subclasses. We will have the ['diagonal'] condition . . . for example, that fails to define a class. Now there will be more such conditions on elements of I [and thus more truths about these conditions] than elements of T. (p. 149n26 to 105)

(Criticism of this complaint of Grim's against VNB affording a class all truths – criticism that goes to the presumed existence of 'diagonal conditions'– is implicit in Section 11 later.) Grim adds a "deep *intuitive* problem" that tells against all truths comprising an *ultimate class* or proper class of any of the alternative set theories he considers, in which paradox is escaped for these classes "precisely because they . . . will not be members of further classes" (pp. 109–10). The deep intuitive problem is that "[a] class of truths . . . , would, appear to . . . be a member of [further classes such as] the class of classes of propositions . . . [and] the class of classes containing one or more truths" (p. 110).

7.3 Capitulation. I think that Grim is right and that prospects are not good for intermediate responses to our arguments that would package the knowledge of an omniscient in "a *nonset something else"* (p. 98). So I move from such responses to describe two ways in which an omniscient being's propositional knowledge might be a Many that makes absolutely no kind of collecting One. Not that I expect everyone to be as disposed as I am to despair of a collecting One suited to an omniscient's knowledge, and as prepared as I am to move on. For the complexity and variety of theories of collectors and theories of truth that presently have advocates is great. And there are possible grounds for assessments contrary to mine with which Grim has not dealt, and with which, being not sufficiently informed of their details, I cannot deal.[5] So I take my opinion 'with a grain of salt' and move on from possible collectors of all truths without *complete* confidence that this is necessary.

8 All truths and possibilities for omniscience

8.1 Omniscience as seamless complete actual knowledge

8.1.1. The simplest response to the challenges of the arguments of Sections 2 and 4, and the threat of related arguments against otherwise contained omniscience, is to say little more than that the many propositions that would be known to an omniscient being – that all truths, in other words – may not be members of a collecting One of any sort. There may be, this response goes, 'innumerably many' true propositions, if numbers of things are always sizes of set-like objects that comprise the things numbered. And an omniscient being would know each of them. According to the present simple proposal, an

omniscient would have every truth always equally in mind. It would have every truth about the world fully and actually in mind as if it were at once present in all places at all times, as if it were 'omnipresent throughout space-time' and perfectly cognizant of all of its immediate surroundings as well as of their multiple temporal and spatial relations, and so had in mind all truths about things in space-time. And it would have many other truths, including many truths about itself, eternally fully and actually in mind. Ordinary minds are bounded in knowledge. The propositions known to any ordinary mind make a set finite or infinite, and only ordinary knowers who know very few things can keep before their minds all the things they know. It makes, Leibniz might say, an excellent privilege of what would be the extraordinary minds of omniscients, that they would know infinitely, perhaps 'innumerably', many truths. On the present proposal they would, amazingly, not merely know all truths, but know them 'all at once' and equally.

This could have been Cantor's way with God's mind. For in 1899 he gave "the totality of everything thinkable" as an example of a multitude he called *"absolutely infinite"* (Cantor 1932, p. 443, as quoted in Rucker 1984, p. 49). And in 1887 he wrote that: "The actual infinite arises in three contexts: *first* when it is realized in the most complete form, in a fully independent otherworldly, being *in Deo*, where I call it the Absolute Infinite" (Cantor 1832, p. 378, as quoted in Rucker 1984, p. 9). It is a fair guess that Cantor would say that absolute infinity is 'realized in God' in two ways, one being that He Himself is in ever so many ways absolutely infinite, and the other being that 'He has in mind' every absolutely infinite multitude, *including* the absolutely infinite multitude of everything thinkable that is true. Of Aquinas's view that God can have in mind only *definite*, by which he meant finite, multitudes (ST I q7,a4, pp. 61–2, quoted in Section 4.4.1 of Chapter V), Cantor, I am suggesting, would say, "No, that is indeed *two* removes from the truth, for He has in mind not only, in addition, all *transfinite* multitudes, but also all *absolutely infinite* multitudes."

8.1.2. "'Innumerably many'," it must be conceded, does, "sound on the face of it . . . a contradiction in terms" (Grim in correspondence, August 8, 1986). But this oxymoron can make the valid point that there are Manys of no 'cardinal number', that there are multitudes *beyond number*. "For what is in a number? Would not a multitude by no number be as multitudinous?" to quote 'the bard' of Section 4.4.1 of Chapter V. *Possibly*, however, numbers can be taken beyond sizes of collecting Ones. A way to numbers for multitudes that are not sets can, according to George Boolos, be found in Frege. Boolos writes:

> Two thoughts about the concept of number are incompatible: that any zero or more things have a (cardinal) number, and that any zero or more things have a number (if and) only if they are the members of some one set. . . . [T]he sets that are not members of themselves cannot be the member of any one set. . . . Let *Frege arithmetic* be the result of adjoining to full axiomatic second-order logic a suitable formalization of

the statement that the Es and the Gs have the same number if and only if the Es and the Gs are equinumerous. Then (1) Frege himself succeeded in deriving arithmetic from Frege arithmetic and (2) Frege arithmetic is equi-consistent with full second-order arithmetic (and is thus consistent, with moral certainty). Frege, we now see, thus provided a consistent theory of natural numbers altogether different from those of Dedekind, Russell and Whitehead, Zermelo. (Boolos 1995, p. 317)

According to Boolos, Frege had a system in which *every multitude of things has a number* (the first thought above) and in which *not only multitudes that are sets have numbers* (contrary to the second thought).

8.1.3 Following Grim, a rule for More-Than relations defined in terms of mappings has been proposed that extends to Many's that do not make collecting Ones: Section 5.3. But that rule *may* leave open that, though there are neither more X's than Y's nor more Y's than X's, there is not a mapping of X's onto Y's, so that one should *not* say that there are *equally* many X's as Y's: The law of trichotomy – $(X \succ Y) \vee (X \approx Y) \vee (Y \succ Y)$ – is at least not an *easy* consequence. It seems for this and other reasons that possibly, while we can speak of there being more and less of one 'uncollected Many' than another, and while Cantor's absolutely infinite multiplicities may 'still be increasable', we should not speak of *numbers* of things that are not collected in Ones.

Rucker reports that "Cantor . . . in the following passage [of 1887], distinguishes between the Absolute Infinite, the physical infinities, and the mathematical infinities: "The actual infinite arises in three contexts: *first* when it is realized . . . *in Deo*, where I call it the Absolute Infinite . . . ; *second* when it occurs in the contingent, created world; *third* when the mind grasps it *in abstracto* as a mathematical magnitude, number, or order type. I wish to make a sharp contrast between the Absolute and what I call Transfinite, that is, the actual infinities of the last two sorts, which are clearly **limited, subject to further increase, and thus related to the finite**" (Rucker 1984, p. 9, *Gesammelte Abhandlungen*, p. 378; emphasis added). In 1899 (see Section 6.2) Cantor wrote in the plural, without mention of God, of multiplicities that cannot be conceived as unities and said that he called such multiplicities absolutely infinite. I assume that he would still have added, taking a thought from the just-quoted 1887 text, that these multiplicities are unlike both finite and transfinite multiplicities in that they are *not* 'limited or subject to further increase'. I suggest that he may have been mistaken in this, and that even if (contrary to Boolos) 'numbers' should be said *not* to go beyond Cantor's transfinite 'Many-that-*are*-Ones' sizing numbers, there are limited, increasable multiplicities that go beyond Cantor's transfinite infinities.

8.2 Stratified, partly collected omniscience. Against a simple and possibly uncollected conception of omniscience stands the thought that things fully and actually present to a mind of any kind, since they would be in this mind and, as it were, circumscribed and collected in reality, should be collected

conceptually in some One of which they can be thought of as members. Rather than simply rejecting premises (1) and (1′) of our Cantorian arguments, there is a compromise-way that responds to this thought and lets these premises hold at least in a limited sense and at a level. The present idea is that the knowledge that an omniscient had actually in or before its mind would always make a set, though at no time would everything that an omniscient knew be actually in or before its mind. This new proposal is that an omniscient's knowledge would be in a certain way divided and stratified. It would be divided between (i) basic knowledge, some of which is at times fully actual knowledge before its mind and the rest of which, while actual knowledge in its mind, need not all be always fully actual; and (ii) the possibly uncircumscribed rest, including items of which this omniscient would have at least virtual knowledge, as well as items of which it had at times actual knowledge.

8.3 Actual and virtual knowledge. To illustrate distinctions that I have uncertainly in mind, take my knowledge of Kolmogorov's axioms for probability. I have actual knowledge of them. I have *fully* actual knowledge of the one I am now thinking about (which happens to be the necessity axiom, $\Box p \supset P(p) = 1$). But I do not in the same way have knowledge of every principle of probability that is entailed by these axioms. I have actual, but not fully actual, knowledge of those of which I have had fully actual knowledge that I can recall but do not presently have before my mind. I have only virtual knowledge of others of which I have never had actual knowledge but could, given time, derive though in some cases only after framing ideas of some of their terms. And, I am sure, that of many principles of probability, I have, and will have, *no* knowledge, actual or virtual.

My remarks relate so far only to what might be termed virtual knowledge 'by deduction'. I also have virtual knowledge of things, for knowledge of which reflection on my states, including my states of fully actual knowledge, would eventually suffice. In sum, I have at least virtual knowledge (i) of things I can deduce from things of which I have actual knowledge, (ii) of things for the fully actual knowledge of which reflection on my states would suffice, (iii) of things I could deduce from the items collected under (i) and (ii), (iv) of things for the fully actual knowledge of which reflection on my states would suffice *given* fully actual knowledge of the things collected under (iii), and so on! My present *virtual* knowledge includes my actual knowledge along with all actual knowledge I could generate from it *a priori* by deduction and reflection. (The example of actual knowledge chosen is not meant to suggest that 'actual knowledge' is only of necessities, or that, when of contingencies, these concern only states of a knowers mind. My actual knowledge includes knowledge of my name and the screen before me. I am not bragging.)

It can seem plausible that at least the actual knowledge of any knower, even an omniscient knower, should be collected in one set, since it is collected in one mind. If that is conceded, then one cannot say that an omniscient would have

actual knowledge of everything. But one can say that an omniscient would have either actual or virtual knowledge of everything. An omniscient, according to the present suggestion, would be with respect to all truths somewhat as Laplace's 'great intelligence' would be with respect to all physical truths. That intelligence, we are told, "for one instant . . . [comprehending] all the forces by which nature is animated and the respective situation of the beings who compose it . . . [and being] sufficiently vast to submit these data to analysis . . . would embrace in [a] . . . formula the movements of the greatest bodies . . . and . . . of the lightest atom; for it . . . the future, as the past, would be present to its eyes" (Pierre Simon, Marquis de Laplace, 1917, p. 4). On the present proposal, an omniscient being knowing certain things in an actual manner would know everything either actually or virtually, and have figuratively but not literally every truth 'present always to its eyes'.

An omniscient being's knowledge according to the present stratified conception would consist in part of omnipresent actual knowledge that composed a set. This knowledge for this omniscient being – given its capacities for total recall and for rendering actual other knowledge by *a priori* operations of deduction and reflection – would be an adequate basis for knowledge of absolutely all truths, knowledge of which would therefore be for it, though always only partly actual, virtually complete.

9 Divine knowledge, a guarded recommendation

9.1. I commend to theologians this conception of stratified omniscience. A Divine Being who was omniscient in this manner would know absolutely everything without being burdened always by fully actual thoughts of absolutely everything, including much that could be for it useless trivia. Its actual knowledge would always be circumscribed in a *set* and, according to the arguments of Sections 2 and 4, thus always incomplete. But its total knowledge – that is, its items of actual knowledge plus those items that it could generate *a priori* as actual knowledge by that superreflection of which only it was capable, and by deductions in that unsystematizable 'theologic' (cf., Grim 1991, p. 76, with reference to Hellman 1981) in which only it was proficient – would be complete and extend to absolutely every truth. It would be a special privilege of the *a priori* means of this being that on actual knowledge of fixed cardinality would be based at least virtual knowledge of every truth and thus, perhaps, of not merely infinitely but innumerably many truths. A Divine Being according to this conception would, both in its actual and virtual knowledge, be vastly removed from intellects such as ours. For a more satisfying conception of Divine Knowledge one can add a condition of availability. One can say that in a Divine Mind thoughts of absolutely all true propositions, though they are never all fully actual, are all always instantaneously forthcoming as fully actual whenever sought. One can say that every bit of a Divine Mind's knowledge would be fully actual 'just like that' when and if needed for *planning* and

action, for *caring and understanding* even when no acts are called for, or *for any other of divine purpose*

9.2 How much of Divine knowledge would be always actual – what knowledge would be in a Divine Mind's omnipresent actual basis? The expected answer from a mathematically minded theologian is, *"As little as is necessary* in order that, given its extraordinary powers of reflection and deduction, it should have actual or virtual knowledge of everything." According to the arguments of Section 2, no sense could be given to the answer, *"As much as possible,"* for it is a corollary of those arguments that, no matter how much fully actual knowledge a being has, there are always truths of which it could have additional knowledge.

9.2.1 THAT CLASSICAL ATOMISM IS NOT THE ANSWER. Can we be more definite in the service of anti-Aristotelian minimality concerning the knowledge that would be always actual in God? Can we say that the always actual knowledge of a divine mind, the omnipresent basis for its complete actual and virtual knowledge, would consist in *all atomic truths together with the negations of all atomic falsehood?* We could if a certain form of 'classical atomism' (Grim 1991, p. 95) were a viable theory of the logical space of possibilities, but it is not, if the arguments of Sections 2 and 4 are sound. According to the theory, a proposition p is *atomic* if and only if it is a member of the set A of propositions such that (i) the propositions in A are fully independent (i.e., for each proposition q in A, every set is consistent that is composed of q and, for every other proposition r in A, either r or the negation of r), (ii) every proposition is a truth function of propositions in a subset of A, and (iii) for each truth function of propositions in a subset of A there is at most one proposition that is that truth function of those propositions. This 'classical atomism' is not sustainable, if, but for their premises-one, the arguments of Sections 2 and 4 are sound. For this definition of 'atomic' implies that if the set of atomic propositions exists, then the set of all propositions exists. If set A is of cardinality a, the theory implies that there are no more than a^2 many propositions (see Section A4 of the appendix). But "[w]e know [by the arguments of Sections 2 and 4, if these are sound but for their premises-one, that] there can be no set of all propositions," from which it follows "by *modus tollens* that there cannot be a set of classical atomic propositions" (Grim 1991, p. 95).

9.3 'Thin' logical atomism. Writing of Frank Ramsey's foundations for values of worlds and probabilities of propositions, Richard Jeffrey says that "Ramsey postulated the existence of *atomic* propositions" (Jeffrey 1990, p. 55) and, without indicating what he has against them, says that he does "not suppose there to be any such things as atomic propositions" (Jeffrey 1990, p. 57). It is explained in Sobel (1998) that Ramsey did not need a *set* of atomic propositions, nor did he need that every proposition should be a truth function of his atomic propositions. It was sufficient for his purposes that each atomic

proposition should be independent of "any or all of the others" (Jeffrey 1990, p. 55) and that no worlds should be alike in their atomic propositions. Ramsey required only a 'thin logical atomism' against *which* I know of no persuasive philosophical arguments. Nor, I confess, have I any idea how his logical atomism might be established. This thin atomism, if sound, could serve a stratified theory of divine knowledge, which could (going beyond Ramsey's requirements) say that there is a set of these atomic propositions and that this would be the actual basis of Divine knowledge.

Given that recourse to the classical *Tractatus*-idea of a set of atomic propositions is not possible, and that a set of the atomic propositions of this thin logical atomism is at best uncertain, one may wonder whether, of the character of the fully actual set-basis of divine knowledge, we could say anything more than that it would be as small as possible while being adequate in a Divine Mind to virtual knowledge of everything. Left open is not only that this may be all that we can say, but also that for all we can know there would not be just one possible minimal basis for a Divine Mind's virtual knowledge, and that reasons, if any, for which is actually its minimal basis are beyond human comprehension.

9.4. I offer stratified omniscience *for consideration*, realizing that the simpler option of uncircumscribed, seamless fully actual omniscience should be preferred by some theologians. For, to the Aristotelian-minded, who would have God be in every way fully actual, stratified largely only virtual divine knowledge would be a major comedown. And what would be the retrieval-character of virtual knowledge, even if instantaneous, would be at best problematic for an out-of-this-world-of-change *eternal* intellect.

PART V. ON THE VERY IDEA OF ALL TRUTHS

10 Taking stock, to move on. The arguments of Sections 2 and 4 establish that there is not a set or a 'totality' that collects precisely the propositions that an omniscient would know, and prospects for there being a One of another sort that collects them are dim. But it seems that they can be a Many for which there is not a collecting One. There are no evident barriers to that, and no possible barriers, if there is the Many of them to think and speak about, as I seem to be doing. But am I thinking and speaking about all truths? Grim might say that I am trying, but that the thing cannot be done, for there are no *propositions* about all truths. A difficult point to make, he would concede, 'in so many words'!! It *seems* that we can think about absolutely all truths. There seem to be propositions, both true and false ones, about all truths, for example, the apparent propositions that every truth is identical with itself, and that every truth is known to some human being or other, and that every truth is expressible in principle in a

language human beings could manage. But, Grim claims on Cantorian diagonal grounds adjacent to those that undo *sets of all truths and all falsehoods*, that there are no *propositions* about all truths and thus nothing to think or say, true or false, about them. Gone is the *very idea* of omniscience as knowledge of any kind of all truths. "For in *saying* it, 'there you go again'," Grim might say. For an argument against *true propositions* being a Many of which we can speak or think, I adapt an argument of Grim's to show that there is no proposition to speak or think that is about all *propositions* (Grim 1991, pp. 119–20).[6]

11 Grim's radical argument against omniscience. Presupposed is that 'whereof there is no proposition, thereof we cannot speak'. *What is to be proved* is that:

(0) There is no proposition that is about all true propositions.

Given my uncertain sense for 'the aboutness of propositions' – more of which will come later – a proposition that was about all true propositions would be about no other propositions, though it could be about other things. Given the object of this argument, its conclusion can hardly be what the argument says it cannot be, a proposition about all true propositions. That, however, is precisely what (0) seems to be! Grim would explain that its *intent* is better expressed metalinguistically along the following lines:

(0) Nothing can fit the description 'a proposition that is about all true propositions'.
(0′) does not seem to be what it says nothing is, namely, a proposition that fits the description 'a proposition that is about all true propositions.' (0′) is about that description and everything else over which unrestricted quantification ranges. Grim writes, "I haven't taken that purer course" (Grim 1991, p. 123). He relies on the reader's ability, if worried about self-reflective problems, to translate as required, as for
(2) An omniscient knower would know every true proposition.

below, which Grim might 'purify' somewhat as follows:

(2′) A knower could be termed 'omniscient' only if a condition expressed by 'knows every true proposition' were true of this knower.

Following Grim I stay away from *explicit* metalinguistic formulations.[7] To proceed with the argument *suppose*, for an indirect proof of (0), that there is a proposition that *is* about all true propositions. *Let P be a proposition that is about all true propositions*. A principle I will not challenge for the existence of *true* propositions about propositions is that, for any *proposition* p about certain propositions, there is a *true* proposition about these propositions: Consider a

proposition p that is about one or more propositions; let the propositions it is about be p-propositions; then it seems that there should be a true proposition that says of precisely p-propositions that they are identical with themselves (and another one that says that they are identical with themselves and different from the false proposition that the screen I see before me is blank). With that principle in hand, given that P is about all true propositions, there is a true proposition about all true propositions. *Let T be a true proposition that is about all true propositions*.

(i) T *IS* about all true propositions.

Let a proposition p be a *T-proposition* if and only if p is a proposition that is amongst the things that T is about. The idea, given my sense for 'about', is *not* that a T-proposition is a proposition that T is *about*. For T may be about all true propositions without being about any propositions in particular. At least that is so, given my uncertain understanding of the difficult notion of what propositions are 'about', according to which the proposition that all universities are elitist, though about all universities, is neither about Oxford nor about Canadian universities.[8] Given that understanding, a proposition is a T-proposition if, and only if (see the comment under (0) above), it is true.

(ii) There *is* a mapping of T-propositions *into* all true propositions. (For each T-proposition is a true proposition, so the identity-mapping of T-propositions onto T-propositions is a mapping of T-propositions into true propositions.)

(iii) But there is *not* a mapping of T-propositions *onto* all true propositions. **(Suppose for a *reductio* that M* is such a mapping. Consider T-propositions such that the true propositions to which M* maps them are not amongst the things they are about: For each such T-proposition t, M*(t) is not amongst the things t is about. THERE IS A PROPOSITION ABOUT *PRECISELY THESE* T-PROPOSITIONS – T is about things including these; we 'reach' a proposition about precisely these propositions *by separating* them from everything else T is about and readdressing it to precisely them; so, as under (0), there is a true proposition about precisely these propositions. *Let T* be such a proposition*: T* is a true proposition, it is itself a T-proposition, such that a T-proposition t is amongst the things T* is about if and only if M*(t) is not amongst the things t is about. Then (a) there *IS* a T-proposition t* such that M*(t*) = T*. (M* maps T-propositions onto all true propositions, and T* is a true proposition.) And (b) for a contradiction, there is *NOT* a T-proposition t* such that M*(t*) = T*. [For suppose there is a T-proposition t* such that M*(t*) = T*. If t* is amongst the things that T* is about, then, by definition of T*, t* is not amongst the things M*(t*), and so, by Leibniz's Law, t* is **not** amongst the things T* is about: Therefore, t* is not amongst the

things T* is about. And if t* is **not** amongst the things T* is about, then, by definition of T*, t* is amongst the things M*(t*) is about, and so, by Leibniz's Law, t* is amongst the things that T* about: Therefore – for a contradiction under our negation of (b) – t* **is** amongst the things that T* is about.]) It follows from (ii) and (iii) by the rule of Section 5.3 that,

(iv) There are more true propositions than there are T-propositions.

(v) So T *IS NOT* about all true propositions. (From (iv), by the general principle that a proposition cannot be about all k's if there are more k's that there are k's it is about.) The contradiction of (i) and (v), drawn as it is from the negation of (0), completes an indirect proof for,

(0) There is no proposition that is about all true propositions.[9]

"Give up omniscience," the argument says, and then to Anselmians it offers the comfort that they can do that without giving up any part of the idea of God, who can still be a knowledgeable being than which no more knowledgeable being can be conceived. The lesson is only that not even such a being would know everything, since a being who would know everything cannot even be conceived.

12 Conclusions

12.1. I am not persuaded by that argument.It depends on an unstated existence-principle for propositions. It depends on a principle that would – for any true proposition T about all true propositions and any mapping M* of T-propositions (i.e., propositions among the things T is about which are all and only true propositions) onto all true propositions – vouchsafe the existence of a 'diagonal proposition' T* such that a proposition t is amongst the propositions T* is about if and only if t is a T-proposition *and* t is not amongst the things M*(t) is about: See the boldfaced lines in the argument under (iii). But any existence-principle for propositions that did that, would, given the 'technology' of Sections 5.2 and 5.3, in vouchsafing the existence of *such* a proposition imply that there are no propositions true *or* false (for there could not be only true ones or only false ones) that are about all true propositions: It would be not only an *existence*-principle, but a *nonexistence*-principle, an *implausible* nonexistence principle.[10]

The argument of Section 11 would impugn candidates for propositionhood such as that every true proposition is true, that no true proposition is false, and that an omniscient being would know every true proposition. And just in this, the *implausibility* of its sweeping conclusion, which is essential to the work it is to do, lies its inadequacy for this work! Grim's last-ditch argument, his 'core argument' against omniscience, rests on a principle, unstated, that would affirm the existence of far more dubious propositions than the candidates for propositionhood whose existence as propositions it would, with the help of that 'technology', impugn. Pending articulation of a theory of existence of propositions and their aboutness powerful enough to correct intuitions, it

is reasonable, when forced to choose, to favor these *prima facie* stronger candidates for propositionhood over *prima facie* weaker candidates such as those troublesome 'diagonal propositions.' Grim says that "the core difficulty" to which suppositions of propositions about all propositions lead is "an 'aboutness' form of Russell's paradox" (Grim 1991, p. 122). I agree. But whereas he thinks that paradox impugns propositions about all propositions, I point to only implicit propositional-existence principles, and their *prima facie* abortive issue to blame.

12.2. I *accept* proposition-existence premises-four – premises (4), (4″), and (4′) – of the arguments of Sections 2 and 4, and to do this reject premises-one – premises (1) and (1′) – which say that *every* knower's knowledge would make a set or totality. The issue here, notice, is not which of several would-be propositions to accept as real. The choice here is between saying, with premises-four, that certain apparent propositions are real and *exist*, or saying, against premises-one, that certain propositions whose *existence* is not at issue are *false*. Furthermore, it is relatively easy to abandon as false these propositions. They lose much of their charm when one stresses (i) that they are about what would be *every* knower's knowledge, and so about even what would be the absolutely complete knowledge of an omniscient knower; and (ii) that they are about sets and totalities understood in technical senses determined in part by The Power Set Axiom and Subtotalities of Totalities, respectively. In contrast, pending articulation of a theory that persuades to the contrary, I *reject* the 'package' of unstated principles – there is, in the argument of Section 11, an allusion to a **Separation Principle for *Propositions***, of which I am particularly suspicious – that would secure involuted and troublesome diagonal propositions *at the cost* of the prosaic putative proposition of earlier arguments that,

(2) An omniscient knower would know every true proposition.

and such *prima facie* innocents as that every true proposition is identical with itself (which is true), and that it is not possible to speak of all true propositions (which Grim would like to think is true).[11]

12.3. "You are," Grim might say, "in denial. Your strategy is to 'deny the diagonal'." He thinks that that strategy, even if successful as far as it goes, does not deliver a lasting solution. He maintains that, even if these crucial-to-the-argument *propositions* can be denied, one is left with more *propositional forms* (empty or not) and thus still with more true propositions than one can handle, since about each form there will be a true proposition that either says that this form is empty or that it is not empty:

A strategy of 'denying the diagonal' would seem no more promising here than before. To deny that there is any proposition that fits the form [that 'involuted and

troublesome' diagonal-form – J. H. S.] would be to claim that it is an empty propositional form of some type. But there will then [by a diagonal argument that focuses on this form] be more propositional forms, empty or otherwise, than there are propositions that [the proposition that is supposed to be about all true propositions would be] . . . , about, and thus ultimately more propositions than P is about. (Grim 1991, p. 153n50 to p. 120)

The rejoinder-argument at which Grim hints would, however, depend on *an existence-principle for propositional forms* that said (i) that each true proposition is of a propositional form different from every propositional form of every other true proposition, and (ii) that for each proposition P about all true propositions there is a propositional form for what would be for P a certain involuted and troublesome diagonal-proposition. Reasons for rejecting these involuted amid troublesome *propositions* would be reasons for rejecting either the existence of *distinctive* propositional forms for all true propositions, or propositional forms for what would be those involuted and troublesome diagonal-propositions (or both). The domain of propositional forms is as negotiable and as yielding to pressures of would-be *reductios* as is the domain of propositions. Grim has doubts concerning propositions and their aboutness that he suspects further work on propositions would deepen and extend (Grim 1991, pp. 18–25, 57–60). He recalls these doubts when he begins to argue against the possibility of a proposition about all propositions (note 49 on p. 153 to p. 119) but does not allow these general doubts to spread to the bold existence-claims concerning propositions on which his argument against the existence of certain innocent-seeming candidates for propositionhood depend, *and to spread* to existence-claims concerning propositional forms on which the 'supplementary argument' he hints in note 50 would depend.[12]

12.4. The argument of Section 11 is an invitation to write off omniscience as a bad pre-Cantorian idea, to write it off on the strength of an unexplained theory of propositions, their existence, and aboutness that would persuade that while, given the assumptions under which it is introduced, T* of the argument *would be* a proposition, premise (2) *is not* a proposition (that is, the sentence for would-be premise (2) does not express a proposition). A more reasonable response, I think, given the present state of theory of propositions, is to decline the invitation, confess uncertainty concerning the existence and aboutness of propositions, and accept (2) as a proposition premise 'on faith.' Regarding what would be Divine Knowledge, it is reasonable to say, as has traditionally been said, that it would in one way or another include knowledge of every true proposition. We should allow that it may amount to even more than that, to some of which more we may be able at least to advert, and regarding the rest, if any, rather than emulating Kant, we should say no more.

APPENDIX. NOTES ON CANTORIAN SET THEORY

A1 Power sets. Briefly, S′ is a *subset* of S if and only if every member (if any) of S' is a member of S. Every set is a subset of itself, and the empty set ϕ (is a subset of every set. The *power set Pow*(S) of a set S is the set of all of the subsets of S. For example, the power set *Pow*({2,7,9}) of the three-membered set of numbers {2,7,9} is the eight-membered set of sets of numbers, {{2,7,9}, {2,7}, {2,9}, {7,9}, {2}, {7}, {9},Φ}. For another example, the power set *Pow*({0,1,2 . . .) of the set of all natural numbers {0,1,2 . . . includes this set itself of all natural numbers, the set of all even numbers, the set of all prime numbers, the empty set, the eight sets in *Pow*({2,7,9}), and many other sets.

According to the Power Set Axiom, for every set S, regardless of its size, there exists a set *Pow*(S) that includes all and only the subsets of S. The Power Set Axiom is partly definitive of what it is to be a set in standard systems such as that of Zermelo. There are concepts of conglomerates of which analogs of this axiom are not true, but analogs of this axiom seem to hold of all 'collections', 'sets', 'classes', 'groups', 'conglomerations', and so on, as these terms of circumscription of discrete items are *ordinarily* conceived. Which is to say that analogs of this principle are at least 'intuitive' for what would be collections, multiplicities, and so on of things known by omniscients.

A2 Cardinalities. Two sets, S and S′, have the *same cardinality* or size if and only if there is a 1–1 correspondence between them; that is, if and only if there is a set C of ordered pairs (x,x′), where x is a member of S and x′ is a member of S, such that each member of S is the first element of exactly one member of C, and each member of S′ is the second element of exactly one member of C. Informally, the requirement for same cardinality is that the members of S and S′ can be paired off without remainder. For example, {1,3,5} and {2,7,9} obviously have the same finite cardinality. For another example, the set of natural numbers has the same infinite cardinality as its proper subset, the set of even numbers, given the 1–1 correspondence:

0	1	2	3	·	·	·	n	·	·	·
|	|	|	|	|	|	|	|	|	|	|
0	2	4	6	·	·	·	2-n	·	·	·

It is a 'privilege' of sets of infinite cardinality that only they have proper subsets of the same cardinality as their own. A set S has a *greater cardinality* than a set S′ if and only if there is a 1–1 correspondence between S′ and a subset of S, but there is no 1–1 correspondence between S′ and S. This means that no set has a 'greater cardinality' than itself. It is, for an easy positive illustration of 'greater cardinality', obvious that {2,7,9} has a 'greater cardinality' than {2,7}. That is, perhaps, expected since {2,7} is a proper subset of {2,7,9}. It is also obvious that the set of natural numbers does not have a 'greater cardinality' than the

set of even natural numbers, though the set of evens is a proper subset of the set of naturals.

A3 Cantor's Theorem

A.3.1 An abstract 'diagonal argument'. For any set S, the power set *Pow*(S) of S has a greater cardinality than S. Every set is in this sense smaller than its power set. This is obvious for sets of finite size, including even the empty set $\phi = \{\}$, whose power set is $Pow(\phi) = \{\phi\}$: The cardinality of $\{\}$ is 0, whereas that of $\{\phi\}$ is 1. Here is a proof, adapted from Hunter (1971, pp. 24–5) of Cantor's Theorem for all sets, whether finite or infinite.[13] Consider any set S. Let the power set of S be *Pow*(S).

(i) There *is* a 1–1 correspondence between S and a proper subset of *Pow*(S). Proof: There is, for example, the 1–1 correspondence in which s of S is paired with {s} of the set *Sing*(S) of singletons of S. *Sing*(S) can be separated out of *Pow*(S). And this 1–1 correspondence – this set of ordered pairs(s,{s}) = {{s},{s,{s}}} – can be separated out of *Pow*(*Pow* [S U *Sing*(S)]), the power set of the power set of the union of S and *Sing*(S). [This proof by example of (i) depends on The Power Set Axiom, Separation, Pairing, and Union. Separation is also termed Specification and is the principle, "roughly speaking, that anything intelligent one can assert about the elements of a set specifies a subset, namely, the subset of those elements about which the assertion is true" (Halmos 1960, p. 4). Pairing says that "for any two sets there exists a set that they both belong to [and to which nothing else belongs]" (p. 9). Union says that "for every collection of sets there exists a set that contains all the elements that belong" to any of these sets (p. 12).]

(ii) There is *not* a 1–1 correspondence between S and *Pow*(S). Proof: *Suppose*, for purposes of an indirect proof (ii), that there is a 1–1 correspondence C between S and *Pow*(S). Each member s of S is paired in C with a set in *Pow*(S). Consider now the 'diagonal set' S^* of members of S includes s if and only if s is not a member of the set C(s) with which s is paired in C: $S^* = \{s{:}s \in S \ \& \ s \notin C(s)\}$. On the present supposition that C is a 1–1 correspondence between S and *Pow*(S), set S^* is well-defined and exists. (I rely here on Separation in the guise of the intuitive principle that, for any 'proper' condition K that separates off certain members of a set S, there is a set S′ of precisely these members of S: K is here 'proper' if it makes no anticipatory reference to S′.) This set S^* is, since it is a subset of S, a member of *Pow*(S). So S^* *is* paired in C with a member of S. But, to complete the indirect proof of (ii), S^* *is not* paired in C with a member of S. (For this last 'line' I offer a subsidiary indirect proof. Suppose that S^* is paired in C with an element s of S. Then element s *is* a member of S^*: For if s is not a member of S^*, then, by the definition of S^*, s is a member of S^*; so s is a member of S^*.[14] But element s is *not* a member of S^*: For if s

is a member of S^*, then, by definition of S^*, s is not a member of S^*; so s is not a member of S^*. Therefore S^* is not paired in C with an element s of S.)

Cantor's Theorem, that *Pow*(S) has a greater cardinality than S, follows from (i) and (ii).

A3.2 A concrete 'diagonal argument' for countable sets. The argument for (ii) is a *diagonal argument*. To explain this label: Suppose that S is countable so that its members can without remainder be numbered 1, 2, 3, and so on. Suppose for a *reductio* that there is a 1–1 correspondence between S and its power set *Pow*(S). Then *Pow*(S) is countable. Let the following be a partial representation of C, a 1–1 correspondence between S and *Pow*(S).

Pow(S)		S																					
		s_1		s_2		s_3		s_4		·		·		·		·		·		·		·	
S_1	−	{+	,	−	,	−	,	+	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·
S_2	−	{−	,	+	,	+	,	+	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·
S_3	−	{−	,	−	,	−	,	−	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·
S_4	−	{+	,	+	,	+	,	−	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·
·	−	{·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·
·	−	{·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·	,	·

A plus sign, in a column or a row indicates that the member of S that heads that column *is* a member of the subset of S on that row, while a minus sign indicates that it is *not*. For example, s_1 is paired in C with a subset of S that contains at least s_1 and s_4 and excludes at least s_2 and s_3.

We now consider $S^* = \{s: s \in S \ \& \ s \notin C(s)\}$. S^* is the set

$$\{\cdot, \cdot, s_3, s_4, \cdot, \cdot\}.$$

S* would be the *diagonal set* relative to C. By design, though S^* would be a subset of S and thus a member of *Pow* (S), it would *not* be paired in C with s_1, s_2, s_3, s_4, or any other member of S. For every s in S, S* *would differ* at least *on the diagonal* from the set that would be paired in C with s. So C is *not* after all a 1–1 correspondence of S with *Pow*(S). This contradicts the supposition made and completes the *reductio* for the negative, that there does not exist a 1–1 correspondence of S with its power set *Pow*(S). Cantor's Diagonal Argument gets its name from the possibility of this display for the demonstration for *denumerable* sets. "By using a general and abstract form of the diagonal argument, Cantor showed that *the power set of a set always has a greater cardinal number than the set itself* " (Hunter 1973, p. 24).

Left to construct is a diagonal argument that shows that there is not a 1–1 correspondence between the set of natural numbers, {1, 2, 3, . . . } and

the set of 'unending decimals' between '0' and '1'. For an argument showing that there are more real numbers in the interval between 0 and 1, one needs to deal with the fact that ordinary mathematical practice has some real numbers in that interval being represented by two 'unending decimals': For example, 1/2 is ordinarily equated with both .4999... and .5000.... [Wilfrid Hodges "write[s] down a proof, not in Cantor's words," in which, for this last business, he "choose[s] the expansion which is eventually 0, not that which is eventually 9" (Hodges 1998, p. 2).]

A4 Cardinalities of power sets. The power set of the set of size 3, {2,7,9}, is

$$\{\{2, 7, 9\}, \{2, 7\}, \{2, 9\}, \{7, 9\}, \{2\}, \{7\}, \{9\}, \phi\}$$

and has 2^3 members. It can be seen in a way that generalizes easily to sets of any finite size why the power set of a size-3 set has 2^3 members. (A tree with 'in' and 'out' branches from element a to element b, from which there are 'in' an 'out' branches to element c, from each of which there are 'in' and 'out' branches, has 2·2·2 branches.) If c is the cardinality of a set, then the cardinality of the set of this set's subsets is 2^c, thus the name 'power set of S′ for the set of all subsets of S. This, as I have said, is obvious for finite sets. It is true also for infinite sets, though, as Geoffrey Hunter cautions, "the justification of [this rule for them] is more complicated . . . ; see, e.g., Fraenkel (1961, chap. II, §7)" (Hunter 1971, p. 40n).[15] While for a cardinal number c, though c^2 is greater than c if and only if c is finite, the case is simpler for 2^c. It is a consequence of Cantor's Theorem and the just-stated general rule that not just for finite cardinal numbers, but for every cardinal number c, 2^c is a greater cardinal number.[16]

. . . . .

Historical notes: "For now, we will use the symbol 'c' to stand for [the cardinality of the real numbers]. Cantor first proved that $\aleph_0 < c$ on December 7, 1873. We know this because he communicated it to his friend Dedekind in a letter the next day. Cantor's first proof of the uncountability of the reals was a bit different from the diagonal argument now used. . . . " (Rucker 1984, p. 246; for a reference to Cantor's publication of his result in 1874, see Hodges 1998, p. 16.) "*Cantor's theorem* of 1892 (that the set of all subsets, the *power set*, of a given set must be cardinally greater than that set) goes further, for it shows that ordinary mathematics must accept indefinitely many different kinds of infinity" (Hallett 1995, p. 70). Cantor proved on that day in 1873 "the first part of his most famous theorem, now known as Cantor's Theorem: For every cardinal k, $k < 2^k$. . . . It is quite easy to see that $k \leq 2^k$. . . . The real difficulty . . . is to prove that $k \neq 2^k$" (Rucker 1984, p. 236).

ARGUMENTS AGAINST THE EXISTENCE OF GOD

XI

Atheologies, Demonstrative and Evidential

1. 'THAT THAT'S SAWCE FOR A GOOSE IS SAWCE FOR A GANDER.'[1]

1.1. Regarding arguments against theisms, here are lines, with which, with certain reservations, I concur:

> The philosophy of religion game has generally been played according to the following rules: a theist puts forth an inference from evidence to the conclusion that God exists: then, a skeptic tries to find fault with the inference or undermine the truth of the evidential claims. . . . [M]ost think that the burden of proof is on the theist, whereas there is no such corresponding burden on the atheist. . . . [But] there is no special burden of proof that the theist must bear. . . . [And] if one takes up the task of providing sound arguments for atheism, formidable difficulties arise. . . . It is much easier to punch holes in theistic arguments . . . than to actually argue for [the] truth [of atheism]. . . . [When the latter is undertaken] we find that the needs of and problems of the atheist are parallel to those of the theist. To impose the same argumentative restrictions upon the atheist is to show that the position is at least as inadequately defended as theism. . . . [W]hatever the reasons for its widespread acceptance, atheism is no less a matter of faith than theism. (Shalkowski 1989, pp. 1, 14)

There is no general presumption against the existence of deities, according to which atheism follows by default in the absence of good arguments for theism. Arguments are no less, and no more, needed to rationalize atheism than to rationalize theism. And of course arguments either way, as all arguments, can run into formidable difficulties, though whether a particular argument does so is a matter of judgment that can be based only on a particular examination. So far I agree with Scott Shalkowski, though I think, with atheists, that there is a presumption against the existence of *some* gods,[2] and though I think, with theists, that it is fair that the atheists should 'go first' in debates. I think this because their task is formally easier than the theist's.[3]

It is not remarkable that some atheists think there is a presumption against the existence of some gods, nor, if true, that some theists think there is a

presumption against the existence of gods other than their own. What is remarkable is that at least one experienced theist insists that, for him, there is a presumption against his own theism. It is a presumption that many, if not all, 'atheists' think runs against that theism, the prevalent theism of believers who find God in *The Bible*. Richard Swinburne is a *perfect-being theist*: "I understand by 'God' a being who is essentially eternal*, omnipotent, omniscient, creator and sustainer of the Universe, and perfectly good" (Swinburne 1998, p. 1. *"I prefer . . . the sense everlasting" (p. 253)). He holds that it is specifically perfect-being theism that is challenged by the spectacle of earthly suffering and evil: "[I]t is that understanding of 'God' which gives rise to the 'problem of evil'" (p. 6). And he holds that *most* perfect-being theists, including himself, in order to deal with the problem and be *justified* in their theism, "**need a theodicy**" (p. 17; emphasis added): They need "to be able to show . . . how, if there is a God, our . . . criteria [that are separately necessary and together sufficient conditions for a perfect being's allowing a bad state to occur] are probably satisfied in respect of **al**l the world's morally bad states" (p. 15; emphasis added). This is partly because of the proper (p. 23) strength, for most perfect-being theists, of impressions of the incompatible between suffering and evils and perfect beings, and partly because of the relative weakness for most such theists of proofs for the existence of God, and of impressions, if any, of encounters of a divine kind. "**The onus of proof** has passed to the [not uncommon] theist [in that need]" (p. 22; emphasis added). When some suffering and evil challenges his theism, it is not for the challenger to establish the incompatibility of this with the existence of this theist's perfect being, but for him, the theist, to establish its compatibility.[4]

1.2. Arguments against, as arguments for, the existence of God, and indeed as arguments in general, are of three forms. Some atheistic arguments would be demonstrations or, in Hume's terms, arguments *a priori*. They would be deductive arguments from necessarily true premises. Many atheistic arguments are deductions that do not claim to be demonstrations – many, Aquinas might say, would be 'demonstrations *quia*' against a particular god as 'cause' from what would be this god's 'effects'. Other atheistic arguments are inductive in spirit and consist of considerations that are held to provide sufficient support for, without entailing, their atheistic conclusions. After brief comments on atheistic 'demonstrations' *a priori* in Part One of this chapter, and a more extended discussion of inductive arguments from evil in Part Two, the next chapter aspires to a comprehensive study of the most important family of atheistic demonstrations *quia*. It studies a series of arguments that contend for a logical problem for perfect beings, first of evil, then of this world's not being a best world, next of this world's not being a best world a perfect being could create, and so on through two additional changes.

PART ONE. ON DEMONSTRATIONS AGAINST

2 Ambitious atheistic demonstrations[5]

2.1. An ambitious atheistic demonstration that would show that the idea of *any* god properly so-called is *a priori* inconsistent might begin with the proposition that God would be a being worthy of worship. It is plausible that that proposition is necessarily true and indeed something like 'analytic.' The argument would be that there is a contradiction in the idea of a being worthy of worship, and so also in the idea of God. John Findlay argues: "[I]f God is to satisfy religious claims and needs [if he is to be a proper object of worship], he must be a being in every way inescapable. One whose existence and whose possession of certain excellences we cannot possibly conceive away. And . . . it is self-evidently absurd . . . to speak of such a Being. . . . " (Findlay 1955, p. 55). The argument could be that (i) God would satisfy perfectly every religious demand and be unqualifiedly worshipful; (ii) only a being that was in every way inescapable could in those ways satisfy these demands and be worshipful; and (iii) it is absurd to speak of such a Being, it is absurd to speak of a *necessarily existent worshipful being*, let alone a necessarily existent *essentially* worshipful being.[6]

Deciding whether the third premise is true calls for study of the possible variety of necessary truths and beings, and of conditions for satisfying 'religious demands' and being worshipful. This premise is plausible if necessary 'beings' are all 'forms' (even if there are normative forms such as Goodness and Beauty) and things such as numbers. This claim is plausible, *if* necessary beings could none of them know us or hear us. But these restrictions on necessary beings need to be argued and, pending compelling arguments, can be rejected by theists who accept the first two of Findlay's premises. Philosophical theists who accept the first and third premises can reject the second. They can deny, with considerable plausibility I think, that it is part of the "religious frame of mind" (Ibid.) that a being that satisfies every religious demand "must be . . . in every way inescapable. One whose existence and whose possession of certain excellences we cannot possibly conceive away" (Ibid.). They can be simply perfect-being theists, such as Swinburne is, without necessitarian trimming or the essentialist gloss. And a philosophical theist can accept the second and third premises, while rejecting the first one and confessing to belief in a limited god who is not worshipful always and everywhere without reservation. He can confess to belief in a god with whom he sometimes commiserates more or less as an equal: "I know, I know. What can even You do with such people!!" Such a biblical god, he might find, satisfies *his* religious and spiritual needs *better* than the omnipotent, omniscient, perfectly good god of classical theologians and philosophers.

2.2. There is the question, what would be a *worthy object of worship*? While it is agreed that God would be 'worthy of worship,' there is disagreement

not only about what would be sufficient to make a being so, but over what is necessary for a being that would be so. At one extreme are opinions of necessary-being essentially-perfect theologians. In the upper middle are opinions that it is sufficient for proper worship that a being, though not necessary and not essentially perfect, would be actually perfect. And there is the lower middle ground opinion that God, though in no way perfect, would be in many nice ways unsurpassed and not approached, that though not an ultimate cause of anything, God would be a common cause of us and much else, and therefore truly awesome, and that these conditions of 'finite excellence', together with historical connections, are sufficient for proper worship. Toward the nether are opinions of those who, if reports are to be taken at face value, 'worship' their ancestors and persons who have 'worshipped' numbers (though I confess to doubts concerning the word 'worship' here). At the bottom is the view (Kant's?) that nothing is 'worthy of worship', that *worship* is seriously undignified and an offense to one's humanity, and wrong regardless of its object.

2.3. The normative sceptic challenges the entire spectrum of opinions: He says that though there are not discernible contradictions in the concept of 'an objectively proper or improper object of worship,' it is not instantiated in any possible world. This reaction goes to an ambitious atheistic demonstration of another kind. The normative sceptic argues that 'differences of opinion' concerning conditions for worthiness for worship are better described as 'differences of attitude' since, regarding what is *worthy* of, what *merits*, worship, there are *only* 'opinions,' only 'attitudes': No *facts* are in dispute, though our words in ordinary god-talk, specifically, the evaluatives 'worthy' and 'proper', imply otherwise. The sceptic says that these words further 'errors of objectification'. When we say that there is a god, what we say is not true, and indeed it is necessarily not true. For what we say when we say that there is a god *presupposes* that there is a being that not only as a matter of fact exists, but that as another matter of fact is worthy of worship. The argument is not that, as the humanist says, there *can* be no *beings worthy of worship*, because worship is necessarily beneath us, but that this humanist and theists assume that the concept of a *being worthy of worship* is possibly instantiated, which is necessarily false. Cf.: "There are no objective values" (Mackie 1977, p. 1). What he means is that there *can* be none.[7]

3 Modest atheistic demonstrations. Modest atheistic demonstrations say that *particular* god-concepts harbor *a priori* atheistic demonstrations. I have shown in Chapter IX that, as there cannot be an unmarried husband, there cannot be an *essentially omnipotent* being. That was a modest atheistic demonstration against a god who would be essentially omnipotent. I have by implication in Chapter X resisted modest atheistic demonstrations that on Cantorian grounds (that Cantor would have rejected) argue against the possibility of

an omniscient being. Those would-be atheistic demonstrations are concerned with the single divine attributes; others are addressed to combinations. For example, it can be argued that perfect justice and perfect charity are not combinable in one being, since perfect justice calls in some cases for punishment of those that perfect charity would forgive and comfort. For another, it may be *argued* that a being who was essentially at all times omniprescient and knew everything about future times, including what it was going to do, could never do anything freely. And it may be argued that freedom and immutability are incompatible, since freedom implies different, and immutability the same, actions in possible worlds. Good and bad modest atheistic demonstrations can be interesting and discussions of them instructive. But, if explicitly modest, they are not yet serious challenges to theism. For that they need to be joined with argued claims concerning conditions that are necessary for worthiness of worship. Such claims are often implicit. The difference between modest atheistic arguments in which such claims are implicit and ambitious atheistic arguments is that in the latter such claims are explicit *and argued*.

PART TWO. IS IT CREDIBLE THAT ALL THIS IS PART OF SOME WONDERFUL ETERNAL PLAN?

Inductive arguments against the existence of God would show that it is *improbable in light of evidence*, usually facts of apparently unnecessary evil, thus, the label 'the evidential argument from evil', which covers several different arguments. Hume's was the first one, and it is one of the best. But then he had the advantage of genius.[8]

4 Hume's argument in Part 11 of the Dialogues

4.1. It is an argument from facts of *apparently unnecessary* evil against the existence of even an only "very powerful, wise, and benevolent Deity" (Hume 1991, Part 11, p. 163), of only "a very perfect being" (p. 164), in which Philo details ways in which it *seems* that the world would have been made a better place by such a Deity.

Philo: There seem to be *four* circumstances on which depend all or the greatest part of the ills that molest sensible creatures.[9] . . . None of them appear to human reason in the least degree necessary [for a nearly perfect being]. . . . The *first* circumstance which introduces evil is that contrivance . . . by which pains, as well as pleasures [punishment as well as reward] are employed to excite all creatures to action, and make them vigilant in the great work of self-preservation. (p. 163)

[T]he *second* circumstance [is] . . . the conducting of the world by general laws; and this seems nowise necessary to a very perfect being. . . . A Being . . . who knows the secret springs of the universe,[10] might easily, by particular volitions [for miracles as accurately defined] turn all . . . accidents [and unexpected events] to the good of mankind, and render the whole world happy, without discovering himself in any

operation. . . . [Even a] few such events . . . regularly and wisely conducted, would change the face of the world; and yet would no more seem to disturb the course of nature or confound human confound, than the present economy of things, where the causes are secret, and variable, and compounded.[11] . . . There may . . . be good reasons, why providence interposes not in this manner; but they are unknown to us: And though the mere supposition, that such reasons exist, may be sufficient to *save* the conclusion concerning the divine attributes [even if they would be infinite and perfect], yet surely it can never be sufficient to *establish* that conclusion. (p. 164)

[T]he *third* circumstance [is] . . . the great frugality with which all powers and faculties are distributed to every particular being. . . . Every animal has the requisite endowments, but these endowments are bestowed with so scrupulous an economy that any considerable diminution must entirely destroy the creature. Nature seems . . . like a *rigid master*. . . . An *indulgent parent* would have bestowed a large stock in order to guard against accidents, and secure happiness . . . in the most unfortunate circumstances (p. 165). (Two hearts, a working one and a spare, would it seems have been better than one.)

The *fourth* circumstance whence arises the misery and ill of the universe is the inaccurate workmanship of . . . the great machine of nature. . . . Thus the winds are requisite [for several purposes] . . . : But how often, rising up to tempests and hurricanes, do they become pernicious? . . . There is nothing so advantageous to the universe but what frequently becomes pernicious, by its excess or defect. (p. 167)

On the concurrence, then, of these *four* circumstances does all or the greatest part of natural evil depend. . . . What then shall we pronounce on this occasion? . . . . Let us allow that, if the goodness of the Deity (I mean a goodness like the human)[11] could be established on any tolerable reasons *a priori*, these phenomena, however untoward, would not be sufficient to subvert that principle, but might easily, in some unknown manner, be reconcilable to it. But . . . [we, at least you, Cleanthes, and I agree that] this goodness is not antecedently established but must be inferred from the phenomena, [and let us assert that] there can be no grounds for such an inference while there are so many ills in the universe. . . . **The true conclusion . . . that the original source of all things is entirely indifferent . . . and has no more regard to good above ill than to heat above cold. . . . seems by far the most probable**.[12] (pp. 168–9; emphasis added)

4.2 It is the vanishing likelihood of 'all of this'. That there is a 'very perfect being', a 'finitely perfect' being (p. 161), lording it over the universe, is said to be, by reason of this evidence of *gross* imperfections, most improbable. How much less probable still that there is in that position an *infinitely* perfect being? Even if "pain or misery in man [such as we find] is *compatible* with infinite power and goodness in the Deity" (p. 160), and their conjunction *possible*, *it* is *absolutely* improbable. Why? Mainly, in Hume's view, because of what would have been 'beforehand' the abysmal predictive power for the spectacle of a perfect-being hypothesis.

What an immense profusion of beings, animated and organized, sensible and active! You admire this prodigious variety and fecundity. But inspect a little more narrowly these living existences, the only beings worth regarding. How hostile and

destructive to each other! How insufficient all of them for their own happiness! How contemptible or odious to the spectator! The whole presents nothing but the idea of a blind nature, impregnated by a great vivifying principle, and pouring forth from her lap, without discernment or parental care, her maimed and abortive children[!] (pp. 168–9)

It is debated whether there are considerations unnoticed or unappreciated that would, if known and appreciated, justify believing that a perfectly good omnipotent and omniscient everlasting being would have allowed particular sufferings of innocents. There should be no debating, however, whether a person would have predicted beforehand such suffering, *any* of it, were he certain then of the existence of such a being: There should be no debating the question whether "the world considered in general and as it appears to us in this life, [is such as] a man . . . would, *beforehand* expect from a very powerful, wise, and benevolent Deity" (Part 11, p. 163). "Certainly not," Hume would say, and he would be right without a doubt. The evidence of suffering and evil rather *obviously* tells against traditional theisms and, Hume says, decisively so in the absence of 'tolerable reasons *a priori* for them' (prospects for which Cleanthes, seconded by Philo, takes a dim view in Part 9 of the *Dialogues*). Taking *liberties* with Hume's 'tolerable reasons *a priori*', to moderate somewhat his conclusion, the '*likelihoods* of suffering and evil' on traditional theisms *beforehand* tell against them, and decisively so in the absence of substantial counterbalancing '*priors*' (whether or not due to tolerable, even if not sound, 'reasons *a priori*') again *beforehand*. The 'beforehand-shift' in the present context sets aside all evidence of unnecessary evil, so that none is particularly probable, while perturbing as little as possible a person's credences for other things such as design, miracles, and theophanies and, from the other side of the ledger, disembodied intelligence and "immediate fulfilments of intention" (Mackie 1982, p. 130). Left unperturbed *can* be a theist's conviction that there is a perfect being, though this conviction can be expected to be *enhanced* in a perspective reached by excising all evidence of unnecessary evil. The atheists's convictions can be expected to be moderated.

4.3 A sceptical challenge

This Bayesian way to the improbability, on the evidence of apparently unnecessary evil, of perfect-being, and very-perfect-being theisms, is, for want of likelihoods beforehand on which we would be entitled to rely, 'not on'. As we should then realize, we would not be up to reliable likelihoods for patterns of good and evil on perfect-being, and very-perfect-being, hypotheses. This is because we should realize that we lacked sufficient reasons for believing that our expectations for various patterns of good and evil, conditional on these hypotheses, were not dramatically 'out of sync' with the 'objective chances' for these patterns were there a perfect, or very perfect, being.[13] We would lack these reasons, because, for all we knew, our understanding and appreciation of kinds of goods and evils, and of the costs in evil that great goods

exact, was drastically inaccurate and incomplete in comparison with the understanding and appreciation of even only very perfect beings. It has been said that we are entitled to claim, on the basis of how things strike us consequent to an exercise of our cognitive faculties, that things it appears that p (for example, that it appears that there is unnecessary in evil), only if it is reasonable for us to believe that, if it were not the case that p, we would have been struck in a discernibly different manner that would not incline us to claim that it appeared that p.[14] I agree, and maintain a similar condition for likelihoods of possible evidence on hypotheses, on which likelihoods we are entitled to rely when reaching, by way of a theorem of Bayes, probabilities for these hypotheses conditional on this possible evidence. The upshot is that, if we are reasonable 'beforehand', either we do not then *have* the likelihoods or conditional expectations required for a Bayesian determination of probabilities for these hypotheses conditional on what from that perspective is possible evidence of evil that is for us now 'old evidence', or we do not then rely on them for the determination of conditional probabilities for the hypotheses on that possible evidence that we are prepared to make unconditional probabilities of the hypotheses, should we acquire the evidence.

AND NOW OUR ATHEOLOGIST'S RESPONSE. Let StrongNCECA be the proposed strong ***n***ecessary **c**ondition for ***e***ntitled-***c***laims, based on exercises of our cognitive faculties that have given rise to 'appearances' (for example, of feet, and valid proofs), to ways things (for example, feet, and proofs) 'appear' to be.[15] I do not agree with StrongNCECA. This condition, related to a claim that it appears that p, which would be based on a person's use of his cognitive faculties, is that this person is entitled to that claim on that basis only if *it is reasonable for him to believe* that his cognitive faculties as used *are reliable* and would not have resulted in an 'appearance' that p if it were not the case that p. I reject this condition *if* we are to understand that in it 'it is reasonable for a person to believe that his cognitive faculties as used on an occasion are reliable' only if it is *not* the case that, for all he knows, his cognitive faculties are *not* reliable in the way described.[16] I reject StrongNCECA understood that way, since I believe that, (a) for all I know I am a footless brain in a vat, and that (b) I am entitled to believe in my feet when, as far as I can tell, I am tying my shoelaces. A condition that I accept is ModerateNCECA, which says that a person is entitled to a claim that it appears that p, which would be based on a use of his cognitive faculties only if *it is not reasonable for him to believe* that his cognitive faculties as used *are not reliable* and *would* have resulted in an 'appearance' that p, even if it were not the case that p. This condition does not make a problem for my feet. I do not have reasons for thinking that I am a brain in a vat and thus regularly deceived by what I take to be my sense. I do not have that or any reason to believe that, even in the circumstances envisioned, 'appearances' of seeing and touching feet are not reliable.[17] And ModerateNCECA does not make trouble for what would be my expectations beforehand for various goods and evils, conditional on perfect-being and very-perfect-being hypotheses.[18] For

I would not have reasons sufficient for believing that my understanding and appreciation of goods and evils and of the cost, in terms of evil, of some goods, was so inaccurate and incomplete as to make these conditional expectations unreliable in the sense, described in the previous section, of being 'out of sync' with corresponding objective chances. To know of the *possibility* that they are unreliable is not to have a reason for thinking that they *are* unreliable. I would not, even if I were a without-a-doubt perfect-being theist, have sufficient reasons for believing that, then, beforehand: I would *beforehand* lack *precisely* the evidence – this being the *point* of the 'beforehand-switch' – that I have now for thinking that there is evil that a perfect being would be justified in permitting for reasons that were beyond my ken. *This, not incidentally, is what explains why, though my probability now, conditional on the existence of a perfect being, for there being evil that a perfect being would be justified in permitting for reasons beyond my ken, should now be* GREAT*, my corresponding conditional probability should then, beforehand, be very very* SMALL. That, I think, should hold of *everyone's* then-and-now likelihoods for evil of perfect-being theisms. The beforehand-switch is *important*.

4.4. Returning to my wind-up of Hume's argument, there should be no debating the question, "Is the world, considered in general and as it appears to us in this life, different from what a man or such a limited being would, *beforehand*, expect from a very powerful, wise, and benevolent Deity?" (Part 11, p. 163). That is a question, upon the answer to which I expect nontheists and all theists, *other than* the sceptical few who think that they would have no expectations either way, to agree. The *gravitas* and logical *clarity* of Hume's argument makes somewhat surprising the recent turn in 'atheistic studies' in which evidential arguments against *INFINITELY perfect* beings have been mounted *and hotly disputed WITHOUT REGARD TO'PRIORS.'* There has been, I think, 'a loss of perspective,' the beforehand-perspective. That, taken with a resurgence of sceptical theism[19] precipitated by recent persistent evidential arguing, explains the dispute regarding what Hume considered 'a no-brainer'.

5 The evidential argument from evil of Rowe (1986). There is in Rowe (1986) an argument that consists centrally of considerations designed to show that *it is very probable that* (A) there are evils that an omnipotent, omniscient, wholly good being O could have prevented such that, had O prevented them, the world would have been better. Given that (B) such a being would have prevented any such evils, it follows according to the argument that it is probable that (C) no such being exists. "Since (A) and (B) entail (C), if (A) is probable and (B) is true, then [(C) is probable]" (Rowe 1986, p. 229).[20]

5.1 The form of this argument. Let 'E' abbreviate 'there are evils that an omnipotent, omniscient, wholly good being O could have prevented such that, had O prevented them, the world would have been better' and G abbreviate 'there exists an omnipotent, omniscient, wholly good being'. ('E' abbreviates A above, '∼G' symbolizes C). The argument takes as 'given' that such a being would have prevented any such evils – it takes as 'given' and true that (G ⊃ ∼E). ('(G ⊃ ∼E)' symbolizes B above.) This conditional is, of course, *prima facie* not merely true but necessarily true: It seems *necessary* that *such* a being would have prevented *such* evils. Rowe, however – mindful of challenges to the necessity of this conditional, rather than deal with them – supposes for his argument merely that this conditional is true (1986, p. 228n3) and does not claim that it is "a conceptual truth" (p. 228). An informal reprise of his argument in 1993 does not mention those challenges and says that (G ⊃ ∼E) "(or something very close to it) is . . . held in common by theists and nontheists. . . . [that (G ⊃ ∼E)] accords with our basic moral principles, principles shared by both theists and nontheists. [And that] if we are to fault this argument . . . we must find fault with its first premise" (Rowe 1993, p. 80).

For a valid argument 'adjacent' to the invalid argument that Rowe seems to have intended in 1986, I propose for the second premise, not the truth of (G ⊃ ∼E), but its *certainty*,

(ii) $$P(G \supset \sim E) = 1.$$

That is entailed by, but not equivalent to, the necessity (G ⊃ ∼E): Were this necessity the second premise of my argument, this argument would be for what I term a 'logical problem of evil' and would belong in the next chapter. The argument's first premise is that E is very probable,

(i) $$P(E) >> \tfrac{1}{2}.$$

From these premises it follows deductively that ∼G is very probable,

(iii) $$P(\sim G) >> \tfrac{1}{2}.$$[21]

The tasks of an advocate of this argument are to provide reasons for E that persuade that it is probable and answers to objections that have been made to (G ⊃ ∼E), so that it should be embraced without a shadow of doubt. This argument, as said, is valid.[22] The neighboring argument, with (ii) replaced by (ii′), $P(G \supset \sim E) \approx 1$, that is, $[P(G \supset \sim E) = 1] \vee P(G \supset \sim E) = (1 - i)$, where i is an infinitesimal (for which see the Appendix to Chapter XIII), is also valid,[23] as is the qualitatively different argument

$$P(E) > 3/4, \quad P(G \supset \sim E) > \tfrac{3}{4} \qquad \therefore P(\sim G) > \tfrac{1}{2}.$$

The validities of these two arguments follow from a theorem that states the strongest rule for a kind of 'probabilistic *modus ponens*': "**Theorem 4.51**.

$$P(A_1) = p, \quad P(A_1 \rightarrow A_2) = q \quad | = P(A_2)\,\epsilon\,[p + q - 1, q]$$

[this interval being optimal, or narrowest consistent with validity]" (Hailperin 1996, pp. 200–203). The argument of a conservative interpretation of the text is "[s]ince [E] and [(G ⊃ ∼ E)] entail [∼G], if [E] is probable **and[(G ⊃ ∼E)] is true**, then it is probable that [∼G]" (Rowe 1986, pp. 228–9; emphasis added),

$$\left[P(E) > \tfrac{1}{2}\right], (G \supset {\sim}E) \therefore P({\sim} G) > \tfrac{1}{2},$$

not valid. For its invalidity, consider that though G is false, its probability for some person can be one.[24] Exploiting this possibility assume that ∼G and P(E & G) = 1. Then it is true that (G ⊃ ∼E), and for this person P(E) $> \frac{1}{2}$, though P(∼G) $\not> \frac{1}{2}$: for this person, P(E) = 1 and P(∼G) = 0.

5.2 Pain and suffering, and heinous crime. For considerations that support (i), that it is probable that E, and that, given (ii), make probable that ∼G, Rowe recalls that he "set forth [in an earlier paper] an example of intense suffering (the intense suffering of a fawn badly burned in a forest fire occasioned by lightning) and observed that as far as we can determine it serves no greater good at all, let alone one that is otherwise unobtainable by an omnipotent being" (Rowe 1986, p. 235). William Alston, having noted Rowe's imagined case [**E1**] of a trapped fawn's "several days of terrible pain before dying" (Alston 1996, p. 100 – first published in 1991), writes that Rowe "[i]n (1988) . . . adds to this a (real life) case [**E2**] . . . of the rape, beating, and murder by strangulation of a five-year-old girl . . . by her mother's boy friend" (Ibid.), a crime that as far as we can tell was of no value whatsoever for anyone, let alone of value that made it worthwhile.[25] Rowe adds, with reference to the agony of the fawn that, "[r]ecognizing . . . that, appearances to the contrary [no matter the length and detail of its description in neutral terms], it might nevertheless serve . . . an outweighing or defeating good, I then claimed that it seems quite incredible that all the instances of suffering that serve no greater good we know or can think of should nevertheless be such that none could have been prevented by an omnipotent being without loss of a greater good" (Ibid.). Shunning what he finds 'quite incredible', Rowe is presumably certain that there are some instances of suffering that could have been prevented by an omnipotent without loss of a greater good; though he realizes that he cannot *prove* this, he cannot doubt it.

5.3 A 'sceptical defence' of theism. "You should believe it," Alston could say. He does say that "we are not justified, and cannot be justified, in judging these [particular] evils [E1 and E2] to be gratuitous" (Alston, as quoted in Rowe 1995, p. 72).[26] According to Alston, we *cannot* be justified in judging that these dreadful evils, or *any* imaginable evils, could have been prevented by an omnipotent being for a better. Though he has "no specific suggestions as to what might be a fully sufficient reason for God to permit those cases [E1 and E2]" (Alston 1996, p. 119), he finds that he is not "justified in holding that God could have no reasons for permitting [these] . . . particular cases of suffering" (Ibid.) that "no one can be" (Ibid.). (Alston should have written 'has' in place of 'could have'. Rowe's position is not that He *could* have no reasons for permitting these suffering, but only that there are not in place any facts that could provide such reasons. He holds only that, if God exists, God *does not* have such reasons.) But that God does not have such reasons in these cases, given the facts of them, is, Alston reminds, a "negative existential claim concerning territory [what could be God's reasons in these cases] about the extent, content, and parameters of which we know little" (p. 121). We do not know what, if any, kinds of things are good, *in addition to the kinds that we know of*, though God would; and we do not know *what will be the total consequences* of the sufferings of E1 and E2, which could reach beyond 'this life', though again God would. "[I]t is [for these two reasons] in principle impossible for us to be **justified** in supposing that God [if He exists] does not have sufficient reasons [in a case] . . . that are unknown to us . . . [even] unknowable by us" (p. 119).

5.4 A 'dogmatic' response to his scepticism. That can sound right, but wait. Alston writes in amplification:

> "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy." . . . The development of physical science has made us aware of myriad of things hitherto undreamed of, and developed concepts with which to grasp them – gravitation, electricity . . . space-time curvature, irrational numbers, and so on. It is an irresistible induction from this that we have not reached the final term, and that more realities, aspects, properties, structures remain to be discerned, and conceptualized. (p. 109)

True. And it is an irresistible induction that much to come will be where we least expect it, and will be of things undreamt, and, but for speculative geniuses, undreamable. Consider, for example, life. Who is to say what life is, what forms it takes, and whether some are unknown, unknowable, or even inconceivable by us? By Alston's strictures it seems that we are not justified, that we *cannot* be justified ever, in supposing there is no life on the moon or at the center of the sun. To oppose Alston's 'sceptical defence' I say that, notwithstanding our great ignorance of science future (not to mention, on a personal note, science present), we are justified in holding that *there is no life on the moon*.

Similarly, I think, though our ignorance of what could be justifying reasons for an omnipotent, omniscient, and perfectly good creator and sustainer of the world may be great,[27] we *can be* justified in holding with some confidence that such a being would not have had sufficient reasons for E1 or E2, when we have looked as long and hard as Alston has, and been left as he "without any specific suggestions as to what might be a fully sufficient reason for God to permit *those cases*" (p. 119), that long-dying fawn and that *child*. That, the perfect-being atheist implies, is the *only* reasonable state of credence for someone who does not have what he considers "strong evidence in its [a perfect being's] behalf" (Rowe 1996, p. 282). An atheist who has looked as long and well as Alston has for reasons that might have been sufficient for a perfect being's permitting these cases of suffering, and not come up with "any specific suggestion" that is at all credible, *is* at least *then* justified in believing that there *are none* in these cases, even though he realizes that 'for all he knows' he is wrong.[28] It is, I maintain, enough for a person to be justified in believing that nothing in these cases would have provided a perfect being with sufficient reasons for permitting the pain in them, that (i) this person is justified in believing that no reasons of which he could ever know would be sufficient for a perfect being's permitting the awfuls of these cases, and that (ii) he is *not* justified in believing that there *are* reasons of which he cannot know that would be sufficient for a perfect being's permitting these awfuls.[29] Relevant to this justification is a great *disanalogy* of the moral and physical sciences. There has been in the moral sciences of axiology and deontology nothing like the exponential development of physics that make "an irresistible induction" (Alston 1991, p. 109) that, relatively speaking, 'we ain't seen nothing yet' of the good and the bad, and the right and the wrong, of which a perfect being would see all. If there is an induction here, it goes the other way: "[T]he discovery [if such there be] of a new property which is morally significant in itself is an extremely rare occurrence. That being so, [it is unlikely that] new morally significant properties will be discovered" (Tooley 1991, p. 115).

6 The evidential arguments from evil of Rowe (1988) and (1996).

6.0 A reprise of the 1988 argument in Rowe (1996)

The latest formulation I have given of the evidential problem of evil ['see (Rowe 1988) and (Rowe 1991),' he says] goes something like this. . . .

P: No good we know of justifies an omnipotent, omniscient, perfectly good being in permitting E1 and E2 [for which see Section 5.2.1 above]; therefore [probably],

Q: no good at all justifies an omnipotent, omniscient, perfectly good being in permitting E1 and E2; therefore [probably],

not-G: there is no omnipotent, omniscient, perfectly good being.

The first inference, from P to Q, is, of course, an inductive inference. My claim was that P makes Q probable. The second inference, from [the probability of] Q to [the probability of] not-G, is deductive[30] . . . Criticisms . . . have focussed mainly on (1) [that none of us is in a position to be justified in believing P] and (2) [that P does not make Q more probable than not]. . . . After discussing two preliminary matters, I will take up . . . some objections to the [first] inference, later turning to some issues concerning P itself. [Rowe 1996, pp. 262–3] I now think [my inductive] argument [for the first inference] is, at best, a weak argument . . . [Rather than attempting] to shore it up . . . I now . . . give what I believe is a better argument for thinking that P makes Q more likely than not. Consideration of this new argument . . . must be postponed until we have discovered . . . [what is] of ultimate interest to us [that P confirms G, both incrementally and absolutely]. [p. 267] [From that order of our discussion it will be] clear that we simplify the argument considerably by bypassing Q altogether and proceeding directly from P to ∼G. And that is what I now propose to do. Our evidential argument from evil . . . now [is] P . . . therefore, it is probable that ∼G. (p. 270)

Section 6.1 takes a dim view of the simplified argument of Rowe (1996). In Sections 6.2 through 6.8, I make the best I can of the abandoned argument of Rowe (1988).

6.1 Regarding the evidential argument of Rowe (1996)

6.1.1 ARCHITECTURE AND MAIN CONCLUSIONS The probabilities of Rowe (1996), with one exception, are all explicitly probabilities that are conditional either on k or on a conjunction of which k is a conjunct. This k is "the background information on which we will rely in forming judgments about how likely P, Q, and G . . . are" (Rowe 1996, p. 267).[31] Rowe stipulates that k entails only "information that is shared by most theists and nontheists who have given some thought to the issues raised by the problem of evil" (Rowe 1996, p. 265), that it does not entail P, even if P is common information in that community (p. 266). One gathers that k entails much information common in that community that is relevant to P: "[w]e will want to include in k our common knowledge of the occurrence of various evils in our world, including E1 and E2 . . . our common understanding of the way the world works . . . along with our knowledge of many of the goods that occur and many of the goods that do not occur" (p. 265). On a special negative note, we are warned that, if information common in that community 'raises Pr(G/k) above $\frac{1}{2}$,' "we will have to exclude it" (Rowe 1996, p. 266).[32] The main conclusions of his new argument are (i) "we have good reason to think P is true" (p. 282); (ii) $\Pr[G/(P \,\&\, k)] < \Pr(G/k)$; (iii) $\Pr[G/(P \,\&\, k)] < \frac{1}{2}$ (p. 269), which answer "the questions that are of ultimate interest to us" (p. 267); and (iv) "it is *irrational* to believe in theism unless we possess or discover strong evidence in its behalf" (p. 282).

6.1.2 THREE PROBLEMS First, k is not well-defined: We cannot start with shared information and simply delete P (if P is shared information) and other things that Rowe would have excised.[33] "**Perhaps** we could think of [the shared

information minus P and other things] as a body of propositions maximally similar to [the shared information] that does not entail P [and these other things]" (Plantinga 1998, 543n8; emphasis added). Second, the nature and soundness of the inference to (iv) is at best obscure. One difficulty in this complicated matter is that if, to go with [$\Pr(G/P \,\&\, k) > \frac{1}{2}$], we have, for some proposition R, [$\Pr(G/R \,\&\, k]) > \frac{1}{2}$]), it does not follow that ([$\Pr(G/(P \,\&\, R) \,\&\, k] > \frac{1}{2}$). 'Evidence' cannot be combined in that fashion. Plantinga may have thought otherwise (Plantinga 1998, pp. 538–40): He implicitly says that "it looks as if we could run" arguments for ([$\Pr(G/P \,\&\, k) = 1/3$] and [$\Pr(G/P^* \,\&\, k) = 2/3$]) in either order, "thus winding up with" [$\Pr(G/(P \,\&\, P^*) \,\&\, k] = \frac{1}{2}$ (Plantinga 1998, p. 540). Against the form of this inference, it could happen that one-third of the purple balls in an urn are graphite, that two-thirds of the prickly balls in the urn are graphite, and that all of the prickly purple balls in the urn are graphite, or that none are.[34] Another problem with the inference to (iv) is the *prima facie* irrelevance of 'probabilities for background information confined to k' to the rational and irrational beliefs of persons of *other* background information: ''[T]is necessary that a reason should be given here for relevance that seems altogether inconceivable." Third, when Rowe's argument is spelled out more fully than he does, it can be seen to be seriously flawed. His argument consists of subsidiary arguments for two probabilities and an inference from these to the conclusion that is another probability:

Survey of evidence in Section VII	*Reasoning in Section IV*	
↓	↓	
$\Pr(P/k) > \frac{1}{2}$	$\Pr[\sim G/(P \,\&\, k)] > \frac{1}{2}$	$\therefore P(\sim G/k) > \frac{1}{2}$

The inference is invalid. To explain, I use the subscript notation described above. What follows for $\Pr_k(\sim G)$ from $\Pr_k(P) > \frac{1}{2}$ and $\Pr_k(\sim G/P) > \frac{1}{2}$ is only that $\Pr_k(\sim G) > \frac{1}{4}$. This is a consequence of the principle for 'probable *modus ponens*',

$$P(h/e) = x, \qquad P(e) = y \qquad \therefore [(xy + (1 - y)] \geq P(h) \geq xy,$$

of Sobel (2002) and Hailperin (1996, pp. 232–4), the bounds of which principle are 'optimal' or 'narrowest consistent with validity' in the exact sense of Hailperin (1996, p. 200). It does follow from $\Pr_k(P) = 1$ and $\Pr_k(\sim G/P) > \frac{1}{2}$ that $\Pr_k(\sim G) > \frac{1}{2}$, but $\Pr_k(P)$ is not to equal 1 (Rowe 1996, p. 267). A 'nice' way to a valid inference is to add the premise

$$\Pr(P/k) \cdot \Pr[\sim G/(P\&k)] > \tfrac{1}{2},$$

which 'says' these probabilities are 'jointly' sufficiently greater than $\frac{1}{2}$ to make $P(\sim G/k)$ greater than $\frac{1}{2}$. Adding this premise, one can delete the original ones.

I have a fourth problem with the conception of conditional probability that Rowe has in mind for his argument. I take it to be a measure of 'objective evidential bearings'. I do not believe in such bearings, and it is unclear to me exactly how, pretending belief, to think about them.[35] There is a fifth difficulty that I get to in Section 6.1.3.3 and follow through Section 6.1.3.5, where it turns 'serious'.

6.1.3 "[A]CCORDING TO BAYES'S THEOREM.

$$\Pr(G/P\&k) = \Pr(G/k) \times \frac{\Pr(P/G\&k)}{\Pr(P/k)},"$$

(Rowe 1996, p. 267). After looking into P, relations between Pr(P/G & k) and Pr(P/k) are studied, and the latter is found to imply lower bounds for the former that depend on Pr(G/k).

6.1.3.1 Premise P. Rowe tells us that the, 'No good we know of justifies a perfect being in permitting E1 and E2′ expresses *the negation of a perfect being exists, and there exists a good we know of, and that good justifies him in permitting E1 and E2* (p. 283n8; 'perfect being' is in place of 'God' which for Rowe abbreviates 'an omnipotent, omniscient, perfectly good being', p. 265.) Rowe is saying that P comes to

$$\sim(\exists x)(G^1x \,\&\, (\exists y)[K^1y \,\&\, J^2yx])$$

or equivalently,

$$\sim(\exists x)G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2yx]),$$

which is equivalent to

$$\sim(\exists x)G^1x \vee [(\exists x)G^1x \,\&\, (x)(G^1x \supset \sim (\exists y)[K^1y \,\&\, J^2yx]^{36}$$

and to

$$\sim(\exists x)G^1x \vee [(\exists x)G^1x \,\&\, \sim(\exists x)(G^1x \,\&\, (\exists y)[K^1y \,\&\, j^2yx],$$

wherein 'G^1' stands for the formula '– is a perfect being', 'K^1' stands for the formula '– is a good we know of', and 'J^2' stands for the two-place formula '– justifies – in permitting E1 and E2'. Henceforth we will use the abbreviations G: $(\exists x)G^1x$), and J: $(\exists x)(Gx \,\&\, (\exists y)[K^1y \,\&\, J^2yx]$ and say that *P 'spelled out'* is

$$\sim G \vee (G \,\&\, \sim J).$$

It may be noticed that ∼J is equivalent to P! So far we do not have a 'proper disjunctive analysis' of the content Rowe 'wants' for his premise (see previous note). A remedy that I will use in Section 6.1.3.5 makes the premise,

$$\sim G \vee \sim J^*,$$

wherein 'J*' abbreviates '*There is a good we know of, y, such that, if there were a perfect being, then there would be a perfect being, x, such that y would justify x in permitting E1 and E2*' , which can be symbolized, using the subjunctive conditional connective, ' $\Box\rightarrow$ ' [$\ulcorner\phi\Box\rightarrow\psi\urcorner$ has the translation $\ulcorner$if it were the case that ϕ , then it would be the case that $\psi\urcorner$]

$$\sim(\exists y)(K^1y\,\&\,[(\exists x)G^1x\,\Box\rightarrow(\exists z)(G^1z\,\&\,J^2yz)]).$$

Proposition $\sim$J* is not equivalent to P, and ($\sim$G $\vee$ $\sim$J*) is not equivalent to ($\sim$G $\vee$ $\sim$J). It is a revision to which Rowe would not object.[37]

*6.1.3.2 The relation of Pr(P/k) to Pr[P/(G & k)].*Rowe assumes that (a) $\mathbf{Pr(G/k) = \frac{1}{2}}$, in order not to beg certain questions with which he wishes not to deal (p. 26), though in his view $Pr(G/k) < \frac{1}{2}$ (p. 283n10). He reports that (b) $\mathbf{Pr(P/k) < 1}$ (p. 267), since k does not entail P (evidently he takes as an axiom, $\Box(p \supset q) \equiv [Pr(q/p) = 1]$). We may let

$$\mathbf{Pr(P/k)} = 1 - \epsilon,$$

with e positive and less than 1, or with P 'spelled out',

$$Pr([\sim G \vee (G\,\&\,\sim J)]/k) = 1 - \epsilon.$$

It follows by the general principles,

$$\sim\Diamond(p\,\&\,q) \supset P[(p \vee q)/r] = P(p/r) + P(q/r),$$

and

$$P[(p\,\&\,q)/r] = P(p/r) \cdot P[q/(p\,\&\,r)],$$

that

$$Pr(\sim G/k) + Pr(G\,\&\,\sim J)]/k) = 1 - \epsilon$$
$$\mathbf{Pr(\sim G/k) + Pr(G/k \cdot Pr(\sim J/G\,\&\,k = 1} - \epsilon$$

From (a), $Pr(G/k) = \frac{1}{2}$, it follows that $Pr(\sim G/k) = \frac{1}{2}$. Substituting for these probabilities, their values yield

$$\tfrac{1}{2} + \tfrac{1}{2} \cdot Pr(\sim J/G\,\&\,k) = 1 - \epsilon.$$

I will return to the bold line for the general case in which P(G/k) = x and P($\sim$G/k) = 1 − x. It follows that,

$$Pr(\sim J/G\,\&\,k) = 1 - 2e.$$

We have, by the first general principle for conditional probabilities, that

$$Pr([\sim G \vee (G\,\&\,\sim J)]/(G\,\&\,k)) = Pr[\sim G/(G\,\&\,k)] + Pr[G/(G\,\&\,k)] \times Pr[\sim J/(G\,\&\,k)]$$

and, by two rather obvious general principles for conditional probabilities, that

$$\Pr([\sim G \vee (G \,\&\, \sim J)]/(G \,\&\, k)) = 0 + 1 \cdot \Pr[\sim J/(G \,\&\, k)]$$

so that

$$\Pr([\sim G \vee (G \,\&\, \sim J)]/(G \,\&\, k)) = \Pr[\sim J/(G \,\&\, k)].$$

From this and the italicized identity it follows that

$$\Pr([\sim G \vee (G \,\&\, \sim J)]/G \,\&\, k)) = 1 - 2\epsilon,$$

which, with P not spelled out, is

$$\mathbf{Pr[P/(G \,\&\, k)] = 1.2\epsilon}.$$

6.1.3.3 Tension in the text. It is of some concern to Rowe that Pr[P/(G & k)] not be very high, that, for example, it not be 4/5. For

$$\Pr[G/(P \,\&\, k] = \frac{\Pr(G/k) \cdot \Pr[P/(G \,\&\, k)]}{\Pr(G/k) \cdot \Pr[P/(G \,\&\, k)] + \Pr(\sim G/k) \cdot \Pr[P/(\sim G \,\&\, k)]},$$

$\Pr(G/k) = {}^1\!/_2$ and $\Pr(P/\sim G \,\&\, k) = 1$; so that, if $\Pr[P/(G \,\&\, k))] = 4/5$, then

$$\Pr(G/P \,\&\, k) = \frac{({}^1\!/_2) \cdot (4/5)}{({}^1\!/_2) \cdot (4/5) + ({}^1\!/_2) \cdot 1} = 4/9.$$

Rowe implies that in a situation in which one initially has "only k to go on" and then 'adds P to k' (p. 272), $\Pr[G/(P \,\&\, k)] = 4/9$ would not be small enough to justify "a shift from 'square agnosticism' to 'square atheism'" (p. 273), which is what he wants. He says that not even $\Pr[G/(P \,\&\, k)] = 1/3$ would be small enough to 'lever' from 'square agnosticism' to 'square atheism' (pp. 273–4). Given that $\Pr(G/k) = {}^1\!/_2$, $\Pr[P/(G \,\&\, k)]$ must be less than ${}^1\!/_2$; for $\Pr[G/(P \,\&\, k)] = 1/3$ when $P(G/k) = {}^1\!/_2$ and $\Pr[P/(G \,\&\, k)] = {}^1\!/_2$. "Well why not, why should Pr(P/G & k) not be *much* less than ${}^1\!/_2$?"

The answer, given present assumptions, is, first, that then Pr[P/(G & k)] would be much less than $(1 - {}^1\!/_2)$, so that, by the result of the previous section, Pr(P/k) would be much less than $(1 - 1/4 = 3/4)$. The second part of the answer is that Section VII of Rowe (1996) 'says' that Pr(P/ k) is considerably *larger* than even 3/4. The concluding lines of Section VII are

I must confess that I know of no way to *prove* that P is true. What we do have is genuine knowledge that many goods we know of are insufficient to justify God in permitting E1 or E2. In addition we have very good reason to believe that many other

goods we know of could be realized by an omnipotent, omniscient being without his having to permit E1 or E2 (or something just as bad). And, finally, we have the failure of theodicists to show how any of the goods we know of can plausibly be held, separately or collectively, to constitute a sufficient reason for God to permit E1 or E2. All this, I believe, gives us good reason to believe that P is true. (Rowe 1996, p. 282)

Rowe does not translate these good reasons to believe that P is true into a *probability for P conditional on them*, but it is not plausible that he would say that they translate into a probability *no greater than 3/4*. His strong summing up suggests a probability much greater than that: It suggests, I think, a probability for P, on *'all of this'*, that approaches 1, and is at least as great as 9/10. And he must say that that is a floor for the probability of P, *on k*, for he must want 'all of this' to be entailed by k. He writes that "we will want to include in k our common knowledge of the occurrence of various evils in our world, including E1 and E2 . . . our common understanding of the way the world works. along with our knowledge of many of the goods that occur and many of the goods that do not occur" (p. 265). A review of the argument of pages 276–82 for the conclusion that we "have good reason to believe that P is true" (p. 282) can confirm that Rowe thinks that he is drawing on "information that is shared by most theists and nontheists who have given some thought to the issues raised by the problem of evil" (p. 265). Rowe wants Pr(P/k) to be *high*. He *needs*, for his final inference to $Pr(\sim G/k) > \frac{1}{2}$, that $Pr(P/k) \cdot Pr[\sim G/(P \,\&\, k)] > \frac{1}{2}$; for the inference that So that $Pr(\sim G/k)$ is *much* greater than $\frac{1}{2}$, he needs that $Pr(P/k) \cdot Pr[\sim G/(P \,\&\, k)]$ is much greater than $\frac{1}{2}$, and for that *each* of Pr(P/k) and $Pr[\sim G/(P \,\&\, k)]$ must be even *greater*.

6.1.3.4. To 'relax' his text, to resolve the tension reviewed in the previous section, Rowe must revisit the assumption that $\frac{1}{2}$, made not to beg certain questions, brings down P(G/k) to secure $Pr[G/(P \,\&\, k)] < 1/3$, and take on those questions. That adjustment will not only reduce the tension, but, though he may not have realized this, 'cut him some slack' for a lower value for $Pr[P/(G \,\&\, k)]$ than $(1 - 2\,\epsilon)$ when $Pr(P/k) = (1 - \epsilon)$. He can be seen to exercise this advantage in a case that would provide him with values for all probabilities of the orders he wants, and in which he believes.

> Suppose that after thorough investigation . . . we were to come to the conclusion that $Pr(G/k) < 0.5$. . . . [That] would mean that $Pr(G/(P \,\&\, k)] < 0.333$. How much less would depend on how low the probability of G on k is and how low the probability of P given G and k is. If $Pr(G/k) = 0.2$ and $Pr(P/G \,\&\, k) = 0.25$, $Pr[G/(P \,\&\, k)]$ would be [less than] .06. (Rowe 1996, p. 274)

Rowe goes for relatively low values for both Pr(G/k) and $Pr[P/(G \,\&\, k)]$. Can he *have* these values *along with* the high value for Pr(P/k) in which he believes, and that he wants for a high value for $Pr[\sim G/(P \,\&\, k)]$? The answer is that he not only can have these low values for Pr(G/k) and $Pr[P/(G \,\&\, k)]$ along with a high value for Pr(P/k), but that they *entail* a high value for Pr(P/k),

specifically, the value 19/20! To see this we may go to the line marked for return in Section 6.1.3.2 to generalize for values of P(G/k) additional to $^1/_2$. Established in Section 6.1.3.2 is that, if $Pr(P/k) = 1 - \epsilon$, then

$$Pr(\sim G/k) + Pr(G/k) \cdot Pr(\sim J/G \,\&\, k) = 1 - \epsilon.$$

Established as well is that, in any case,

$$Pr[P/(G \,\&\, k)] = Pr(\sim J/G \,\&\, k)$$

so that

$$Pr(\sim G/k) + Pr(G/k) \cdot Pr[P/(G \,\&\, k)] = 1 - \epsilon.$$

And so, for the general case, we have, if $Pr(P/k) = 1 - \epsilon$, $Pr(G/k) = x$, and $Pr[P/(G \,\&\, k)] = y$,

$$(1 - x) + xy = 1 - \epsilon.$$

'Solving for ϵ', we have, given those values, that

$$\epsilon = x - xy,$$

and thus

$$Pr(P/k) = 1 - (x - xy).$$

For $x = 0.2$ and $y = 0.25$, as said,

$$Pr(P/k) = 1 - [2/10 - (2/10) \cdot (1/4)] = 1 - 1/20 = 19/20.$$

"No problem, tension resolved?"

6.1.3.5. "'Yes problem', for only *that* tension has been resolved." Additional to tension due to the assumption that $Pr(G/k) = {}^1/_2$ for probabilities of the text's argument, there should be tension, *regardless of the value of Pr(G/k)*, between the high value that Rowe (1996) wants for Pr(P/k) and the low value he wants for Pr[P/(G & k)]. For, or so I will argue, Pr[P/(G & k)] *should not be less than* Pr(P/k) if P is 'spelled out', *not* by $[\sim G \vee (G \,\&\, \sim J)]$, but by the disjunction $(\sim \mathbf{G} \vee \sim \mathbf{J^*})$ of Section 6.1.3.1, as I think Rowe would be happy for it to be (see note 38), with $\sim J^*$ such that it does not entail (E1 & E2). It is a matter of evidence in k for $(\sim G \vee \sim J^*)$ that it is convenient to abbreviate by '$P^{\#}$'.

The evidence to which Rowe refers in his conclusion that, "all this, I believe, gives us good reason to believe that $[P^{\#}]$ is true" (Rowe 1996, p. 282), is in k, and, new point, is evidence for $\sim G$ not 'directly', but only 'through $\sim J^*$, or the possibility of $\sim J^*$'. It is all evidence for $P^{\#}$ *whether or not G. All of this evidence in k for* $P^{\#}$ *is evidence that is also in (G & k)*. It seems, furthermore, that *all* evidence *in k* for P *should* go against G only 'by way of $\sim J^*$, or the possibility of $\sim J^*$'. What does it mean? It means that evidence e in k for P should be

either directly for ∼J*, so that Pr(∼J*/k) = Pr[∼J*/(G & k) = Pr[∼J*/(∼G & k)], *or* like the evidence (E1 & E2): While presumably Pr(G/[(E1 & E2) & k]) < Pr(G/k), no one would say that Pr(G/[(E1 & E2) **& J*** & k]) < Pr(G/k) : J* should 'screen off' (E1 & E2) from G; (E1 & E2) should not be of relevance to G 'independent of the possibility of ∼J*.' It seems that all evidence e in k for (∼G ∨ ∼J*) that is not evidence directly for ∼J* should be like (E1 & E2) and evidence against G only by way of the possibility of ∼J*. If that is right, then all evidence in k for P should still be in (G & k), and if *this* is right, $Pr[P^{\#}/(G \ \& \ k)] = Pr[(\sim G \vee \sim J^*)/(G \ \& \ k)] = Pr[\sim J^*/(G \ \& \ k)]$ should *not* be less than $Pr(P^{\#}/k) = Pr[(\sim G \vee \sim J^*)/\ k]$.

"But Rowe believes that there is *abundant* evidence in k for (∼G ∨ ∼ j*) that is of and about evil *other than* the particular evils E1 and E2. This evidence concerning other evil has no relevance to ∼J*, which is specifically about reasons that could justify a perfect being's permitting those particular evils. This evidence in k of other evil would be for (∼G ∨ ∼J*) only 'by way of ∼G'. None of *this* evidence in k for (∼G ∨ ∼J*) would remain to be drawn from (G & k)." Cf.:

> My own view is that Pr(G/k) < 0.5. For I think the information we possess concerning the abundance of various evils in the world renders G unlikely. And I do not think the other information in k manages to counterbalance the weight of our information about the abundance of evils in the world. (Rowe 1996, p. 283n10)

To sort out the theoretical situation, I start with the question, "What is evidence of and about *other* evil *doing* in the 'background information' of this argument against the existence of a perfect being whose sole premise is concerned precisely with evidence about the particular evils E1 and E2?" My suggestion, in a note to the 'first problem' of Section 6.1.2, was that this evidence concerning *other* evil does not *belong* in the 'background information' of Rowe's argument. My thought is that this argument should 'want' evidence of other evil assessed *similarly* and, if and when under consideration, addressed in a premise analogous to (∼G ∨ ∼J*). We should want the argument for Pr[(∼G ∨ ∼J*)/(G & k)] << Pr[G ∨ ∼J*] not to depend even *in part* on the presence in k of evidence concerning *other* evil that directly supports ∼G. Similarly for the other particular evil: For an argument like the present one from there not being known goods that would justify a perfect being's permitting it, we should want to exclude from *its* background information evidence concerning E1 and E2. It is only the argument from there not being known goods that would justify a perfect being's permitting evil that will have background information from which no shared information concerning evil is excluded.[38]

I have argued that when k is trimmed in a manner appropriate to an argument that would run a high value for $Pr(P^{\#}/k) = Pr[(\sim G \vee \sim J^*)/k]$ to a high value for Pr(∼G/k), then, unfortunately for the argument and contrary to Rowe (1996), $Pr[P^{\#}/(G \ \& \ k)]$ should *not* be less than $Pr(P^{\#}/k)$. Stephen Wykstra might say that $Pr[P^{\#}/(G \ \& \ k)] = Pr[(\sim G \vee \sim J^*)/(G \ \& \ k)] = Pr[\sim J^*/(G \ \& \ k)]$

should in fact be *greater than* Pr(P#/k). This on the ground that, even if ∼J* is not "**just** what one would expect" (Wykstra 1984, p. 91; emphasis added) given k which entails (E1 & E2), there is *enough* in the analogy of what would be our relation to God throughout our lives to the relation of a child to adult caring parents (p. 88) to make *somewhat* "expectable – given what we know of our cognitive limits – that the goods by virtue of which this Being allows [this] known suffering should . . . be beyond our ken" (p. 91). Objection: "But a loving parent 'is there' for his puzzled suffering child, whereas the victims in E1 and E2 'suffer alone' as far as we can tell." Response: "The parent–child analogy is being used here to argue the positive relevance of G *specifically to* ∼*J**, which is *silent* on matters of the presence or absence of a 'comforting presence'. Why are you changing the subject? Your remark has no differential bearing on Pr(∼J*/ k) and Pr[∼J*/(G & k)]. It has no differential bearing on Pr[(∼G ∨∼J*)/ k] and Pr[(∼G ∨∼J*/(G & k)]."

I think that Wykstra is right. I think, for the reason I have given, and this one of his, that the new argument of Rowe (1996) is in serious trouble regarding Pr(P#/k) and Pr[P#/(G & k)]. It wants Pr(P#/k) to be great and for Pr[P#/(G & k)] to be small, whereas Pr[P#/(G & k)] should be, if not equal to Pr(P#/k), then *greater* than it. Crash! Plantinga writes, "This new argument . . . is, if anything, weaker than the old" one of 1988 (Plantinga 1998, p. 532). It is, I think, considerably weaker.

Back to the argument of 1988 to revise and revamp.

6.2 Cosmetic revisions and an assumption. Several revisions are incorporated in the statements below of P and Q introduced in Section 6.0. These revisions avoid the suggestion that P entails the existence of a perfect being; make quite plain that P is entailed, as Rowe wants it to be (Rowe 1996, pp. 264–5), by there not being a perfect being; make clear that P entails E1 and E2; and bring out the 'thoroughly indicative' intended character of P.

P: E1 and E2, and either there is not a perfect being, or there is a perfect being, and no good we know of justifies a perfect being's permitting *either E1 or E2*.

(E1 and E2) & (∼there is a perfect being ∨ [there is a perfect being & ∼(∃x)(we know that x is a fact, that it is good that x, and that x justifies a perfect being's permitting E1 or permitting E2)])

Q. E1 and E2, and either there is not a perfect being, or there is a perfect being, and no good at all justifies a perfect being's permitting E1 or permitting E2.

(E1 and E2) & (∼there is a perfect being ∨ (there is a perfect being & ∼(∃x)[(x is a fact, and it is good that x) & x a perfect being's permitting E1 or permitting E2)]))

Framework Assumption I. We know that (E1 and E2), have good grounds for (E1 and E2), and have a probability of one for it.

6.3 A major makeover. Now come *replacements* for P and Q that are in some ways substantially different.

P*: E1 and E2, and either there is not a perfect being, or there is no x such that *we have a reason* for thinking that x is a fact, that it is good that x, and that x would justify a perfect being's permitting E1 or permitting E2, *which is not a reason for thinking that there is an omnipotent, omniscient, perfectly good being*.

and

Q*: E1 and E2, and either there is not a perfect being, or there is no x such that *there is a reason* for thinking that x is a fact, that it is good that x, and that x would justify a perfect being's permitting E1 or permitting E2

I understand 'reasons *we have* for thinking something' and 'reasons *there are* for thinking this thing', so that reason we have are reason there are, but not necessarily vice versa. The *reasons we have* depend on what we know. We *have* a reason *r* for thinking that *p* if and only if, (i), we know that *r*, and, (ii), *r is* a reason for thinking that *p*.

I assume for the argument I am assembling that it is certain that if Q* then there is not a perfect being:

Substantial Assumption. $\Pr(Q^* \supset \sim G) = 1$.

This agrees with my reconstruction of in Section 5 of the argument of (Rowe 1986), in which I give Rowe the premise, $\Pr(G \supset \sim E) = 1$. Let Q*, somewhat spelled out, be [(E1 & E2) & (∼G ∨ ∼RsnJstngGood)]. A compound thought that can recommend the present assumption is that surely, given E1and E2, if there is a perfect being, then there is a good that justifies its permitting E1 and E2; and that if something is a fact, then there is a reason for thinking that it is a fact, if this reason is only the fact itself. Taking from this thought that [(E1 & E2) ⊃ (G ⊃ RsnJtngGood)], given Q* it follows that ∼G: one may assume, for a short indirect argument, ∼∼G. An inference to the Substantial Assumption can then be by the principle that if $\Pr(p) = 1$ and $\Box [q \supset (p \supset r]$, then $P(q \supset r) = 1$.

6.4 To a high probability for P.* It seems that, given that we know E1 and E2, we have *excellent* grounds for believing that P*, and according it high

probability. Here *come* these grounds. Here comes an argument available to us for P*.

> Suppose that ∼P*, that is, ∼[(E1 & E2) & (∼G ∨ ∼R)], wherein 'R' is short for 'there is an x such that we have a reason for thinking that x is a fact, that it is good that x, and that x justifies a perfect being's permitting E1 or permitting E2, which x is not a reason for thinking that there is an omnipotent, omniscient, perfectly good being'. Then, since it is given by Framework Assumption I that (E1 & E2), it follows that R: one may use DeMorgan to infer ∼(E1 & E2)∨ ∼ (∼ G ∨ ∼)R, use ∼∼ (E1 & E2) to detach ∼(∼*G* ∨ ∼R, and reach R with DeMorgan, simplification, and double negation. Now we are confident that *none of us makes R highly probable*, for surely no one is keeping a secret until he can publish it a reason he has for thinking that a certain x is a fact, that it is good, and that it would justify a perfect being's permitting E1 or permitting E2, which x is not a reason for thinking that there is an omnipotent, omniscient, perfectly good being. But given how long the controversy has been on, it is very likely that some of us *would* by now think we *knew* that R, if R was the case: That is, *if it were the case that R, then some of us would make R highly probable*. From which it follows by *modus tollens* that ∼R. That completes an indirect argument for P*.

Excellent grounds, don't you think, for believing that P* and according it high probability?

6.5 A nudge to the argument's premise. To close *one* obvious gap between P* and Q*, I add

S: We have no reason for thinking that there is a perfect being. [39]

not as a premise, but as a conjunct of (P* & S), with which I replace premise P*. Premise (P* & S), partially spelled out, is

[(E1 and E2) & (∼G ∨ there is no x such that *we have a reason* for thinking that x is a fact, that it is good that x, and that x justifies a perfect being's permitting E1 or permitting E2, *that is not a reason for thinking that G*)] & (we have no reason for thinking that G).

Premise (P* & S) entails

T: There is no x such that *we have a reason* for thinking that x is a fact, that it is good that x, and that x justifies a perfect being's permitting E1 and E2.

which contrasts with a part of Q*.

Q*: E1 and E2, and either there is not a perfect being, or **there is no x such that *there is a reason* for thinking that x is a fact, that it is good that x, and that x justifies a perfect being's permitting E1 or permitting E2.**

6.6 To the extra burden of the argument. Conjunct S does not 'beg the question' of the argument whether or not G. It does not entail that ∼G. But many, if not all, will want S justified. They will want to be shown that 'there is nothing in' *any* of the arguments for the existence of a perfect being. Not everyone believes that! I do. And many will want to be persuaded that, setting aside 'the arguments', they have no reason for believing G. Not everyone believes *that*! A second 'framework assumption' for the argument being built includes that this preliminary persuasion, and more, has been successful.

Framework Assumption II. We agree that we have no reason$^{\#}$ for thinking G, and that, considerations for evil and suffering E1 and E2 aside, we have no reason$^{\#}$ for thinking that ∼G. ($^{\#}$Reasons here and below are 'reasons on balance'.[40])

This assumption relieves the extra burden of the argument being assembled that lies in conjunct S by positioning this argument in a two-stage argument against the existence of a perfect being. In the first stage one finds that considerations of suffering and evil E1 and E2 aside, we have no reason for thinking that G or for thinking ∼G. In the second stage one finds that, *given the result of the first part*, we have a good reason *against* believing that G, *everything* considered, so that one's probability 'at the end of the day' for ∼G is high. Here, by the Framework Assumption II, the second part is isolated to see whether it delivers that conclusion, on the assumption that the first part has delivered its conclusion.

6.7 On the first inference of the argument, and moving on. Is there, as the first inference requires, a 'transfer of probability' from (P* & S) to Q* so that a high probability for (P* & S) entails a high probability for Q*:

(1) □[our probability for (P* & S) is high ⊃ our probablity for **Q*** is high]?

We have seen that (P* & S) entails T. This means that,

(2) □[the probability of (P* & S)is high ⊃ our probability for **T** is high]

That is by The Consequence Principle, which says that for any p and q, necessarily

$$\Box(p \supset q) \supset [P(q) \geq P(p)].$$

What the argument requires, however, is (1). It is by no means obvious that this entailment obtains. I think it does not, and so, 'without removing (P* & S) from the screen', take it out of the argument, which is then left with the sole premise Q*.

6.8 *On the new argument, Q* ∴ ∼G*

6.8.1 TO A HIGH PROBABILITY FOR Q* It is, I think, not that the probability of (P* & S) 'makes' the probability of Q* high, but that *something like* what makes P* highly probable makes Q* highly probable. Here is a way in which someone might explain his confidence in Q* while endeavoring to bring out the confidence of others in Q*. "We agree that there are no grounds for believing that G. (This explanation and persuasion that we are 'presently' into *presupposes* discussion that has secured this agreement.) So a reason for thinking that something x is a fact, that it is good that x, and that x would justify a perfect being's permitting E1 or permitting E2 cannot be a reason for thinking that G. And grounds for our believing Q* must be identical with grounds for believing (let us call it) *Q*.

> (E1 & E2) & (∼G ∨ there is no x such that *we have a reason* for thinking that x is a fact, that it is good that x, and that x would justify a perfect being's permitting E1 or permitting E2, *that is not a reason for thinking that there is an omnipotent, perfectly good being*).

Given, by Framework Assumption II, that we have no grounds for believing that ∼G, our grounds for believing the second conjunct of *Q* must be grounds for believing its second, unabbreviated disjunct. Let this disjunct be abbreviated by **'Y'**. We have the following grounds for believing Y. This controversy has been going on for over 20 years. There is no *such* reason for thinking that there *is* (as against 'might be') a justifying good for E1 or E2, and this not for want of diligent efforts to find such a reason. No one expects such a reason to be found. We can assume that we are therefore *close* to having the best ground possible for us for believing that there *is* no such reason, which would be the fact that, no matter how much talented time and energy were expended on the project, no such reason would be found. It is a reasonable prediction that, as no such reason for thinking has been found, so, though the debate will continue for some time, none will be found. We do not have what would be the best ground possible for our believing Y, namely, knowledge that, Z, no matter how much talented time and energy were expended on the project, no such reason would be found. But we do have *excellent* grounds for believing Y, grounds nearly as good as we would have if we knew that Z. So we have excellent grounds for believing *Q*, and we have no grounds for *doubting* *Q*. A general nervousness at the thought of 'the great unknown' is not a ground for doubting *anything*. On all this we can agree, and so agree that Pr(*Q*) is high.

Recalling that our grounds for believing Q* must be the same as our grounds for believing *Q*, we can agree that Pr(Q*) is high.[41]

6.8.2 ON TO 'PERFECT-BEING ATHEISM' With *the probability of its premise secured*, the single inference of our argument to the probability of its conclusion proceeds by way of the Substantial Assumption that $Pr(Q^* \supset \sim G) = 1$. For transparency this can be added as a premise[42] for an inference from Pr(Q*) is high, and $Pr(Q^* \supset \sim G) =$, to Pr(~G) is at least as high, by the principle that, for any p and q, $[Pr(p \supset q) = 1] \supset Pr(q) \geq Pr(p)$.

7 A Bayesian issue for evidence of evil

7.1. Let 'G' abbreviate 'there exists an omnipotent, omniscient, and perfectly good being' and '*Æ*' abbreviate '*AS-FAR-AS-WE-SEE there has been and is evil and suffering that could have been avoided by an omnipotent being, and is such that, if it had been avoided, the world would have been the better for that*' or '*there has been and is APPARENTLY unnecessary and avoidable-by-an-omnipotent-being-with-net-gain-to-the-world evil and suffering*'. Consider a person for whom P(*Æ*) = 1. *Æ* is not Rowe's E of Section 5, which does not have the qualifier 'as far as we can see' or 'apparently'. And Rowe does not presume that his E is for many persons *certain* evidence. He seeks to persuade that on reflection most would find it *nearly certain evidence* and relies only on most finding it more probable than not. Let TI be this person's total information or evidence, so that TI is the strongest proposition p such that, for this person, P(p) = 1 (or equivalently, the weakest proposition p such that P(p) = 1 and p entails every proposition q such that P(q) = 1). Let K be a proposition logically equivalent to (TI ∨ ~*Æ*). K shall in my proposal 'stand in' for what would be this person's total evidence, that it is, for what would be the strongest proposition, K′, of which he would be certain, *had he never come to be certain Æ*. I assume that K and K′ are equivalent *for present purposes* to establish a Bayesian condition for evidential arguments from evil.[43] The assumption is an unimportant gloss on the solution I offer for its case of the Problem of Old Evidence.[44] Though I proceed explicitly in terms of K, nothing is made of its identity with (TI ∨ ~*Æ*), and I could as well have 'left K out of this' and proceeded simply in terms of K′ .

7.2. My main proposal is that,

$$\text{Æ is sufficient to establish} \sim G,$$

if and only if

$$P'(\sim G/Æ) > \tfrac{1}{2},$$

where P′ would be the person's probability function were his total evidence not TI but K. Since K does not entail *Æ*, P′(*Æ*) could be less than 1, indeed, *much*

less than 1. I assume – this is required for the coming application of a version of Bayes's Theorem, given that I am operating with 'standard' conditional probabilities P′ only that P′(Æ) would 'beforehand' still have been greater than 0. The displayed condition is for a very weak form of 'establishment', a minimal 'standard of proof'. Stronger forms use a larger fraction than $\frac{1}{2}$ and make adjustments required in what follows. Alternatively, the displayed condition can be taken as only necessary for establishment. 'Establishment' is of course intended in a person-relative subjective sense that does not entail the truth of what is 'established.' Applying Bayes's theorem for a hypothesis and its negation, we have,

$$P'(\sim G/Æ) = [P'(\sim G) \cdot P'(Æ/\sim G)]/[P'(\sim G) \cdot p'(Æ/\sim G) + P'(G) \cdot P'(Æ/G)].$$

So Æ is sufficient to establish ∼G, that is, $\mathbf{P'(\sim G/Æ) > \frac{1}{2}}$, if and only if

$$P'(\sim G) \cdot P'(Æ/\sim G) > P'(G) \cdot P'(Æ/G),$$

or equivalently, if and only if

$$\mathbf{P'(Æ/\sim G)/P'(Æ/G) > P'(G)/P'(\sim G)}.$$

This inequality frames the issue in evidential arguments from evil. It provides discipline for the issue whether it is, for a person, more probable than not on the evidence Æ of apparently unnecessary, avoidable-by-a-perfect-being-with-a-net-gain-to-the-world evil that, ∼G, there is not a perfect being. The probabilities in those inequalities are, recall, what would be this person's probabilities were his total evidence not T but K.

. . . . . .

Caveat. While this Bayesian framing of 'the evidential problem of evil' is formally correct, I do not recommend that the challenge to theisms by the evidence of evil be managed in this way. Better, I think, for reasons stated in a note to the last paragraph of the appendix to Chapter VII, to follow Hume's lead and bring the problem of this evidence into a free-ranging 'particular Bayesian discussion of the evidence, *all* of it' for and against theisms. Better, that is, to bring the problem of this evidence into an application of Bayes's theorem for *many* hypotheses in which the *total* evidence, naturally so-termed, relevant to a challenged theism – evidence of apparently intentional design of parts of the world, of apparently intentional fine-tuning for life of the basic physical parameters of the world, of *consensus gentium*, for miracles, for theophanies, of apparently unnecessary for the greater good evil, of apparently imperfect natural design, of extinct species, of a lawless Big Bang, and of everything else seen as evidence relevant to that theism – is treated *as* evidence, rather than sinking all but the evidence of suffering

and evil into 'background information' to affect only from there the 'priors' and 'likelihoods' of the theism of primary interest, and alternatives to it.

7.3. Projecting what I think would be my own probabilities, I propose that for almost everyone P′(Æ/∼G) would be, on reflection, limited, as well as extensive, *much* greater than P′(Æ/G). I think that for almost everyone "the world . . . as it appears to us in this life [complete with Æ] . . . would, *beforehand* [be 'the last thing' he would] expect from a very powerful, wise, and benevolent Deity" (Hume 1991, Part 11, p. 163). "Perhaps," a critic might say, "but that is mere speculative psychology. The question is not what *would*, but what *should*, these probabilities be for everyone. A prior question is whether *anyone* should have *any* such probabilities." In my view, roughly stated, a person's relevant probabilities, if he has any, are reasonable if and only if they are what they would be on full ideal reflection, and that in this people can differ. That is why I favor 'would' over 'should' in the present context. The latter, without explanation to the contrary, can suggest an 'objective' one-size-fits-all requirement, whereas in my view it is for each person to dispose for his own case what I propose by projection from my own. But if I am right that, for almost everyone P′(Æ/∼G) would be much greater than P′(Æ/G), then for almost everyone Æ is sufficient to establish ∼G, *unless* P′(G) is at least that much greater than P′(∼G). Let me add that this condition can be satisfied for persons of faith without reasons.[45] And it can be satisfied for persons who think they have *reasons* (for example, good arguments or personal encounters with God) for believing G that outweigh reasons in Æ for believing ∼G that come from the very low likelihood, for them as for most everyone, of Æ conditional on G.

8 Another skeptic. One might guess that Peter van Inwagen is such a believer:

> My . . . position . . . is . . . that there are reasons for believing in God [an omnipotent, omniscient, and morally perfectly being, different from the now largely discredited 'arguments for the existence of God' – reasons of the general kind recently advocated by William Alston, Alvin Plantinga, and Nicholas Wolterstorff] . . . that . . . are strong enough [for those who have them] to override any conceivable prima facie case against theism. (van Inwagen 1995, p. 70n6)

But he is not such a believer. He *would be* if, for him, P′(Æ/∼G) were greater than P′(Æ/G), but, one may gather, he is persuaded by a metaphysical/moral story that he "is not in a position to assign **any** probability to [Æ on G]" and "to refuse to make any judgment about [even] the **relation** between the probabilities of [Æ on G and on ∼G]" (pp. 73 and 75; emphasis added – I 'gather' this from his position on S, all suffering there has ever been, T, theism, and HI, 'the hypothesis of indifference'). He urges "extreme modal and moral

skepticism (or, one might say, humility) in matters unrelated to the concerns of everyday" (p. 84). Cf.:

> [M]itigated scepticism which may be of advantage to mankind . . . is the limitation of our enquiries to such subjects as are best adapted to the narrow capacity of human understanding. . . . [It says that a] correct Judgment . . . . confines itself to common life, and to such subjects as fall under daily practice and experience. [Hume 1902, p. 162; emphasis deleted] To be a philosophical sceptic is, in a man of letters, the first and most essential step towards being a sound believing Christian. . . . (Hume 1991, Part 12, p. 185).

Van Inwagen's position regarding conditional credences $P'(Æ/\sim G)$ and $P'(Æ/G)$ – which, I gather, is that *no one* is *in a position* to have them – is consistent with my claim that almost everyone *does* have such conditional credences or opinions, and that for almost everyone $P'(Æ/\sim G)$ would beforehand have been much greater than $P'(Æ/G)$. Van Inwagen would not deny that people have, and would have beforehand, conditional credences for Æ on $\sim G$ and on G. He maintains, I gather, that people are not *entitled* to any such conditional credences, that "no one is in a position to know whether [Æ] is what one should expect if [G, or if $\sim G$]" (van Inwagen 1995, p. 72). His stand is that no one can have reasons or grounds that position him to judge what *should be* our conditional credences here, and that therefore we *should not have* conditional credences here: We should not "assign any . . . probability to [Æ] on [G]" (p. 73) or on $\sim G$. He tells "a story that is true for all [he] knows" to which "[he] believe[s] we have no reason to assign any probability or range of probabilities" (p. 79).

The story is in part modal: It 'goes' in this part that every possible world in which there are higher sentient beings is either massively irregular or contains suffering morally equivalent to that in ours. He holds "[w]e should have reason to reject this [part of the] story" only if we can " 'design' a world" that is a counterexample, which we can do only if, for a start, we can describe "in some detail [its] laws of nature" (p. 79). His story is in another part moral: It 'goes' in this part that being massively irregular is at least as great a cosmic defect as is containing suffering morally equivalent to that in our world. We can be, he is sure, in no position to assess this part of the story. Returning to Æ and G, he would say that we have reason to assign probabilities to Æ on G, and Æ on $\sim G$, only if (a) we can hold in mind in some detail these two cosmic states (one of massive irregularities, the other of suffering equivalent to ours), carefully compare them, assess their relative values; and (b) *supposing* (what is not possible for us) that we do all that, we have reason to think that our assessments of "the relative values of states of affairs of literally cosmic magnitude . . . that have nothing to do with the practical concerns of everyday life" *are at all reliable* (p. 83)!! He is sure that *neither* condition can be satisfied.

Against this demanding (a)–(b) condition, I think that persons' sundry credences and conditional credences are *reasonable* to the extent that they agree with those they would eventually have, and from then on

keep, during the course of ideally exhaustive and scrupulous reflection that ran to everything, including all possibilities for being moderately or radically limited cognitively, without learning anything other than what is delivered by this reflection.[46] I defend the reasonableness (as I understand that condition) of what I believe are almost everyone's credences that make P′(Æ/∼G) *much greater* than P′(Æ/G) for them, not by detailing common reasons and grounds for these conditional credences, but by citing what would be their *robustness*, their *persistence* on extended reflection that includes attention to attempts such as van Inwagen's to submerge them in a solvent of "extreme modal and moral skepticism" (p. 84). Persistence on reflection that marshals every capacity for rational criticism is what reasonableness is for persons' credences and conditional credences.

Van Inwagen's strategy is to stop evidential arguments from evil before they would start.[47] He will not allow conditional probabilities for comparison. "If . . . consequences of modal and moral scepticism are accepted, then there is no reason to believe that the probability of [Æ] on [G] is higher than the probability of [Æ] on [∼G]" or, more to the point, that that would have been so *beforehand*, when nothing directly bearing on Æ was known: "[T]he evidential argument from evil cannot get started" (p. 85). That all of this *would have been* 'the last thing' anyone would have expected beforehand from an omnipotent, omniscient, and perfectly good creator and sustainer is, I assume he would say, beside the point, *if*, as he thinks, those consequences of scepticism are to be accepted and we put no store in such expectations now or beforehand. For my part, I reject van Inwagen's radical modal and moral scepticism, and I reject the 'idea' of completely objective norms of reasonableness that it implicitly deploys. It is, in my view, van Inwagen's response to the evidential argument from evil, framed as this response is in terms would-be objective norms, that is the nonstarter, not evidential arguments framed as they can be in terms of what we have good reason to believe would be 'beforehand' robust on reflection widely shared likelihoods of ∼G and G for Æ and 'priors' of ∼G and G.

. . . . . .

Postscript to skeptical theists. A theorem derived in the appendix is relevant to the situation of radical sceptical theists. According to this theorem, E, the presence of some evil, or the absence of some good, is *evidence against* their theism, T, if and only if ∼E, the absence of this evil, or the presence of this good, is *evidence for*, T:

$$[\Pr(E) > 0 \;\&\; \Pr(\sim E) > 0] \rightarrow [\Pr(T/E) < \Pr(T) \leftrightarrow \Pr(T/\sim E) > \Pr(T)].$$

As radical skeptical theists say that no apparently 'gratuitous' evils can be evidence against T, so they must say that no apparently 'gratuitous' goods such as those glorified in many religions can be evidence for T. Letting T′ come from T by stripping its deity of 'moral qualities', they must see evidence

for T confined to that which is as good for T′, and for the rest, as "[a] person, seasoned with a just sense of the imperfections of natural reason . . . fly to revealed truth" (Hume 1991, p. 185), enjoy revelatory personal theophanies, or, as Ronald Reagan used to say, confess, of the good part T, that it is simply what 'they happen to believe'.

9 It can be different strokes for different folks. We have in the Bayesian framing I have proposed for the issue, not an answer to the question of where the balance of reasons lies for persons on full reflection. It is only a clarification of the issue, and in particular of the way in which Æ tests a person's faith and the reasons he finds *for* G in K, that is, in his evidence 'the facts of evil in Æ aside'. People can differ here not only when they differ in 'experience and knowledge,' that is, in their total evidence (TI cf., Rowe 1993, p. 88), but also even when they agree in that. For people can differ in what they make of the same evidence, no matter how much cooperative thought they give to it. But once a person's mind is made up in certain ways, consistency can require that it be made up in other ways. The principle developed above for the evidential argumentation regarding perfect beings and evil,

$$[P'(\sim G/Æ) > 1/2] = [P'(Æ/\sim G)/P'(Æ/G) > P'(G)/P'(\sim G)],$$

states such a consistency-requirement. It gives a necessary and sufficient condition for a decision against G. It says that someone for whom what would been beforehand the *likelihoods* of G and ∼G for appearances of evil and suffering that a perfect being could have avoided for a better world are related in the manner of that condition to what would have been beforehand his *priors* for G and ∼G, must on pain otherwise of inconsistency consider ∼G more probable than not on the evidence Æ. If the inequality on the right is *great*, he must, on pain otherwise of inconsistency, consider of the two, perfect-being theism and perfect-being atheism, the latter 'by far the most probable hypothesis' on the evidence Æ.

APPENDIX. PROMISED DERIVATIONS

Section 6.1.3.1. "P comes to,

$$\sim(\exists x)(G^1x \,\&\, (\exists y)[K^1y \,\&\, J^2yx])$$

or, equivalently,

$$\sim(\exists x)G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2yx]),$$

which is equivalent to

$$\sim (\exists x)G^1x \vee [(\exists x)G^1x \,\&\, (x)(G^1x \supset\sim (\exists y)[K^1y \,\&\, J^2yx]).$$

A derivation for the first equivalence – see Appendix C of Chapter II for rules of inference and forms of derivation.

1.	~~*SHOW*~~ $\sim(\exists x)(G^1x \,\&\, (\exists\, y)[K^1y \,\&\, J^2yx]) \equiv$ $[\sim(\exists x)G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2yx])]$	DD
2.	~~*SHOW*~~ $\sim(\exists x)(G^1x \,\&\, (\exists y)[K^1y \,\&\, J^2yx]) \supset$ $[\sim(\exists x)G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2\, yx])]$	CD
3.	$\sim(\exists x)(G^1x \,\&\, (\exists y)[K^1y \,\&\, J^2yx])$	assumption
4.	~~*SHOW*~~ $\sim(\exists x)G^1x \vee (x)(Gx \supset \sim(\exists y)$ $[K^1y \,\&\, J^2yx])$	ID
5.	$\sim [\sim(\exists x)\, G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1\, y \,\&\, J^2yx])]$	assumption
6.	$\sim \sim(\exists x)G^1x \,\&\, \sim(x)(G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2yx])]$	5, DeMorgan $[\sim(\Phi \vee \Psi)/$ $\therefore (\sim \Phi \,\&\, \sim \Psi)]$
7.	$\sim(x)\, G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2yx])$	6, S
8.	~~*SHOW*~~ $(x)\, (G^1x \supset \sim(\exists y)[K^1y \,\&\, J^2yx])$	UD
9.	~~*SHOW*~~ $G^1x \supset \sim(\exists y)\, [K^1y \,\&\, J^2yx]$	CD
10.	G^1x	assumption
11.	$\sim (G^1 \,\&\, (\exists y)\, [K^1y \,\&\, J^2yx])$	3, QN, UI
12.	$\sim (\exists y)\, [K^1y \,\&\, J^2yx]$	11, DeMorgan, 10, DN, MTP
13.	~~*SHOW*~~ $[\sim (\exists x)G^1x \vee (x)\, (G^1x \supset \sim\, (\exists y)K^1y$ $\&\, J^2yx])] \supset \sim(\exists x)(Gx \,\&\, (\exists y)\, [K^1y \,\&\, J^2yx])$	CD
14.	$\sim(\exists x)G^1x \vee (x)(Gx \supset \sim(\exists y)K^1y \,\&\, J^2yx])$	assumption
15.	~~*SHOW*~~ $\sim(\exists x)(G^1x \,\&\, (\exists y)[K^1y \,\&\, J^2yx])$	ID
16.	$(\exists x)(G^1x \,\&\, (\exists y)\, [K^1y \,\&\, J^2yx])$	assumption
17.	$G^1a \,\&(\exists y)\, [K^1y \,\&\, J^2ya]$	16, EI
18.	$(\exists y)\, [K^1y \,\&\, J^2ya]$	17, S
19.	$(x)\, (G^1x \supset \sim(\exists y)\, [K^1y \,\&\, J^2yx])$	17, S, EG, DN, 14, MTP
20.	$G^1a \supset\ \sim(\exists y)\, [K^1y \,\&\, J^2ya]$	19,UI
21.	$\sim (\exists y)\, [K^1y \,\&\, J^2ya]$	17, S,20,MP
22.	$\sim(\exists x)(G^1x \,\&\, (\exists y)\, [K^1y \,\&\, J^2yx]) \equiv [\sim(\exists x)G^1x$ $\vee(x)(G^1x \supset \sim(\exists y)\, [K^1y \,\&\, J^2yx])]$	2, 13, CB

A derivation for the second equivalence using Sentential Substitution. For formulas ϕ and ϕ' such that ϕ' comes from ϕ by uniform substitutions of formulas for some or all sentence letters in ϕ, if ~~*SHOW*~~ 'ϕ' occupies an available line for which no lines were 'entered from without,' so that ϕ was derived

'from its own resources' and is a theorem,

$$\phi \; / \phi'.$$

1.	~~*SHOW*~~ $[\sim (\exists x)G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1y \; \& \; J^2yx])]$ $\equiv [\sim(\exists x)G^1x \; \& \; (x)(Gx \supset \sim(\exists y)[K^1y \; \& \; J^2yx])]$	DD
2.	~~*SHOW*~~ $(\sim P \vee Q) \equiv [\sim P \vee (P \; \& \; Q)]$	DD
3.	~~*SHOW*~~ $(\sim P \vee Q) \supset [\sim P \vee (P \; \& \; Q)]$	CD
4.	$\sim P \vee Q$	assumption
5.	~~*SHOW*~~ $\sim P \vee (P \; \& \; Q)$	ID
6.	$\sim[\sim P \vee (P \; \& \; Q)]$	assumption
7.	$\sim\sim P \; \& \; \sim(P \; \& \; Q)$	6, DeMorgan
8.	$P \; \& \; Q$	7, S, DN, 4, MTP, Adj
9.	$\sim(P \; \& \; Q)$	7, S
10.	~~*SHOW*~~ $[\sim P \vee (P \; \& \; Q)] \supset (\sim P \vee Q)$	CD
11.	$\sim P \vee (P \; \& \; Q)$	assumption
12.	~~*SHOW*~~ $\sim P \vee Q$	ID
13.	$\sim(\sim P \vee Q)$	assumption
14.	$\sim\sim P \; \& \; \sim Q$	13, DeMorgan
15.	Q	14, S, 11, MTP, S
16.	$\sim Q$	14, S
17.	$(\sim P \vee Q) \equiv [\sim P \vee (P \; \& \; Q)]$	3, 10, CB
18.	$[\sim(\exists x)G^1x \vee (x)(G^1x \supset \sim(\exists y)[K^1y \; \& \; J^2yx])] \equiv [\sim(\exists x)G^1x \vee [(\exists x)G^1x \; \& \; (x)G^2x \supset \sim(\exists y)[K^1y \; \& \; J^2yx])]$	2, Sentential Substitution, P: $\sim(\exists x)G^1x$ Q: $(x)(G^1x \supset \sim(\exists y)[K^1y \; \& \; J^2yx7)$

Postscript to Section 8. 'Evil or the absence of good would incrementally disconfirm a particular theism if and only if that good or the absence of that evil would confirm this theism:

$$[Pr(E) > 0 \; \& \; Pr(\sim E) > 0] \rightarrow [Pr(T/E)] < Pr(T)$$
$$\leftrightarrow Pr(T/\sim E) > Pr(T)].'$$

1.	~~*SHOW*~~ [Pr(E) > 0 & Pr(∼E) > 0] → [Pr(T/E) < Pr(T) ↔ Pr(T/∼E) > Pr(T)]	CD
2.	Pr(E) > 0 & Pr(∼E) > 0	assumption
3.	~~*SHOW*~~ Pr(T/E) < Pr(T) → Pr(T/∼E) > Pr (T)	CD
4.	Pr(T/E) < Pr(T)	assumption
5.	~~*SHOW*~~ Pr(T/∼E) > Pr(T)	ID
6.	∼[Pr(T/∼E) > Pr(T)]	assumption
7.	[Pr(T/∼E) < Pr(T)] ∨ [Pr(T/∼E) = Pr(T)]	6:∼(x > y) entails [(x < y) ∨ (x = y)]
8.	~~*SHOW*~~ Pr(T/∼E) < Pr(T) → 1 ≠ 1 (!)	CD
9.	Pr(T/∼E) < Pr(T)	assumption
10.	Pr(∼E & T)/Pr(∼E) < Pr(T)	9, 2, S, definition of conditional probability
11.	Pr(∼E & T) < Pr(T) · Pr(∼E)	algebra
12.	Pr(E & T) < Pr(T) · Pr(E)	4, 2, S, definition of conditional probability, algebra
13.	Pr(∼E & T) + Pr(E & T) < Pr(T) · Pr(∼E) + Pr(T) · Pr(E)	11, 12: (x < y) and (w < z) entail (x + w) < (x + z)
14.	Pr(T) < Pr(T) · Pr(∼E) + Pr(T) · Pr(E)	13: □(p ≡ (q ∨ q'))and ∼◇(q & q') entail P(p) = P(q) + P(q')
15.	1 < Pr(∼E) + Pr(E)	14, algebra
16.	1 < 1	15: Pr(∼E) = 1 − P(E)
17.	1 ≠ 1	16
18.	~~*SHOW*~~ Pr(T/∼E) = Pr(T) → 1 ≠ 1	CD
	Similar to (9) − (17).	
19.	1≠1	7, 8, 18, SC
20.	~~*SHOW*~~ Pr(T/∼E) > Pr(T) → Pr(T/E) <Pr(T)	CD
	Similar to (4) − (19).	
21.	[Pr(T/E) < Pr(T) ↔ Pr(T/∼E) > Pr(T)]	3, 20, CB

XII

The Logical Problem of Evil

> **Philo:** *Epicurus*' old questions are yet unanswered. Is he willing to prevent evil, but not able? then is he impotent. Is he able, but not willing? then is he malevolent. Is he both able and willing? whence then is evil?
>
> (Hume 1991 Part 10, p. 157)

The problem of evil 'proper' is the *prima facie* logical incompatibility of the existence of a perfect being – a being who would be omnipotent, omniscient, and perfectly good *inter alia* – and evil.[1] Other problems challenge perfect-being theism with plausible contingencies other than its bare existence, evidence for which is observed, for example, that this is not the best of all possible worlds. I count all of the original problem of these other problems as problems of evil and identify 'the problem of evil' with the family of them. In this chapter arguments in a series of this family are considered, starting with 'Epicurus' old questions' made into the Argument **from Evil**, proceeding to the Argument from **the World's Not Being a Best World**, next to the Argument from **the World's Not Being a Best Divinely Creatable World**, after that to the Argument from Not Being a Best Divine Bet World, then to the Argument from **There Being a Better World Than This One**, and finally to a rearrangement of that argument into a **Dilemmatic Argument** that rearranges the previous one under the disjunction, either there is a best possible world or there is not. Movement from one argument to the next, except to the last one, is consequent to concessions, never forced, to objections made to the previous argument.

Each argument is a 'logical' challenge: Each maintains that some contingent proposition is plausible, and that it is *logically incompatible* with perfect-being theism. "What a surprise," some may say. "Isn't the logical problem of evil supposed to be yesterday's news?" Yes. One hears that "[l]ogical arguments from evil are a dying (dead?) breed" (Draper 1996, p. 175). "It is," Alston reports, "now acknowledged on almost all sides that the logical argument is bankrupt" (Alston 1996, p. 97). "Alvin Plantinga's 'Free Will Defense' is intended to establish that it is logically possible both that God exists and

that evil exists. Given certain moral and metaphysical assumptions (including incompatibilism), Plantinga's argument for this conclusion is, I believe, fairly compelling" (Rowe 1998b, p. 115). "So far as I am able to tell, this thesis [that evil is incompatible with the existence of God] is no longer defended" (van Inwagen 1996, p. 151). "It is now widely acknowledged that the Free Will Defense [tricked out by Plantinga's hypothesis of the logical possibility that every creaturely essence suffers from transworld depravity] adequately rebuts the logical problem of evil" (Peterson 1998, p. 47). The word has gotten around, "The Logical Problem of Evil is no longer a problem."[2] I do not believe it.[3]

1. THE ARGUMENT FROM EVIL

(1) *Evil exists*: Evl. (2) The existence of evil is incompatible with the existence of a perfect being:
$(\Diamond \text{Evl} \ \& \ \Diamond \text{PrfBng}) \ \& \sim \Diamond(\text{Evl} \ \& \ \text{PrfBng})$[4] $\therefore$ (3) There does not exist a perfect being: $\sim$PrfBng
(Evl: evil exists; PrfBng: a perfect being exists)

1.1. The argument is valid. Premise (2) entails that it is necessary that if evil exists, a perfect being does not exist: $\Box(\text{Evl} \supset \sim\text{PrfBng})$. It is obvious that that and premise (1), Evl, entail (3), $\sim$PrfBng. Premise (1) is viewed by all but desperate opponents of the argument as incontestable, but it is not necessarily true. For there is evil only if there are things that suffer, do, or are evil. Necessary things – numbers, forms, abstract entities, and so on – can neither suffer, do, nor be evil. It is not necessary that there are contingent beings. Therefore it is not necessary that there is evil.[5] Premise (2), if true, is necessarily true, for its symbolization is 'fully modalized'. Each sentence abbreviation in it is 'covered' by either the necessity or the possibility operator. This premise, if not true, is necessarily not true.

1.2 *Critical discussion.* First come desperate challenges to premise (1), and then a possibly good objection to premise (2) that will move us to the next argument in our series.

1.2.1. To raise an 'epistemological objection' to premise (1) it might be said that there is no evil, that all appearances of it are *illusory*. A person could believe this if he was sure that (i) a perfect being exists and that (ii) evil is incompatible with the existence of a perfect being. Such a person would need to think that all is well, not merely on balance, but through and through. He would need to think that there is not merely no unnecessary and avoidable evil, but that there is no evil at all. Not denying that there are *appearances*

of evil, this believer would not be a *sceptic* regarding 'the testimony of his senses', moral or natural, who *doubted* the veridicality of every appearance of evil. He would believe that every appearance of evil was *nonveridical*. (Madness!)

Resisting in a somewhat different manner the first premise, it might be said – to raise a curious 'semantical objection' – that while not all is well by *human* standards, and in *human terms*, all would be well and good by God's standards and in That One's terms, which are *authoritative* for what is really well and good. This objection is, however, if anything, weaker than the previous crazy one. For if God's standards would be very different from ours, so different that the many things we count as bad, horrible, abominable would all be *good and pleasing* in God's eyes, then it is not true that in our terms, and by our standards, God would be *good*. Far from it!! And it is of course our term 'good' and our standards that are *relevant*, for it is with this term and these standards in mind that we *speak* of a perfectly good being whose compatibility with 'evil,' our term, our standards, again, is at issue. I do not suggest that any defender of theism would find tenable that all of this that we abhor would be, item by item, pleasing to God's eyes, and thus really good. The objection floated for comment in this paragraph is almost certainly not available on the least reflection to any actual theist or believer in a *worshipful* being, for I think that The Devil would not be 'worshipful' properly speaking.

A third 'metaphysical objection' to the first premise could be that Evil is *nothing real*, that it is only an *absence*, a *privation*, of good. All that *is*, is good as *far* as it *is*. The way to deal, without wasting time, with this silly line is not to say that it does not work against the argument from evil, but to observe that it is of no avail against 'the argument from the absence of goodness' that we had not thought to raise instead, and maintain by Epicurean questions, so: "Why is there so much awful and painful *deficiency of being*, not only in forests during fire storms in which innocent fawns are consumed, but everywhere one looks? Does He intend it? No, for He is benevolent. Is it contrary to His intention? No, for He would be almighty. Is it by unanticipated accident? No, for He would know everything always."

1.2.2. Not surprisingly, all serious resistance to the argument has gone to its second premise. Here is one line that we will allow to move us to the next argument in our series.

A perfect being would necessarily make sure that the world is a *best* of all possible worlds, but *not* necessarily that it is *perfect*, and without the least tincture of moral or natural evil. Indeed, a perfect being would make sure that the *world* was *not* perfect, for the simple reason that every best possible world has some evil in it, since there are great goods such that, (i), any possible world without them is inferior to some possible world with them, and, (ii), they cannot be in the absence of certain evils. These evils are *necessary* for a best possible world, that is, it is necessary that a best possible world have some. Premise (2) of your argument is false. Far from its being

not possible that both evil exists and a perfect being exists, though each is alone possible; it is not possible that a perfect being exists *unless* some evil also exists. You could count on some from such a one. (That there is evil is some *evidence* that there *is* a perfect being! I could not resist tossing this in.)

1.2.3. Perhaps a best of all possible worlds *would* have some evil in it. That is by no means obvious. It is not at all obvious that the world is a better place for having in it evils that provide occasions for struggle and remedy, and sinners who can repent and mend their ways. Even if evils can be enhancing parts of wholes, it is not obvious that they would be enhancing parts of best whole worlds. I doubt that this is possible, but I think that it is 'epistemically possible' that it is possible, which is to say that I do not know that evil is *not necessary* for a best world. Let us concede that it is so, and concede on this ground that, contrary to premise (2), evil in the world is *not* incompatible with the existence of a perfect being.[6] Let us make these concessions to move on.[7]

2. THE ARGUMENT FROM THE WORLD'S NOT BEING A BEST WORLD

(1′) *The world is not a best of all possible worlds*: [(∃x)Bst(x) & ∼Bst@].[8]

(2′) That the world is not a best of all possible worlds, is incompatible with the existence of a perfect being: (◊[(∃x)Bst(x) & ∼Bst@] & ◊ PrfBng) & ∼◊([(∃x)Bst(x) & ∼Bst@] & PrfBng).

∴ (3) There does not exist a perfect being: ∼PrfBng

(Bst: *a* is a best of all possible worlds; @: the world (this world of ours, the actual world); PrfBng: a perfect being exists.)

I begin in the following with certain radical challenges to premise (1′) open only to those who run 'scratch arguments' for the existence of a perfect being. Next 'omniscience *cum* free will challenges' to premise (2′) are noticed to postpone detailed discussions of them to Appendix A. After that, premise (1′) is revisited. Certain pros and cons are set out concerned with sinners, and premise (1′) is found to be very plausible. Following that, two objections due to Alvin Plantinga and Robert Adams to premise (2′) are examined. Each involves the 'play' in our ideas of free agency, of subjunctive counterfactuals. After Plantinga's objections, comes to deal with them, The Argument from the World's Not Being A Best Divinely Creatable World. After Adams's, comes to deal with them, The Argument from the World's Not Being a Best Divine Bet World.

2.1. Radical resistance to the reasonableness of premise (1′). We do not have the best imaginable reasons for believing premise (1′). We have not compared the world in complex entirety with even one other possible world spelled out

in detail, and seen that this other world is *better*. Only an 'infinite mind' could be up to cosmic value judgments informed by such views. But we do have what are the best reasons for believing premise (1′) that are possible for finite minds. For we have that there are ever so many ways in which *in so far as we can see* the world could have been and be better than it has been and is. And we have no reason to think, were we to look and think longer, that we would change our minds and see the world as unimprovable.

There are *interesting* grounds for "extreme . . . moral scepticism . . . in matters unrelated to the concerns of everyday life" and in particular when it comes to judgments of truly cosmic proportions (van Inwagen 1995, p. 84). But they do not persuade that it is unreasonable to think that the invasion of humanity by the AIDS virus is not on balance a good thing. Of course we do not know what *in detail* would be the state of the world were this plague never to have been, but we *think* it would be *better*. *As far as we can see* it would be better in some ways, and worse in none. And we believe that, no matter how long we were to think about what we have reasons to believe about the natural world itself, it would be for us that as far as we could see the world have been better without AIDS. Which means, I contend, that it is not only *natural* but *reasonable* for us to think that it *would have been better without it*, that it would have been better had AIDS never been. Thinking that is as reasonable as thinking that *the world* ***will*** *be better once rid of it, if that day comes*, and better not mainly for the value of our triumph over it, if that is how it ends, but mainly because it is over and gone. My guess is that *everyone* thinks that, and thinks that it is *reasonable* to think that, though in 'substance' this thought is still 'subjunctive': Other words for those italicized are 'were the day to come that the world is rid of AIDS, it would be well rid of it'. Or should we wonder, "But what would we do with our compassion without it, what would challenge us so?"

2.2 *A radical stand against the truth of premise (1′).* It is not easy to quarrel with the first premise, though some have. Leibniz says that it is demonstrably false:

> [T]he world looks like a confused chaos [rather than a metaphysically and morally most perfect world in which minds are granted the greatest possible happiness and joy]. . . . But on closer inspection, the contrary must be stated. It is certain a priori, by the very reasons we have adduced, that all things and especially minds, obtain the greatest possible perfection. (Leibniz 1965, p. 91: 'closer inspection' makes an unkept promise)

Having argued for an ultimate reason for the world that exists of metaphysical necessity, Leibniz explained "a sort of Divine Mathematics or a Metaphysical Mechanism" (p. 87) that entails the greatest quantity of essence and perfection in which is manifest at once God's power and "his goodness and wisdom"

(p. 90). Leibniz thinks that we have 'tolerable reasons *a priori*' (Hume's words: 1991, p. 168) not for rejecting as false every appearance of evil, but for rejecting as false the appearance that this is not 'as good as it gets', that this is not a best of all possible worlds. Swinburne (1998) labors to detail reasons *a posteriori* for thinking of each kind of evil that exists in the world, that a perfect being would have been wise to risk it. He can be seen (by an inference that is at best problematic) from these reasons, a reason *a posteriori* for thinking of the evil that exists in the world, that a perfect being would have been wise to risk all of it. That is not far from Leibniz's conclusion that this is a best possible world. It is difficult for a perfect-being theist to distance himself significantly from this Panglossian bad joke.

2.3 *Alleged incompatibilities between the omniscience of a perfect being and freedom.* I make out of these alleged incompatibilities an objection to premise (2′). However, given the *unlikeliness* of this objection, discussion of the intrinsically interesting alleged incompatibilities themselves is left to Appendix A.

2.3.1 The objection. It starts with the following'finding':

> **IncFrOm** (incompatibility of freedom and omniscience). *That this world features free agents who from time to time exercise their freedom is incompatible with the existence in it of a being that is omniscient in it in the manner of a perfect being.*

This is the main thing that needs to be *established* for this objection: Possible arguments for IncFrOm are assessed in Appendix A. From IncFrOm it follows that there cannot be both freedom and a perfect, and therefore omniscient, being in this world.

> **IncFrPrfctn** (incompatibility of freedom and perfection). That this world features free agents who from time to time exercise their freedom is incompatible with the existence in it of a perfect being.

Premise (2′) entails (see its symbolization in Section 2, The Argument from the World's Not Being a Best World),

> (*): It is not possible that both the world is not a best possible worlds, and there exists in it a perfect being.

Given IncFrPrfctn, and the following two assumptions, we can reach the negation of (*).

> **PossPrfctn** (possibility of perfection). It is possible that there exists in the world a perfect being.

> **FrAgtsBstWrlds** (free agents in best worlds). It is necessary that every best world features free agents who from time to time exercise their freedom.

Most opponents and defenders of perfect-being theism accept FrAgtsBstWrlds. Considerations can persuade a metaethicist that, if it is true, then it is necessarily true. Ethical objectivists, such as Plato, Leibniz, and G. E. Moore, think that every value principle, framed as this one in only 'universal' terms, is, if true, necessarily true. We will assume that FrAgtsBstWrlds is necessary. PossPrfctn is of course accepted by every perfect-being theist, and it is not questioned by opponents to their theism who think there is a job for 'demonstrations *quia* against' to do. PossPrfctn, if true, is also necessary. It follows from IncFrPrfctn and FrAgtsBstWrlds that, (a), it is necessary that, if the world is a best world, then there does not exist in it a perfect being. It follows from (a) and PossPrfctn that *it is possible* that both the world is not a best possible world, and there exists in it a perfect being, or equivalently that,

> ∼(*): *It is not the case that it is not possible* that both the world is not a best possible world and there exists in it a perfect being.

A derivation of ∼(*) from IncFrPrfctn, FrAgtsBstWrlds, and PossPrfctn is given in Appendix B.

2.3.2. Arguments for the alleged incompatibility of perfect-being omniscience and freedom have been left to Appendix A for several reasons, led by the reason that this objection to premise (2′) founded on them is an *unlikely* objection. Here is why. Someone who not merely mounts for its interest, but sincerely makes this objection to premise (2′), must accept both PossPrfctn, that it is possible that a perfect being exists in this world, and IncFrPrfctn, that freedom and perfect beings are incompatible. Who then can be expected to make these objections to premise (2′)? For two reasons, not persons who believe that *there does not exist* a perfect being, that is, not persons who accept the conclusion of The Argument from the World's Not Being a Best World. First, it is unlikely that persons who *accept* the *conclusion* of the argument would be moved to object to this argument in any way! And second, someone who believes that there does not exist a perfect being should believe that, contrary to PossPrfctn, it is *not* possible that there exists in this world – this very world of ours – a perfect being. For it *is* possible that *there exists in this world a perfect being*, that is, it is true that ◊P@ – where 'P' is here a predicate of *worlds*, 'P@' says that there exists in @ a perfect being – if and only if it is true that P@, and indeed that □P@: It is a general principle of *propositions about particular worlds* that what is true of a particular world at *some* world is true of *that particular world* at *every* world![9] Nor can such objections be expected from persons who reject

the argument's conclusion and believe that *there exists* a perfect being. Such believers can be expected to believe that, contrary to IncFrPrfctn, freedom is compatible with perfect-being omniscience. Believers can be expected to be interested in *defending* this compatibility. They cannot do that, and, 'on the next page', be against it in order to meet the challenge of The Argument from the World's Not Being a Best World.

2.4 An argument from moral evil for premise (1′)

2.4.1 Mackie makes it. We, theists and nontheists, are apt to agree that a best world would feature free agents, and it seems that, furthermore, a best world would feature good and well-behaved ones. It is a *possibility*, Mackie contends, that "beings who . . . act freely . . . always go right" (Mackie 1973, p. 213), and that it is not only a possibility, but "the obviously better possibility" (p. 213). He allows that one can, without *a priori* contradiction, say that God's excellent purposes would be best served by free sinners. There is, he naughtily reminds, scripture for the idea that sinners would figure in His plans: "'[J]oy shall be in heaven over one sinner that repenteth, more than over ninety and nine just persons, which need no repentance' [Luke 15:7]." But, Mackie continues,

> however good the authority for it, this is a very strange view . . . it would be hard . . . to endorse the sober evaluation that sin plus repentance is, as an organic whole, better than sinlessness. . . . [A] father may rejoice more over the return of a prodigal son than over another's merely constant good behaviour. . . . But it does not follow that the father prefers on the whole to have a prodigal son who ultimately returns than to have a constantly well-behaved one. (Mackie 1982, p. 159)

What would we think of a father who *had* that preference? What would we think of a father who *did something* to serve that preference?! It is not *a priori* impossible that God's excellent purposes would be served by there being some free sinners, but, Mackie tries to persuade, it is *really* impossible. The truth for theists must be that it is necessary that there are no sinners in best worlds. From that, and the hardly questionable fact that there are sinners, it follows that the world is not a best possible world. That is premise (1′).

N.B. Mackie mounts his argument against perfect-being theism on an implication of that position, namely, that there are objective values. It is part of the position of perfect-being theism that God would be perfectly good, *objectively*, and would want the world to be as objectively good as it can be. Mackie frames his argument against it in terms licensed by this position, though these are terms in which he cannot consistently frame positive theses of his own. He does not believe in objective values. He thinks that the sentence 'there are no sinners in *best* worlds,' which would predicate objective values to worlds, does not express a proposition that is true or false at any world. In his opposition to theism, Mackie speaks with the theists in the language of ordinary thought and talk about values to simplify his opposition, which could have been *deeper*. His

objection could have been that (1) theism has certain metaphysical presuppositions to do with objective values; (2) if these presuppositions are false, then theism is not true; and (3) if they are true, then theism is false, because of the presence in the world of moral evil. When framing and discussing objections to perfect-being theism, to keep things simple, I follow Mackie's lead and use the language of objective values, though I agree with him and do not believe in them. Further, with him, I take for granted G. E. Moore's view of objective value supervening with logical necessity on natural properties, as spelling out part of what objective values would be, if there were any.

2.4.2 Hicks's opposition. Hicks argues that all's well that ends well, and that soul-making, not happiness and virtue, would have been God's goal for our lives. Therefore – I am leaving out details – best worlds feature moral *failures* as well as *cherished triumphs*. So Mackie is wrong. In *best* worlds free agents do *not* always go right.

2.4.2.1 DETAILS IN HICKS'S WORDS. The first stage of the creative process that culminated in

> the development of man as a rational and responsible person capable of personal relationship with the personal Infinite who has created him . . . was, to our anthropomorphic imaginations, easy for divine omnipotence. [Hicks 1994 , p. 178a] But the second stage of the creative process [in which man advances 'towards that quality of personal existence that is the finite likeness . . . revealed in the person of Christ'] is of a different kind altogether. [p. 178a [177b]] For personal life is essentially free and self-directing. It **cannot** be perfected by divine fiat, but only through uncompelled responses and willing co-operation of human individuals in their actions and reactions in the world in which God has placed them. Men may eventually become the perfected persons whom the New Testament calls 'children of God,' but they **cannot** be created ready-made as this. The value-judgment that is implicitly being invoked here is that one who has attained to goodness by meeting and eventually mastering temptations . . . is good in a richer and more valuable sense than would be one created *ab initio* in a state either of innocence or of virtue. In the former case . . . the individual's goodness has within it **the strength of temptations overcome**, a **stability based upon an accumulation of right choices**, and a **positive and responsible character that comes from the investment of costly personal effort**. I suggest, then, that it is an ethically reasonable judgment, even though in the nature of the case not one that is capable of demonstrative proof, that human goodness slowly built up through personal histories of moral effort [and – I interject – occasional actual sin] has value in the eyes of the Creator which justifies even the long travail [and – witness history – havoc] of the [sin-ridden] soul-making process. (pp. 178a–179b; emphasis added)

2.4.2.2 CRITICISM. Hicks does not say that God could not by *fiat* have created man perfect, that we could not have been created 'ready-made' for Himself perfect children. Taking words in their contexts, which can out of context suggest otherwise, we see that what he thinks is that it is *better* that men

should progress through pain, **and through their own actual moral errors**, to "the perfected being[s] whom God is seeking to create" (p. 178b). *But why?* Hicks's reasons are that moral strength and a positive and responsible character "slowly built up through personal histories of moral effort," *and occasional error and sin*, which Hicks conveniently neglects *explicitly* to mention, have more "value in the eyes of the Creator" than would precisely that strength and positive, responsible character attained without pain, and again, this being essential to his argument, *without actual moral error* (p. 178b). Really? Well this is what he 'says.' He concedes that the comparative value-judgment he invokes, and on which his argument hinges, is not capable of demonstrative proof. To this I add that it is on its face *implausible*, to my mind absolutely implausible. *Why* would God prefer men and women who work their way *through careers of evil-doing* to states of moral perfection, to men and women who *work* their way through "an accumulation of right choices," and *only* right choices, to the same end? Is God supposed to think that people can 'learn' from, and grow through, only actual mistakes, the more and the bigger, better for the growth they can occasion?!

To reprise and elaborate, I will comment on this retailing of Hicks's line: "[I]t is not clear that God can instantaneously create morally mature persons, since moral maturity almost certainly requires the experience of temptations and, according to some, the actual participation in evil" (Peterson et al., p. 108). For clarity and certainty regarding these matters, we may consider *morally mature persons* in relation to *processes* of moral maturation, and to their ideal *end-state*. Let it be analytic that a *morally mature*$_1$ person is not only *in* an ideal state of moral maturity, but has *reached* it by a process of moral maturation. Being morally mature$_1$ is like *being educated*. Let it be analytic that a *morally mature*$_2$ person is *in* an ideal state of moral maturity, whether or not reached by a process of moral maturation. Being morally mature$_2$ is like being *knowledgeable and intellectually skilled*. God *could not* have *instantaneously* created morally mature$_1$ persons, but presumably omnipotent God *could have* instantaneously created morally mature$_2$ persons. Also, perhaps both moral maturity$_1$ and moral maturity$_2$ – *supernatural agency aside* – require for their realizations encounters with temptations and actual participation in evil. However, even if growing up does involve, if not getting, then at least wanting to get, dirty, assuming that supernatural divine agency makes sense and is possible, there is no reason for thinking either that God could not, by supernatural means, have instantaneously created morally mature$_2$ persons, or that God could not have created persons and arranged for their maturations into morally mature$_1$ persons *without* having them ever *actually participate in*, or even *experience temptations to*, evil. They could read books of edifying imaginative fiction. (Compare with Plato's different views regarding medical and judicial wisdom, and experience in one's own person of illness and wickedness. He says that "[p]hysicians **would** prove most skilled if . . . they themselves had suffered all diseases and were not of very

healthy constitution" (*Republic* 408d–e, Shorey translation; emphasis added), but that for the soul of "a judge . . . [far from being required] it is not allowable for [it] . . . itself to have run the gauntlet of every kind of wrongdoing and injustice" (409a).

Assuming the *logical point* that God *could* have created persons who would be, when they grew up, morally mature$_1$ though they had never actually participated in evil, the substantive *value-issue* is whether God would have seen that it was *better* to do that, without creating any persons who actually sinned. Theodicizing apologists such as Hicks must say, *most implausibly*, that God would *not* have seen that, because as a matter of value-fact it is not *true*. The problem for this theist's value judgment is not that people experience *temptations*. Nor is the hard problem even that they encounter serious and sometimes *potentially* soul-destroying *adversities*. The *hard* problem for this theist's value judgment is that souls are sometimes destroyed and, worse, that good people in their progress toward moral perfection *succumb* to temptations and *participate in evil*. It is not *believable* that that is the only *logically* possible way to moral perfection. (Also recall that probably no one has ever made it to moral perfection, and that some have made their good what, for much of the world, is evil.)

2.4.3 A pass for premise (1′). Creating only beings whose lives always freely go right *might*, for all we know, not have been the best possibility open to a perfect creator: That is an 'epistemic possibility'. But it is hard to believe that, if it was a possibility open to a perfect creator, there was a *better* one open in which some beings' lives freely go wrong, badly wrong, sometimes. More simply, and without inviting questions concerning options open to a perfect creator, it is hard to believe that, while there are worlds in which free agents behave well all of the time, there are *better* worlds than any of those in which at least some free agents lead morally chequered careers. *Suppose* that deliberately and with foreknowledge creating only blameless free agents to love and be loved by *would* have been a possibility for a perfect creator. Few persons will, on sober reflection, find that they are enthusiastic for the idea that even though a possibility, a perfect creator would not have actualized it. Few will be enthusiastic for the idea that in This One's mysterious wisdom the choice would *not* have been to create only blameless and happy free spirits, but would instead have been for several miserable free sinners among whom some would repent and get good. Few will be enthusiastic for the suggestion that the best of all possible worlds includes for that, or *any* other reason, free sinners. And so most, surveying the presence in history and today of not a few 'seriously evil agents', of not a few 'megabad agents', will consider premise (1′) to be almost certainly true. (Thanks to John Skorupski for these phrases.)

2.4.4 'Back to premise (2).' Mackie has an argument for premise (1′). It is (a) *best* possible worlds all feature free agents who always go right, and none

who ever go wrong; and (b) there are in the world free agents who sometimes go wrong, that is, there is 'moral evil' in the world; therefore, premise (1′), the world is not a best of all possible worlds. Hicks opposes (a). He says that best worlds feature free agents who mature through their moral struggles and sometimes lose. He does not, however, make plausible that not only their struggles, but their sometime failures, are important to their worth, and the worlds's values. Furthermore, even if he had, that would mean only that Mackie's *argument* for premise (1′) is unsound, not that premise (1′) is not true. So we are back to the point reached earlier, that to fault The Argument from the World's Not Being a Best World, one needs to fault its *second* premise and explain why the existence of a perfect being is *not* incompatible with the world's not being a best world. Plantinga and Robert Adams, attending to 'the logic of counterfactuals of freedom', offer explanations. They mount logically sophisticated **Free-Will Defenses** that go to our argument's second premise. According to Plantinga, it is *possible* for free-will–related reasons that no one, not even a perfect being, *could have made* this world a best world. According to Adams, though God could have done that, that is, though that is how His creative efforts could have worked out, not even He *could have made sure* that He did that, and that that was the way His creative efforts were going to work out.

2.5 Plantinga's touted objection to premise (2′). Here comes an introduction to it, in my words.

> Best worlds would feature free beings who, while exercising their freedom in morally significant ways, never misused it. So premise (1′) is true. To this you add that a perfect being would have made sure that the world contained free beings with such careers. Your argument, given that evaluative judgment on which we agree, is that (a) (necessarily) a perfect being *could have* created any possible world, and (b) (necessarily) a perfect being would have created the best world it could create. Your conclusion is premise (2′). On that we disagree, *and* we disagree on the 'subsidiary premise' (a) that takes you to premise (2′).[10] I am confident that there *are* worlds that it would have been beyond the power of a perfect being to create. What persuades me that it is *possible* that amongst them are all worlds in which free agents unerringly exercise their freedom would be one and all beyond the power of a perfect being to create is that it is possible that every possible person is 'transworld depraved'. Let me explain. (See Plantinga 1974a, pp. 34–53 [Plantinga 1974b, pp. 169–89]. References below to these texts will be distinguished here by the absence or presence of brackets.)

I have addressed Plantinga's objection to premise (2′) of The Argument from the World's Not Being a Best World. It can be addressed as well to the corresponding premise of what could be *an argument from moral evil* that comes from The Argument from Evil by putting 'moral evil' in place of 'evil'. In my explanation of Plantinga's objection, I will exercise, without I think prejudice, some license. The objection I will discuss, while not word perfect for

Plantinga's, is in essentials his. He concedes, I think, that best possible worlds are worlds in which there are free beings who never misuse their freedom, while maintaining that it is *possible* that none of these are worlds into which God could have made ours, because it is *possible* that persons in those worlds are in a manner depraved relative to this world of ours. The logic of 'counterfactuals of freedom' and some metaphysics are keys to this possibility, by being keys to the possibility of *universal transworld depravity*. (Oh my!)

2.5.1. 'Counterfactuals of freedom', the power of God, and 'Leibniz's Lapse'

2.5.1.1 'COUNTERFACTUALS OF FREEDOM'. Suppose that Paul has an aardvark, that you offer him $500 for it, and that of his own free will he turns you down. What if you had offered him $700 and he had been free to accept your offer or turn it down? Is it true that he would have of his own free will accepted that offer? The question is whether the following '**counterfactual conditional of freedom**' (my term) is true.

> (*Offer* $\Box\!\!\rightarrow$ *Accept*). If you had offered Paul $700 and he had exercised his freedom to accept or not to accept your offer, then he would have of his own free will accepted it.

[**N.B**. (*Offer* $\Box\!\!\rightarrow$ *Accept*) abbreviates 'If it were the case that *Offer*, then it would be the case that *Accept*)'; '*Offer*' abbreviates 'you offered Paul $700 and he exercised his freedom to accept or not to accept; and '*Accept*' abbreviates 'Paul, of his own free will, accepted your offer'.] That conditional is true, Plantinga assumes, if and only if a *related* counterfactual conditional is true. It is the counterfactual conditional that has as its antecedent, instead of the antecedent *Offer*, a proposition that affirms the state of affairs that is '**the maximal world segment for*(Offer*$\Box\!\!\rightarrow$***Accept)***.' This state of affairs – for short, *MWS(Offer*$\Box\!\!\rightarrow$*Accept)* – is to be the state of affairs that (a) includes your offering Paul $700 and his exercising his freedom to accept or not to accept your offer, (b) includes neither his accepting your offer nor his not accepting it, and (c) is "[f]or the rest . . . as much as possible like the actual world" (p. 42). "What you're really asking," Plantinga assumes for the counterfactual of freedom of his discussion, when you ask whether or not *(Offer*$\Box\!\!\rightarrow$*Accept)* is true, "is whether, under a *specific set of conditions*, Paul would have sold it [of his own free will]"[11] (p. 40). The state MWS (*Offer*$\Box\!\!\rightarrow$*Accept*) entails the state hypothesized in the antecedent of *(Offer*$\Box\!\!\rightarrow$*Accept)* and those conditions. It entails what one might 'intuitively term' **the initial segment** of the actual world to the time of Paul's of-his-own-free-will acceptance or nonacceptance of the offer [Plantinga 1974b, p. 176ff]; it entails "causal or natural *laws*" [p. 178] of the actual world, all of them I think Plantinga would say; and it entails more, though perhaps nothing important to the argument. It entails precisely what Plantinga would say was that 'specific set of conditions' recently mentioned, which, intuitively, is everything of possible relevance to

the truth-value of *(Offer□→Accept)*, 'and then some that makes no difference' (cf., [p. 176]). The counterfactual conditional that he assumes is equivalent to *(Offer□→Accept)* has, instead of the antecedent *Offer*, the antecedent *MSW (Offer □→Accept)*; it has the same consequent, *Accept*. This counterfactual conditional is as follows:

If MWS(*Offer□→ Accept*) had obtained, Paul would have of his own free will accepted your offer of $700. or, for short, [*MSW(Offer□→Accept)* □→ Accept] or, for shorter, *(Offer+ □→ Accept)*. It is, relative to *(Offer□→Accept)*, a 'loaded' counterfactual of freedom, in that its antecedent entails everything of possible relevance to the truth-value of this simpler conditional of ordinary discourse.

Plantinga *assumes* that some counterfactuals of freedom can be 'nonvacuously' true, which is to say true ones that have possibly true antecedents. Suppose that these counterfactuals of freedom, (*Offer □→ Accept*) and *(Offer+ □→ Accept)*, are true and that their antecedents are present possibilities. This supposition entails several 'metaphysical assumptions' (Rowe's term: l998b, p. 115), or better, 'metaphysical/axiological/logical assumptions'. First comes application to these conditionals of a supposed condition for the truth of any subjunctive conditional of the form, (if it were the case that p, then it would be the case that q), regardless of the character of its antecedent condition: This is for the conditionals in hand that there *is* a unique 'maximal world segment' for *(Offer □→ Accept)* – it is the condition that *the* maximal world segment for *(Offer □→ Accept) exists*. This logical assumption, let it be ***[MA1]***, rules out two things for a counterfactual of freedom that satisfies it. It rules out that there should be *several* ways, or worlds, in which (a) and (b) are met, *and* also (c), which is that everything else is as much like the way things are in the actual world as is possible given that (a) and (b) are met. And it rules out that there is *no* such way or world, but only an endless progression of ways or worlds in which everything else is more and more like the way things are in the actual world. Second comes a condition that explains somewhat the intended freedom of these conditionals, which is to be *valuable* freedom, the freedom that might be said to be for its value present in any best world. The metaphysical/axiological condition of freedom is that, if the antecedent of *(Offer+ □→Accept)*, which includes Paul's *exercising his freedom to accept or not to accept an offer*, presents a possibility, there is a possible world in which the antecedent-state of (*Offer+ □→Accept*), namely, MWS(*Offer□→Accept*), obtains, and Paul freely accepts the offer, and *another* possible world in which that antecedent-state obtains, in *which* world Paul does *not* accept the offer. While if *(Offer+ □→Accept)* is true, Paul of his own free will accepts the offer in the 'closest worlds' in which MWS(*Offer □→Accept*) obtains, there must be other worlds (they will be 'farther out') in which this antecedent-state obtains and he, of his own free will, does not accept the offer, *if he is to be free in the intended sense*. Freedom in the intended sense includes *freedom of choice* and entails *the possibility of choosing and acting otherwise*. That such freedom

is *possible* for humans is a metaphysical assumption of Plantinga's argument. That it is valuable to the point of its being necessary that there is some of it in any best world is an axiological assumption of his argument. Let this metaphysical/axiological assumption be ***[MA2]***. This assumption, which goes to the sense in which Paul would be free with respect to accepting or not accepting, and would act of his own free will, 'says' that the laws and the initial segment of the actual world to the time of Paul's decision to accept the offer or not, do *not* entail what he will do, accept it or not accept. It may be recalled that MSW(*Offer* $\Box\!\!\rightarrow$ *Accept*) includes the laws and that initial segment. Cf.: "If S is free with respect to [an] action, then it is causally or naturally possible both that *U* [the state of the universe up to its time] holds and S *take* (or decide to take) the action, and that *U* hold and S *refrain* from it"[12] [p. 166, cf., p. 170]. A third condition that goes to the intended freedom of these conditionals is coming.

2.5.1.2 "THE POWER OF GOD" (PLANTINGA 1974A, P. 41). Plantinga assumes that there are nonvacuously true counterfactuals of freedom, and that a nonvacuously true 'loaded' counterfactual of freedom is true that antecedent of which by design includes everything of possible relevance to the resolution of the agent's choice.[13] For definiteness, we may let the 'loaded' counterfactual of freedom *(Offer+* $\Box\!\!\rightarrow$*Accept)* be true. "We are now in a position to grasp an important fact," (Ibid.), namely, that it follows from our assumptions that, supposing that God exists, there is a possible world "beyond the power of God to create" (Ibid.). The stipulation for definiteness that ***(Offer+*** $\Box\!\!\rightarrow$ ***Accept)*** is true entails, by *[MA2]*, that there is a possible world in which it is true that ***(Offer+ & ∼Accept)***. [**N.B**. '∼*Accept*' abbreviates 'Paul, of his own free will, *did not* accept your offer'; it does not abbreviate '*it is not the case that* Paul, of his own free will, accepted your offer', the negation of what '*Accept*' abbreviates.] That stipulation entails that any such world is 'beyond the power of God'. *Suppose*, for purposes of an indirect argument to the contrary, that *it is* ***not*** *beyond the power of God to make a world in which it is true that (Offer+ & ∼Accept)*. Suppose, indeed, that God were to do that, *that is*, suppose that God were, starting early, indeed working away throughout time, to make over the world (our actual world in which He resides) so that it instantiated such a possible world. Then he would have made true that ***∼Accept***, *and* that *Offer+*. But that means, since it is true in the actual world that *(Offer+* $\Box\!\!\rightarrow$*Accept)*, he would have made true ***Accept***. *Contradiction*. Could not God have 'undone' that counterfactual of freedom and put in its place the contrary counterfactual of freedom *(Offer+* $\Box\!\!\rightarrow$ *∼Accept)*? No. To suggest metaphorically why not, it is because this would then not be a counterfactual of *Paul's* freedom, but of *God's*.

Look at it this way. *Offer+* includes Paul's being free to accept or not to accept and exercising this freedom. If God actualizes *Offer+*, then the counterfactual *(Offer+* $\Box\!\!\rightarrow$ *Accept)* says that Paul exercises that freedom in acceptance. "If, on the other hand, God had [actualized the rest of Offer+

and] *brought it about* that Paul didn't sell [accept] or had *caused him to* refrain from selling, then Paul would not have been free with respect to this action . . . [and Offer+] would not have been actual (since [Offer+] includes Paul's being free with respect to [that action]) . . . " (pp. 41–2). In that case God would not have actualized *Offer+* and *~Accept*. Suppose, however, that, *per impossibile*, God had done *that*. Then it would be not Paul, but God, who 'first chose' Paul's response to your offer, which is not consistent with the sense of freedom intended in *Offer +*, which involves the choice of a response to your offer being Paul's *alone*. This part of Plantinga's argument for limits on God's power uses a feature of the freedom Plantinga intends *other than* that it entails the possibility of acting otherwise. The intended freedom would be not only, as already said, 'freedom of choice', but also *freedom of authorship*, that freedom in which a person's actions are *his own alone*, that freedom in which his will, *and no one else's before it*, is a sufficient cause of his action. 'Frankfurt-cases' are designed to prize apart these freedoms while demonstrating that of the two only the latter is important for moral responsibility. That freedom of authorship is possible for humans, and that it is valuable to the point of being present in every best world, is, we may say, a metaphysical/axiological assumption ***[MA3]*** of Plantinga's argument. It is important to Plantinga's defense that the freedom of it is *both* 'freedom of choice' *and* 'freedom of authorship'.

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All so far has supposed that God exists. If God does not exist, but is possible, then there is a world that is beyond God's power in every world in which he exists. On the supposition that God does not exist in this world, *this* world is beyond God's power in worlds in which he exists. "Clearly the only worlds within God's power to actualize [would be] those that include his existence" [p. 170]. This point having been made, we may, as Plantinga does, let it disappear much of the time: "[W]e [may much of time] restrict our attention to these worlds" in which God exists (Ibid.). In Plantinga's view that is not a "real restriction" (Ibid.).

2.5.1.3 "LEIBNIZ'S LAPSE" (P. 44 [PP. 180FF]). "It is, of course, up to God whether or not to create Paul and also up to God whether or not to make him free with respect to the action of [accepting your offer]. . . . But if He creates [Paul] and creates him free with respect to this, then whether or not he actually performs this action is up to [Paul] – not God" (p. 44), and whether or not he actually performs it, there is, by the argument of the previous section, a world in which Paul freely acts otherwise, that is, a world that God cannot create. Leibniz held that God, in His omnipotence, "could have created any possible world He pleased. We now see that this contention – call it 'Leibniz's Lapse' – is a mistake" (p. 44). Plantinga is saying that, in the possible world that this world of ours instantiates, if there are true counterfactuals of freedom, and

God exists, He did not, contrary to Leibniz, have it in His power to create just *any* world or *cosmos*, just any aggregate of all contingencies. Counterfactuals of freedom *constrained* his choices. They are in a manner fixed *independent* of His choices. That is the hard part, the 'hard core,' of Plantinga's argument. *God's operating to make a counterfactual freedom true that is contrary to a (nonvacuously) true counterfactual of freedom would so far destroy the freedom of its subject, and so* FAIL *to make true a contrary counterfactual* OF FREEDOM. The conclusion reached, that it is *possible* that a perfect being *could not* have created *just any* world, spoils the argument sketched in the first paragraph of Section 2.5 for premise (2′): That argument uses the premise, (a), that necessarily a perfect being *could* have created just any world. Of course, to spoil that argument is not to show that it conclusion, premise (2′), is false. This is Plantinga's next order of business, for which he has made up *the idea of 'transworld depravity'*.

2.5.2 The idea explained. Mayor Curley Smith did a bad thing. He accepted a bribe from L. B. Smedes of $35,000. Suppose that Curley could have been bought by Smedes for less. Suppose it is true that:

> (*S-Offer* $\Box\!\!\rightarrow$ *C-Accept*): If Smedes had offered Curley a bribe of $20,000 and Curley had exercised his freedom to accept or not to accept that offer, then he would have of his own free will accepted it.

Then, given our metaphysical assumptions, it is true as above that,

> (*S-Offer*+ $\Box\!\!\rightarrow$ *C-Accept*): If MWS(*S-Offer* $\Box\!\!\rightarrow$ *C-Accept*) had obtained, Curley would have of his own free will accepted Smedes offer of a bribe of $20,000.

And, as above, it would have been beyond the power of even an omnipotent to have brought it about that (S-Offer+ & ~C-Accept). Curley was a bad boy. But so far, according to our story, not 'modally speaking very bad.' There is a world in which Curley does a certain right action, which world not even an omnipotent could bring about. But that is not to say that Curley was 'incorrigible', as he would have been if not even an omnipotent (of our world) could have brought about any world in which Curley of his own free will does right actions, and only right actions (for example, a world in which he lives a short happy life out of politics). We have not said that Curley was that bad. We have not said he was 'transworld depraved.'

Stipulations for an explanation. Let X be a possible free performance of Curley's such as his at a certain time accepting of his own free will a bribe from Smede of $20,000. Suppose X obtains in world W. Let **the maximal world segment of W relative to X** – $MWS_{W}(X)$ – be the state of affairs that (a) includes Curley's exercising his freedom with respect to X, (b) includes neither X's

obtaining nor X's not obtaining, and (c) is otherwise as much as possible like W. Let a possible performance be **morally significant** in a world if and only if its obtaining in this world would be wrong, and its not obtaining in this world would be right, or vice versa. Curley's accepting Smede's bribe of $35,000 was, we have been taking for granted, morally significant in the actual world. Lastly, let **the agent of a possible performance go wrong with respect to it** in a world if and only if this performance does not obtain and its not obtaining consists of his doing something that is wrong, or it obtains and its obtaining consists in his doing something wrong. With these stipulations in place, we say that Curley was *transworld depraved* if and only if, for *every* world W in which he does of his own free will some right actions and no wrong ones, there is a performance X of Curley's such that (i) Curley does X in W, (ii) X is morally significant in W, and (iii) *it is true in the actual world*, this world of ours, that if $MWS_{\mathrm{W}}(\mathrm{X})$ were to obtain, then X would go wrong with respect to X. It is a consequence, as above, that if Curley was transworld depraved, then no one other than he himself could have put him right. It is a consequence that it would have been beyond even God's power (for Curley, though powerful, was no god) to create a world in which Curley did some right actions of his own free will and no wrong ones.

The idea, somewhat roughly put, is that if Curley was transworld depraved, then for every possible world W in which he is good and pure, and does right actions of his own free will, and no wrong actions freely, there is *some* action X that he does in W of his own free will such that, if all circumstances relevant in world W to his virtuous choice of X were to obtain in the actual world, he would go *wrong* in the actual world: If God were to lead him to virtuous choice X – and that is all that God could do toward realizing this good free choice in the actual world – Curley would make not it, but some bad choice instead. Curley, were he transworld depraved, would have been 'incorrigible' in the sense of not being perfectly corrigible by anyone other than himself.

. . . . . .

The idea defined (cf., p. 48 [186])

> For any possible person P[14] and possible world W, **P is transworld depraved relative to W** if and only if, for every world W′ in which P does of his own free will some right actions and no wrong ones,[15] there is in W′ a performance X by P such that (i) X is morally significant in W′, and (ii) it is true in W that if $MWS_{\mathrm{W}'}(\mathrm{X})$ were to obtain*, then P of his own free will would go *wrong* with respect to X,* for short, $[MWS_{\mathrm{W}'}(\mathrm{X}) \mathrel{\Box\!\!\rightarrow} \mathit{Wrong}(\mathrm{X})]$ is true in W. (*Strictly, 'if conditions were to obtain in W that answered to the full description $MWS_{\mathrm{W}'}(\mathrm{X})$ in W′'. Similarly for X, which is a concrete performance in W′.

I assume that there can be in W only a performance exactly like it.)

. . . . . .

ABOUT THE IDEA AS DEFINED. Transworld depravity is *relative to a world*. The definition allows that a possible person who is transworld depraved relative, say, to this world of ours, may not be transworld depraved relative to certain other worlds. It indeed 'predicts' that: No possible person who does right actions of his own free will and no wrong ones in a world is transworld depraved relative to *it*; and the definition 'contemplates' possible persons who are transworld depraved relative to a world, though in some worlds they do right actions of their own free wills, and no wrong ones. Transworld depravity is not '*essential* transworld depravity'. A possible person P who was essentially transworld depraved would be transworld depraved relative to every world, for there would be no world in which this person existed, did something, anything, did some right actions of his own free will, and no wrong ones. Though Plantinga may have supposed that some possible persons are essentially transworld depraved, there is not clear evidence that he did. One last point, a possible person can be transworld depraved relative to a world without existing in this world. So not only every person that exists in a world, but every possible person, can be transworld depraved relative to it.

If P suffers from transworld depravity relative to W, as defined, then W is not itself a world in which P would of his own free will do some right actions and no wrong ones. Indeed, if P is transworld depraved relative to W, there is a restricted *sense* in which no such world is 'accessible' from W: No such world is accessible by actions of anyone in W *other than P*; if P is not in W, no such world is accessible by actions of *anyone* in W. "What is important about the idea of transworld depravity is that if a person suffers from it, then it [would not have been] within [even] God's power [if God exists] to actualize any world in which that person is significantly free but does no wrong – that is, a world in which he produces moral good but no moral evil" (p. 48 [188]). Adams says something similar: "Roughly speaking, a possible free creature (or its essence) has trans-world depravity, in Plantinga's sense, if and only if that creature *would* do some wrong if God created it and permitted it to act freely, **no matter what else God did**" (Adams 1987, p. 89; emphasis added.).

2.5.3 The transworld-depravity argument against premise (2′). There are, Plantinga is confident, "any number of possible worlds" in which agents exercise free will, which worlds are beyond the power of God to create (p. 44). That is not merely a possibility but a fact: It was 'Leibniz's Lapse' to suppose that in His omnipotence "God . . . could have actualized just any world He pleased" (Ibid., [cf., p. 184]). That takes away a premise that atheologians

such as Mackie use to argue for premise (1′), and it presages an argument against their premise (2′). The fact of the Lapse *suggests* that perhaps among worlds an omnipotent *could have* actualized, there is not one that is nontrivially morally perfect, one in which there is some moral good but no moral evil. The *idea* of transworld depravity is for the demonstration that that *is* a possibility: It is explained to "demonstrate the possibility that among the worlds God could not have actualized are all the worlds containing moral good and no moral evil" (p. 45). The *demonstration* has as its sole *premise* that Universal Transworld Depravity – transworld depravity *relative to the actual world* of *every* possible person – is a *logical possibility*: ◊**(UTD)**. The *conclusion* of the demonstration, ◊**Beyond**, that *it is possible* that it was *beyond God's power* to create a morally perfect world in which there is moral good and no moral evil follows, *given that* universal transworld depravity *entails* that it was beyond God's power to create such a world, □**(UTD** ⊃ **Beyond)**. The argument – ◊**(UTD)**, □**(UTD** ⊃ **Beyond)** ∴ ◊**Beyond** – is valid in the Sentential Modal Calculus of Appendix B to Chapter III. Now comes a *subsidiary* demonstration of the entailment, □**(UTD** ⊃ **Beyond)** (cf., pp. 51–2).

Suppose, for a 'conditional proof' of (UTD ⊃ Beyond) at a possible world W, that (i) UTD at W – that is, suppose Universal Transworld Depravity relative to W. Now consider *any* world W′ such that W′ is a morally perfect world in which there is moral good and no moral evil, and thus a possible person P who does of his own free will some right actions and no wrong ones in W′. We have from (i) that (ii) P is transworld depraved at W. So, (iii) there is in W′ a performance X by P such that (a) X is morally significant in W′, and (b) it is true in W that if $MWS_{W'}(X)$ were to obtain, then P of his own free will would go *wrong* with respect to X, for short, [$MWS_{W'}(X)$ □→ *Wrong*(X)] is true in W [from (ii), and the recent definition of transworld depravity at or relative to a world]. Therefore, (iv) a perfect being could not 'create W′ in W′, that is, 'make-over' W into W′: In other words, W' is beyond the power of a perfect being to create in W, or, for short, *W′-beyond*. Why? Because were a perfect being to do that, he would establish in W both $MWS_{W'}(X)$ & *Right*(X), where the latter is short for 'P *of his own free will* goes *right* with respect to X, and *no one other than P* can establish *that*. Therefore, (UTD ⊃ W′-Beyond) at W, and, since W' is *any* a morally perfect world in which there is moral good and no moral evil, (UTD ⊃ Beyond) at W. Finally, since W is any possible world, we may conclude that □(UTD ⊃ Beyond), which is the object of this subsidiary derivation.[16]

From, ◊Beyond, the possibility that a perfect being could *not* have actualized a morally perfect world in which there is moral good but no moral evil, *and* the premise upon which atheologians (other than Mackie) themselves insist, namely, that only a world in which there are free persons exercising their freedom can be a *best* world, *it follows* – and this is the *object* of Plantinga's Free-Will Defense – that, *contrary* to premise (2′), that the world is not a best of all possible worlds, is *compatible* with the existence in it of a perfect being.

'Creators' create from within worlds: "God does not, strictly speaking *create* . . . possible worlds. . . . What He creates are the heavens and the earth and all that they contain" (p. 38 [169]). So He can, in His 'world-making,' be somewhat restricted. A perfect being in a world W relative to which all possible creatures suffer from transworld depravity, though he could *contemplate* possible worlds in which free agents do good and never practice evil, would be only *tantalized* by them. He would see that it was beyond His power to 'create' any one of them. While it would be open to a perfect being in this world W to 'make it over' in many ways, *that* way of perfection would not have been one of them. Like everyone else in W, a perfect being's capacities to 'make over' W would be constrained by counterfactuals of freedom that are true at W and, say, what would be the upshots in terms of free actions in various circumstances. If all possible creatures suffer from transworld depravity relative to this world W, no one, not even Him, could make effective arrangements for a world populated exclusively of unerring active free agents. This is what makes *interesting* the possibility of universal transworld depravity, and makes it *contrary* to premise (2′) of the Argument from the World's Not Being a Best World.

2.5.4 But *is it* possible? *That is the question*

2.5.4.1. Plantinga *says* that it is possible: "[T]he interesting fact here is this: it is possible that every creaturely essence – every essence including the property of being created by God – suffers from transworld depravity" (p. 53 [188]). He *says* that it is possible that every person God could have realized suffers from transworld depravity relative to this world of ours. The trouble is that he *only* says this. He says that "we must demonstrate the possibility that among the worlds God could not have actualized are all the worlds containing moral good but no moral evil" (p. 45 [185]). His demonstration is that *that* possibility is entailed by the possibility of universal transworld depravity. He does not in any manner demonstrate the possibility of universal transworld depravity. He offers *no argument*, *no persuasion* to incline the intellect, for this possibility *at all*. He merely explains the idea of universal transworld depravity, and, in his silence in effect says, "See, you understand that, which settles that it is possible." One gathers that he thought that he had established the *possibility* of it, merely by explaining the *idea* of it. He did that, however, *only if* his explanation not only does not harbor *a priori* a contradiction, which is already, given its complexity, something that is not entirely obvious, but *also* only if it does not harbor a *real contradiction*. It is one thing for a condition *not to be a priori impossible*, so that no contradiction is derivable *a priori* from concepts involved in an expression of it, and *another thing* for it to be *not logically impossible*, or equivalently, *logically possible*: the point of Section 8 of Chapter III.

2.5.4.2. *Is* this awful condition not only understandable (with difficulty), but really possible? I doubt it. I doubt that it is possible that every creaturely

essence should suffer from transworld depravity. I doubt that that is possible for a *single* creaturely essence. Consider Adam, the *first* sinner. Could he have been transworld depraved relative from 'day one'? Might it have been that, no matter to what lengths an omnipotent had gone, he could not have arranged things so that when *Adam's* times to sin or not came, he never did? Is it *possible* that not even an omniscient and omnipotent being, working 'from the beginning' on the project, could not have arranged things so that Adam did the right thing of his own free will his '*first* time out', and then continued on a good roll begun, through to his very last free act, which he would manage as he had all the rest in an exemplary fashion? Where could the difficulty be in that? Where *could* the difficulty be in that for a perfect being?

There *is* a world in which there is a perfect being, a person Adam, and no one else, in which world Adam is of an exemplary moral character who, on every morally significant occasion, of his own free will acts in character and does the right thing. Plantinga insists on it: "[T]here are possible worlds in which God and ***Curley*** both exist and in which the latter is significantly free but never goes wrong" ([p. 185], cf., p. 47). Let PAW (for Perfect Adam World) be a 'God-world' in which Adam is morally perfect, and let X_1 be Adam's first in a long series of of-his-own-free-will good deeds in PAW. God in this world of ours could have realized absolutely all of the relevant 'effable' conditions in which Adam does X_1 of his own free will in PAW. Plantinga gives us that. God could have realized all the conditions that it is natural to assume are relevant to Adam's freely doing X_1 in PAW. For an omnipotent could have 'realized' – he could have 'actualized' [p. 173] – the 'maximal world segment of PAW *to* X_1,' $MWST_{\text{PAW}}(X_1)$, which would be $MWS_{\text{PAW}}(X_1)$ 'minus' the bit that is the history of PAW from the time of X_1. $MWST_{\text{PAW}}(X_1)$ will include the 'initial segment' of PAW to the time for X_1, the laws of PAW, that Adam is free with respect to X_1, and that Adam is of an exemplary moral character. This 'maximal world segment of PAW to X_1' does not include that Adam acts in character when he exercises this freedom with respect to X_1. But it includes everything that it would be natural to suppose is relevant to whether or not he acts in character. And that it *seems* should be sufficient to Adam's freely doing X_1 in our world, so that as in PAW it is true, so in our world it should be true, that $[MWST_{\text{PAW}}(X_1) \,\Box\!\!\rightarrow \text{freely}X_1]$.

Suppose for a moment that it would be *false* in this world of ours that $[MWST_{\text{PAW}}(X_1) \,\Box\!\!\rightarrow \text{freely}X_1]$, as Plantinga says that it *could* be? Consider what could *make* it false. *Something about Adam*? But he would be in our world at the time for X_1 in all describable ways as he is in PAW at that time. He would be *of the character* he is in PAW, *under the influences* he is in PAW, and *of the opinions* that he is in PAW *that provide his reasons* in PAW for X_1. *Everything* on Adam's side of unproblematic relevance to his freely doing X_1 would be as it is in the world PAW in which that is what he does. *Could it be something on the world's side*, something about our world, that makes it false that he would in our world do the same? But what has just been said of Adam would be true *mutatis*

mutandis of it, for in settling everything about him that is of unproblematic relevance to whether or not he does X_1 we do the same for it. *What then* could ground, to what could we allude as harboring an explanation of, the falsity in this world of ours of that 'counterfactual of freedom'? Something in the *futures* of Adam and the world, after the time for X_1? Really? Does that *make sense*? The 'ineffable centers' the secret essences of this world of ours, and of Adam? Their 'quiddities'? Does *that* make sense?![17]

Not being able to understand how, given its truth in PAW, this counterfactual of freedom could fail to be true in this world of ours, I do not believe that it does fail to be true. *Similarly* for $[MWST_{\mathrm{PAW}}(X_2) \,\Box\!\!\rightarrow \mathrm{freely}X_2]$, and so on for each of Adam's significantly free acts in PAW. I cannot 'make *sense*' of *singular* transworld depravity, the transworld depravity of one possible creature, *let alone* the *universal* transworld depravity. For this reason I doubt the possibility of it.

2.5.4.3. Mackie gets wrong Plantinga's idea of 'transworld depravity'. He thinks that Adam is, in Plantinga's sense, 'transworld depraved' if and only if "in whatever world he exists, if he is significantly free he commits some wrong actions" (Mackie 1982, p. 174). He confuses 'transworld depravity' with 'essential conditional depravity' where that is being 'essentially depraved-if-significantly-free'. 'Essential conditional depravity' is, given what I take to be Plantinga's understanding of what it is to be 'significantly free' in a world (for which see pp. 29–30 [pp. 165–6]), equivalent to 'essential transworld depravity'. Mackie then wonders "how [it is] possible that every creaturely essence" should be in that condition, he wonders how it is possible that God was "faced with [this] limited range of creaturely essences" (Ibid.). Mackie's challenge to this possibility that Plantinga does not maintain happens without intending to be relevant to the possibility that Plantinga does maintain. His challenge is that the explanation would need to run in terms of a "logically contingent state of affairs, *prior to the creation and existence of any created beings with free will*, which an omnipotent god would have to accept and put up with" (Ibid.). That "suggestion," he says, "is simply coherent" (Ibid.).[18]

2.5.4.4. "That even one creaturely essence suffers from transworld depravity in relation to some world. What an extraordinary suggestion! That *every* creaturely essence suffers so. Well really!" "Still it is possible," Plantinga would calm incredulous me, "and that is enough to solve once and for all the problem of evil, as many think that I have done." "But," I come back, "how do we *know* it is possible? What reason do we have to believe that it is possible?" We have, for what it is worth, the evidence that universal transworld depravity is *prima facie* not *a priori* impossible, that there is not evident contradiction in the idea of it. But that evidence, given the difficult ideas involved, and the manner in which the hypothesis of universal transworld depravity in the presence of God 'pushes the envelope of our ideas', is not compelling even for this condition's not being *a priori* impossible. Furthermore, in the case of such a hypothesis, of a hypothesis that is as far as it is from the assertions and speculations of

ordinary thought and talk, and from fine-textured and 'tested' areas of ideas and vocabulary, even firmly established freedom from *a priori* contradiction would not be much evidence for *real logical possibility*. Finally, that this hypothesis seems not to be an *a priori* impossibility is the only evidence to which we can aspire for its real possibility. There is nothing that could count as 'conceiving of' this awful state, in the sense of conjuring an 'appearance of its possibility'. Reflecting on the strong evidence *against* it found in its 'not making sense' and the paucity of evidence for it, we cannot, in my view, when we 'get real and reasonable' believe in the possibility of universal transworld depravity.

2.5.4.5. 'We'? Well not all of us. Free-will defenders who put store in the possibility of universal depravity can be perfect-being theists who find that they 'must' believe in some of this possibility. Plantinga may be, or have been, such a perfect-being theist. There is, I think, even *some* evidence that he took such a dim view of God's possible creatures, that he considered universal transworld depravity to be a *fact*. There is the evidence of his tease: "I leave as homework the problem of comparing transworld depravity with what Calvinists call 'total depravity'" (p. 48 [p. 186n1]). Klaas Kraay has wondered whether anyone has done that homework. So have I, for from Plantinga's reference to it I expected the Calvinist doctrine of 'total depravity' to entail universal transworld depravity. But, according to my only source, it may be inconsistent with it.

Depravity, Total. The Reformed doctrine, given classical expression by John Calvin, of the 'depravity and corruption' of all man's faculties . . . as a consequence of the Fall. Thus Calvin claimed that since the Fall man is depraved 'by nature', though he recognized a primary sense of 'nature' – before the Fall – in which man is good. (Richardson and Bowden 1983, p. 153.)

If the doctrine includes that there were *men and women* before the Fall *who did not survive it*, they were thoroughly good in this world. But perhaps it says that there were no such men or women. If it goes further and says that the Fall was *logically necessary* and that *necessarily* there were *no* men or women before it who did not survive it, then I think total depravity entails the *actuality* of universal transworld depravity. Personally, I do not believe in total depravity *neat*, let alone thus heavily modalized.

2.5.5. Immediately after writing that "the interesting **fact** . . . is [that] it is possible that every creaturely essence . . . suffers from transworld depravity" (p. 53 [188]; emphasis added), Plantinga writes: "But now **suppose** this is true" (Ibid.). This sequence is curious. Hearing that something is a fact, one expects not the word 'suppose' but in sequels the word 'given'. It is as if, as he wrote these lines, Plantinga realized that, if a fact, the possibility of universal transworld depravity is not an entirely obvious fact. If so, it was a singular moment. For the most part Plantinga shows himself satisfied with his progress to the fact of this possibility. Adams is confident that, *given*

Plantinga's assumptions, the hypothesis of universal transworld depravity is logically possible.[19] Mackie, at the other extreme, was in no doubt that it is not possible that "every creaturely essence suffers from transworld depravity" (Mackie 1982, p. 174 – Mackie misunderstood transworld depravity, but his reasons for rejecting the possibility of what he took it to be are, as said, reasons for rejecting the possibility of what it is). The suggestion, he felt compelled to say, "is simply incoherent" (Ibid.). My view is to Mackie's somewhat as Plantinga's seems to have been to Adam's. I do not say that the idea of universal transworld depravity in the presence of God is 'simply incoherent'. Since I can discern no contradiction in it, I say that it is as far as I can see coherent, that the possibility of universal transworld depravity is for all I know possible, that for me it is 'epistemically possible' that this condition is logically possible. But, since it does not 'make sense' to me, I make its *logical* possibility 'barely possible epistemically'. As I have said, I *doubt* it. While Mackie was sure, I am only nearly sure that universal transworld depravity is not possible.

2.5.6 On Satan and his cohorts, and the idea that perhaps all *evil is moral evil.* It can seem that objections from freedom to premise (2′), even if successful, leave the problem of imperfection largely intact. For while, if successful, they would protect the existence of a perfect being from logical challenges posed by the world's moral imperfection; they would seem to leave in tact challenges posed by suffering attendant to forest fires, earthquakes, plagues, and other 'acts of God' that argue that the world is 'naturally imperfect.' However, objections from freedom, if they work against logical challenges posed by moral evil, can be stretched to work against such challenges posed by these evils, and indeed all evil, since every evil can with imagination be put down to the bad acting of the 'invisible' and extraordinarily powerful free agents of fertile religious imagination. For example, all the bad turns of nature can be put down, don't you know, to the work of "Satan and his cohorts" (p. 58 [p. 192]). No one who is prepared to accept the logical possibility of universal transworld depravity of possible persons should hesitate to accept the logical possibility of all 'natural evil' being 'supernatural moral evil'. "It is Satan and his busy little cohorts, those persons, who make the wind blow unfortuitously, who set forest fires, who create and shepherd in their early days nasty viruses, and so on. Never mind how they do all of this bad stuff. Attend simply to the logical possibility of their doing it, and you have a complete defence to problems posed by all kinds of evil. For you now see that it is possible that it is all moral evil. You have seen that it is possible that not even a perfect being could create free agents who would not do evil things. And you can see that it is *possible* that no crew God could have installed would been and done better than the ragged crew that is actual. You can see that that is *possible* can't you?" "No, but let's move on."

3. THE ARGUMENT FROM THE WORLD'S NOT BEING A BEST DIVINELY CREATABLE WORLD

3.1. Plantinga's objection is that possibly a perfect being who exists in this world could not have made it a best world, because such a world would feature free agents who never misused their freedom, and it is possible that every possible person suffers from transworld depravity relative to this world. I doubt that this is possible, but, not knowing how to settle the issue, I leave it to raise an argument that is not challenged by Plantinga's modal imagination. The idea of it is that a perfect being could have created a best world a perfect being could have created (!), and a perfect being could have done better than this sorry old world of ours: Now to spell it out.

(1″) The world is not a bet possible world that a perfect being in it could create: [(∃x)BstCrrtbl(x) & ∼BstCrtbl@].

(2″) *That the world is not a best possible world that a perfect being in it could create, is incompatible with the existence in it of a perfect being:* (◊[(∃x)BstCrtbl(x) & ∼Bst@] & ◊PrfBng) & ∼◊ ([(∃x)BstCrtbl(x) & ∼Bst@] & PrfBng).

∴ (3) There does not exist a perfect being: ∼PrfBng.
(BstCrtbl: – is a best possible world that a perfect being could create – is a best divinely creatable world); @: the world (this world of ours, the actual world); PrfBng: a perfect being exists.)

About possibilities for creation: I shall say that a being *can create* a possible world if and only if he exists and is active and has a free choice that if taken would result in that world. That makes each of us a 'creator of worlds', if we have free choices, but not in a big way! I think of beings as 'more or less creators' in their possible worlds, according to the *range* of their free choices, which will depend on their *powers* of choice. Our powers of choice are, alas, rather limited. 'Creators' (upper case) would be *much more* powerful; perfect-being Creators would be omnipotent. *Regarding the argument*: Premise (1″) is nearly as credible as premise (1′). Resistance to the argument goes naturally to premise (2″). It is by design not bothered by the possibility of universal transworld depravity, if it is possible.[20] There is, however, a way to resist premise (2″), which, incidentally, is another and better way to resist premise (2′). It is better because it rests on a more plausible possibility-claim. Now comes a 'middle knowledge' objection to premise (2″) that I reject, followed by another that I accept.

3.2 Adams's objection to Premise (2″) in my words

Best worlds that a perfect being could have created would feature free beings who collectively made as good use of their freedom as free beings in any world he could create would do. Let us agree to that. To this you add that, for sure, a perfect being would have created a best world he could have created. On this we disagree. My

reason for parting is that a perfect being could not have *made sure* that the world that resulted from his creative activity was a best world that could result from it. That is because to make sure of that he would have had to have known not only what world of free beings and actions was going to result from his creative activity, which, in his omniscience, he would have known. He would need *also* to have known what worlds of free beings and actions (these being the only worlds with which he need have been concerned) *would have* resulted *had he taken other* creative measures. He would need to have had what Iberians called 'middle knowledge' concerning how free beings *would have exercised their freedom*, *had he taken other creative measures*. But such would-be knowledge is *impossible*. No one can know what anyone *would* do with their freedom, on *any* counterfactual supposition. 'Middle knowledge' would be knowledge of counterfactuals of freedom that said what agents would do. Such knowledge is *impossible*. This is because, though counterfactuals of freedom of the form

(If it were the case that p, then it *MIGHT* be the case that q) : $(p \Diamond \rightarrow q)$

are *some of them true*, counterfactuals of freedom of the form

(If it were the case that p, then it *WOULD* be the case that q) : $(p \Box \rightarrow q)$

are *all false*.[21] Not even an omniscient could know what, were he to take creative measures other than those he will take, a free agent *would* do with his freedom *for there is nothing true here to know*. An omniscient would know what free agents have done, are doing, and will do with their freedom, nothing of what they *would* do on this or that counterfactual supposition, for there are no 'woulds' of their freedom to know. There are only 'mights'."[22]

3.3 'Middle knowledge'

3.3.1 Adams is against it. Plantinga assumes that 'would-counterfactuals of freedom' can be true. Adams maintains that they are all false (Adams 1987, p. 91n4). He thinks that there is nothing that can *make* them true, nothing *in virtue of which* they can be true. Grounds for such conditionals that come to mind are intentions, beliefs, desires, and habits of their subjects. But, Adams maintains, grounds for these conditionals are never sufficient to make them true. The propositions that such grounds establish are, he says, never of the form, (if it were the case that P, then it would be the case that Q). Only counterfactuals of freedom of the form, (if it were the case that P, then it might be the case that Q), are ever well-grounded and true. The *possibility* for God of knowledge of 'would'-counterfactuals of freedom was affirmed by sixteenth-century Iberian Jesuits. "Such knowledge," Adams informs, "was called '**middle knowledge**' because they thought it had a middle status between other kinds of knowledge – between God's knowledge of the merely possible and his knowledge of the actual; or between his knowledge of necessary truths, which follow from the divine nature, and his knowledge of his own will and everything that is causally determined by his will"

(Adams 1987, pp. 77–8 [91n2]; emphasis added). Adams says that this middle knowledge is *impossible*. It is impossible "even for God" because 'would'-counterfactuals of freedom' "cannot be true" (p. 78). God, since He would be omniscient, would have complete conceptual and factual knowledge, including complete foreknowledge of His creatures' actual free conduct, but not even He would have one bit of this middle knowledge, for there is none possible to be had.

3.3.2. I am persuaded by examples of kinds that bother Adams that he overreaches in his put-down of all 'would'-counterfactuals of freedom. "There does not normally," he concedes, "seem to be any uncertainty at all about what a butcher, for example, would have done if I had asked him to sell me a pound of ground beef, although we suppose he would have had free will in the matter. . . . What makes us regard it as certain? Chiefly his character, habits, desires, and intentions, and the absence of countervailing dispositions" (p. 88). And these, I think, can make *true* what we would confidently believe. Adams's mistake, let me tentatively say, may lie in his thinking that the conditionals, *If I were to ask the butcher to sell me a pound of ground beef, then he* ***would****, acting* ***in*** *character, freely serve up a pound*, and, *If I were to ask the butcher to sell me a pound of ground beef, then he* ***could****, acting* ***out*** *of character, freely* ***not*** *serve up a pound*, are logically incompatible. In fact they can both be true, or so it seems to me. Though "[a] free agent [always] **may** [i.e., can] act out of character" (p. 80; emphasis added), it does not follow that a free agent never *would* act *in* character or *out* of character, but always only *might* act in character, (FrCh$\Diamond\rightarrow$ freelyA), and might *not*, (FrCh $\Diamond\rightarrow$ freely$\sim$A).

It is not plausible that opposed pairs of 'would-counterfactuals of freedom' – (FrCh $\Box\rightarrow$ freelyA) and (FrCh $\Box\rightarrow$ freely$\sim$A) – are *necessarily* always *both* false. However, it is I think very plausible that they are *sometimes* both false, and that Adams is not 'all wrong' about them, though, regarding cases in which both are false, I think that more is true than Adams may think. I think that in these cases things more definite than 'might'-counterfactuals of freedom will be true, namely, 'chance-counterfactuals'. Speaking generally of counterfactuals, I believe that there are always either true 'counterfactuals with chancey consequents' (Lewis's way of bringing chances in), or true 'chancey counterfactuals' (my way): that is, either true 'would'-counterfactuals with 'weighted-might' consequents, suggestive notation, ($P \Box\rightarrow \Diamond_x Q$), or true 'weighted-might'-conditionals, suggestive notation, ($P\Diamond_x\rightarrow Q$), $0 \leq x \leq 1$. (I make ($P\Diamond_1\rightarrow Q$) and ($P_0\rightarrow Q$) equivalent to ($P \Box\rightarrow Q$) and ($P \Box\rightarrow \sim Q$), respectively.)

Of classical solutions to the problem of divine middle knowledge, I would adapt Luis Molina's (1535–1600) and say that God would fully and perfectly comprehend His possible creatures characters, whereas our comprehensions of ourselves and others are always incomplete and imperfect. My adaptation would, however, leave open what Molina and Francisco Suarez

(1548–1617) were concerned to close, namely, the possibility of opposing 'would'-counterfactuals of freedom both being false. I would leave open that sometimes there *is* no 'what *would* happen' to be known, that sometimes there are only *chances* to be known.

3.4 'Diminished middle knowledge' against to premise (2″). The argument now is not, as Adams's is, from the purported *impossibility* of middle knowledge, but from the *possibility* that middle knowledge even of an omniscient would be *limited*. *Premises* for this objection are

(i) It is necessary that there is a free agent in every best of all possible worlds that a perfect being in this world @ of ours could create.
(ii) It is possible that this world @ of ours is such that, if a perfect being B exists in @, then, for every best possible world W that B could create, there is a person P in W who is not in @, a morally significant free action A of P in W that contributes to the value of W, and a 'counterfactual of freedom', CF, such that (*a*) CF says that, were P created by B in @, then P would freely do A; and (*b*) **CF is false in @**. [We could elaborate by adding that only a '*chance*-counterfactual of freedom' that assigns a chance less than 1 to P's freely doing A, were P created by B in @, is true in @.] The conclusion of this objection is that,
(iii) It is possible that a perfect being B in @, no matter which possible persons it created, would not *know for sure* that they would behave as required in order for the world B created to be a best world B could create

The idea of the argument against premise (2″), still Adams's idea, is that it is possible that a perfect being would not know *exactly* how his several alternative creation-efforts *would* work out in actions of free agents created, *because* there *is* no exact and complete way in which they ***would*** work out in that respect, but only several ways in which they *might*, with various chances, work out. Given that *possibility*, the condition of the world's not being a best possible world a perfect being could have created is, contrary to premise (2″), *compatible* with the existence of a perfect being. Though B 'took its best creative shots', not knowing how they were going to work out, because there was no 'how they were to going to work out' to know, the upshot could fall short of the best world possible as an upshot of his creative choices. Think of a roulette wheel balanced to be, for every possible 'twirl', *undetermined* over some range of stops, so that not even the best twirl *would for sure* have the best possible result.

3.5. It is not obvious that premise (ii) of this challenge to premise (2″) is true. It is relevant to my 'not obvious' assessment of premise (ii) that, unlike Adams, I think that mundane 'would'-counterfactuals of freedom are often

true. Also relevant is the 'epistemic possibility', the 'for all I know possibility', best divinely creatable worlds are not 'chancey worlds', that a world's not being creatable for sure is a defect of which best worlds would not partake. It has been said that 'the Lord does not play dice'.[23] That *could* be because, for the reason just stated, the Lord does not *need* to get out of His creative choices the best world He can create. But then it is certainly not obvious that premise (ii) is *false*, and so, no surprise by now, I will concede it to the theist, in order to move to an argument against perfect-being theism that *accommodates* the possibility that 'middle knowledge' is in significant ways limited.

4. THE ARGUMENT FROM THE WORLD'S NOT BEING A BEST DIVINE BET WORLD

Suppose that not even a perfect being *could* know exactly how his possible creative choices *would* work out, because there *is* not for *any* of them a complete and exact way in which it *would* work out to know. Even so, it seems that there *must* be the ways in which his various possible creative choices *might* work out, and for each way a creative choice might work out, the *chance* of its working out in that way, with the chances of its working out in the various ways in which it might summing to 1. If that is right, then *a perfect being would know about these chances*. He would know not only how his various possible creative choices *might* work out, but would know also the *chance*s for these ways they might work out, and the atheologian is back in business with the following argument. Here is an argument run in terms of 'best divine bet worlds.'

(1‴) The world is not a best divine bet world: [(∃x)BstDbt(x) & ∼BstDbt(@)]
(2‴) *That the world is not a best divine bet world is incompatible with the existence of a perfect being:*
(◊[(∃x)Bst(x) & ◊ ∼BstDbt(@)] & PrfBng) ∼ ◊([(∃x)Bst(x) & ∼BstDbt(@)] & PrfBng).
∴ (3) There does not exist a perfect being: ∼PrfBng
(BstDbt: – is a best divine bet world;
@: the world (this world of ours, the actual world); PrfBng: there exists a perfect being.

Let a world be a **best divine bet world** if and only if it is a possible result of a sequence of creative choices made by a perfect being, who in his creative choices *maximizes expected value*.[24] Let the *expected value* of a choice be a weighted average of the values of worlds that are possible upshots of it, the weight for the value of a world being the objective chance of this world given the choice.[25] For a formula for the expected value of choice Ch, let Chance(W/Ch) = c if and only if (Ch $\Diamond_C\rightarrow$ W) or (Ch $\Box\rightarrow\Diamond_c$W). (Nothing presently depends on

which theory, mine or Lewis's, is used for chance-counterfactuals.)

$$\text{Expected Value of Ch} =_{\text{df}} \sum_{\text{W such that Chance(W/Ch)}>0} \text{Chance(W/Ch)} \times \text{Value(W)}.$$

Let a choice *maximize expected value* if and only if no alternative choice its agent could make has greater expected value. Best divine bet possible worlds are defined in terms of what may be *several* choices taken one after the other, in which sequence each creative choice maximizes expected value in terms of at-the-time-of-this-choice chances for worlds. Maximizing later choices would take into account how, by their times, chances for worlds of earlier choices had been somewhat resolved. If there exists a perfect being in the actual world, it, the actual world, could be a work in progress in which there are many divine creative choices to come.

Premise (1‴), that this is not a best divine bet world, is only somewhat less plausible than premise (1′), that this is not a best world. Who, with the knowledge of a perfect being of chances for characters, would have chosen Hitler, knowing what he was going to become, and what he was freely to do?[26] Who, having run the risk of his turning out badly, would stand by and watch as he carried on as he did? Is it plausible that a best divine bet entailed the risk of AIDS, and Alzheimer's disease, and Bambi's immolation? Adams *may* accept premise (1‴), for he writes not only that "[n]o matter how shrewdly God acted in running . . . risks . . . his winning *every* risk would not be antecedently probable," but that he finds it very plausible that "if God had acted differently in certain ways, he would *probably* have had better behaved free creatures, on the whole, than he actually has" (Adams 1987, p. 91). The present argument from the world's not being a best divine bet world is not bothered by 'middle knowledge' problems that upset the argument from the world's not being a best divinely creatable world, nor 'transworld-depravity' problems that worry the argument from the world's not be best world. So is the work of the atheologian against perfect-being theism done; can he rest his case with this argument from the world's not being a best divine bet world? Not yet, a perfect-being theist may say, for what if there is not a best divine creative bet, and thus no best divine bet worlds?

5. THE PROBLEM OF THE BEST

The move from The Argument from Evil was made consequent to the concession that, though a perfect being would make sure that the world is a *best* possible world, best worlds all feature some evil. Part of that concession was of course that there *is* a best possible world. That became an 'official presupposition' of the premise that *the world is not a best possible world*. This premise is naturally understood and was explicitly intended 'with existential import' so that, spelled out, it is the premise that (i) *there is a best possible world*, and

(ii) this world is not a best of all possible worlds. Its intent is *explicit* in the symbolization provided for it, viz., '[(∃**x**)**Bst(x)** & ∼Bst@]'. Similarly for the first premises of the arguments from this world's not being a best divinely creatable world and from its not being a best divine bet world. The first premises of these arguments presuppose 'bests' of the kinds of worlds with which they are concerned. Now comes an objection to these presuppositions, and thus to the first premises themselves of these arguments.

There are infinitely many possible worlds, and just as there is not a *greatest number*, so there is not a *best possible world*. Just as numbers get bigger and bigger without upper bound, so worlds can be arranged in an order in which their values increase without upper bound. Similarly for a best world that a perfect being could create: Here too there are only better and better worlds without upper bound. And similarly for imagined bets by Creators of worlds: They too get only better and better without ever reaching a best.

Understanding *worlds* to exclude God and be possible nonempty totalities of contingent beings, a metaphysician/theologian might maintain that it is of their nature to be, contrast with God, *imperfect*, that just as it is of the nature of numbers that they are *enlargeable*, so it is of the nature of *worlds* that they are *improvable*. It might be said to be the special privilege of God, that He alone of things, all things, is best of each of His kinds.[27]

The claims that there are *not* best worlds, best divinely creatable worlds, best divine bet worlds are somewhat plausible. It *would be obvious* that there is not a best possible world, and nearly as obvious that there are not best divinely creatable or best divine bet worlds either, if it were obvious that necessarily 'the more goods the better'. However, when one thinks about it, it is not obvious that necessarily 'the more goods the better'. For one thing, it seems possible that some goods could derive some of their value from being rare, even unique. Suppose, however, that there is not a best possible world, a best divinely creatable world, or a best divine bet world. Then the arguments from the world's not being a best world, a best divinely creatable world, or a best divine bet world lose their first premises (1′), (1″), and (1‴). Furthermore, if it is *necessary* that there is not a best possible world (divinely creatable world, best divine bet world), then the existence of a perfect being is *not incompatible* with there not being a best possible world (a best divinely creatable world, a best divine bet world). *Nothing* is incompatible with what is *necessary*: Only *possible* things can be *incompatible*, properly speaking, with one another. (Cf., footnote 5.) Therefore, if it is necessary that there is not a best world, *etc.*, our arguments lose their second premises as well.[28]

So is it 'game over' for the atheologian who would in one way or another argue from the evil of the world against its being the work of a perfect being, *unless* he can preface his argument or arguments with demonstrations of the existence of relevant 'bests'? No at all, for while having his argument still concerned, much as it has been with best worlds, best divinely creatable

worlds, or best divine bet worlds, he can free it of the assumption that there *are* such 'bests.'

6. THE ARGUMENT FROM THERE BEING A BETTER WORLD THAN THIS ONE

6.1. A *perfect* being would be not only omnipotent, omniscient, and perfectly good, but **perfectly rational in choices and actions**, and would have a 'hand' in the creation of the world (either he would have chosen the actual world or made a bet on worlds of which it was a possible result). Certainly a perfect being could not create a world or place a bet on worlds for *no* reason. A perfect being would act only for the best of reasons. ("*That's* for sure," Leibniz might chime in.) And so there are problems posed by the evidence of evil *whether or not* there is a best world, a best divinely creatable world, or a best divine bet world, and there are arguments as good as those we have considered addressed respectively to perfect-being theists who are not persuaded by either Plantinga's or Adams's objections, to perfect-being theists who are persuaded by Plantinga's but not by Adams's objection, and to perfect-being theists who are persuaded by Adams's objection and not by Plantinga's. Here is an argument addressed to perfect-being theists of the first 'nonpersuasion'.

(1*) *There is a better world than this one, i . e, the world is a bettered-world*: Bttrd@.
(2*) That the world is a bettered-world is incompatible with the existence of a perfect being:
($\Diamond$Bttrd@ &$\Diamond$PrfBng) & $\sim\Diamond$(Bttrd@ & PrfBng).
$\therefore$ (3) There does not exist a perfect being: $\sim$PrfBng.

For ready reference, here is The Argument from the World's Not Being a Best World.

(1′) *The world is not a best of all possible worlds*: [($\exists$x)Bst(x) & $\sim$Bst@].
(2′) That the world is not a best of all possible worlds, is incompatible with the existence of a perfect being:
($\Diamond$[($\exists$x)Bst(x) & $\sim$Bst@] & $\Diamond$PrfBng) & $\sim\Diamond$([($\exists$x)Bst(x) & $\sim$Bst@] & PrfBng).
$\therefore$ (3) There does not exist a perfect being: $\sim$PrfBng.[29]

6.2. The argument – (1*), (2*) $\therefore$ (3) – can work for perfect-being theists who are persuaded by neither 'transworld depravity' nor 'middle knowledge' objections to The Argument from the World's Not Being a Best World. The coming 'demonstration' for the argument proceeds under the assumption that a perfect being could create any possible world he pleased (*pace* Plantinga), and of any

world he could make sure that he created it (*pace* Adams). Perfect-being theists persuaded by one of those objections can, for arguments addressed to them, replace occurrences of 'bettered-world' by occurrences of either 'bettered-divinely-creatable world' or 'bettered-divine-bet world.'* 'Demonstrations' for these arguments would proceed on assumptions acceptable to their target audiences. (*The world is a *bettered-divinely-creatable-world* if and only if there is a better divinely creatable world. It is a *bettered-divine-bet* world if and only if there is a 'bet on worlds' that is a better bet than any in which this world has a chance.)

6.3. Few who are not persuaded by Plantinga's/Adams's objections quarrel disingenuously with premise (1*). Certainly the world could have been better as far as we can see, even if there is no way in which it could have been best and unimprovable, because there is no best amongst possible worlds. Even if all moral evil is made worthwhile ultimately by valuable acts of repentance and moral improvement (a repugnant suggestion to many minds), and even if 'natural evils' too are made worthwhile as costs to be paid for the possibilities of valuable moral effort and brave endurance, it is, as Rowe claims, "quite incredible that all the instances of suffering that serve no greater good we know or can think of should nevertheless be such that none could have been prevented . . . without loss of a greater good" (Rowe 1986, p. 235). As with previous arguments, so with this one, its burden is borne mainly in the second premise, (2*): (◊Bttrd@ &◊PrfBng) & ∼◊(Bttrd@ & PrfBng). Presently targeted perfect-being theists are not prepared to grant that there is a best possible world. And so they must grant that it is possible that @ is bettered, ◊Bttrd@, since if this is not possible, @ is a best world. And they of course must grant that a perfect being is possible, ◊PrfBng. The atheologist need therefore *persuade* only that ∼◊(Bttrd@ & PrfBng), in words, only that *it is not possible that both there is a better world w than @, and a perfect being exists*. This should seem to presently targeted theists 'analytic' of perfect practical rationality when that is joined with perfect goodness, omnipotence, and omniscience. For a perfectly good being would *prefer the better to the worse*; an omniscient being would know the relative values of worlds; and it certainly seems that no perfectly rational being chooses, or so much as allows, what it knows it disprefers to something it knows it can choose, or allow, instead: It seems *hardly* remarkable that a "rational agent never wittingly picks an inferior option" (Sorensen 1994, p. 150). To detail, we have that ∼◊(Bttrd@ & PrfBng) is equivalent to □∼(Bttrd@ & PrfBng), to demonstrate which it is sufficient to demonstrate without aid of contingent premises ∼(Bttrd@ & PrfBng). We may assume for an indirect demonstration (i) (Bttrd@ & PrfBng). Then (ii) there is a possible world w *better* than the actual world @, and (iii) that a perfect being P exists. From (ii) and (iii) it follows that, (iv) P prefers w to be rather than @ (for P is perfectly good and omniscient – P prefers the better to the worse, and knows the difference). Thus, (v) given a choice which

world was to be that include as options @ and w, P did not choose @ (for P is rational). (vi) P had a choice which world to exist that included as options @ and w (for, as granted by presently targeted perfect-being theists, his options ran to every possible world). From (v) and (vi) it follows that, (vii) *P did not choose @ to be*. But, (viii) *P did choose @ to be* (for @ is the actual world, and choosing a world to be the actual world is part of the 'hand' that presently targeted perfect-being theists assign to a perfect being). Contradiction. So, contrary to (i), we have that ∼(Bttrd@ & PrfBng).

But, an objector might observe, if there is not a best world, and a rational being never chooses something when there is something he can choose instead that he prefers, then that an omnipotent, omniscient, and perfectly good and rational being would not choose, or allow, *any* world to be. *Just so*, a defender of the argument can respond, while reminding that eminent theists have agreed.

> [I]f there were no best possible series, God would have certainly created nothing, since he cannot act without reason, or prefer the less perfect to the more perfect . . . [From a letter to Des Bosses, 1711: Leibniz 1951, p. 95] Against those who think that God might have made things better than he has . . . if comparative perfection were sufficient, then in whatever way God had accomplished his work, since there is an infinitude of possible imperfections, it would always have been good in comparison with the less perfect; but a thing is little praiseworthy when it can be praised only in this way. (*Discourse on Metaphysics*, p. 293.)

If there is not a best possible world, then, by a simple principle of rational choice, the mere *existence* of a world, never mind its details and how good or bad it is, is inconsistent with His existence, and there is, for those to whom the present argument is addressed, a *cosmological* argument *against* the existence of God, a perfect being. The argument at hand would indeed be an *ontological* argument against the existence of God, if it is necessary that some world, if only an 'empty of contingents world', must exist and necessary that there is not a best possible world for the simple reason that there is not a best possible world.[30]

7. A DILEMMATIC ARGUMENT TO THE WORLD'S BEING IMPROVABLE

7.1. An atheologian who is confident of premise (2*) can leave open whether there is a best of all possible worlds. As just demonstrated, he can argue for premise (2*) without violating that neutrality. To highlight this neutrality, the following rearrangement of his argument puts it 'up front' to serve a constructive dilemma. To symbolize it I capitalize on the equivalence of 'the world is improvable' and 'the world is bettered'.

(0) Either there is a best possible world, or there is not a best possible world: $[(\exists x)\mathrm{Bst}(x) \vee \sim(\exists x)\mathrm{Bst}(x)]$.

(1**i) If there *is* a best possible world, then the world is improvable: $[(\exists x)\mathrm{Bst}(x) \supset \mathrm{Bttrd@}]$.
(1**ii) If there *is not* a best possible world, then this world is improvable: $[\sim(\exists x)\mathrm{Bst}(x) \supset \mathrm{Bttrd@}]$.
(2*) That the world is improvable is incompatible with the existence of a perfect being: $\sim\Diamond(\mathrm{Bttrd@} \wedge \mathrm{PrfBng})$.
$\therefore$(3) There does not exist a perfect being: $\sim\mathrm{PrfBng}$

This argument is again for perfect-being theists who are not persuaded by Plantinga's/Adams's objections to premise (2′) of the argument from the world's not being a best world. The conjunction of premises (1**i) and (1**ii) is logically equivalent to (1*) of the previous argument. Indeed, premise (1**ii) is necessary, which means that premise (1**i) is equivalent to (1*). Premise (0) is a tautology that can be suppressed. The present argument is the previous argument, somewhat rearranged and 'teased out'.

7.2 *On the principle of choice behind premise (2*).* The argument in Section 6.2 used a restriction to perfect beings and choices of worlds of a perfectly general principle of rational choice, namely that, *necessarily, for any rational person p, if p has a choice between things including x and x′, then if p prefers x′ to x, p does not choose x*. Let a person p be *in a choice-situation that presents options in set* O *for choice* if and only if he knows his options (the things presented to him for choice) constitute O, and it is necessary that he makes a choice from O (it is not an option to exit the choice-situation without choosing an option from its 'choice-set'). A corollary of this principle that can give pause is that there are choice-situations in which a rational person can be, though an irrational person who has the same information and preferences can be in them. Now come considerations to persuade that the principle is solid, and with it this corollary.

7.2.1. Assume that a person's preferences for sums of money are in the cases of this paragraph simple. He *likes* money, *the more the better*. Suppose a choice between \$1 and \$2. It would be irrational for him to choose \$1. "[N]ow suppose one adds infinitely many more options {\$3, \$4, \$5, . . . }" (Sorensen 1994, p. 147). Would that make his picking \$1 rather than \$2 (not to mention \$3) rational, as if the addition of those infinitely many options had "washed out differences" between \$1 and \$2, and the irrationality of his choosing \$1 when he can choose \$2 instead? Surely not! Choosing \$1 – so much as accepting \$1 – when \$2 can be had instead would remain inconsistent with his being perfectly rational. Now consider a choice from the set {\$1, \$2, \$3} and see that choosing less than \$3 would be irrational for him and would remain so were the infinitely many options {\$4, \$5, \$6, . . . } added here. *And so on* for every finite set of options {\$1, . . . , \$n}. No matter how great the greatest option \$n, choosing even one dollar less would be irrational for him *and would remain*

irrational after the expansion of his choice-set to infinity. Of course! Why bother to say these things? Because it *follows* that, for no number k, would his choosing $k from the infinite choice-set {$1, $2, $3, . . . } be rational (because, 'reverse repetition,' it would not be rational to choose it from the finite choice-set {$1, . . . , $k, $(k +1)}. That means that a rational person cannot be in a situation in which the choice-set is {$1, $2, $3, . . . }, though someone like him, except for being a touch irrational, could be in it (*if such a choice-situation is possible*) and walk away with a bundle.

The parenthetical 'reality-check' can suggest that this case in not of proper concern, since it is *not* possible. "There is not that much money in the world." (Cf., Richard Jeffrey's 'avoidance' of the St. Petersburg Paradox: Jeffrey 1990, pp. 154–5.) Perhaps there is no cause for concern about such situations in ordinary life. But in philosophy we are taking time out, and there is for our *immediate* subject possible concern, since not everyone will agree that there are not that many *worlds* in the 'space' of all possible worlds, and an argument like that of the previous paragraph leads as certainly as that argument led to its conclusion, that a *perfect being* could not be in the situation of choosing worlds if there is not a best world, but only always better and better, and the situation presents him with an unrestricted choice.

7.2.2. To look at it another way, suppose a being of limited powers who can create only finitely many different worlds. Suppose this being has created a world W less good than a world he could have created. This Creator of limited power could not be perfectly good, perfectly knowledgeable, and perfectly rational. How then could the choice of that world W be consistent with the perfection of a being of *unlimited* power who could create any of infinitely many worlds including the *infinitely many* that are better than W?! It has been said that "failing to do the best you can is a flaw or manifests an incompleteness . . . only if doing the best you can is at least a logical possibility" (Morris 1993, p. 244). That is not right *exactly* as stated, but no matter, for whatever it comes to when fixed, doing *less well* than an 'incomplete' being would do, who, *because less powerful*, had only a proper subset of your options, *would* 'manifest incompleteness' on your part. It might "not impugn God's goodness to say that He could have acted better than He in fact did. . . . [But it would impugn] His goodness to say that some other imaginable being could have done so" (Grover 1988, p. 223).[31]

7.2.3. To spell out an argument of the sort just indicated, let what is to be proved 'from scratch' be that it is not the case that both (i) there is a world better than the actual world @, and (ii) there is a perfect being: ∼(Bttrd@ ∧ PrfBng). Relating the coming proof to premise (2*): It∼◊(Bttrd@ ∧ PrfBng), which is equivalent to □∼(Bttrd@ ∧ PrfBng), which is proved when ∼(Bttrd@ ∧ PrfBng) is proved 'from scratch'. For an indirect argument, I assume (i) and (ii). Arguments of this Section 7, as of Section 6, are, recall,

for perfect-being theists who are not persuaded by certain objections of Plantinga and Adams. I take from this that it follows from (i) that, (iii) there is a world better than the actual world @, *that a perfect being could have chosen instead of* @. We may 'stipulate from (iii)' (by two existential instantiations and a simplification) that, (iv) PB is a perfect being. Then, (v) @ *was the object of a rational choice by PB*. Being a perfect being would here include having options for what world is to be actual, and exercising these rationally. We may 'stipulate from (iii)' (this would be part of the 'dividend' of the mentioned process of 'stipulation') that, (vi) W is a world better than @ that PB could have chosen instead. I also take from the address of the argument to perfect-being theists not persuaded by those objections that, (vii) there could have been instead of PB an only 'somewhat perfect' being such that his 'creative options' would have been exactly W and @, who was otherwise like PB. We may stipulate that, (viii) SPB is such a somewhat perfect being. Then, (ix) SPB would have chosen W, and (x) @ WOULD NOT *have been the object of a rational 'creative choice' by SPB*. However, (xi) the worlds SPB could have chosen to be actual are a *subset* of those that PB could have chosen to be actual, and (xii) the preferences of SPB for worlds he can choose to be actual are the same as the preferences for these worlds of PB. And also, (xiii) for *any* choosing beings X and Y, if S′, the set of exactly the items that X can choose, is a subset of S, the set of exactly the items Y can choose, and X and Y agree in their preferences for items in S', then if the object of a rational choice by Y from S is in S′, it is identical with the object of a rational choice by X in S′. It follows from (vi), (xi), (xii), and (xiii), for a contradiction with (xi) that concludes the present indirect argument, that, (xiv) @ WOULD *have been the object of a rational 'creative choice' by SPB*.

7.2.4. In the spirit of Thomas Morris's remark – "failing to do the best you can is a flaw or manifests an incompleteness . . . only if doing the best you can is at least a logical possibility" (Morris 1993, p. 244) – are these words of Peter Forrest: "If [for every possible world there could be a better one] . . . God could not create [a] best possible world, just as he could not name [a] greatest integer" (Forrest 1981, p. 52.). That is true, but, contrary to Forrest's implicit intent, the cases are very different. For perfect-being theists not bothered by Plantinga's and Adams's objection, though there are no *prima facie* plausible reasons why, if God were to exist, He would name a greatest integer, there *are* such reasons why, *if God were to exist, He would have created a best possible world*. Here, for such perfect-being theists, are plausible premises for that conclusion.

If God were to exist, then: (i) of every possible world, God would have known that he could create that world; (ii) God would have created the actual world; and (iii) if there is a better possible world than the actual world, then there is a world that God would have preferred to the actual world. If God were to exist, God would

be a rational agent. A rational agent does not create something to which he prefers something he knows he can create instead.

These premises are plainly consistent with there not being a best possible world, as is the recently italicized conclusion they entail. To this conclusion and these premises, whether or not there is a best world is *irrelevant*. Whether or not there is a best divinely creatable world, and whether or not there are best divine bets for worlds, are similarly irrelevant to the following premises.

If God were to exist, then: (i) of every possible world God could have created (or, of every bet for worlds God could have 'made' at times), God would have known that he could create that world (or, God would have known at these times what bets for worlds he could 'make'); (ii) God would have created the actual world @ (or God would have made bets for worlds of which the actual world was a possible outcome); and (iii) if there is a possible world that God could have created that is better than the actual world (or if there are better bets for worlds that God could have 'made'), then, of some world W that God could have created (or bets for worlds β that God could have 'made'), God would have preferred W to @ (or, God would have preferred β to the bets for worlds he 'made'). If God were to exist, God would be a rational agent. A rational agent does not create something such that there is something he prefers to it that he knows that he can create instead (or, a rational agent does not 'make' bets to which there are bets he prefers that he knows he can make instead).

Since it is neither here nor there, as far as these premises go, whether or not there is a best world that God could have created, and whether or not there are best bets for worlds that God can 'make', these issues are irrelevant to the conclusions entailed by these premises, which are that if God were to exist, he would have created a best possible world he could have created, or he would have 'made' best bets for worlds that he could 'make'.

7.3. There can seem to be a 'high-tech' way around the problem for perfect-being Creators, if there is not a best world. The idea is that available to a perfect being will be *mixed creation-strategies*, that there are best mixed creation-strategies whether or not there is a best *pure-creation strategy* (which is to say, a best possible world), and that these are better than pure-creation strategies, if there is not a best one of these. Now to spell this out idea out, and, after that, to consider it.

7.3.1. A perfect being would have the where-with-all to assign the 'actual re-alization' of one world or another irrevocably to chance, with various possible worlds being assigned various chances. He would have available not only 'pure' creation-strategies, but also 'mixed' creation-strategies. Why not? Being perfect a perfect being would be omnipotent. We sometimes have ways of choosing mixed strategies. He would have then always. Now, if there is not a best world, then amongst his mixed creation-strategies would be some that have *infinite*

objective expected values. Here the *objective* expected value of a mixed strategy with *finitely* many possible outcomes is a weighted average of the values of these outcomes, in which weighted average *the weights are their* CHANCES *in this strategy*. The idea of the objective expected value of a mixed strategy with infinitely many possible outcomes is an extension of that simple averaging idea. For a mixed strategy with denumerably many possible outcomes, there is a *limit-rule*: The objective expected value of such a mixed strategy can be the limit of a denumerable sequence of partial sums, if this sequence has a limit, or infinity if not, and every partial sum is greater than its predecessor without an upper bound. So a perfect being would not be flummoxed by there not being a best possible world. Assuming that there is not a best possible world, he could not choose from only pure creation-strategies. Each, we are assuming, has some finite value, and none has value greater than or equal to every other. But he could choose from his creation-strategies, pure and mixed, since there would be available to him mixed creation-strategies of *infinite* objective expected value than which none had greater objective expected values.[32] They would be in that sense *best* strategies. For a rational choice, he could choose one of these. End of problem.

7.3.2 Or is it? Two cases say no

7.3.2.1. . I suppose for the *first case* that a perfect being would *not* know 'just before' he committed to a mixed creation-strategy of infinite objective expected value how this strategy that he was about to make *was going to work out*.[33] I suppose that he would know only that the mixed creation-strategy he was about to make had its appointed *chances* for working out into this world or that, equivalently, he would 'then' know of its possible outcomes only their chances, that is, only the chances that the strategy was going to establish for them.[34] What these suppositions come to is that ***his*** *expected value* for a mixed creation-strategy would equal ***its*** *objective expected value* and be similarly infinite. What does it mean? *His* expected value for a mixed strategy is something like a weighted average of the values of its possible outcomes, in *which* weighted average the weights are *his* SUBJECTIVE PROBABILITIES *for the mixed strategy having this or that outcome*. In this case in which he has knowledge of the objective chances of outcomes, and nothing more about them, his subjective probabilities for outcomes equal what he knows are their objective chances. The *question* I press from two angles is whether the infinity of his expected value for this mixed strategy makes his choice of it *rational*. First, I worry about the *relevance* of his infinite expected value for this strategy to his choice of it. Second, I argue that he, this perfect-being Creator, would have available strategies that he preferred to this strategy, so that, notwithstanding his infinite expected value for it, his choice of it would be irrational.

For the matter of relevance, I begin with a curiosity of this mixed strategy on the assumption that has been in place that the values of particular

worlds, and of pure strategies for these, are finite. 'When' he chooses this mixed strategy, his *expected value* for it is infinite. This is the case we are considering. But 'when' he chooses this strategy he knows that its '*end value*', which is the value of the world he is going to get out of his creative labor, is *finite* and so *considerably* less than this strategy's expected value. Cf.: "[I]f a random variable has infinite expectation, then its true value is bound to be less than it expectation value" (Castell and Batens 1994, p. 49). Given that, how much should this mixed strategy be *worth* to him? Supposing that it 'costs him', how much should he be willing, how much is it *reasonable* that he should, 'pay' for it?! Certainly not an amount equal to his expected value for it, for he knows that, if he pays that much, *he must lose* a great deal. The *relevance* to his choice of his infinite expected value for this mixed strategy is a problem that is nothing like what this strategy is *worth* to him.[35]

For the matter of irrationality, there is why, even if his expected value for a mixed strategy is infinite, we should take for granted that he has available to him strategies he must rationally prefer to it. For definiteness (and relevance to ideas afloat), let this strategy be, from a denumerable set S of worlds whose members can be strictly ordered according to ascending goodness without upper bound, to select by "a very intricate device that, at the push of a button, will **randomly** select a number and produce the corresponding world" (Howard-Snyder 1994, p. 260; emphasis added). Let that be mixed creation-strategy J.[36] Now comes a strategy J′ that we must assume is available to our Creator if J is to select from set S by a device that at the push of a button selects a number and produces the world that corresponds to that number's *successor*. The expected values for these two mixed creation-strategies are 'infinite' (that is, here *greater than finite*). Even so, it seems that he should *prefer* the second strategy. Look at it this way: If there is a single intricate device that randomly selects a number, and the two strategies differed only in the way that number is used in the production of a world – if they differed only in their ways of linking the selection of a world to the outcome of a single 'random-number selection experiment' – our Creator would know that the second strategy way was *bound* to result in a better world, and of the two strategies he would prefer it.[37] Of course there is a strategy he would prefer to this successor-strategy. It seems that there are available to him countless mixed creation-strategies for which his expected values are 'infinite' and the same, and that each must be *dispreferred* by him to another, so that not one is a strategy he can rationally choose.

7.3.2.2.. So much for the 'first case,' in which it is supposed that a perfect being would *not* know 'just before' he made a mixed creation-strategy how this strategy that he was about to make was going to work out. Now, for the *second case*, I suppose that though he "has not **control** over what number his randomizer will deliver" (Howard-Snyders 1994, p. 263; emphasis added), he would *know* what number it was going to deliver. That,

indeed, is required by his omniscience, which includes his *foreknowledge* at all times of all truths concerning future times. It includes 'seeing through chance processes to their outcomes'. Suppose he knows he is going to choose a mixed creation-strategy M the objective expected value of which is infinite. In this case, *his* expected value for it is *not* infinite. Suppose that M will result in world W, whose chance in M is c and whose value V is finite. Our perfect-being Creator knows that the objective chance for W in M is c. But he also knows this result W of M. (How? *Somehow*. Perhaps by 'foresight'![38]) His probability for W's being the outcome of M, his confidence in that, is thus not c, but 1. And *his* expected value for M is not infinite, but finite V. In this, my 'second case,' there is no possibility of relief for perfect beings from problems if there is not a best world. While there can be infinite, and maximal, *objective* expected values of some mixed creation-strategies, our perfect Creator's expected value is, in this *second* case, in which he has foresight of the outcomes of chance events, for *every* creation-strategy, pure or mixed, *finite*, and, given that there is not a best possible world, *not maximal*.

8. MIGHT LOVE BE THE ANSWER?

It is, Adams maintains,

> consistent with God's being perfectly good . . . [that he] desire to create and love all of a certain group of possible creatures [even though they are] not . . . the best of all possible creatures, or included in the best of all possible worlds. . . . The desire to create *those* creatures is as legitimate [and consistent with his perfect goodness] . . . as [would be] the desire to create [instead] the best of all possible worlds. (Adams 1987, p. 55; article first published in 1972)

God though perfectly good need not, Adams says, prefer the better possible creatures to the worse.

> Here [in a 1979 paper] it can be added [to claims of that 1972 paper] that he could be perfectly good and cause or permit *evils* that are necessary for good ends that he loves, **even if those goods are not [parts of] the best states of affairs obtainable by him**. That there are . . . goods for which God's causing or permitting the evils that happen was necessary, I think the theist must believe . . . I am suggesting, in effect, that the existence of creatures such as we are . . . may . . . be a good of the relevant sort that is loved by God. (Adams 1987, p. 72: article first published in 1979; bold emphasis added).

I think that love for the acknowledged worse over the acknowledged better 'makes sense,' that it makes psychological sense, *for human beings*. It happens. And perhaps *preferences*, in the sense of *higher valuations* that can provide bases for choices and preferential behavior, for the loved worse over the unloved better and more deserving, are consistent with *human* goodness. (I personally hope so.) But preferences out of love for worse over better worlds

would not, I think, be consistent with the perfect goodness *of a perfect-being Creator*.

Almost every human prefers the preservation of the human race to its ultimate replacement by a more excellent species and thinks none the worse of himself for that preference (Adams 1987, p. 72). Many good people favor the "preservation and internal development" of their own civilizations and cultures over their dissolutions into more excellent ones (pp. 71–2). People are naturally attached to their cultures; they identify with them, they think of their histories, and wish for the sake of those who have been before that their cultures, their civilizations, should never end. Good people can, for the last reasons, favor the perpetuation of traditions *other* than their own, and this too over dissolutions into better ones. "A good person accepts significant costs . . . for the sake of what he loves, and not only for the sake of what is best" (p. 72). True. But again the position of a *perfect-being Creator* of worlds out of whole clothe is different. Suppose that he loved human beings 'in the beginning' before there were any human beings, and that he did not then love other *better* beings that he was capable of creating instead of human beings. His condition might be compared with that of an ordinary person who is captivated by the thought of a possible culture in another galaxy while being left cold by the thought of what he realizes would be a better culture there. *That* difference of affections, in contrast with the human ones cataloged, would be *strange*. And, for a separate point, preferences and consequent to them preferential behavior, for the acknowledged worse over the better that purported to be based on such affections would not merely be inconsistent with perfect goodness, but *incoherent*. Remember, to return to the problematic case, the Creator is to realize 'in the beginning, before he has done any creating', that the human beings he not only loves but prefers and 'values' more highly than certain other beings are less valuable than these other beings!

Ancient Jews are said to have thought of themselves as 'the chosen people'. Jahweh chose them. Why? "Perhaps," an ancient theologian might have said, "not because we are better than all other people, but because before He *chose* us, He *loved* us – *before* He chose us, He loved us – more than He loved other people, if He loved any others." Now *even if* that pre-choice *affection* would be consistent with Jahweh's goodness – perhaps because he was not responsible for it, because he could not help it, much as we are often not responsible for affection that simply happens to us, perhaps surprising us – His *preference* based only on it for the Jews would *not* be consistent with his goodness. Similarly, though perhaps more clearly, for a Creator's love for and preference for human beings over better beings before He created human beings and not these others, and similarly *certainly* for worlds: "*Why did you create this world rather than that better one*?" "Well, as soon as I thought of it, I don't know, I simply fell in love with it. Does that answer your question?" "*Who did you say you are*? *I may have mistaken you for someone else*?"

If the idea that God *suffers affections* is considered theologically incorrect, then so, I think, must be Adams's love-line. For his argument cannot begin, if one insists that God can only *love for reasons* and according to how much things *deserve to be loved*. It cannot begin if it is assumed that He is in *control* of His emotions as well as His judgments and actions. The idea would be that the human race might be 'out of God's antecedent love for it', that our race might be for this and no other reason His chosen, over better species that could be in its place, and but for His love would some day be in its place. Adams says that an *imperfect world* of erring humans could be, out of God's love specifically for *these* human beings, his choice over *better worlds*, including, if such there be, *best worlds*. But it seems to me that *even if* this love would be consistent with His perfect goodness, autonomy, and nonpassivity, neither divine preferences nor choices based on them would be.

CONCLUDING OPINIONS ABOUT PERFECT-BEING THEISMS, AND THE PHILOSOPHY THEREOF

What are we to make of all this? Shall we, because of the problem of evil in its myriad devolutions, give up on the God of the philosophers and classical theologians, that omnipotent, omniscience, benevolent being, that perfect being? I think so. I think that on those grounds we should set that pretender to the office down. Which is not to say that we should on those general grounds give up on God entirely, including 'the God of the Bible and Rabbis' (Howard Wettstein) 'or something very like Him' (Arthur Hugh Clough). Regarding God, thus understood, and therefore regarding God as understood I suspect by most believers, there is no *logical* problem of evil, and though there are for the world-wise and reflective evidential problems of evil, these can be for particular persons manageable without confusion, and without extraordinary 'priors' for their theisms.

The logical problem of evil is a problem for perfect-being theologies *only*. Most philosophy of religion, classical and modern, is for perfect-being theologies. That makes that problem and this philosophy not very important for the general population, or the particular population of believers, if, as I think, God would be, or could be, for these populations something else. Still this philosophy is 'very interesting for those who are interested', and I am not on this late page perversely disparaging or discouraging its practice. Personally, I love it.

APPENDIX A. ON ALLEGED INCOMPATIBILITIES OF DIVINE OMNISCIENCE AND FREEDOM

There are objections to the Argument from the World's Not Being a Best World that feature one argument or another for,

IncFrOm (freedom incompatible with omniscience): That this world features free agents who from time to time exercise their freedom is incompatible with the existence in it of a being that is omniscient in it in the manner of a perfect being.

These arguments, 'predicted' in Section 2.3, differ in what they 'take' from the qualifier 'in the manner of a perfect being'. To simplify my discussion and to ease contact to the literature, I assume that an omniscient who was a perfect being would be a *divine* being, indeed, God, and would be *everlastingly* omniscient. Different 'takes' from that qualifier for arguments to be considered in coming sections are that a perfect being would be (1) *simply* everlastingly omniscient, (2) *merely* (that is, not essentially) everlastingly omniscient, (3) *essentially* omniscient, and (4) *necessarily* everlastingly omniscient, or essentially omniscient and a necessary existent. The first argument uses *part* of (1): This argument, in effect, derives IncFrOm from the impossibility of foreknowledge, *anyone's*, of exercises of freedom. The other three arguments are concerned *specifically* with what would a perfect being's foreknowledge. These arguments share a format that comes from that of an argument of Nelson Pike's.

All four arguments are for incompatibilities of foreknowledge and *freedom of choice* that would entail *possibilities of acting otherwise*.[39] David Hunt explains that St. Augustine solved at best, not the problem of omniscience and freedom of choice, but (only?) the problem of omniscience and a certain 'choiceless freedom' that, without entailing possibilities of acting otherwise, is sufficient for moral responsibility. Hunt somewhat misleadingly writes that if we unevasively approach

> the problem . . . there are basically two tacks that can be taken . . . show how infallible foreknowledge is in fact compatible with avoidability; or explain why even a libertarian can deny that avoidability is a condition of free agency. Ockham and his modern followers have provided by far the most thorough and interesting case for the form, while Augustine and the anti-PAPists [the anti-Principle-of-Alternative-Possibility-ists] have made the most powerful case for the latter. These are the main options for anyone who eschews the easy out provided by theological and anthropological revisionism. (Hunt 1999, p. 21)

Misleading are the reference to *the* problem and the suggestion that these are exclusive-of-one-another tacks or options. There are two problems: There is the problem of divine omniscience and free choice; and there is the problem of divine omniscience and that 'freedom' that suffices for moral responsibility. And the two tacks can be merged, and always are, at least implicitly, by those that follow the first. They can be seen to want all the 'freedom' that is needed for moral responsibility, and *more*, in case choice and possibilities of acting otherwise are *not* needed, for they want that too. The arguments to be considered are all designed to deliver only this something more.[40]

A1 An argument from the purported impossibility of foreknowledge of exercises of freedom

A1.1. The argument of this section is to show that IncFrOm proceeds from

> EvrlstOm (everlasting omniscience): It is necessary that a perfect being would be everlastingly or at all times omniscient.

The 'main argument' here is that, since,

> ImpKnFtFr (knowledge of future free acts is impossible): No one (and thus not an everlasting omniscient) could know in advance what a free agent was going to do with his freedom.

it follows, given EvrlstOm, that IncFrOm. To see how, suppose that a world features free agents who from time to time exercise their freedom, and that there exists in it a perfect being. Then, by EvrlstOm, there exists in this world a being who knows in advance what at least some free agents in it will do with their freedom – let that be KFF. KFF is not just contradicted by ImpKnFtFr, but said by that principle to be impossible. From which it follows – given that EvrlstOm and ImpKnFtFr, since necessity and impossibility propositions, are themselves necessary – that the supposition made entails an impossibility, and so is itself impossible. That impossibility is, in other words, what was to be proved, namely, the incompatibility of freedom and the omniscience of a perfect being.

A1.2 ImpKnFtFr. IncFrOm follows from EvrlstOm and ImpKnFtFr. Now comes the heart of this argument for IncFrOm; it is a 'subsidiary argument' for ImpKnFtFr.

If it is known that a person will do a thing, then he will of necessity do it. But this means that if it is known that a person will do a thing, he will not do it freely. As said, he will do it of necessity. He will have to do it, since otherwise, if he did not do it, it would not be known that he will do it, but only believed. Furthermore, he will not do it freely because it is impossible that he not do it; he has no choice. It is impossible that he not do it, because, as already said, it is necessary that he will do. Indeed, and with apologies for 'furthermore' the two considerations are really one, since for any proposition, it is necessary if and only if its negation is impossible.

This argument may be compared with reasoning considered, but not endorsed, by Plantinga:

If God knows in advance that Paul will have an orange for lunch tomorrow, then it must be the case that he'll have an orange tomorrow; and if it *must* be the case that he'll have an orange tomorrow, then it isn't possible that Paul will *refrain* from so doing – in which case he won't be free to refrain, and hence won't be free with respect to the action of taking the orange. (Plantinga 1974a, p. 66)

If *anyone* knows that in advance, the rest follows, or does not follow. Nothing is made in this argument of the foreknowledge being God's. One response to this argument for ImpKnFtFr is to find in it a reason against EvrlstOm. If no one can know what free agents will do with their freedom, then it can seem unreasonable to require omniscience – knowledge of all truths – of a perfect being. What can recommend itself is a world-and-time-indexed requirement according to which a perfect being would in a world and at a time know only every truth of that world that is then knowable. A *better* response, however, is to find *fault* with this argument for ImpKnFtFr.

A1.3. This argument trades on an amphiboly of a kind that is regrettably, and curiously, common in philosophic prose. Freed of that amphiboly, the argument collapses. It is said that if an act will be done freely, then it cannot be known that it will take place. And, in case this is not sufficiently obvious to be used without a reason in an argument against premise (2′), a reason is given. The reason comes to this: If it is known that something will be done, then that thing has to be done. The trouble with this explanation is that the sentence in which it is conveyed is 'amphibolous in the scope' of the necessity signaled by 'has to be'. That scope can be wide and cover the entire conditional, or narrow and cover only its consequent. For concreteness let A be my driving home tonight, and consider the sentence, 'if it is known that A will be done, then A has to be done'. This sentence can mean either that it is necessary that if it is known that A will be done, then A will be done, $\Box[K(A) \supset A]$, or that if it is known that A will be done, then it is necessary that A will be done, $[K(A) \supset \Box A]$. The *first* of these is true. It is a trivial consequence of the general and unremarkable principle that, for any proposition p, if p is known, then – by definition of 'known' as distinct from 'believed' – p is true. From this we have that if p is known, then p is so. The conditional $[K(A) \supset A]$ is true by definitions of knowledge and truth, and so, as always with things that are true by definitions, it is necessarily true, which is to say that $\Box[K(A) \supset A]$. The *second* of those constructions, $[K(A) \supset \Box A]$, is rather obviously false. "Willa knows that I will drive home tonight (she knows these things!). So, (i) it is true that K(A). But it is absurd to suggest that it is logically necessary that I will drive home tonight, that it is impossible that I will walk, or run, or skip. So, (ii) it is false that $\Box A$. From (i) and (ii) it follows that it is false that $[K(A) \supset \Box A]$." (More or less as said in a night course some years ago.)

When the 'subsidiary argument' for ImpKnFtFr *uses* 'if it is known that something will be done, then that thing has to be done', it intends it in the second of the ways, distinguished, $(p)[K(p) \supset \Box(p)]$. But in that sense what it says is not necessarily true. In that sense what this sentence says is simply *not true*, since from time to time we know things that are not necessary truths. The argument gets what plausibility it has entirely from the ease with which that use of the sentence can be confused with a use in which it 'says' that $\Box(p)[K(p) \supset p]$. In this sense the sentence expresses a necessary truth, but not something

that is up to its work in the argument. The sentence 'if it is known that p, then it has to be p' is supposed to say something that, when conjoined with K(A), entails □(A). K(A) and □(p)[K(p) ⊃ p] certainly do not entail □(A), for K(A) and □(p)[K(p) ⊃ p] are true, and □K(A) is certainly false, if modal intuitions count for *anything*. K(A) and □(p)[K(p) ⊃ p] entail only A. Necessity attaches to this entailment, of course, but not to the thing entailed.[41]

*A2. **Mere** everlasting omniscience and freedom: An argument 'after' one of Nelson Pike's*

A2.1. The argument to be considered works with the premise,

> **MrEvrlstOm** (merely everlasting omniscience): It is necessary that a perfect being would be merely everlastingly omniscient.

'Merely' here means *not essentially*. The coming argument is adapted from Pike (1989, pp. 61–4): Pike's own argument is for the incompatibility of God's existence with freedom, where 'God' would name an essentially everlasting omniscient diving being (see pp. 58–60). I place quotation marks around the argument I have made up to discuss, to distance myself from it: I shall say that there is something wrong with it. "*To be derived from MrEverlstOm*.

> **IncFrOm** (freedom incompatible with omniscience): That this world features free agents who from time to time exercise their freedom is incompatible with the existence in it of a being that is omniscient in it in the manner of a perfect being.

Suppose, for purposes of an indirect derivation, the following instances of existential generalizations that say, respectively, that there are free agents who from time to time exercise their freedom, and that there is a being that is omniscient in the manner of a perfect being.

(i) G is omniscient in the manner of a perfect being.
(ii) A is a free agent A who, exercising his freedom, will do X not Y at some time t, though he could instead do Y.

To complete this derivation, I derive from (i) and (ii) a contradiction.

(iii) G is a merely everlastingly omniscient being. from (0) and (i)
(iv) A will not do Y at t. from (ii)
(v) A could do Y at t. from (ii)

I assume that knowledge of a perfect being entails belief, and that it follows from (i) and (iii) that,

(vi) At some time t′ earlier than t, G believed that A was not going to do Y at t.

It follows from (iii), (iv), (v), and (vi) that:

(vii) **Either, (a) if A were to do Y at t, this would bring it about that G believed at t′ that A was going to do Y at t; or (b) if A were to do Y at t, this would bring it about that G did not exist at t′; or (c) if A were to do Y at t, this would bring it about that, though G *was* merely omniscient at t′, G *was not* merely omniscient at t′.**[42]

It is plainly too late at t for (a) and (b). Since G is merely everlastingly omniscient – since though everlastingly omniscient G is not essentially so – there is nothing that anyone can do at t that would bring it about that G, who did hold a certain belief at t', either held a different belief at t′ or did not exist at t′. G's past beliefs and existence are in this respect no different from those of ordinary persons: They are 'over and done with', 'fixed forever'. For G is only less erring (G is unerring) and temporally greater (G is always there) than ordinary persons, not *essentially* less erring and temporally greater. 'Modally speaking' G is just like us. As analoges of (a) and (b) would be 'out' for us, so they are 'out' for G. So:

(viii) Neither, (a) if A were to do Y at t, this would bring it about that G believed at t′ that A was going to do Y at t; nor (b) if A were to do Y at t, this would bring it about that G did not exist at t′.

But it follows from (v) that,

(ix) It is not the case that, if A were to do Y at t, this would bring it about that, though G was merely omniscient at t, G was not merely omniscient at t'.

So (c) is 'out' too. For this inference we have that the possibility affirmed in (iv) is 'real agent possibility,' and it is a valid principle of well-known logics for subjunctive-conditionals that, in a very weak sense of possibility (for Robert Stalnaker, logical possibility, for Lewis, entertainability for purposes of 'subjunctive speculation'), for any propositions p and q, POSS(p) entails $\sim [(p \,\Box\!\!\rightarrow (\sim q \,\&\, q)]$.

(x) Neither (a), nor (b), nor (c). from (viii) and (ix)
(xi) **It is not the case that [either (a), or (b), or (c)].** from (x)

The contradiction in (vii) and (xi) completes an indirect derivation of IncFrOm from MrEvrlstOm."

A2.2 Criticism. While (a) and (b) are 'out' for the reasons stated, (c) is not out, though it has been expressed in a manner that can make it seem so. The 'trick' lies in the 'parenthetical amphiboly' of its sentence 'if A were to do Y at t, this would bring it about that, *though G was merely omniscient at t′*, G was not merely omniscient at t,' which is equivocal between,

(c1) Though G was merely omniscient at t′,
if A were to do Y at t, this would bring it about that G was not merely omniscient at t′.
(Om(G,t') & [Y(A,t) □→ ∼Om(G,t')])

and

(c2) If A were to do Y at t, this would bring it about that
O was not merely omniscient at t' though O was merely omniscient at t′.
(Y(A,t) □→ [∼Om(G,t') & Om(G,t′)])

Premise (c) needs to be taken for (c1) in the inference from,

(iii) G is a merely everlastingly omniscient being.
(iv) A will not do Y at t.
(v) A could do Y at t.

and

(vi) At some time t′ earlier than t, G believed that A was not going to do Y at t.

to the disjunction, (vii), [(a) ∨ (b) ∨ (c)]. For we have seen that those lines entail ∼(a) and ∼(b), which means that they entail (vii) only if they entail (c). **These lines do entail (c1)**: They entail that G was merely omniscient at t′; and they entail that G would still have believed at t that A was not going to do Y at t, were A to do Y at t, were A to do Y at t; so they entail that *G would not have been omniscient at t′, were A to do Y at t*. There is no problem with that. It is not as if G was *essentially* omniscient at t′.[43] **But these lines cannot be said also to entail (c2) without begging the question of this derivation**. For (v) entails ∼(c2) so the four lines entail (c2) only if they are inconsistent. And they are inconsistent only if 'freedom is incompatible with merely everlastingly omniscience,' which is what is to be proved. Thus (c) needs to be (c1) in (vii). *However*, (c) needs to be (c2) in (ix), which is ∼(c), in order for (ix) to follow as it is said to from (v). Step (v) does entail ∼(c2), but it does not entail ∼(c1). Step (v), which is POSS[Y(A,t)], is consistent with (c1), which is (Om(G,t′) & [Y(A,t) □→ ∼Om(G,t′)]). Depending on whether (c) is interpreted as (c1) or (c2), the derivation stops at (vii) or at (ix) before it is done.

A2.3 Moving on. We are not out of the woods, however, for perhaps a perfect being would, by proper definition, not be merely everlastingly omniscient, as well as omnipotent, perfectly good, and the rest, and not essentially so. Perhaps a perfect being would be essentially everlastingly omniscient and the rest. Perhaps, beyond even all that, a perfect being would be a necessary existent. That is not an unheard of opinion. Such strengthenings of the idea of perfectness can be expected to change the argumentative context in significant ways. The deep

difference between nontheological foreknowledge arguments for fatalism that nearly everyone sees are frauds and delusions, and theological arguments on at least some of which much of the jury is still out, has been said to depend entirely on the supposed fact that God, in contrast with every possible ordinary person, would be essentially omniscient if not necessarily existent. (Consider Fischer 1989, p. 15, and Widerker 1989a, pp. 105–8.) Reflection on the dissolution of the argument of the Pike-style argument of the previous section can suggest that, if not already in essential everlasting omniscience, then certainly in essential everlasting omniscience coupled with necessary existence there *is* a condition that closes every way out, and that is incompatible with freedom.

A3 An adaptation of the argument to ***essential*** *everlasting omniscience* ***without*** *necessary existence*

A3.1. The argument would derive IncFrOm the premise,

> (0′) EssEverlstOm-NcEx (essential everlasting omniscience without necessary existence): It is necessary that a perfect being would be essentially everlastingly or at all times omniscient, but not a necessary existent.

The argument of Section A2.1 adapted to this premise can seem to go through. Here are its 'lines', with subsidiary argumentation for two of them. It is to be understood throughout that G is *not* a necessary being. Suppose:

(i) G is omniscient in the manner of a perfect being.
(ii) A is a free agent A who, exercising his freedom, will do X not Y at some time t, though he could instead do Y.

Then:

(iii′) G is an essentially everlastingly omniscient being. (from (0′) and (i))
(iv) A will not do Y at t. (from (ii))
(v) A could do Y at t. (from (ii'))
(vi) At some time t′ earlier than t, G believed that A was not going to do Y at t. (from (iii'), (iv), (v))

It follows from (iii′), (iv), (v), and (vi) that

(vii′) **Either, (a) if A were to do Y at t, this would bring it about that G believed at t′ that A was going to do Y at t; or (b) if A were to do Y at t, this would bring it about that G did not exist at t′; or (c′) though G was essentially everlastingly omniscient at t, if A were to do Y at t, this would bring it about that G was not essentially everlastingly omniscient at t′.**

Alternatives (a) and (b) are 'out' for the reasons that expelled them in the derivation of Section A2.1. One supposes that it is too late at t for anyone to do anything about G's beliefs, and of course too late for anyone to do anything about G's *existence*, at earlier time t′. So:

(viii) Neither (a) nor (b).

And this time (c′) is 'out' too. It is not, as (c) was in the previous derivation, amphibolous. Alternative (c′) says that G was essentially everlastingly omniscient at t′, *and* that if A were to do Y at t, G would not have been essentially everlastingly omniscient at t′. Given (v), that A could do Y at t, so that there is a world in which A does Y at t that is 'near enough' for purposes of counterfactual speculation, the conjunction (c′) cannot be true. For, according to its first conjunct, there is no world in which G exists and is not always essentially everlastingly omniscient. Whereas the second conjunct is true only if, at 'nearest' worlds in which A does Y at t, G exists and is not essentially everlastingly omniscient at t′.[44] So:

(ix) It is not the case that, though G was essentially omniscient at t, if A were to do Y at t, this would bring it about that G was not merely omniscient at t′.
(x) Neither (a), nor (b), nor (c). (from (viii) and (ix))
(xi) **It is not the case that [either (a), or (b), or (c)].** (from (x))

The contradiction on lines (vii′) and (xi) completes an indirect derivation of IncFrOm from EssEvrlstOm. -NcEx.

A3.2 Criticism. This argument runs into trouble with its exclusions of (a) and (b). While these are 'out' for merely factually everlasting omniscients who are like ordinary knowers in not being *essentially* of their knowledgeable characters, it is a fresh question whether they are 'out' for our present *modally* extraordinary, because *essentially* everlasting omniscient. To leave something for the next hypothesis that adds necessary existence to essential everlasting omniscience, I will not question the exclusion of (a) here and will resist only the more easily understood exclusion of (b). Assume, as it is easy to do, that (a) is 'out' and that it is not the case that, if A were to do Y at t, this would bring it about that G believed at t′ that A was going to do Y at t. Assume, indeed, that A's doing y at t would bring it about that, if G existed at t, A would still believe that A was not going to do Y at t. Assume beyond that, if you like, that there would be no change at all in anyone's past beliefs. Then G would not *exist* at t′, since if he did, he would believe a falsehood, and G – *essential* everlasting omniscient that he is – does not ever exist in a possible world in which he believes a falsehood! 'Actions otherwise' in worlds inhabited by essentially everlasting omniscient *deities* without necessary existence would be acts of 'ontological deicide'!! (Cf., comments on would-be 'ontological suicides' of essential omnipotents in Sections 5.3 and 5.4 of Chapter IX.)

Facts of the past existence of essential omniscients without necessary existence would, according to most accounts of the soft/hard fact-distinction, not be 'hard facts.' And they would, assuming as I am presently, that all past beliefs are 'fixed' relative to future choices and actions, not be fixed in this manner.

What would happen if A were to do Y at t? Would there be a spectacular disappearance of G throughout all time? Probably not. What would be the case is what is the case in 'nearest worlds' to the actual world in which A does Y at t. Given that past beliefs are fixed relative to actions – we are presently assuming this – it seems that through all time there would be a being G′ amazingly similar to G in place of G. This G′ would not *be* G, since G′ would believe at t′ that A was not going to do Y at t, which would be false, and have many other beliefs that G has though they would be false, and essential everlasting omniscient G does not in any world in which he exists believe anything false. But this G′ would, it seems, be in every other way a ringer for G.[45]

A.3.3 Remarks on soft and hard, and fixed and unfixed facts. According to McCord Adams (1989, pp. 75–6): (1) Statement p expresses a *hard fact* about time t if and only if p expresses a fact that is at least in part about t, and not at least in part about any time future relative to t. (2) A statement expresses a *soft fact* about a time if and only if it expresses a fact that is at least in part about that time, and is not a hard fact about that time. (3) A statement p is at least in part *about* time t if and only if the happening or not happening, the actuality or nonactuality, of something at t is a necessary condition for the truth of p. This account is "not fatally flawed as [some critics] think" even if it is at points (e.g., 'something' and 'necessary condition') "stated rather obscurely" (Widerker 1989b). Of subsequent proposals for the exact terms of the soft/hard fact-distinction, the account in Zemach and Widerker (1989, pp. 113–7) is I think most promising. For one virtue, it does not depend on the identification of atomic or simple facts as do certain other proposals.

Fischer might have objected in 1989 that the Zemach-Widerker account would get wrong F2 (that exactly seven persons believe at t that A will do X at later time t′) in a world in which F2 along with F1 (an essentially omniscient necessarily existent being believes at t that A would do X at t′) were facts. F2 would be hard relative to t on the Zemach-Widerker account, though F1 was soft in that world. According to John Fischer, either neither or both should be hard relative to t (Fischer 1989, p. 37). But in my view, the Zemach-Widerker account is on a good track (see Section 4.3.2 of Chapter IX for possible improvements), and its authors can take comfort in the thought that in the world described (if there is such a world, featuring as it would a truly extraordinary, a modally extraordinary, believer), though F2 would be hard relative to t, it would not be 'fixed' relative to all possible choices at t, including A's. It is, they might agree with Fischer, "very

important to distinguish two sets of issues: first, temporal nonrelationality and relationality (i.e., hardness and softness), and second, fixity and nonfixity (i.e., being out of one's control and being in one's control" (Fischer 1994, p. 115). In saying that, according to the Zemach-Widerker account of the hard past, F2, if a fact, though hard, *may* not be fixed relative to this choice – in saying that it is not, if a fact, necessarily fixed, *if* F1 is logically possible (as it may very well not be; see Section 5.4 of Chapter IV for room for doubting that an essentially omniscient *necessarily existent* believer is so much as logically possible) – I am anticipating a view retailed in Section A4.2 later. According to this view, another view of Zemach and Widerker: F1, if true, is both soft and not fixed, *because* if A were not to do X at t′, then that omniscient being, without being in any way different in its intrinsic nature, would not believe at t that A will do X. Connectedly, F2, if true along with F1, is though hard, not fixed, for if A were not to do X at t′, then exactly six persons (not seven) would believe at t that A will do X. (I do not agree with Wierenga that it is "a *defect* " of the Zemach and Widerker account of the hard past that "[i]t is not clear that on [this account] no one has it within his or her power so to act . . . that a hard fact [true proposition] about the past would have been false [and not a fact]" (Wierenga 1989, p. 108).

Fischer himself, it may be noted, in 1994 adopts an account – he calls it the Entailment View – that makes F2 a hard fact in every world in which it is a fact and F1 a soft fact in any world which it was a fact (Fischer 1994, p. 113). His account of 1994 is open to the criticism that, in 1989, he directed against the Zemach-Widerker account. Fischer stresses in 1994 that on his, and all intuitive accounts, facts can be soft and yet fixed relative to possible future actions. I have pointed out that given a certain view (the one explained later in Section A4.2) that Fischer himself rejects (pp. 121ff), in a world in which not only F2 but also F1 were facts, F2, though hard, according to his Entailment View would not be fixed.

A3.4. It is well known that it has been held that the existence of an essentially everlasting omniscient would be a soft fact that was not fixed. This is the position of McCord Adams's influential paper, "Is the Existence of God a 'Hard Fact'?" "In sum, I have argued that the existence of an essentially omniscient and everlasting God is not a 'hard' fact . . . " (McCord Adams 1989, p. 85). It is therefore curious that this position is more often than not either forgotten or ignored in discussions. Have versions of it any public defenders other than McCord Adams and me? Little has been said against the line. The only objection to it of which I am aware is that it is 'theologically implausible' and would be 'unseemly' for God's existence to be dependent on His creatures' free actions. Further to the assertion of 'theological implausibility' (Fischer 1989, p. 34), Fischer writes that "God [would be], after all, the supreme being, a being 'than which nothing greater can be conceived,' in St. Anselm's words"

(Fischer 1989, p. 9). "Further to the charge of unseemliness," John Fischer might wish to say, "such a God would hardly be worthy of worship" (p. 96). But St. Anselm's authority is of doubtful use regarding the assertion of theological implausibility, since he of course maintained that God would be a necessary existent, and such a god could not, whatever its essential attributes, be dependent for its existence on anything. As for worshipfulness, though it reflects Fischer's opinion, that opinion is not universally shared. It was apparently not McCord Adams's opinion, who we must assume found no difficulty in worshipping a god she believed she had the power to eradicate merely by 'sleeping in' on a morning in which she does not do that. Perhaps she was impressed by the consideration that He is never worried – that He always knows that no one ever will undo Him – that He always knows that He is, that He is essentially, everlastingly.

> And God said unto Moses, I AM THAT I AM. (*Exodus* 3:14)[46]

Isn't that enough to get to worshipping?

*A4 A similar argument for **essential** everlasting omniscience **with** necessary existence*

A4.1. The argument of this last section would derive IncFrOm from,

(0′) EssEverlstOm+NcEx (essential everlasting omniscience with necessary existence): It is necessary that a perfect being would be essentially everlastingly omniscient, and a necessary existent.

The Pike-style argument adapted to essentially everlasting omniscients with necessary existence features the lines:

(iii″) G is an ***essentially*** everlastingly omniscient being who is a ***necessary existent***.

(vii″) Either, (a) if A were to do Y at t, this would bring it about that G believed at t′ that A was going to do Y at t; or (b) if A were to do Y at t, this would bring it about that G did not exist at t′; or (c") though G was an essentially everlastingly omniscient necessary being at t′, if A were to do Y at t, this would bring it about that G was not an essentially everlastingly omniscient necessary being at t′.

It is clear that the way around the previous argument taken in Section A3.2 is now closed. That way was to observe that (c′) of (vii′) is out, and, on the assumption that (a) is out, to affirm (b) and say that it is not out for an essentially everlastingly omniscient G who does not exist necessarily. But (b) certainly is out for our present essentially everlastingly omniscient G, for it is given that he exists necessarily. From that it follows that there is nothing that anyone can do at any time that would bring it about that this G did not exist at t′.

For, the essentially everlasting necessary being that G is, in every possible world G exists always. So both (b) and (c″) are out for essentially everlastingly omniscient and necessarily existent G. That leaves (a), which it may be recalled was only assumed to be for the argument of Section A3.1. I say that it is not out for the present argument, or, less assertively, that no compelling arguments to the contrary exist in the literature, and that there can be found in it a sketch of how (a) can be countenanced. But, one wants to say, "Surely the fact that someone *believes* at some time t that something will be done at a later time is a fixed fact about t. How could a person's beliefs at a time be counterfactually dependent on what happens at later times?"

A4.2 Ways in which an everlastingly essentially omniscient necessary being's beliefs might be malleable. Suppose G is such a being. G's beliefs in a possible world, about the world apart from G, could in two ways be depicted as *functions of* the state of this possible world *apart from him*. One Way:

> This position might be supported by extending Putnam's point that meanings and beliefs ain't in the head. According to Putnam, my belief that water is wet – the state of my mind that constitutes in fact, my believing that – would have been a different belief – the belief that XYZ is wet – if lakes and oceans on earth had been filled with XYZ rather than water. (Fischer 1989, p. 94)

Though I would call it 'water,' it, by hypothesis, would not be water, but XYZ. And my beliefs expressed using 'water' would be not about water, but about this XYZ stuff. So one sees that my beliefs are partially constituted by what happens to be the case outside me. Somewhat similarly, the suggestion goes, for G, an essentially everlasting omniscient who is a necessary existent. Taking it to be possible that A do Y at t – supposing, that is, that A has this power to act otherwise – the current suggestion concerning G's beliefs says that if A were to do Y at t, then of course G would be in a different world at t′. As a consequence, though the same 'in himself' if 'transported by my action-otherwise to another world' (as Putnam would have me be 'in my head' if transported to Twin Earth), his 'intrinsic mental state' at t′ would, in this other world, come to somewhat different beliefs. In particular, this state would not, as it does in the actual world, constitute amongst countless other beliefs and attitudes the belief that A will not do Y at t, for that would be a false belief. Rather, it would constitute amongst countless other beliefs and attitudes the belief that A *will* do Y at t. As my beliefs are 'in their wide contents' partially constituted by what happens, so, the hypothesis goes, G's beliefs at a time would be partially constituted by what happens later. It is a *prima facie* coherent theoretical option.

It makes Putnam-sense; it could for all we know, be that the internal state of G would constitute, amongst other things, a belief that p in a world, if and only if it is the case that p in this world. (Cf., Zemach and Widerker 1989a, p. 118, and Wierenga 1989, pp. 107–8.) Why not? G could be in all ways God-like and yet "not a member of any linguistic community" (Zemach and Widerker, p. 119).

Who knows, therefore, in virtue of what the unlearned mental words in which He would believe would "have this or that definite meaning" (p. 119) in this or that possible world? "We suggest that the item in virtue of which some state of God's [would mean] p, [would be] the fact that p" (Ibid.). Soon I will take up possible objections to this Zemach-Widerker account of how, without G's intrinsic state at a time being ever counterfactually dependent on what happens at later times, G's beliefs at a time could be thus dependent. First, however, let me enter a simpler and possibly better account.

Another way: A theorizing theologian might *reject* "the idea that God's beliefs involve . . . mental representations" (Fischer 1994, p. 124) and inner, in contrast with wide, content that underlies the Zemach and Widerker account. (I will now allow that the G of whom I have been speaking would be God.) A theorist might say that, for God, believing facts about the world's future and past, as about its present, would be a matter of unmediated-by-representations 'cognitive possession'. That seems the natural thing to say about the knowledge and belief of a would-be essentially *everlasting* necessarily existent perfect being God, as it would be the only thing to say about the knowledge and belief of a would-be essentially *eternal* necessarily existent perfect being God to whom, it can be seen, Pike's argument cannot be extended. Few, if any, philosopher/theologians think that believing facts about the world would be for Him mediated by mental representations that were *caused* by those facts, or that believing would be for Him mediated by mental representations from which those facts could be only *inferred*. As He would act on the world directly, so, the present suggestion goes, would He apprehend it directly. Direct-apprehension theologians see would-be divine mental representations of facts about the world as 'fifth wheels' and do without them in their accounts of divine believing. Connectedly, their theories afford no distinction between the narrow and wide contents of Divine beliefs. Nor do they afford a distinction between Divine Knowing and Divine Believing: Each is for these theorists a matter of direct apprehending.

A4.3 Possible, but I think not persuasive, objections. Fischer has written against the Zemach-Widerker account that "God's omniscience would be seriously attenuated if the same state of God's mind at t_1 would constitute different beliefs about Jones, depending on Jones's behaviour at t_2" (Fischer 1989, p. 94). One wonders, however, how it would 'seriously attenuate' God's omniscience for Him to be, though not 'in Himself', still in his beliefs, 'transworld mutable' in the manner that Zemach and Widerker say He might be. Would Fischer say there is a problem for God's omniscience if His beliefs at t depend on Jones's behaviour at t?! "But is it not unseemly – is it not even 'blasphemous' – to suggest that we might have this kind of power over God's beliefs?" See Zemach and Widerker (1989, pp. 120–1) for this question, and for an implied claim, No, as its answer. Fischer's charge of attenuation may harbor the idea that God's being 'in Himself' the same in every world would make absolutely mysterious

differences in His *behavior* in different worlds. One supposes that in a world in which A does X at t, God might warn of dangers that A's doing X courts, even though in another world in which A does not do X, God would not issue that warning. "But how to understand this difference in behaviour, given no difference in God himself, so that for one thing 'the narrow content' (Fischer 1994, p. 123) of His beliefs are the same? Would not differences between their 'wide contents', since 'not reflected in any 'internal' differences' (p. 123) be inaccessible to God, and so not reflected in His behavior?" Presumably not, for by hypothesis God is omniscient, and nothing that is the case is inaccessible to Him. That, it must be conceded, makes a *mystery*, for we are apt to suppose that when someone X knows some contingent fact F, this fact *makes* a difference 'to and in X'. But that there is mystery here should not surprise, especially if, as I think, this mystery arises for an *essentially* everlastingly omniscient who is furthermore a *necessary existent*. Such an *extraordinary* being – such a *modally* extraordinary being – would be in His existence, and one supposes might well be in His beliefs, behavior, and 'psychology', removed by a *vastness* from *ordinary* beings such as ourselves, and certainly *very* strange and mysterious to our ways of thinking. Fischer in the end graciously concedes that "greater mysteries have been associated with God," and that he does not claim to "have presented a knockdown argument for the [practical] relevance [even in His case] of [only] the narrow sense of belief content" (p. 123). It can be seen, incidentally, that Fischer's challenges to the Zemach and Widerker way of making Divine beliefs about future facts counterfactually dependent on these facts are *specific* to that way. They are not adaptable to the way of direct-apprehension theologians. This is one reason for thinking that the direct way is a better way, even if, as I think, the Zemach and Widerker way is good enough to disarm Fisher's basic-version argument for the incompatibility of divine omniscience and the possibility of actions otherwise.

A4.4. We have accounts of how, if G were transported by A's doing Y at t to another possible world, G's mental state, though unchanged in itself, would *constitute* true, and thus somewhat *different*, beliefs at t′ that would include the belief that A will do Y at t. That these accounts are sketchy should not surprise, given that they are *inter alia* accounts of an essentially omniscient and necessary existent's mind, and its relation to contingent beings who can 'act otherwise'. However, sketchy though they are, they allow us with *some* (not much!) comprehension to affirm (b) for such an omniscient. They are ways around the most promising of Pike-style arguments, namely, the argument that would show that the essential everlasting omniscience of a necessary existent cannot cohabit a world with free agents (itself included?!!). Certainly these accounts propagate mysteries, but then that, to some minds, can be a further recommendation. For if there is one thing on which all can agree, an essentially perfect God who existed of necessity would be wondrous and mysterious indeed.

Compatibilist Conclusions

'Actions otherwise' would bring about the nonomniscience of merely everlastingly omniscient beings: Section A2. We can say that 'actions otherwise' would bring about changes in the beliefs of essentially everlastingly omniscient beings who were necessary existents: Section A4. And we can either say (with Zemach and Widerker) that 'actions otherwise' would do that to essentially everlastingly omniscient beings who were not necessary existents, or say (with McCord Adams) that they would bring about that these omniscients had never existed: Section A3. I conclude that freedom is compatible with the existence of a perfect being no matter in what manner it would be omniscient, merely everlastingly, essentially everlastingly, or essentially everlastingly as a necessary being, or, less assertively, that I know of no good reasons for thinking otherwise.

APPENDIX B. A DEDUCTION IN SECTION 2.2.3 SPELLED OUT

To detail the deduction of ∼(*) from IncFrPrfctn, FrAgtsBstWrlds, and PossPrfctn, I symbolize using the following patterns of abbreviation. B: - is a best world; P: a perfect being exists in -; F: there are in – free agents who from time to time exercise their freedom; @: the actual world. The possible world that is actual, @, 'exists in itself': That is, @ is in the domain of existents of @. Indeed, *every* possible world is in the domain of existents of *every* possible world: We have thus, for use in the derivation to come, the *additional* premise that $\Box(\exists x)(@ = x)$, or equivalently that $\Box$**E!**@. "But this business of worlds in worlds is absolutely wild!!" Not at all. For 'worlds' here are 'possible worlds' or comprehensive ways that things might have been. They are not massive 'concrete individuals' but comprehensive *possibilities*. And that these, that all possibilities, should be the same no matter what 'world' is the way things actually are, is exactly right: As numbers are necessary existents, so are possibilities. Now comes the deduction (it is in SMC of Appendix B to Chapter III) of,

$$\sim(*):\sim\Diamond(\sim B@ \;\&\; P@)$$

from

$$\text{IncFrPrfctn: } \sim\Diamond(F@ \;\&\; P@),$$
$$\text{FrAgtsBstWrlds:}\Box(w)(Bw \supset Fw),$$
$$\text{PossPrfctn: } \Diamond P@,$$

and (additional premise),

$$\Box E!@.$$

1.	~~*SHOW*~~ ~(*): ~~◊(~B@ & P@)	DD
2.	~~*SHOW*~~ ◊(~B@ & P@)	ID
3.	~◊(~B@ & P@)	assumption
4.	□~(~B@ & P@)	3, MN
5.	◊ P@	**PossPrfctn**
6.	~◊(F@ & P@)	**IncFrPrfctn**
7.	□(~B@ ⊃ ~P@)	6, interchange of logical equivalents
8.	~~*SHOW*~~ □(B@ ⊃ ~P@)	ND
9.	~~*SHOW*~~ B@ ⊃ ~P@	CD
10.	B@	assumption
11.	□E!@	additional premise
12.	□(w)Bw ⊃ Fw)	**FrAgtsBstWrlds**
13.	E!@	11, N
14.	B@ ⊃ F@	12, 13, N, FUI
15.	□~(F@ & P@)	6, MN
16.	~P@	10, 14, MP, DN, 15, N, DeM, MTP
17.	~~*SHOW*~~ □ ~P@	ND
18.	□(~B@ ⊃ ~P@)	7, R
19.	□(B@ ⊃ ~P@)	8, R
20.	~P@	18, N, 19, N, separation of cases
21.	~◊P@	17, MN (This contradicts line (5).)
22.	~~◊(~B@ & P@)	2, DN

PRACTICAL ARGUMENTS FOR AND AGAINST THEISTIC BELIEFS

XIII

Pascalian Wagers*

A person can have good practical reasons for believing in God even if he is convinced that he does not have good intellectual reasons for doing so. Whether a person does have good practical reasons for believing in God depends on his probabilities and values for consequences of that believing, in which probabilities and values persons can vary. Pascalian wagers founded on a variety of possible probability/value profiles will be examined from a Bayesian perspective central to which is the idea that states and options are pragmatically reasonable only if they maximize subjective expected value. Attention will be paid to problems posed by infinite values. Part One explains the distinction between intellectual and practical reasons, and presents Pascal's Wager as cast by William James. Part Two comments on characteristics of belief options in Pascalian wagers to be studied. Part Three contains discussions of several wagers. It explores the variety of probabilities and values that can be brought to the betting table, and sets out wagers determined thereby as problems in Bayesian decision theory. A series of notes and an appendix bring to our subject a theory of 'hyperreal' numbers.

PART ONE. GROUNDS FOR BELIEFS

1 Theoretical and practical reasons. What is the proper response of a person who is convinced that there are no good arguments for or against the existence of God, that on balance his evidence and experience testify sufficiently neither way, and that he cannot decide the issue of God's existence on intellectual grounds? It can seem (to borrow and change the concluding speech in Hume's *Dialogues*) that such a person, if inquisitive and contemplative, can do nothing other than to withhold assent from both theistic and atheistic hypotheses, as often as either occurs, and accept that, when discounted by objections against them and by arguments opposed to them, arguments for neither are sufficient to sustain belief. Convinced that the arguments for and against God's

existence are deficient, reasonable persons, exercising responsibility for their opinions, would, an intellectualist (he might be William Clifford) maintains, form none on this subject. For at least on matters such as God's presence, the wise proportion and trim their beliefs to the evidence and reasons. When convinced that neither side of such an issue enjoys a sufficient balance of evidence and reasons in its favor, they do not take sides. The wise say of such matters not merely, "I do not know," but also, "I do not believe, nor do I disbelieve."

Austerity of opinion in the perceived absence of intellectual grounds for belief can strike one as healthy and reasonable.[1] It can strike one as exclusively reasonable. For surely if in a person's view there is not sufficient evidence to decide some issue, then there cannot in his view be sufficient reasons for a belief either way on the issue. Suppose, for example, that Mary believes that her lover is trustworthy. Then surely, if she is minimally rational – if she is so much as consistent – she cannot both think that she has no grounds for this belief that he is honest and think that she has very good reasons for her belief in his honesty. I say, however – here begins a response to radical intellectualists such as Clifford – that she *can* without contradicting herself think these two things about her reasons, since the reasons she thinks that she lacks, and those that she thinks that she has, can be of different kinds. If you say that Mary's belief that her lover is trustworthy is true, your assessment is of the *proposition* that you imply that Mary believes, while if you say that Mary's belief that her lover is trustworthy is unshakable, or that it is important to their relationship, your assessments are of *a state or condition of her mind*. If you say that her belief is ill-advised, you may have in mind neither the proposition nor the state, but *activity related to the state*; you may have in mind the *acquisition* or *maintenance* of that state. Intellectual reasons for or against beliefs *qua* propositions believed go to the issue of their *truth* and take the form of arguments, experiences, and analyses that persons perceive to bear on this 'objective' issue. Practical reasons for beliefs *qua* states of minds go to their *desirabilities*, as do practical reasons for beliefs *qua* acquisitions and maintenances of these states. Assessments of beliefs as reasonable can be in terms of either believers's intellectual reasons or their practical reasons. Mary's belief *qua* proposition can be assessed in terms of her evidence for it; it can be assessed in terms of its theoretical or intellectual reasonableness for her. Her belief *qua* acquisition and maintenance of a belief-state of mind can be assessed in terms of her probabilities and values for their possible consequences broadly understood to include these activities themselves as well as the state of mind they serve, which activities and state she might consider to be of intrinsic value or disvalue. Her belief *qua* these activities can be assessed according to its desirability and, as can any activity, according to its pragmatic or practical reasonableness for her. I have spoken of Mary's belief, understood one way or the other, as being reasonable, in one way or another, *for her*. The categories of intellectual and practical reasonableness with which I am concerned are subjective and function, on the intellectual side,

of the evidence and experience of a person that in his view bears on the truth of the proposition he believes and, on the practical side, of his probabilities and values for consequences of a person's belief acquisition and maintenance activities. Mary's belief *qua* proposition can be intellectually reasonable for her, though it would not be intellectually reasonable for her lover (who may know that he is not trustworthy). Such differences are possible even when there are no differences between the evidence and experience of persons, and such differences can persist after any amount of reflection by either or both parties on identical evidence and experience. Similarly, Mary's belief *qua* activities of acquisition and maintaining – *qua*, one might say, her being actively *believing* – can be desirable and practically reasonable for her, though it would not be for her lover (who may place no value on being trustworthy, let alone on *his* believing that he is, the cad!), and these differences are possible without differences in evidence and experience, or thoughtfulness.

Mary can without contradiction think that in one way she has no reasons at all for her belief that her lover is trustworthy, and that in another way she has every reason to believe this. She can think both that she has no evidence for his trustworthiness, and that for the sake of their relationship she should trust him implicitly, and believe that he is trustworthy (cf., Holton 1994, p. 68). Her well-based answers to the theoretical question, *Is he, or is he not, faithful?,* and the practical question, *Should I, or should I not, trust him?,* can match or mismatch. They can be not only (yes, yes) and (no, no), but also (no, yes) and even (yes, no). They can be (don't know, yes) and (don't know, no) as well as (yes, it doesn't matter) and (no, it doesn't matter). There are no restrictions on the ways in which a person's intellectual and practical reasons for or against a belief can fall out. The case for beliefs in God is similar. Here too there is a theoretical question, *Does He, or doesn't He?*, and there is for a person who has a choice in the matter the practical question, *Should I, or should I not, believe in Him?* And a person who lacks, or thinks he lacks, intellectual grounds sufficient to justify an answer to the first question can have, and realize that he has, ample practical grounds for an answer to the second. James defends the thesis that "our passional nature . . . lawfully may . . . decide an option between propositions, whenever it is a genuine option that cannot by its nature be decided on intellectual grounds. . . . " (James 1956, p. 11). I say that it can be practically reasonable for a person to cultivate beliefs in such cases, and that it is not only in such cases that it can be practically reasonable to 'will to believe'. It can be practically reasonable to believe one way, even in the presence of decisive intellectual grounds not to believe that way.

2. THE WAGER

For a person who can choose whether or not to believe in God, the question whether or not to believe can be important, and even if he has what for him is excellent evidence against God's existence, he can have powerful pragmatic

reasons for believing in His existence. If that is a person's situation, then I think that Pascal's wager or something like it can, in James's terms, be for him "a regular clincher" (James 1956, p. 11) for belief in the face of excellent evidence against what he would believe.

Infinity – Nothing: Pascal's Wager According to James

In Pascal's *Thoughts* there is a celebrated passage known in literature as *Pascal's wager*. In it he tries to force us into Christianity by reasoning as if our concern with truth resembled our concern with the stakes in a game of chance. Translated freely his words are these: You must either believe or not believe that God is – which will you do? Your human reason cannot say. A game is going on between you and the nature of things which at the day of judgment will bring out either head or tails. Weigh what your gains and your losses would be if you should stake all you have on heads, or God's existence: if you win in such case, you gain eternal beatitude; if you lose, you lose nothing at all. If there were an infinity of chances, and only one for God in this wager, still you ought to stake your all on God; for though you surely risk a finite loss by this procedure, any finite loss is reasonable, even a certain one is reasonable, if there is but the possibility of infinite gain. Go, then, and take holy water, and have masses said; belief will come and stupefy your scruples – *Cela vous fera croire et vous abêtira*. Why should you not? At bottom, what have you to lose? (James 1956, pp. 3–4)

Recourse to games and bets to negotiate religious beliefs can seem comical, and degrading. James writes:

[if] the Mahdi were to write to us, saying, "I am the Expected One whom God has created in his effulgence. You shall be infinitely happy if you confess me; otherwise you shall be cut off from the light of the sun. Weigh, then, your infinite gain if I am genuine against your finite sacrifice if I am not!" His logic would be that of Pascal; but he would vainly use it on us, for the hypothesis he offers us is dead. . . . The talk of believing by our volition seems, then, from one point of view, simply silly. From another point of view . . . it is vile . . . how besotted and contemptible seems every little sentimentalist who comes blowing his voluntary smoke-wreaths, and pretending to decide things from out of his private dreams! (pp. 6–7)

But, James reminds, beliefs and attitudes are often in large part products of our passional natures.

We find ourselves believing, we hardly know how or why [in all sorts of things] – in molecules . . . democracy . . . Protestant Christianity . . . all for no reasons worthy of the name. (p. 9) Evidently . . . our non-intellectual nature does influence our convictions. There are passional tendencies and volitions which run before and others which come after belief, and it is only the latter that are too late for the fair; and they are not too late when the previous passional work has been already in their own direction. Pascal's argument, instead of being powerless, then seems a regular clincher, and is the last stroke needed to make our faith in masses and holy water complete. (p. 11)

I agree. Depending on the direction of previous passional work, Pascal's argument can *be* a real clincher for having recourse to which one need not apologize. Previous passional work, together with cogitation or previous intellectual work, deliver 'to a moment' a person's probabilities and values. And, when these are right, Pascalian arguments can mobilize decisions for, and commitments to, belief – they can, for persons already inclined to believe, lead to full belief. It all depends on the direction that has been taken by previous passional work and cogitation. It all depends on the probabilities and values that are delivered to the moment.

It is said that "[i]n 1746, Denis Diderot dismissed the wager with the observation that, 'An Imam could reason just as well this way'" (Sorensen 1994, p. 145; cf., Jordan 1994, p. 101). Perhaps challenging Pascal's own wagers in this way is in order, since Pascal does seem sometimes to take himself to be offering "a theologically neutral proof for belief in God" (Sorensen 1994, p. 145). But when he does, he is confused. Diderot's one-liner, far from being a criticism of the reasoning of a Pascalian argument, is merely fair notice that the directions such a argument provides a person – whether it tells him to believe something and, if it does, what it tells him to believe – depends entirely on the probabilities and values this person brings to the argument.

3. PART TWO. BELIEF-OPTIONS

3.1. With one exception belief-options discussed in Part Three shall be between choosing to believe some proposition and choosing not to believe it. The exception is Case 5, where alternatives are choosing to believe one or the other of two propositions and choosing not to believe either. James counts as belief-options not only, as his official account suggests, "decision[s] between two hypotheses" (p. 3), but also, as in Pascal's Wager, decisions whether to believe single hypotheses. I count as a *belief-option or choice* a situation in which an agent is sure that he can, if only in time and with effort, believe any one of a number of hypotheses, sure that he cannot believe more than one, and sure that he can at once withhold belief from all of them. A belief-option between n hypotheses, for $n \geq 1$, consists of $(n + 1)$ states, n of which are willful belief-states. The other one is a willful state of no belief in those hypotheses.

None of my cases will involve precisely the belief-option addressed by Pascal, the alternatives in which are to believe that God is and to believe that He is not. To believe neither of these propositions is to refuse to wager in the game Pascal says is under way. Of that game he says "you must wager. It is not optional. You are embarked" (Pascal 1989, p. 81). Pascal's own wager is addressed to a choice between beliefs in two propositions, that God exists and that He does not exist. My simple options involve belief and nonbelief in one proposition, and not to believe a proposition is, of course, not necessarily to disbelieve it or to believe its negation. It is possible not to believe a proposition by having no opinion upon it. But what is it to believe a proposition?

In particular, how are beliefs related to subjective probabilities? I agree with Daniel Hunter that to believe a proposition is to have for it a sufficiently high probability, where "what counts as a sufficiently high probability is vague" (Hunter 1996, p. 87) and context-dependent (p. 95). Belief in this ordinary sense contrasts with "belief without doubt or reservation, which may be identified with a subjective probability equal to, or infinitesimally close to, one" (p. 95). In what follows 'belief' can be understood in either that ordinary or this "refined philosophical sense" (p. 96). Either way, to believe a proposition is to have for it a probability greater than $\frac{1}{2}$, which is the only connection between belief and probability on which I depend.

3.2 Alternatives in options are 'live'. Alternatives in belief-options are choices to do things. In simplest belief-options they are *choosings or willings* to secure belief in a proposition, and choosings or willings to avoid belief in it, where each course is perceived by the agent as a real possibility, a live alternative that he is sure he can undertake and complete even if only with effort and time. James requires of a live belief-option that each hypothesis in it should "appeal as a real possibility" (p. 2) for belief and nonbelief and be for us 'live not dead.' In this spirit, I require that belief and nonbelief in each should be perceived as attainable, if only with considerable effort and discipline. It is not, however, part of my idea of liveliness that hypotheses in a belief-option should appeal in the sense of being of some already formed interest or probability.

Since it is part of my idea that, in belief-options, states of belief and nonbelief be perceived as attainable, *certainly* attainable (which is not to say that states of certainties must be perceived as certainly attainable) by directed choice and exercise of will even if only with considerable time and effort, some theorists might claim that clear-headed thinkers would never confront what I call belief-options. An argument that I will not discuss, and by which I am not persuaded, could go like this: We cannot simply decide to believe things and, 'just like that', believe them. Similarly, we cannot decide 'just like that' not to believe things. *Nor* can we ever by choice, with time and effort, follow a course of action that *guarantees* belief or nonbelief. All we can do is take steps that make more or less probable, and never certain, beliefs and nonbeliefs. Clear-headed thinkers would realize their limitations in these respects, and so would never view choosing to believe this or that proposition as an option. (Cf., Duff 1986, p. 108. I indicate in note 27 later how this objection to Pascal's own arguments can be taken out of play. This matter, though important, is dealt with in a note, rather than the main text, pursuant to a policy concerning the numbers of this chapter, stated in its introductory paragraph and recalled in Section 4.4 when I begin to practice it.)

3.3 Options will be 'momentous', and in a sense 'forced'. In all belief-options to be considered the agent has perceptions of possible substantial loss from postponing his decision. All belief-options to be considered are to be in this

sense momentous. And these belief-options will be forced in the sense that no significant practical differences are perceived by the agent between *disbelief* in the proposition (that is, belief in its negation) and *mere nonbelief*. Regarding 'the religious hypothesis' as he understands it, James writes that "[w]e cannot escape the issue by remaining sceptical . . . because . . . *in that way* . . . *we lose the* good [promised by religion], *if it be true*, just as certainly as if we positively chose to disbelieve" (p. 26). Taking alternatives in a simple belief-option to be *believing and not believing*, James maintains that simple belief-options are forced not merely in the way I have explained, but that they are also logically forced and such that there is "no standing place outside" (p. 3). Since I take the alternatives in belief-options to be *choosing or willing* to believe one or another hypothesis, and choosing or willing to believe none of them, my belief-options are never *logically* forced. 'Outside' of the alternatives of my belief-options there is always the logical possibility of neither choosing to believe nor choosing not to believe, but of instead letting belief be or come or not as it may. The one case, Case 9, in which that is an option is thus not strictly a case of one last belief-option in the sense upon which I have settled.

3.4 Most options considered will feature infinite values. For definiteness and continuity with James and Pascal, issues will concern belief in a Christian god, or, in Case 5, in alternative Christian gods that differ in their policies for the granting of heavenly rewards. For continuity especially with Pascal, as well as for intrinsic interest and to encounter certain difficulties, all cases with the exception of Case 8 feature infinite values. Swinburne, though he realizes that "for [Pascal] . . . the ratio of worldly gains and losses to gains in the life to come, if there is one, is of finite to infinite," says that "[i]t is more convenient for the purposes of calculation to measure Pascal's comparative evaluations on a scale on which [the value of a Christian life followed by eternal Heaven] $= 1$, [worldly life followed by eternal hell] $= -1$, [the value of a Christian life without eternal heaven] $= 0$," [the value of an unChristian worldly life] $= 0$, and ≥ 1 [the value of a Christian life followed by '?' – maybe Heaven, maybe Hell] ≥ -1 [(Swinburne 1969, p. 221). Pascal's comparative evaluations cannot be measured on a bounded scale. It is sufficient to note that Pascal differentiates amongst ordinary worldly valuables, holds that the infinite quite annihilates the finite, and says that highest eternal valuables are infinite. This means that on a bounded scale his ordinary valuables, in order to fix their relations to highest valuables, would all have measure 0, which, however, would misrepresent their relations amongst themselves. (It is possible to help out Pascal by giving him infinities that do not 'quite annihilate the finite.' This is done in notes coming that work with 'new numbers'.) Swinburne eschews infinite values and the difficulties that they make for decisions that would get the best out choice situations. I court these difficulties not only so that cases considered should be relevant to persons (if any) who have infinite values,

and to contact with traditional and contemporary discussion, but also for the interest of issues they raise.

3.5 Decision problems. I view a decision problem as a choice that is at least *subjectively* forced, as a choice that is between "things [an agent] is sure he can do that 'cover' the things he thinks he might do" (Sobel 1983b, p. 162). A decision problem is a choice from a set such that the agent is sure "he must do some action from this set perhaps because there is an action in this set . . . such that he is sure that not doing any other action in the set would be [for him] tantamount to doing it" (Ibid.). It follows that belief-options as defined need not, without augmentation, be decision problems, since, in addition to choosing-to-believe and choosing-not-to-believe alternatives, the alternative of *not* choosing and letting belief come or not as it will, can be live. However, in all cases to be considered except Case 9, assumptions make it possible to ignore this additional alternative and to work with analyses in which it is not made explicit. This is done in these cases to simplify and to isolate issues. Also ignored will be 'mixed strategies', in which one's choice amongst options is committed to a chance-device.

4. PART THREE. ON THE VARIETY OF POSSIBLE PASCALIAN WAGERS

> Pascal's system of evaluation whether [one] . . . ought to become a Christian is perfectly workable, but it does not necessarily yield Pascal's results. . . . [It] depends on how [one] estimates the [value of] the different outcomes and how [one] assigns probabilities. (Swinburne 1969, pp. 221, 223)

Everything depends on the values and probabilities a person brings to the table when 'wagering' how to 'manage' his beliefs. Everything *includes* here a person's *belief-options*, for these will feature beliefs and nonbeliefs in 'live' hypotheses in which *he is sure* he has the means to guarantee belief. Pascalian wagers cannot pretend to instruct one and all regarding what they should believe, but only what various people should believe, *given* their beliefs on certain matters, and their values and probabilities for various things. "But cannot wagering be prefaced by demonstrations of what would be everyone's relevant subjective probabilities and values 'at the end of the day,' so that, since these would have for everyone authority, by studying the relevant-for-all wager, instruction for one and all – whether, and if so in what, to believe – can be found." In my opinion, No. I think that ultimate agreements in the values and probabilities for Pascalian wagers is merely something in which, for some reason, some people like to believe. Now come a few Pascalian wagers. They have been chosen for contact with the literature of Pascalian wagers, for possible relevance to subjective states (probabilities and values) of some persons, and for occasions to comment on some of the 'logic' and these wagers.

Case 1: Gain, even in this life, is anticipated

4.1. 'I', the agent (who is not necessarily me in these cases), do not believe in a Christian God (for definiteness in these cases, God is always a Christian God), even though 'I' think that if God existed, believing in Him would lead to eternal bliss, and think, as did James and Pascal, that, whether or not he exists, believing in him would lead to gain in this life. Cf.: "[R]eligion affirms that we are better off even now if we believe her" (James 1956, p. 26), and, Blaise Pascal, "I tell you that you will gain in this life."[2] Also, 'I' am convinced that believing in God leads to nothing beyond this life if He does not exist. This last thing indicates that 'I' place absolutely no credence in the idea that a god exists who punishes Christian believers and rewards others. To continue specifications of the case, 'I' value eternal bliss incommensurably more than every quite other gain that 'I' think 'I' just might at some time secure. For 'me' the slightest chance for this prize would be worth not merely every risk but the 'for all practical purposes certainty' of every worldly sacrifice.[3] Finally, 'I' see not believing in God as a 'nothing ventured, nothing else gained or lost' option.

4.2 How shall we evaluate 'my' options?

The basic idea (probably Pascal's) was published at the end of Arnaud's *Port-Royal Logic* (1662): "To judge what one must do to obtain a good or avoid an evil one must consider not only the good and the evil in itself but also the probability of its happening or not happening, and view geometrically the proportion that these things have together." (Jeffrey 1995, p. 179)[4]

To view geometrically the proportion of these things is to combine them in EXPECTED VALUES, where these are certain probability-weighted averages of values of possible consequences. In general, for an option a probability-partition[5] of possible circumstances $\{c_1, c_2, \ldots, c_n\}$, which partition is 'sufficiently exclusive' for the analysis of the expected value of *a*,[6] this expected value is a weighted average of the value of *a* in these several circumstances, the weight for its value in a given circumstance being the probability of that circumstance's obtaining were *a* done:

$$\begin{aligned} ExVal(\mathrm{a}) = {} & Prob(c_1, \text{if a}) \cdot ExVal(\text{a and } c_1) + Prob(c_2, \text{if a}) \cdot ExVal(\text{a and } c_2) \\ & + \cdots + Prob(c_n, \text{if a}) \cdot ExVal(\text{a and } c_n).[7] \end{aligned}$$

Expected values are relevant to issues in cases to be considered of practical reasonableness and rational desirability at least to the extent that in these cases it is sufficient for the practical reasonableness of an option, or the rational desirability of a state, that its expected value be *greater than* that of every alternative to it. Left open for discussion in cases to come is whether the expected value of an option or state's being *equal to* or greater than that of every alternative to it is also always (and thus even when infinite) sufficient for

pragmatic reasonableness, and, further, whether its being *equal to or greater than* that of every alternative to it is necessary and sufficient. It remains, however, that in every case (and not only in all cases to be considered) expected values of options, even when not decisive, are relevant to reasonable choices. We may thus proceed to their determinations in the present case, even though, as will be explained, this happens to be a case in which exact determinations and calculations of expected values are not needed.

I have elsewhere challenged the sufficiency of maximization for rational options in decision problems. Say that an option *a* is *strongly ratifiable* if and only if *a* exclusively maximizes on the epistemic condition that *a* is done: For expected values relevant to the strong ratifiability of *a* one uses probabilities (and expected values – Sobel 1997b) revised by conditionalization on *a*. I have by implication maintained that not exclusive maximization alone, but only exclusive maximization together with strong ratifiability, is sufficient for pragmatic reasonableness. (Sobel 1983b, 1990.) This further condition of strong ratifiability is not relevant to the cases studied in this book, since in them possible circumstances (for example, that God exists) are taken to be not only certainly causally, but also, evidentially, independent of options (for example, that I choose to believe in God). Also, Sobel (1997b) values of options in circumstances are not sensitive to beliefs concerning the circumstances. A consequence is that in these cases an option maximizes if and only if *a* choice for it would be ratifiable. There is a case of some interest in which God's existence, while of course being certainly causally independent of an agent's state of mind, would *not* be evidentially independent of it. A person might, David Hume has suggested, view possible news that he was going to attain belief in God as news of an impending miracle. He might believe that he would believe in God only if moved by Faith, and that "whoever is moved by *Faith* to assent . . . is conscious of a . . . **miracle** in his own person" (Hume 1902, p. 131; bold emphasis added). For this person, news that he was going to believe in God could be evidence (possibly his first) of this belief's truth.

4.3. To study the problem of the present case, and to assess (under some numerical assumptions) expected values, I use a system of matrices due to Richard Jeffrey (Jeffrey 1990, Chapter 1 "Deliberation: A Bayesian Framework"), and begin by identifying options. Let us assume that my options 'come down to' choosing to believe in God and choosing not to believe with other options either coming to the same things or being worse than both. These are assigned to rows in matrices to come. We then distinguish circumstances that, according to the story, may obtain and affect in significant ways consequences of these options. The very simple division of possible total circumstances between those in which God exists and those in which He does not covers these ways. These head the columns. Cells in the first matrix, the 'consequence-matrix,'

contain indications of salient differentiating consequences of 'my' options in these circumstances.

	God exists	God does not exist
by choice to believe in God	eternal bliss, and gain in this life	gain in this life
by choice not to believe in God	status quo	status quo

Other words for the two options could be 'to will to believe in God' and 'to will not to believe in God', though these words might suggest that the options consist in believing in God 'just like that', and similarly for not believing, whereas the options of the case consist in somehow or other (but presumably not 'just like that') making up one's mind on this matter by choice. Sentences for the options, sentences that realizations would make true, are 'I shall by choice believe in God' and 'I shall by choice not believe in God'. *On simplifying assumptions*. The second alternative in the present case is not the simple negation of the first one. 'My' live and importantly different options could include not only the two of this case but also the course of neither willing to believe nor willing not to, but instead letting belief come or stay as it may, naturally or by the grace of God. To simplify and to isolate issues, in every case until Case 9, such a further course is taken to be either a course 'I' cannot choose (which would be strange) or a course that is inferior to another course open to 'me' because 'I' am of a decisive nature and averse to leaving things to nature or God when 'I' can settle them by choice. 'Mixed strategies' are ignored here and in *all* cases to come since, (1) the 'expected value' of a mixed strategy cannot exceed that of each strategy mixed in it, and (2) it is part of 'my' (the agent's) decisive nature to be averse to leaving to chance things that can settled by choice. There are no important differences between versions of options explicit in the consequence-matrix, which versions are live for 'me' of these options. 'I' have no live options (including ones that would relate to 'other gods') that are superior to options made explicit in the consequence-matrix for 'my' problem. Similar assumptions are intended for all cases in this chapter.

4.4. The next step in a fully articulated analysis is to provide numerical representations of 'my' values for consequences (or, more accurately, for 'my' expected values for option-circumstance conjunctions), for which purpose I use in the text standard real finite numbers and standard Cantorian transfinite cardinals or set-sizing infinities. These numbers, and in particular these infinities, are used for familiarity and to make contact with other discussions of the wager. For these reasons I use them notwithstanding the strangeness of Cantorian infinities. Sets are equal in size when there is a 1–1 correspondence

between their members (Section A2 of the appendix to Chapter X). Cardinal numbers, set-sizing numbers, are equal when the sets they number are equal in size. Thus, to record one well-known anomaly of Cantorian infinities (related anomalies will be noticed below), though the *set* of even numbers is a *proper subset* of the set of natural numbers, the *number* of even numbers, *card*(even numbers), is *equal* to the number of natural numbers, *card*(natural numbers), by the 1–1 correspondence, {(0,0), (2,1), (4,2) . . . }. For substance and truth, instead of standard reals and Cantorian infinities, *Robinsonian finite and infinite hyperreals* would recommend themselves. Therefore, though for the reasons stated only standard real numbers and Cantorian infinities are used in the main text, in a series of asterisked notes I comment on hyperreals and dramatic differences to arguments that their employment, instead of standard numbers, can make. With them all the 'paradoxes' are gone, and all 'cheap shots at Pascal' are duds. This hyperreal subtext culminates in the appendix, in which I elaborate on the theory of hyperreals and on their advantages for decision theory. Now comes the first of these counterpointing notes.[8]*

4.5. To reflect the value that 'I' attach to eternal bliss both alone and with any other good or evil 'I' think 'I' might secure, we may employ ∞ for the value of eternal bliss and gain even in this life (and for the expected value of the conjunction of 'my' choosing to believe and God's existence), where ∞ is the number of natural numbers. This number – '$\aleph_0$' in Cantor's notation – is the smallest Cantorian infinity. (Hunter 1973, p. 18. A set of smaller transfinite cardinality could be put into 1–1 correspondence with a subset of the natural numbers. But every subset of the natural numbers can be ordered from smallest to larger and larger, and so every infinite subset of natural numbers can be put into 1–1 correspondence with the set of all natural numbers.)[9]* It is natural and convenient to set the value of the status quo at zero. Any positive finite number would do for gain in this life: To facilitate mathematical points I have chosen 10/7. These numerical assumptions are collected in the 'value-matrix' for our case,

	G	~G
Bel(G)	∞	10/7
~~Bel~~(G)	00	

Abbreviations – **G**: '*God exists*'; '**Bel(G)**': '*I shall by choice believe in God*'; and '**~~Bel~~(G)**': '*I shall by choice not believe in God*'. ~~Bel~~(G) is not equivalent to ~Bel(G), nor is ~~Bel~~(G) equivalent to Bel(~G). I have decided against the more articulated system of abbreviations: Ch: I shall by choice make true that; Bl: 'I' shall believe that; G: God exists. The following equivalences may illuminate the scheme I have adapted:

$$\text{Ch}[\text{B1}(\text{G})] \equiv \text{Bel}\ (\text{G})$$

and

$$\mathrm{Ch}[\sim \mathrm{B1(G)}] \equiv \text{~~Bel~~}(\mathrm{G}).$$

4.6. For a full Bayesian analysis (I ignore for now a shortcut available in the present case) numbers are found for 'my' degrees of confidence for action/circumstance conditionals to the effect that, if 'I' perform this or that option, then 'I' would be doing so in this or that circumstance. Taking for granted that 'I' am sure that God's existence is independent of whether or not 'I' believe in God, 'my' probability for the conditional that if 'I' were by choice to believe in God, then God would exist,[10] is the same as 'my' probability for the conditional that if 'I' were by choice not to believe in God, then God would exist: Each is the same as 'my' probability for the categorical proposition that God exists. Similarly for probabilities in the second column. For definiteness let this probability be .3. Since $Prob(\mathrm{G}) + Prob(\sim\mathrm{G})$ is necessarily 1, we have for our case the following 'probability-matrix'.

	G	~G
Bel(G)	.3	.7
~~Bel~~(G)	.3	.7

4.7. "View[ing] geometrically the proportion that all these things have together" (*Port-Royal Logic*), we may, using numbers in the value and probability matrices for the case, compute expected values for 'my' two options:

$$\begin{aligned}
ExVal\,[\mathrm{Bel(G)}] &= Prob[\mathrm{G, \text{ if Bel (G)}}] \cdot ExVal\,[\mathrm{Bel(G)\ \&\ G}] \\
&\quad + Prob[\sim\mathrm{G, \text{ if Bel(G)}}] \cdot ExVal\,[\mathrm{Bel(G)\ \&\ \sim G}]; \\
ExVal\,[\mathrm{Bel(G)}] &= .3(\infty) + .7(10/7) = .3(\infty) + 1 = \infty; \\
ExVal\,[\text{~~Bel~~(G)}] &= .3(0) + .7(0) = 0.
\end{aligned}$$

Regarding the identity,

$$.3(\infty) + 1 = \infty,$$

it may be noted that for, every positive rational r, the product $\mathrm{r}\cdot\infty$ equals ∞, and, for any natural number n, the sum $(\infty + \mathrm{n})$ equals ∞, given that ∞ is a standard Cantorian infinity. *For substantiation of* $\infty + n = \infty$, n a natural number, consider first $\mathrm{n} = 1$ and note that 1 is the cardinal number of the set that contains exactly -1. That is, $1 = card(\{-1\})$. We have $\infty + 1 = card$(the set of natural numbers) + $card(\{-1\}) = card$(the set of natural numbers U $\{-1\}) = card$(the set of natural numbers) $= \infty$. For the penultimate identity we have the 1–1 correspondence, [(−1,0), (0,1), (1,2) . . .]. It can be seen similarly that $\infty + 2 = \infty$, that $\infty + 3 = \infty$, and so on. *For substantiation of* $n \cdot \infty = \infty$, n a positive natural number, consider the union of n nonoverlapping sets of size ∞. It can be seen that there will be a 1–1 correspondence of its

members with the members of a set, any set, of size ∞.[11*] Substantiation for $\infty/m = \infty$, m a positive natural number, and $(n/m)\infty = n \cdot (\infty/m)$ are left to the reader. So Bel(G) exceeds ~~Bel~~(G) in expected value. That makes believing in God willfully a dictate of practical reason.

4.8. That conclusion can be reached in this case with less work. In this case the quantities of probabilities for option/circumstance conditionals are not relevant to the how expected-values of options compare, since:

(i) Bel(G) *strongly dominates* ~~Bel~~(G)] – in the value-matrix entries for Bel(G) exceed those for ~~Bel~~(G)] in both columns;

and

(ii) the two circumstances G and ~G are, 'I' am sure, *independent* of 'my' options – this is reflected by sameness of rows in the probability-matrix.

Suppose 'my' probability for G is g. Then

$$ExVal\,[\mathrm{Bel(G)}] = g(\infty) + (1 - g)(1)$$

and

$$ExVal\,[\sim\!\mathrm{Bel(G)}] = g(0) + (1 - g)(0)$$

from which it follows that, no matter what the magnitude of g,

$$ExVal\,[\mathrm{Bel(G)}] > ExVal\,[\sim\!\mathrm{Bel(G)}].$$

Bel(G) would maximize expected value in *this* case and be pragmatically reasonable, even if 'I' were sure that God does not exist, so that g was 0. Also, Bel(G) would excel in expected value, even if the value of willing to believe when God exists were not infinite, as long as it is positive.

4.9. It is important to the Pascalian/Bayesian approach that belief in God be an option and be viewed as something that, even if only with effort, can be maintained or brought about by choice. But it is not essential to the work of wagers in a case that God be somewhat probable initially. Previous passional work can be in the direction of belief without its having established a positive probability for the God-hypothesis. The wager can work not only as a "regular clincher" to settle and fix already present inclinations to belief, but also to recommend projects that will culminate in changes of mind from complete disbelief to full belief.

The argument of this case is like "Lacheliber's defensive reformulation of Pascal's argument," which Rescher says "just is not Pascal's" (Rescher 1985, p. 117). Rescher acknowledges that Pascal writes "toward the end of the *infini-rien* passage: 'I tell you, you will thereby profit in this life'," but maintains that "this consideration plays no rôle in the argument" of that passage

(pp. 149–50, n. 125). True, but this afterthought of Pascal's could have played a large simplifying role for an argument to bolster his own faith. It not only makes chances and probabilities irrelevant, but, as recently observed, makes insistence on the infinite value of heaven's rewards unnecessary to Pascal's personal wager. Perhaps Pascal made little of this consideration, and brings it in only as reinforcement, because he viewed it as contentious and sought an argument that was independent of it.

4.10. It is assumed for this case, and to keep things simple is assumed for all cases to come, that the agent's mind is in certain ways completely made up, so that his preferences and credences for possible consequences of options are represented by 'point' value and probability functions that enable calculations of 'point' expected values for options. In more realistic cases, those preferences and credences would not be so definite and would be represented by sets of such 'point' functions that led to sets of point expected values for options. Complications in development of a general theory of expected values adequate to such more realistic cases, of principles of reasonableness for such cases, and of Pascalian wagers more realistic in these ways, are left for another day.

4.11. In this case, (i) Bel(G) *strongly dominates* ~~Bel~~(G), and (ii) circumstances are *certainly independent* of options. That ensures that the expected value of Bel(G) is greater than that of ~~Bel~~(G). Strong dominance alone provides no such guarantee. Here is a take-your-medicine case to this point. Assume for the case the following combined consequence and value-matrix.

	You willl get well: W.	You will not get well: ∼W.
You take your bitter medicine: T	recovery with a bitter aftertaste 9	bitter demise −10
You do not take your bitter medicine: ∼T	recovery 10	demise −9

Not taking your medicine strongly dominates. But perhaps the medicine would be good for you. Perhaps it would at least improve the odds. Assume the following probability-matrix for the problem.

	You will get well: W.	You will not get well: ∼W.
You take your bitter medicine: T	.9	.1
You do not take your bitter medicine: ∼T	.5	.5

Then, though ∼T strongly dominates in the value-matrix, you should take your medicine, for the expected value of that option is greatest.

$$ExVal(\mathrm{T}) = .9 \cdot 9 + .1 \cdot -10 = 7.1$$
$$ExVal(\sim\mathrm{T}) = .5 \cdot 10 + .5 \cdot -9 = .5$$

Jeffrey makes the point of this paragraph with a fallacious nuclear disarmament argument (Jeffrey 1991, pp. 2, 8–9). A similar argument could 'go', "We ought to rid ourselves of nuclear weapons, for if there is peace, we save money, and if there is war, we avoid mutual mass destruction."

5. Case 2: Believing would have only other-world rewards

5.1. This case is like Case 1 except that in it 'I' do *not* think that by believing 'I' would gain even in this life. The case, given by consequence and value-matrices combined, is that:

	God exists. G	God does not exist. ∼G
by choice to believe in God Bel(G)	eternal bliss ∞	status quo 0
by choice not to believe in god ~~Bel~~(G)	status quo 0	status quo 0

Bel(G) does not strongly dominate ~~Bel~~(G) in this case, though it does weakly dominate it: It is as good in every column, and better in one. So probabilities matter a little since

$$ExVal[\mathrm{Bel}(\mathrm{G})] > ExVal[\text{~~Bel~~}(\mathrm{G})]$$

if and only if

$$Prob(\mathrm{G}) > 0.$$

When $Prob(\mathrm{G}) = 0$,

$$ExVal[\mathrm{Bel}(\mathrm{G})] = ExVal[\text{~~Bel~~}(\mathrm{G})].$$

5.2. In this case the dictate of pragmatic reason is clear as long as 'I' do not quite write off God. But what if $Prob(\mathrm{G}) = 0$? Should we say that belief in God is exclusively pragmatically reasonable even when $Prob(\mathrm{G}) = 0$, since, though expected values are then equal, belief is distinguished from nonbelief by the fact that it affords the *possibility* of a gain, and indeed the possibility of an infinite gain, albeit a possibility to which 'I' assign no positive probability? James must think that weak dominance is decisive when there is "the possibility of **infinite** gain" (Section 2; emphasis added): For he says that in that case "any finite loss is reasonable" (whether or not, we may add, there is dominance).

It *may* have been Pascal's view that weak dominance *without* the possibility of an infinite gain is decisive, that it is decisive even when expected values are equal,[12] a view that Nicholas Rescher seems to endorse: "If you win, you win everything; if you lose, you lose nothing. Don't hesitate then" (Pascal 1962, p. 134). Regarding these lines, Rescher says, "If the choice stood on this basis, its resolution would be straightforward" (Rescher, p. 12). Regarding these lines, however, it is likely that Pascal had in mind for 'everything' a possible infinite gain, and thus only in a special case of weak dominance with equal expected values does one have nothing to lose.

. . . . .

And what if 'I' thought "there were an infinity of chances, and only one for God" (James). Then there is a question of what, in strict mathematical terms, 'I' would be thinking. There is a question of what measure of confidence in God's existence 'I' would be expressing in mathematical metaphor.[13*]

5.3 Bare possibilities and weak dominance. What is the truth here? Is the bare possibility of a gain, or at least the bare possibility of an infinite gain, decisive for choice when expected values are equal? No, it cannot be. For, to be decisive for choice from options, a condition needs to distinguish them, it needs to attach to just one of them. And that is something that bare possibilities of gains can never do. Just as it is possible that God exists and rewards believers, it is *possible* that He exists and rewards *disbelievers*. Perhaps you say, "No, that is not possible. It would not be consistent with His nature." No matter, since the point is still made by the possibility of an 'Anti-God' who rewards disbelievers in God. Bare possibilities are not possible tie-breakers. These need to be subject in some way to the discipline of plausibility. *Bare* possibilities – that is, zero-probabilities – are irrelevant to choice.

5.4. Weak dominance coupled with certain independence of circumstances from options, under a sufficient division of possible circumstances, is not decisive for choice. For, if it were, it would, *per impossibile*, in some cases be decisive for incompatible options. Let a *distinguishing column* for an option under a division of possible circumstances be a column in which its value exceeds that of another option. We may, to make the point of this paragraph, consider a decision problem in which all distinguishing columns under sufficient partitions of circumstances are headed by circumstances whose probabilities are zero. For a minimal case of this kind, I assume that {C1, C2, C3} is a sufficient logical partition of possible circumstances, and $P(C2) = 1$, and that each of C1, C2, and C3 is certainly independent of both O1 and O2. From these assumptions it follows that each of {C1, C2} and (C3, C2} is a sufficient probability-partition of possible circumstances. I assume values for conjunctions of options and circumstances displayed

in the following value-matrices for it:[14]

		circumstances	
		C1	C2
	O1	∞	0
options			
	O2	0	0

		circumstances	
		C2	C3
	O1	0	0
options			
	O2	0	∞

Were weak dominance decisive for choice in this case, it would be decisive for O1 (according to the first matrix) and for O2 (according to the second), which is impossible. If a rule is 'decisive for an option', then this option is *exclusively* reasonable. Only *restricted* versions of the weak-dominance-with-certain-independence-of-circumstances-from-options rule can be valid. When there is *strong* dominance under a sufficient division of circumstances, *every* column is a distinguishing column. That is why, whereas weak dominance conjoined with certain independence of circumstances from options is *not* decisive for choice, strong dominance *is* decisive for choice when conjoined with certain independence of circumstances from options.

6. Case 3: Belief is not considered to be cost-free

6.1. Now comes a case like Case 1, save that, far from thinking as 'I' do in Case 1 that 'I' would gain even in this life by believing, 'I' am convinced that 'I' would lose. Perhaps 'I' think that coming to believe would take time and effort, and that sustaining a belief won by effort would be further time-consuming. Or perhaps 'I' am convinced that the evidence is clearly against the existence of God, so that measures sufficient to belief would either change 'my' mind about the state of the evidence, and this not for good intellectual reasons but by self-manipulation, or, if not change 'my' mind about the evidence, somehow suppress the natural effect of 'my' view of it. 'I' might, agreeing with Pascal, think that successful measures to believe would dull and damage 'my' rationality and thus (shades of Aristotle) 'my' humanity.

> "Naturallement même cela vous fera croire et vous abêtira."
> Naturally, it will at once make you believe, and make you stupid as a beast.

'I' might view means to willful belief as assaults on 'my' person or on 'my' essence. Suppose 'I' do. Suppose 'I' view, as an inseparable concomitant of 'my' managing to believe, effects on 'my' rational soul of negative value, though

still of only finite negative value that are as nothing when compared with the value of eternal bliss. Does Pascal suppose this? Does he view these effects in a negative light, or does he merely acknowledge them in a matter of fact spirit? He has an interlocutor say, in response to the observation that taking holy water, having masses said, and so on, will stupefy in the manner of an uncritical submissive calm beast ("vous abêtira), that that is what he, this interlocutor, *fears*. To this Pascal answers, "And why? What have you to lose?" (Pascal, p. 82). Pascal's rhetorical question can be taken to say that, in his view, there is nothing wrong with beastlike stupefaction. A more likely reading, however, takes this question in the context of its immediate sequel and has it say that, though beastlike stupefaction is bad, it is a price worth paying not only for a chance for eternal bliss, but for gains even in this life that belief yields ("[y]ou will be faithful, honest, humble, grateful, generous" – Pascal 1989, p. 82a). On this reading his interlocutor is right to think that he loses something beyond "glory and luxury" (p. 82b). But, Pascal seeks to persuade, he is wrong in thinking that he loses anything *on balance*.

6.2. Swinburne claims that "[m]any would consider that it is highly immoral to choose to believe propositions . . . when the evidence is now known not to support them" (Swinburne 1969, p. 225). And he implies that "[i]f a man claimed that there was nothing immoral in inducing in oneself . . . beliefs . . . not supported by present evidence," then he could not think that there was anything bad about, or in, doing that (Ibid.). Regarding the first claim, I say that it does seem that some people (for example, Clifford) disapprove 'morally' of strategic, induced believing, though I doubt that *many* people have these attitudes.[15] Regarding Swinburne's second implied point, I suspect that most people would agree that strategic induced believing, even if not in the least *immoral*, is *unhealthy* and *bad* for persons, and thus is not to be indulged in lightly. It is, I suspect almost everyone thinks, at least *usually* unwise. The agent, 'me', of the present case is certainly of this view.

6.3. 'My' value-matrix, given that 'I' think that I have something to lose by compromising 'my' intellectual reason, though 'everything' to gain, can be

	G	∼G
Bel(G)	∞	−1
~~Bel~~(G)	0	0

In this case, there is not even weak dominance, and belief is the dictate of pragmatic reason,

$$ExVal\,[\mathrm{Bel(G)}] > ExVal\,[\sim\!\!\mathrm{Bel(G)}],$$

if and only if, for 'me',

$$Prob(\mathrm{G}) > 0.$$

'My' value-matrix, given that 'I' think that I have something to lose by compromising 'my' intellectual reason, though 'everything' to gain, can be

	G	∼G
Bel(G)	∞	−1
~~Bel~~(G)	0	0

6.4. In this case, there is not even weak dominance, and belief is the dictate of pragmatic reason,

$$ExVal\,[\mathrm{Bel(G)}] > ExVal\,[\text{~~Bel~~(G)}],$$

if and only if, for 'me',

$$Prob(\mathrm{G}) > 0.$$

In standard transfinite arithmetic, for any positive probability for God, g, no matter how minute,

$$\mathrm{g}\cdot\infty + (1-\mathrm{g})(-1) = \infty,$$

and

$$\mathrm{g}\cdot 0 + (1-\mathrm{g})(0) = 0.^{16*}$$

But if 'I' am sure that God does not exist, if, for 'me', the probability that G is zero,

$$ExVal\,[\mathrm{Bel(G)}] = 0\cdot\infty + 1(-1) = -1 < ExVal\,[\text{~~Bel~~(G)}] = 0\cdot 0 + 1\cdot 0 = 0,$$

and the clear dictate of pragmatic reason is that, notwithstanding the possibility of an infinite reward for belief, since 'I' discount that possibility completely, 'I' should not believe.[17*]

7. Case 4: Alternative reward-policies for salvation are taken seriously

7.1. I now complicate the previous case by supposing that 'I' make positively probable two hypotheses concerning God's policy for rewards of eternal bliss. According to the first traditional one, God rewards all and only believers with eternal bliss; according to the second, God rewards with eternal bliss all and only those who are in His connection intellectually reasonable and believe or not according to their views of the evidence.[18] Let it be a part of the present case that 'I' am convinced that the evidence is clearly, but not completely, against the existence of God, and that 'I' place some credence in each of these hypotheses concerning God's policy. (Left open is how much credence 'I' place in each.) And suppose, as in the previous case, that 'I' see believing as burdened by this-world costs that pale completely for 'me' when compared

with eternal bliss. Combined consequence and value-matrices for this case can be as follows:

	God exists, and saves precisely believers.	God exists, and saves precisely the theoretically rational.	God does not exist.
by choice to believe in God Bel(G)	G & ReBel eternal bliss, and this-world costs ∞	G & ReRat this-world costs -1	$\sim$G this-world costs -1
by choice not to believe in God ~~Bel~~(G)	status quo 0	eternal bliss ∞	status quo 0

7.1.1. In this case, since both *Prob*(G & ReBel) and *Prob*(G & ReRat) are positive, expected values of Bel(G) and ~~Bel~~(G)are equal. As a matter of standard transfinite arithmetic, products of finite factors and an infinity are still infinite and indeed the same infinity, as are remainders from finite subtractions.[19*]

7.1.2. Perhaps there is a valid tie-breaker for this case of equal standard Cantorian infinite expected values. There are rationales for breaking its tie in different ways.

(1) It can seem that what is pragmatically reasonable in it should depend on which is greater, *Prob*(G & ReBel) or *Prob*(G & ReRat), and so on which option makes more probable the infinite gain that each makes somewhat probable. Cf.: "[W]hen each possible outcome carries an infinite . . . value, it is rational to bet on the outcome most probable to occur" (Schlesinger 1994, p. 89). Why? "Because [when] neither expected utilities nor magnitude of the prize can serve as one's criterion, by elimination it should be reasonable to be guided by the value of the probability: [to] wager on the outcome that is most likely to materialize" (p. 90).[20*] *If Prob*(G & ReBel) > *Prob*(G & ReRat), this line recommends Bel(G).

(2) It can also seem that what is pragmatically reasonable in this case is decided by the greater probability of a loss on Bel (G), given that possible relative gains discounted by their probabilities are equal, even if their discounting probabilities are not equal:

$$Prob(\text{G \& ReBel}) \cdot \infty = Prob(\text{G \& ReRat}) \cdot (\infty + 1).$$

It can seem that, since neither expected values nor discounted magnitudes of *gains* can serve as one's criterion, by elimination it should be reasonable to be guided by the likelihoods of possible relative *losses*. In the case, given that [*Prob*(G & ReRat) + *Prob*(~G)] > *Prob*(G & ReBel)], this line recommends ~~Bel~~(G), even if *Prob*(G & ReBel) > *Prob*(G & ReRat).

7.1.3. The suggestion of rationale (1) – which I favor (for reasons in note 18) – is not that if there is a tie in infinite expected values, then, since the two options are both pragmatically reasonable, reasonableness *tout court* of belief depends on which belief would be the more intellectually reasonable – a suggestion of Lycan and Schlesinger (1989):

> [I]f EUs are *equal*, then by Bayesian [prudence] principles it doesn't matter what one does. . . . Moreover, so long as prudence does not rule on the matter, one would do best on *epistemically*-rational grounds to go with probabilities. . . . Indeed, in this case we think a person should be rationally faulted for failing to prefer an objectively more probable god, so long as EUs are equal. (Lycan and Schlesinger 1989, p. 86)

I do not have a theory of reasonableness *tout court* in which practical and intellectual considerations are merged, so that in some cases intellectual considerations take precedence (for example, Lycan and Schlesinger suggest, when an issue cannot be decided on practical grounds), and in some cases practical considerations take over (for example, James suggests, when an issue cannot be decided on intellectual grounds). I do not have an *intuitive idea* of reasonableness *tout court* of beliefs in which intellectual and practical reasonableness are somehow ordered and intertwined. My rationale (1) concerns pragmatic reasonableness only. It suggests a way in which that reasonableness might depend on more than expected values of options, and depend in particular on *patterns* of probabilities and expected values of consequences – $Prob(c_1,\text{if a})\cdot ExVal(\text{a and } c_1) + Prob(c_2,\text{if a})\cdot ExVal(\text{a and } c_2) + \cdots + Prob(c_n,\text{if a})\cdot ExVal(\text{a and } c_n)$ – that go into expected values of options – $ExVal(\text{a})$.

8. Variants of Cases 3 and 4

8.1 Case 3′. For a variant of Case 3 one could suppose that 'I' am convinced that God, if he exists, blesses with eternal bliss *everyone* (if only eventually, after some 'soft' or 'hard' time). Let this variant of Case 3 have the combined consequence and value-matrices,

	God exists, and saves everyone.	God does not exist.
by choice to believe in God	eternal bliss, and this-world costs ∞	this-world costs -1
by choice not to believe in God	eternal bliss ∞	status quo 0

8.2 Case 4′. For a related variant of Case 4, let the second God-hypotheses that 'I' make positively probable posit a God who saves everyone. And let it

be that, while 'I' assign a very low probability to this second God-hypothesis, 'I' assign to it a much greater probability than 'I' assign to the first traditional one. Let the matrices be

	God exists, and saves precisely believers.	God exists, and saves everyone.	God does not exist.
by choice to believe in God	eternal bliss, and this-world costs	eternal bliss, and this-world costs	this-world costs
	∞	∞	-1
by choice not to believe in God	status quo	eternal bliss	status quo
	0	∞	0

Assessments of options in these cases are left to the reader.

9. Case 5: Competing God-hypotheses are taken seriously. It has been claimed that "other things being equal, one should go for the deity that offers the more attractive afterlife" (Lycan and Schlesinger 1989, p. 85). To illustrate, I consider a belief-option addressed to just two God-hypotheses. Each posits a God that rewards believers precisely in Him with eternal bliss, the promised blisses, however, being perceived to be of different orders of wonderfulness.

God_1 promises eternal bliss to precisely believers in Him, and God_2 promises even greater eternal bliss to His followers. God_1 and God_2 differ in their natures, and not merely in their promises. They differ in their promises *because* of differences in their natures (cf., Lycan and Schlesinger 1989, p. 89): Eternity in the presence of God_2 would for 'me' be more valuable because of what 'I' consider to be God_2's greater magnificence. (For definiteness, assume that God_2 would be, in Alvin Plantinga's terms, maximally excellent whereas God_1 would be merely maximally great (Plantinga 1974a, p. 107) and that for 'me', though eternal communion with maximal greatness would be wonderful, it pales in comparison with the prospect of eternal life in the presence of maximal excellence.) Let 'me' be sure that, if there is a god, then He is either God_1 or God_2. Let 'me' be almost sure that there is not a god, and that if there is a god, He is almost certainly the lesser God_1, the greater God_2 being too good to be true: For definiteness, let 'my' probabilities for these possibilities be, respectively, .90, .09, and .01. So "other things are not equal": Indeed, possible payoffs and probabilities run in different directions. Let the value-matrix for the case (complete with indications of 'my' probabilities for possible circumstances that remain certainly independent of 'my' options) be as follows, wherein 2^∞ is the cardinality of the power set (or the set of all subsets) of a set of size ∞: 2^∞ is greater than ∞ (Hunter 1973, pp. 21–4). According

to The Continuum Hypothesis, 2^{∞} is the smallest infinity that is greater than ∞.

	.09	.01	.90
	G_1	G_2	$\sim G$
Be1(G1)	∞	0	0
Be1(G2)	0	2^{∞}	0
~~Bel~~(G1 v G2)	0	0	0

~~Bel~~ (G_1 v G_2) is the option in which 'I' choose not to believe either God-hypothesis or, equivalently assuming consistency on 'my' part, of each choose not to believe it, [~~Bel~~ (G_1) & ~~Bel~~ (G_2)]. In the more articulated notation of Section 4.4, ~~Bel~~ (G_1 v G_2) is equivalent assuming consistency on 'my' part to,

$$Ch[\sim B1(G_1 \text{ v } G_2)]$$

and

$$Ch[\sim B1(G_1)] \text{ \& } Ch[\sim B1(G_2)].$$

In this case, though *Prob*($\sim$G) is much greater than *Prob*(G_1), which in turn is much greater than *Prob*(G_2), *ExVal*[Bel(G_2)] is incommensurably greater than ExVal[Bel(G_1)]. (This is because $2^{\infty} > \infty$, and for any real number r, $r \cdot 2^{\infty} = 2^{\infty}$.) So, even though other things are not equal – even though probabilities are far from equal – 'I' should opt for Bel(G_2) and implement allegiance to the god that offers the more attractive afterlife.[21*]

10. Case 6: Alternative policies not only for rewards, but also for punishments, are taken seriously

10.1. For this case I modify the God-hypotheses of Case 4, so that each possible God would reward according to the policy stated for it in that case and would *punish* with eternal damnation in accordance with a complementary policy.[22] A traditional God would punish nonbelievers, and a God biassed to theoretical rationality would, in this case in which, as in Case 4, it is given that 'I' am convinced there is good evidence against the existence of God, punish 'me' were 'I' to believe. To recall another feature that we retain for the present case, each of these God-hypotheses is positively probable, and no other God-hypothesis is positively probable. Assume finally that just as eternal bliss is by 'my' lights incommensurably better than every merely worldly good, so eternal damnation is to the same measure incommensurably worse than merely worldly evils. Pascal may have viewed eternal damnation as, though bad, not infinitely bad. "There is not so great disproportion between our justice and that of God, as between unity and infinity. The justice of God must be vast

like His compassion. [But] justice to the outcast is less vast . . . than mercy towards the elect" (Pascal 1989, p. 80). For consequence and value matrices we may assume

	God exists, saves believers, and punishes non-believers	God exists, and saves theoretically rational believers, and punishes theoretically irrational believers	God does not exist.
	G, ReBel, & PunNon	G, ReRat, & PunIrr	$\sim$G
by choice to believe in God Bel(G)	eternal bliss, and this-world costs ∞	this-world costs, and eternal damnation $-\infty$	this-world costs -1
by choice not to believe in God ~~Bel~~(G)	eternal damnation $-\infty$	eternal bliss ∞	status quo 0

10.2. In contrast with Case 4, in which expected values are equal, in the present case they are not *defined*. Consider Bel(G). Its expected value, if defined, would be the sum,

$$Prob[\text{G \& ReBel \& PunNon}](\infty) + Prob[\text{G \& ReRat \& PunIrr}](-\infty) + Prob(\sim\text{G})(-1),$$

and, since the probabilities involved are all positive and finite, this sum would equal the sum,

$$\infty + (-\infty) + -Prob(\sim\text{G}).$$

Within standard transfinite arithmetic, this would-be sum is not defined (cf., Jeffrey 1990, pp. 153–4, where such sums are said to be "indeterminate"). Sums involving positive and negative Cantorian infinities are not defined: For such infinities '$\infty - \infty$' has no natural sense. Negative inverses of Cantorian transfinite cardinals are not defined: For such infinities, '$-\infty$' has no natural sense.[23*] We would want $(\infty + (-\infty))$ to equal $(\infty - \infty)$, and we would want $(\infty - \infty)$ to number *every* set that comes from an ∞-sized set by removing all members of an ∞-sized subset. There is no such number. But suppose there were. Then it would be 0, since every set is a subset of itself. However, on the supposition being tested, it would not be 0 but 1, since, for every ∞-sized set S, there is an ∞-sized subset S′ that comes from S by removing just one member. And it would be ∞, since, for every ∞-sized set, there is ∞-sized subset S′ that contains 'from S every other member'. (I recall that ∞ is here Cantor's $\aleph_0$.)

Ways in which expected value considerations might be supplemented by other considerations when expected values are equal have been considered. There are closely related ways in which other considerations might be viewed as pragmatically decisive when, because of mathematical constraints, expected values are not defined. For example, it can seem that in the absence of defined expected values, relative probabilities for the two God-hypotheses should be pragmatically decisive in the present case, so that Bel(G) is uniquely pragmatically reasonable if and only if

$$Prob(\text{G, ReBel, \& PunNon}) > Prob(\text{G, ReRat, \& PunIrr}).$$

For a contrary line, we have the somewhat plausible suggestion that, if $Prob(\sim\text{G}) > 0$ in this case, then quite regardless of the status of the inequality just displayed, ~~Bel~~ (G) is uniquely pragmatically reasonable, since, while both options make positively probable both infinite gains and losses, only Bel(G) makes positively probable a defined relative loss.

11. Case 7: Reason itself is considered another great thing – the cost of belief is reckoned as infinite. For what doth it profit a man, if he gain the whole world, and suffer the loss of his own soul?[24]

11.1. In this case, as in Case 3, belief is perceived as costly. But, in contrast with Case 3, the cost of belief is now viewed as 'infinite'. 'I' am convinced that the evidence is decisive against the existence of God, and though 'I' think that 'I' can even so believe in God if 'I' will, 'I' see processes to that belief doing 'me' great harm. Contemplating the project of adding "a tinsel splendour to the plain straight road of . . . life . . . to drown the common sorrows of our kind by a self-deception," 'I' see this project as allowing these sorrows "not only to cast down, but also to degrade" and humiliate (Clifford 1989, p. 78)![25] 'I' value reason, including responsiveness to perceived evidence, not merely instrumentally as a means to ends such as truth, but intrinsically and for deep personal reasons. 'I' see it as essential to 'my' humanity; so may have Pascal, who said that induced strategic believing would make 'me' stupid *as a beast* ("vous abêtira"). 'I' see reason in both its practical and theoretical deployments as essential to, as indeed constitutive of, 'my' being. Concerned as 'I' am with 'my' self, reason, both practical and theoretical, in 'my' own person is for 'me' beyond any this-world price. But 'I' do not scorn heaven, and indeed make it incommensurably more valuable than every worldly good that is quite other than this good of reason itself. (Were 'my' rational soul 'my' highest good *without qualification*, were it for 'me' incommensurably greater than *every* other good, then, if there are kinds of truth that by their natures cannot be decided on intellectual grounds, a rule of thinking that would prevent 'me' from believing in truths of these kinds would *not* (*pace*, James, p. 107) be an irrational rule. It would not be for 'me' an irrational rule.) Here is a consequence-matrix

and a partial value-matrix for the case.

	God exists. G	God does not exist. ∼G
by choice to believe in God Bel(G)	eternal bliss, and injury to 'my' rational soul (to 'my' intellectual reason) ?	injury to 'my' rational soul (to 'my' intellectual reason) $-\infty$
by choice not to believe in God ~~Bel~~(G)	status quo 0	status quo 0

11.2. What remains is to decide on a measure for the importance to 'me' of eternal bliss conjoined with injury to 'my' rational soul. 'My' value for this conjunctive whole is not a simple function of 'my' values for its parts. (Cf., Moore 1993, Sections 18–20.) Its measure is certainly not the sum of theirs, for that would be $(\infty + (-\infty))$, which would-be sum is not defined. There are four possibilities for the value of this compound. (1) It could be negative, in which case ~~Bel~~(G) would strongly dominate and be uniquely pragmatically reasonable. (2) It could be zero, in which case ~~Bel~~(G) would weakly dominate and, provided only that $Prob(\sim G) > 0$, would be uniquely pragmatically reasonable. (3) The value of this whole could be positive and finite, in which case again ~~Bel~~(G) would be uniquely pragmatically reasonable without question, provided only that $Prob(\sim G) > 0$. In these cases, 'I' should choose not to believe, which is an easy choice in the case, for 'I' am convinced the evidence is decisive against the existence of God, so choosing not to believe will be choosing to let the evidence as 'I' perceive it 'have its way with me' (this may include taking care not to be manipulated by others to believe against the evidence as 'I' see it). (4) The value of this compound could, notwithstanding the equality of the infinite values and disvalues of its parts, be infinitely valuable. That could give rise to problems like those discussed in connection with Case 6, if the infinite value of [Bel(G) & G] equaled the infinite disvalue [Bel(G) & ∼G]. In that case, *ExVal*[Bel(G)] would not be defined, if 'I' consider both G and ∼G somewhat probable. If those infinite values and disvalues were unequal, the probability of G would determine which of Bel(G) and Bel(∼G) was uniquely pragmatically reasonable. For example, if $ExVal[\text{Bel}(G)] > -ExVal[\text{Bel}(\sim G)]$, then $ExVal(G) > ExVal(\sim G)$ if and only if $P(G) > 0$.

11.3. 'I' (and as it happens, I myself) value being rational, and assent to imperatives to proportion 'my' beliefs to the evidence as 'I' see it, and to maximize expected value. But 'I' do not commit the dread Naturalistic Fallacy. 'I' do not (as a Thomist might) mistake natural tendencies to proportion beliefs to evidence, and to maximize expected values, for "ethical imperative[s], or

for . . . evaluation principle[s] of 'wisdom' or something of the sort" (Mavrodes 1981, p. 65). Nor does 'my' concern that 'my' belief be proportioned to 'my' evidence have anything to do with Cartesian abhorrence of error. 'I' (and still, as it happens, I myself) consider proportioned responsiveness in belief to perceived evidence, and in action maximizing responses to 'my' beliefs and desires, to be not merely natural tendencies for humans, but projects in which 'I' have very deep personal stakes. 'I' think (shades of Plato) that as we have stakes in the health of our bodies and in natural processes constitutive thereof, we have great stakes in the health of our rational souls. 'I' think (shades of Kant?) that everyone has a stake in the rational coherence and governance of his deeds and opinions, as conditions and processes essential to and constitutive of being an integrated functioning person who thinks and acts.

11.4. It has been said, and I agree, that

> [o]ne's own good comprises not only one's states but also the possession of one's self as a mind. . . . among the duties of self-preservation is the conscientious man's commitment to live without evading any issue – to seek out and weigh what cogent reasons would lead him to do [and to believe], and to submit himself without self-deception or evasion to their determination. . . . acceptance of the principle of non-escapism . . . has the most intimately personal reason. It rests on an individual's inmost concern to preserve himself intact as a living and functioning self; mentally in possession of himself and of his world, able to look at himself . . . without hiding from himself. The penalty for slighting this need is his undoing as a person. (Falk 1968, pp. 373–4)

I think that cogent reasons well-weighed operate to proportion beliefs to perceived evidence, and to cause actions that maximize expected values given one's beliefs and desires. So I think that 'I', the agent, am on to something, though 'I' value it more than I value anything, including eternal bliss.

11.5. I do not agree with Bernard Williams, who implies by rhetorical questions that

> [w]ith regard to no belief could I know – or, if all this is to be done in full consciousness, even suspect – that I had acquired it at will [as the culmination of a project to acquire it, or 'just like that'] . . . . [W]ith regard to every feat of this kind which I had performed I necessarily . . . believe that it [has] not taken place. (Williams 1973, p. 148)

Jonathan Bennett accepts a restricted version of Williams's thesis with which I also disagree. Bennett implies that a person cannot maintain a belief that he knows that he acquired at will, whether immediately or as the culmination of a belief-acquisition project, unless "since voluntarily acquiring it he has encountered evidence for it" (Bennett 1990, p. 11). There are, I think, no *impossibilities*, whether logical or merely psychological, here. (In contrast, I think it is impossible to believe 'just like that' or 'immediately' at will. A question that is for me open, partly as a result of reading Bennett's extended report

of his failure to close it, is whether this impossibility is conceptual or logical, as Williams thinks, or some kind of biological or psychological impossibility.) There are, however, *difficulties* that yield practical reasons against acquisitions of beliefs by will. Having acquired a belief somehow by will and without regard, for example, to the absence of justifying evidential reasons, or worse to the presence of refuting evidential reasons, entails, in 'my' view, unless deficiencies in the evidence have been subsequently rectified, the curtailment in this belief's connection of natural processes for the integration of a person's beliefs that involves proportioning to evidence and the lack thereof. Acquiring a belief somehow by will requires in these cases, if you are the person, that you in this way "deaden your acuteness" (Pascal 1989, p. 82 – "vous abêtira" – another translation). Knowing that one has acquired a belief somehow by will compounds the damage in these cases by entailing complicity in this 'dehumanization'. Not knowing compounds the damage by entailing forgetting and cutting oneself off from a part of one's self.

12. Case 8: All goods and evils are considered commensurable

Infinite things alone – for example, eternity and salvation – cannot be equaled by any temporal advantage. We ought never to place them in the balance with any things of the world. (*Port Royal Logic*, p. 357.)

12.1. Attempts to apply the Bayesian framework to gambles in which figure infinities conceived in the manner of Cantor run into problems. To get quite away from these without going to Robinsonian 'hyperreal infinities', we consider an infinity-free Pascalian wager, *a wager in which all goods and evils are commensurable*. This will not only avoid those difficulties, but make a wager that can be relevant to people so far left out. Many persons, perhaps most people, would discover on reflection that they do not have values, not even values for heaven and hell, for representations of which infinities are required. This, I think, is true of me. (See note 3.)

12.2. I take, for a case, the just previous case with values for eternal bliss and personal rationality scaled down from being incommensurably greater than quite other this-world goods to being merely much much greater. 'I' value eternal bliss (eternal lightness of heart and joy in the presence of God), and value it greatly, but 'I' do not value it incommensurably more than everything worldly. It is *not* for 'me' so valuable that 'I' would pay any price not to lose the smallest chance of winning it. For example, 'I' would trade *away* a small chance for eternal bliss for 'myself' in order that 'my' dog not be tortured or 'my' daughter mutilated. Indeed, leaving chances out 'I' would make a straight trade at least in the second case. So 'I' do not value eternal bliss incommensurably more than 'I' value every good that others might enjoy. And 'I' do not value it incommensurably more than every personal good. 'I'

would not, for every chance no matter how small for eternal bliss, cut off 'my' arm or suffer stupefaction or madness. Turning now to rationality, 'I' value it greatly too, but (we suppose for the case) not as greatly as eternal bliss. Let there be in 'my' view parity between a 1/3 chance of eternal bliss and a certainty of not detracting from my rationality to the extent that a willful belief in God would do, so that 'I' value eternal bliss twice as much as 'I' disvalue that detraction. Let an expected value-matrix for the case be,

	G	∼G
Bel(G)	2−1	−1
~~Bel~~(G)	0	0

In this case I have let 'my' (the agent's) value for the conjunction of eternal bliss and injury to his rational soul be the sum of the values, respectively, of eternal bliss (2) and this injury (−1), which is of course 1. It is evident that,

$$ExVal[\text{Bel(G)}] > ExVal[\text{~~Bel~~(G)}],$$

if and only if

$$Prob(\text{G}) > Prob(\sim\text{G}).$$

In this case, 'my' willing to believe in God is pragmatically reasonable if and only if 'I' think it more probable than not that God exists.

13. Case 9: God would frown upon willful believing

13.1. Previous passional work can qualify Pascal's Wager as a "regular clincher" in a case, but it will do so only if previous work has not also established views regarding the employment of strategic thinking to decide how and what to believe that argue decisively against this employment. All Pascalian wagers may be off, if 'I' am sure that though the Deity, if He exists, rewards uncalculated, unstrategic faith, and that "faith . . . adopted willfully after . . . a . . . calculation" would *not* be well-received (James, p. 6) – they may all be off, if am sure that "the Deity [if He exists would] . . . take particular pleasure in cutting off believers of this pattern from their infinite reward" (James, p. 6)! And all bets can be off for 'me' if 'I' make intellectual reasonableness so important that for 'me' "it is wrong always . . . to believe anything upon insufficient evidence," as it could be if willful believing is for 'me' infinitely bad *in itself* (Clifford, p. 79).

13.2. Let Case 1 be modified by the addition of a view of the first sort. Suppose 'I' am sure that God, if He exists, rewards faith with eternal bliss when faith, and only when faith, either comes naturally and unsought to open inquiring minds or is given by Him and comes by His grace. Suppose 'I' am sure He would quite cut off, along with nonbelievers, all willful calculating believers

and deny them entry into heaven. Then 'I' should *hope* and perhaps *pray* that 'I' naturally or by divine grace believe. For it is not only when judging what he ought to *do* that a person must consider both values and probabilities of consequences, and view geometrically the proportion they have together, but also when the question is what he "ought to fear or hope" (*Port-Royal Logic*, p. 357). Here too it is not only advantages and disadvantages but also their likelihoods that matter. (I have written "and perhaps pray" with the thought that possibly 'I' cannot only hope for belief that comes naturally or from God, but also *pray* for this belief, only if 'I' already believe in the person to whom 'I' pray, and so either have, or cannot get, that for which 'I' would be praying.)

We have for the *attitudinal problem* not what to do, but what to hope for, the following matrix that comes from Case 1's matrices. Let 'an unchosen belief in God' be a state of belief that is not by choice, but rather comes naturally and unsought or as a gift from Him, and similarly for 'an unchosen nonbelief in God'. The question is for which it is rational to hope. A sentence made true by the occurrence of this state is 'I am in a state of belief in God, not by choice or act of will, but naturally and unsought or as a gift from Him' Understand 'an unchosen nonbelief in God' similarly.

	God exists. G	God does not exist. ~G
an unchosen belief in God Bel(G)	eternal bliss, and gain in the life ∞	gain in this life 1
an unchosen non-belief in God ~~Bel~~(G)	status quo 0	status quo 0

To simplify, I have rescaled to measure the value of an unchosen belief in the absence of God by 1. The expected value of belief by nature or grace exceeds that of nonbelief. Belief is, given 'my' values and probabilities, the more desirable of these states, and a condition for which it is reasonable that 'I' should hope and perhaps pray. If $Prob(G) > 0$, belief is a much more valuable state for which 'I' should fervently hope.

13.3. But though belief by nature or grace is a state for which 'I' should hope, belief may not recommend itself as the end of an act. For example, 'I' should choose not to choose either alternative in 'my' binary belief-option, if 'I' think that in this way 'I' have a finite (noninfinitesimal) chance of coming either naturally or by grace to believe, and if $Prob(G) > 0$. The pragmatic reasonableness under these conditions of choosing not to choose between alternatives of the belief-option can be brought out this time, not by ignoring, but by making explicit the third live alternative in 'my' decision problem that features as well the alternatives of 'my' binary belief-option. I assume the following consequences

and values:

	God exists. G	God does not exist. G
by choice to believe in God Bel(G)	gain in this life 1	gain in this life 1
by choice not to believe in God ~~Bel~~(G)	status quo 0	status quo 0
neither by choice to believe in God, by choice not to believe in God ~[Bel (G) v ~~Bel~~ (G)]	a chance for eternal bliss, and a chance for gain in this life ∞	a chance for gain in this life x such that $0 < x < 1$

The value x reflects not only the chance for gain in this life, but also 'my' averseness due to 'my' decisive nature – which is part of Case 1 – to leaving things to nature and God that 'I' can settle by choice. In the more articulated notation of Section 4.4, ~[Bel (G) v ~~Bel~~ (G)] is equivalent to

$$\sim(\mathrm{Ch}[\mathrm{Bl}(\mathrm{G})] \text{ v } \mathrm{Ch}[\sim\mathrm{Bl}(\mathrm{G})])$$

and thus to

$$\sim\mathrm{Ch}[\mathrm{Bl}(\mathrm{G})] \,\&\, \sim\mathrm{Ch}[\sim\mathrm{Bl}(\mathrm{G})].$$

Given values just assumed, 'my' expected values can be ranked, depending on whether or not $Prob(\mathrm{G}) > 0$, without settling on a precise value for this probability. It can be seen that, if $Prob(\mathrm{G}) > 0$, then

$$ExVal(\sim[\mathrm{Bel}(\mathrm{G}) \text{ v } \sim\!\!\mathrm{Bel}(\mathrm{G})]) > ExVal[\mathrm{Bel}(\mathrm{G})] > ExVal[\sim\!\!\mathrm{Bel}(\mathrm{G})].$$

While if $Prob(\mathrm{G}) = 0$, then

$$ExVal[\mathrm{Bel}(\mathrm{G})] > ExVal(\sim[\mathrm{Bel}(\mathrm{G}) \text{ v } \sim\!\!\mathrm{Bel}(\mathrm{G})]) > ExVal[\sim\!\!\mathrm{Bel}(\mathrm{G})].$$

13.4. In this case, 'I' will, if reasonable, *hope* for belief. If 'I' think that God is at all probable, 'I' will fervently hope. *But if 'I' do think that God is at all probable* – **Prob(G) > 0** – then *'I' will take no steps toward belief* other than inquiring into the issue of His existence with an open mind. 'I' will, assuming $Prob(\mathrm{G}) > 0$, for example, take care *not* to choose 'my' companions, 'my' mentors, or 'my' discussants in order to encourage or facilitate belief rather than nonbelief or disbelief. And 'I' will take care *not* to "think hard about [only] **certain** kinds of evidence" (Swinburne 1969, p. 223; emphasis added). For belief consequent to manipulative and deck-stacking steps would not have been unsought, and would not have come naturally to 'me' as an open inquiring mind. It would not be the kind of belief for which 'I' hope. In another case, in which there are in 'my' view net finite *costs* in this life to beliefs in God, *by-choice* belief, or

'willful belief', would be a kind of belief 'I' would, whatever 'my' probability for God, be concerned to avoid. *On the other hand, if 'I' am sure that God does not exist* – if, for 'me,' **Prob(G) = 0** – *then if 'I' am reasonable, 'I' will take steps to change that, and believe!!*

But wait. Would 'I' not have reasons, if 'I' managed, pursuant to that recommendation to believe by choice and so to make 'my' probability for G greater than 0, to *undo that effort* – to 'take it back' – and to let belief come naturally by grace, or not at all? 'I' would have such reasons *if*, but only if, it would be *possible* for 'me' to 'take it back'. By hypothesis, 'I' would have by choice actualized a state of certainty – a *belief* – that God exists, and 'I' might be sure that there was nothing 'I' could do about that. If 'I' *cannot* 'take it all back,' then the third option, of not by choice either believing or not believing, would no longer be open to be recommended, and 'I' would be stuck with Bel (G). Poor 'me'. If, however, 'I' realize that 'I' *can* 'take it all back,' then the third option would be open and recommended. Suppose, however, that 'I' were to pursue it with a vengeance so that for 'me' once again *Prob*(G) = 0. Well then 'I' would once again be looking at a recommendation to change that and believe, though if 'I' do that again without finality. . . . ?! Evidently, 'my' situation in this case *can* be one in which '*I' cannot make up 'my' mind what to do*. If it *is*, then I think there is nothing rational for 'me' to do.[26]

13.5. Rescher sees Pascal's reasoning as proceeding in two stages. First, for persons of certain orientations (of certain values and probabilities), the desirability, indeed the infinite desirability, of a Christian life is established. Second, it is said that, since belief is an integral part of this life, its pragmatic justification follows for persons of these orientations (Rescher 1985, pp. 18–9, 22–3). The present case shows that for persons of certain other, in one way more Jamesian, orientations, though a Christian life is infinitely desirable, it does not follow from that that this life, essential to which is belief in God, is pragmatically justified. That infinite desirability cannot provide a "pragmatic justification of a [Christian] praxis" (Rescher 1985, p. 23) for persons of the more Jamesian orientation of 'my' present case. According to *this* orientation, a Christian life that would include Christian belief is infinitely desirable *only if* not engaged in *for* its infinity desirability.[27*,28]

THE LESSON

Lycan and Schlesinger conclude their defence of Pascal's Wager with the invitation,

Let us pray.

I append to my examination of Pascalian wagers the invitation, the advice,

Let us reflect.

We should, in so far as we can, make up our minds as our probabilities and values would have us do, and thus with reflection on these. And as our probabilities and values change, we should reflect and make up our minds again and accordingly. We should, I think, not only look to our credences and preferences for direction, but also 'look into' our credences and preferences, so that we may be guided by credences and preferences that themselves stand up on reflection on their intrinsic merits, to reflection that proceeds quite without regard to consequences of having them, reflection that attends exclusively to their terms and objects. We should beware of deacons and pundits who claim to provide guidance and answers for everyone regardless of their probabilities and values.

Whether persons should believe or not believe, and if they should believe what they should believe and how, willfully or not, depends on their credences and preferences, their values and probabilities. Informed and thoughtful people can and do differ in these, especially with regard to other-world matters of heaven and hell and to inner-world matters having to do with selves and rational souls. James would agree. Writing in his Preface of December 1886, with "The Will to Believe" particularly in mind, he says,

> I do not think that any one can accuse me of preaching reckless faith. I have preached the right of the individual to indulge his personal faith at his personal risk. I have discussed the kinds of risk; I have contended that none of us escape all of them; and I have only pleaded that it is better to face them open-eyed than to act as if we did not know them to be there. (James 1956, p. xi)

Agreed. Pascalian wagers are major issues. In them souls and selves and lives are on the line. Let us therefore think things over to identify our personal wagers, our options, and our values and probabilities for things at stake, so that, by viewing geometrically all these things together, we may wisely compose our minds regarding things divine or leave these matters to intellectual reason and, if such there be, grace.[29]

APPENDIX. HYPERREALS AND DECISION THEORY

A1 Hyperreals. I adapt here explanations of Henle and Kleinberg (1979) and Hunter (1988).

A.1.1 R – a theory for nonstandard reals.*

A1.1.1. Let **R** be a first-order theory for the real numbers such that:

(i) R features "constant symbol[s]. . . . for. . . . real numbers . . . , for . . . function[s] . . . on the real numbers . . . function symbol[s] . . . , [and] for . . . relation[s] on the real numbers. . . . relation symbol[s]" (Henle and Kleinberg 1979, p. 21). R is to be a first-order theory and so its language contains countably many constants (see Hunter 1973, pp. 137, 173).

I include all constants needed for our work, but, perforce, not constants for all real numbers and relations and functions thereon as Henle and Kleinberg do "to play it safe" (p. 21). I pretend prescience concerning our needs.

(ii) R has "axioms . . . true of . . . the real numbers" (Hunter 1973, p. 4) that entail everything of interest to us. At Hunter's suggestion 'adequate for' has been replaced by 'true of'. In a note to the offprint of his paper that he sent me, he observes that the real numbers cannot be completely characterized by first-order axioms. Since I include predicates for integers and natural numbers, and for all familiar operations thereon, R cannot have axioms that are complete for truths expressible in R.

A1.1.2. **R*** shall be R plus a new constant '$\odot$', the axiom, '$\odot > 0$', and the denumerable sequence of axioms, '$\odot < 1/2$', '$\odot < 1/3$', and so on. R* is a first-order theory.

A1.2 Models for R – hyperreals great and small, and middling*

A1.2.1. R* has a model. "For any finite subset of the new set of axioms you can find a real number which has the properties ascribed to [$\odot$] by that finite subset of axioms, and which also has all the properties ascribed by the axioms of R, understood in their ordinary intended sense. . . . So by the Compactness Theorem (*If every finite subset of the proper axioms of a first-order theory K has a model, then K has a model*) R* has a model" (Hunter 1988, p. 4). Indeed, R* has a model that includes standard reals, or, better, surrogates thereof, along with elements "that behave in the ways mathematicians have wanted infinitesimals to behave" (Hunter 1988, p. 5). Cf.: "The compactness theorem . . . states that there is a nonstandard universe containing pseudo-reals [**hyperreals**] . . . including . . . [an] infinitesimal . . . such that . . . any true statement about the standard reals that you can state in the formal language [of R and derive from the axioms of R] is true also about [these pseudo-reals]" (Davis and Hersh 1972, p. 84).

A1.2.2. Let system S be such a model for R*. This system includes *the infinitesimal* $\odot$. Let any element of S of which the new axioms of R* are true be **an infinitesimal**. The truths in the formal language of R are true of the elements of S; let these elements be 'hyperreal numbers', including this one $\odot$. There are *other infinitesimals* in S, for example, $\odot/2$ is a smaller infinitesimal, and $2 \cdot \odot$ is a greater one. (Suppose $2 \cdot \odot$ were not a greater infinitesimal. Then there would be a 'rational hyperreal' r such that $2 \cdot \odot > r$, and it would be $\odot > r$. The new axioms entail that, for every 'rational hyperreal' r, $\odot \not> r$.) There are also positive **infinite hyperreals** – let these be positive **infinimals** – that are greater than all real hyperreals. For example, $1/\odot$ is an infinimal. (Suppose $1/\odot$ were smaller than a real hyperreal r. Then the real hyperreal 1/r would be smaller than $\odot$, in contradiction of a theorem of R*.) There are also negative infinimals.

A1.3 Infinimal integers. "We come now to one of the great curiosities of the hyperreal numbers" (Henle and Kleinberg 1979, p. 36). Some infinimals are integers ('integer hyperreals'). I picked axioms for R to secure the theorem that every real number is exceeded by an integer; so it is a theorem of R* that every hyperreal is exceeded by an integer. Similarly, some infinimals are odd and some are even: I picked axioms to ensure that every hyperreal integer is either odd or even. Pascal would be surprised: "[I]t is untrue to say that it [infinity] is even, untrue to say that it is odd" (Pascal 1962, pp. 132–3).

*A1.4 What might hyperreal numbers **'be'**?* What might members of the domain of a model for R* be? There are many possibilities, just as there are many possible 'constructions' of the standard reals. There is a point in saying that 'the set of hyperreal numbers' is not 'well-defined', for that definite description is 'improper.' However, contrary to Hunter (1988), there is a point – indeed essentially the *same* point – in saying that 'the set of real numbers' is not well-defined. Hunter observes that models for R^* "are [not] all isomorphic to one another" (p. 5). That is true of models for first-order theories for standard real numbers. It has been said that "God made the [positive] integers, all else [zero, negative integers, rationals, reals, imaginaries, and so on] is the work of man" (Leopold Kronecker, quoted in Henle and Kleinberg 1979, p. 3). Suppose we accept this metaphor. Then we can say that there is not just one way of doing that work, and of creating objects that behave as we would have various kinds of numbers behave. Let me confess to a penchant for Platonic realism and the inclination to replace that metaphor by one that says that 'God made *all* the numbers, not from others already made, but from scratch' ('numerical creationism, not evolution'). The real numbers, for example, are what they are, and not other things. Standard real numbers are, for example, *not* Dedekind cuts, those ordered pairs of sets of rational numbers that are the domain of a model for R. And similarly for the hyperreals.

A1.5. Henle and Kleinberg explain one 'construction of the hyperreals'; they give one model for R^*. In it, as standard reals can be 'identified' with denumerable sequences of integers, so the hyperreals of S are 'identified' with denumerable sequences of standard reals, two sequences being counted as the same hyperreal if and only if they are identical at a 'quasi-big' number of positions. (More exactly, therefore, hyperreals are 'identified' with sets of such sequences equivalent under the relation of sameness at a quasi-big number of positions.) Henle and Kleinberg explain how predicates and operators defined for standard reals can be redefined for such sequences in a manner that ensures that precisely the sentences that are true of the standard reals, given standard definitions of those constants for them, are true of these sequences when these constants are redefined. For examples of hyperreals so constructed we have: (1,1,1 . . .), which is the hyperreal surrogate of the standard real 1; (1, ½, 1/3, 1/4 . . .), which (given those redefinitions – they also run in terms

of quasi-bigness) is greater than (0, 0, 0 . . .) and less than (r, r, r . . .) for every standard real r, so that (1, ½, 1/3, 1/4 . . .) is an infinitesimal hyperreal; (1, 2, 3 . . .), which is a positive infinimal, either odd or even, depending on details of that construction (given in particular the sequence of subsets of the natural numbers used in one's definition of quasi-big sets of natural numbers – see Henle and Kleinberg, p. 126); and (−1, −2, −3 . . .), which is a negative infinimal – (−1, −2, −3 . . .) is the negative inverse of the positive infinimal (1, 2, 3 . . .).

While in the construction just described there are hyperreals that are naturally identified as surrogate reals, there may not be in terms of this construction any natural identification or correspondence of sets of hyperreals with transfinite cardinals, for example, Cantor's $\aleph_9$. Hyperreals are identified in this construction with sequences of real numbers, and transfinite cardinals are not real numbers. Constructions of the hyperreals aside, it may be that no sense can be given to the 'position' of Cantorian transfinite cardinals 'along' the hyperreal line. (They of course cannot be 'on' it: Their arithmetic is different.) It may be that Cantorian transfinites are not simply another but an unrelatedly other kind of number.

A1.6. Surreals – another mathematical option. Thanks to Alan Hájek, who, in a valuable and imaginative essay, calls attention to John Horton Conway's system of "[a]ll numbers great and small" (Conway 1976, p. 3) that, like Robinson's system of hyperreals, features well-behaved infinitesimals and infinities. In Conway's construction, all "so-called *surreal numbers*" (Hájek 1994) are made from scratch (well, from sets) in one continuous process (we do not first make the integers, then the rationals including 'reconstructed integers', and so on). The surreals made include ones that can be viewed as extending and filling in the standard real line much as Robinson's hyperreals do. A surreal number x is an ordered pair (L,R) of possibly empty sets of surreal numbers in which no surreal number in L is greater than or equal to any surreal number in R. For these numbers '$\geq$' and '$\leq$' (and then '=', '>', and '<'), '+', − (negative inversion), and multiplication are defined. The surreal (∅, ∅) appears in the first stage of the construction: It is surreal 0. At the next stage we meet surreal 1 and –1, which are, respectively, ({0},∅) and (∅,{0}). At the omega, stage we meet omega, the first infinite ordinal surreal, which is ({0,1,2, . . . },∅), and also its negative inverse. There is no last stage. The construction secures closure and proper behavior "under the usual operations (addition, substraction, multiplication, division, exponentiation, extracting roots, and so on), and [that] all numbers can be compared in size" (Hájek 1994).

Conway's surreal and Robinson's hyperreal infinitesimals and infinimals are alike in their *major* advantages for decision theory. Relative merits for decision theory of these two systems of numbers – of which relative merits I am not qualified to judge – thus turn on fine and subtle points toward which

I offer only two tentative observations. It may be an advantage of Conway's numbers that "infinite cardinals like" Cantor's $\aleph_0$ can, "[i]f we . . . adopt the axiom of choice[,] . . . be identified with the least corresponding [surreal] ordinal numbers" (Conway 1976, p. 3): Conway, recall, would have his system encompass all numbers great and small. Conway contemplates the identification of $\aleph_0$ not with a surreal number but only with a certain *set* of surreal numbers. It is, looking the other way, an advantage for their use in decision theory that Robinson's hyperreals are made to behave in all ways as standard reals not *by work*, which is often for the inexpert tricky, but *by definition*. A related point is that, from my Platonic perspective, Conway's surreals compare not so much with Robinson's hyperreals, whose theory is R^*, but with various 'constructions' of Robinson's hyperreals for models of R^*.[30]

A2 Hyperreals in decision theory

A2.1. Hyperreals, including infinimals, are compactly ordered by the less-than-or-equal-to relation. Between any two there is another. Indeed, between any two infinimals (as between any two reals) there are uncountably many infinimals. Infinimals are not set-sizing numbers. The smoothly continuous order of infinimals seems better suited than set-sizing numbers can be to measure things such as incommensurably-greater-than-ordinary intensities of preference, things that are not countable quantities or subject only to 'quantized' increases and decreases. Similarly for finite hyperreals and ordinary intensities of preference. "[T]he hyperreal line is a lot denser than the real line, that is, more numbers are packed into the same space" (Henle and Kleinberg 1979, p. 36). Hyperreals seem to the imagination more smoothly continuous than standard reals, as they are certainly more continuous than transfinite cardinals.

Infinimals are intuitively better suited to represent 'infinite values' than are Cantorian infinities, and *also* – as has been observed in asterisked notes – infinimals lack the features that move some expected value theorists to view infinite expected values as the stuff of a "monstrous hypothesis" (Nalebuff 1989, p. 178) that should be banned from decision theory (cf., Jeffrey 1983, pp. 153–4). There are thus several aspects of hyperreals in general, and of infinimals in particular, that recommend them over standard finite reals and transfinite cardinals for use in decision theory. *And we can have them for nothing*! There can be no obstacle simply to reinterpreting foundations for real-number decision theory as foundations for hyperreal-number decision theory, given that hyperreals can be defined in a manner that insures that everything true about standard reals, on which given foundations depend, is true of hyperreals (see (ii) in Section A1.1).

It is *true* that "[t]he very axioms [whether they be those of Richard Jeffrey or Leonard Savage] . . . that ensure that rational choice is choice that maximizes expected utility, logically rule out infinite utility" (McClennen 1994, p. 126), *if*

by 'infinite utility' is meant 'standard Cantorian infinite utility. But if what is meant by 'infinite utility' is 'nonstandard Robinsonian infinimal utility,' then that is false.

A2.2 "It is time to think big!" (Sorensen 1994, p. 139). Roy Sorensen seeks "to motivate the extension of decision theory to infinite quantities" (Ibid.). Having reviewed problems that standard theories of numbers make for this extension, he draws an applaudable conclusion. "Math just offers models. If we do not like the results, then just continue shopping – or fashion your own model" (p. 155). However, Sorensen's "diagnosis is that **our decision principles** need to be revised or remodelled to accommodate the transfinite" (Ibid.; bold emphasis added). That is the strategy tentatively pursued in my main text. My considered diagnosis is that we should instead make use of an accomplished elegant remodeling of the transfinite that happens to suit it to our decision principles, and to their foundations, unrevised.

Sorensen's particular idea – not worked out in detail or shown to be adequate to all of decision theory's traffic with infinite values – is "that an adequate overhaul will turn on two readings of 'better.' Under one reading, things are better when goodness is added (or badness subtracted). Under another reading, 'better' means a greater amount of goodness (or a less amount of evil)" (Ibid.). It is remarkable that he does not consider the hyperreal option by which decision theory can, without any adjustments, be reinterpreted to accommodate the very big and, for that matter, the very small. Of that option, I sing, Let it be, for it works and is done.

Notes

Chapter I

1. I am not alluding to the idea that, when introduced, 'YHWH' is preglossed by the line, *ehyeh asher ehyeh* (I am that I am, in the King James translation), which some scholars say means "'He who causes (things) to be' or perhaps 'He who is'," (Everett Fox in *The Five Books of Moses* 1995, p. 270). Fox himself translates this – "one of the most enigmatic and widely debated statements in the Hebrew Bible" (Ibid.) – thus, "I will be there howsoever I will be there." Now comes all of *Exodus* 3:14 as translated by Fox: "God said to Moshe: Ehyeh Asher Ehyeh/I will be there howsoever I will be-there. And he said: Thus shall you say to the Children of Israel: Ehyeh/I-will-be-there sends me to you." I think that God is here depicted not as saying what His name is, or what it means, or who He is, but rather as saying to Moses how they can know him: "When the people ask for my name, tell them not to worry. Tell them, as I have been telling you [in the same word, 3:12], that I am there with them [cf., Rashi below] then, and will ever be there with them ['there for them,' in modern idiom]. They will not need a 'true name' with which to call on me, for I will be there, present with them, then and always. Let them know me as one who will be there. Let them call me 'I-will-be-there' to remind them of my dependable presence." God is then depicted as further instructing that His historical connections with 'the people' that they know, be recalled: in Fox's translation – "And God said further to Moshe: Thus shall you say to the Children of Israel: YHWH, the God of your fathers, the God of Avraham, the God of Yitzhak, and the God of Yaakov, sends me to you" (3:15). Rashi's cryptic 'gloss' on *ehyeh asher ehyeh* is (in translation): "I will be with them in this sorrow – I who will be with them in the subjection . . . of other kingdoms" (Rosenbaum and Silbermann, *Exodus*, p. 12; Rosenbaum and Silbermann, pursuant to a policy announced in their Preface, *Genesis*, p. i, translate *ehyeh asher ehyeh*, both as it occurs in 3:15 and in Rashi's commentary when he mentions it, by 'I am that I am'.) Thanks to Arnold Silverberg for putting me on to Fox's translation with commentary, and for conversation about what would be God's names.
2. Are there other instances of common names having, in this fashion, been made into a proper names? Willa Fowler Freeman Sobel has suggested that there may have been a time when one could have said, informatively of an emerging

convention of language, that 'The World' names the one and only world, and that 'The Universe' names the one and only universe. It may seem fitting and suggestive that there are these possible parallels between 'God' and 'The World.'

There may be more instances of the reverse process in which proper names are 'made into' common names. I conjecture that proper name counterparts in ancient languages of 'the Sun' were made into common names. Making a common name out of a proper name as in 'She is a regular Pollyanna', in 'He may be another Wayne Gretzky', and in "You are no John Kennedy" (said by Lloyd Benson to Dan Quayle during a televised debate). Bernard Katz wonders whether such uses of proper names as common names are merely metaphorical, or elliptical positive and negative similes.

Katz observes that there is another way in which proper names may all be common names in the waiting. Consider 'There are only three Bernard Katzs listed in the phone book'. A Bernard Katz is here not a person with Bernie's special qualities, but is any person of the same name. Only persons? Would cats named 'Bernard Katz' be Bernard Katzs?

3. Everett Fox renders the line "I am YHWH your God" (*The Five Books of Moses* 1995, p. 369.) I wonder about his decision (and that of the King James translators) to capitalize 'god' here, rather than let it agree with the plural in his (their) translation of 20.3: "You are not to have any other gods before my presence." He resumes (as do they) capitalization for 20:5, "I, YHWH your God, am a jealous God." The second capitalization in this line is particularly curious.
4. 'He', 'Himself'?! Well yes, when I am commenting on The Bible, as well as when I am 'conversing' with other writers who hue to traditional forms; otherwise not. Thanks to Shirlee Goldman-Herzog and Roger Herzog for helping me to this policy. Cf.: "Readers who are uncomfortable with the maleness of God in these texts may wish to substitute 'God' for 'he' in appropriate passages. While, as a translator, I am committed to reproducing the text as faithfully as I can, it is also true that the ancient Hebrews viewed God as divinity beyond sexuality, and modern readers as well may see fit to acknowledge this." (Fox 1995, p. xxx). Further, while I accept on authority that in "the narrative presentation of the Holy Scriptures, divinity is assigned masculine grammatical gender" (Hook and Kimel 1995, p. 220), imagery and metaphor are sometimes unequivocally feminine. Thus: "The Rock that birthed you, you neglected, you forgot the God that produced-you-in-labor" (*Deuteronomy* 32:18; Fox 1995, p. 1003).
5. Cf.: "... *Elohim* meaning 'God'" ("To the Reader," *The New Oxford Annotated Bible*, Oxford: Oxford University Press, 1991).
6. "... meaning 'Lord'" (Ibid.).
7. "Hearken O Israel: YHWH our God, YHWH (is) One!" (*Deuteronomy* 6:4; Fox 1995, p. 880). "Despite the centrality of this phrase as a rallying cry in later Jewish history and thought, its precise meaning is not clear. It most likely stipulates that the Israelites are to worship YHWH alone" (Ibid.). Proximate passages are, I think, not consistent with reading the line as an affirmation of monotheism, according to which YHWH would be the one and only god not just for the people of Israel, but period. "Now you are to love YHWH your God" (6:5). "You are not to walk after other gods ... for a jealous God is YHWH"

(6:14–15). "Know that YHWH your God, he is God, the trustworthy God, keeping the covenant of loyalty with those who love him and with those who keep his commandments" (7:9) – YHWH, among gods, is the trustworthy one who keeps the covenant made with His chosen who have entered into a covenant with Him. "[F]or YHWH your God, he is the God of gods and the Lord of lords, the God great, powerful, and awe-inspiring" (10:17), but, by implication, not, for all that, the only god. There comes in *Leviticus* this report: "YHWH spoke to Moshe, saying: Speak to the entire community . . . and say. . . . Do not turn-your-faces to no-gods, and molten gods you are not to make yourselves, I am YHWH your God!" (*Leviticus* 19:4; Fox 1995, p. 601). "**No-gods**: Heb. *elilim*, a popular play on *el/elohim* ('God, gods') and *al*, 'nothing.' Greenstein (personal communication) suggests 'little-gods' as another possibility" (Ibid.). 'Lesser gods' would cohere with the message elsewhere that in any case YHWH is the greatest god, the god of gods.

8. There is, perhaps, this much in the Strawsonian presupposition thesis for proper names: A sentence in which a proper name 'has widest scope' (but for the scope of other names) that would express a proposition about the world (and not part of a myth or story) expresses a proposition, true or false, only if this name names something that exists in the world. 'God', in 'God does not exist' as used by atheists, does not 'have widest scope': in suggestive symbolizations, this sentence goes into '∼[G]E(G)' rather than '[G]∼E(G)'. So if 'God exists', '[G]E(G)' does not express a proposition true or false, then atheists' use of 'God does not exist', '∼[G]E(G)', can and does express a true proposition. Stirton likes the 'line' that says that "an atomic statement containing a non-denoting singular term will always be false, in the sense . . . of having a true negation" (Stirton 1995, p. 47). My suggestion is that a better, though more complicated, 'line' does not equate 'expresses a false proposition' with 'has a negation that expresses a true proposition'.
9. That a 'proper name' does not designate in this world, or indeed in any possible world, is consistent with its being a 'rigid designator', if that is a term that, in each world in which it designates, designates the same thing. According to that rule 'God' can be a 'rigid designator' even if God does not exist. 'God' is (vacuously) a rigid designator, if it is necessary that God does not exist. I do not say that that is the rule Saul Kripke intended, for he warned that, "Concerning rigidity: . . . I deliberately ignore delicate questions arising from the possible nonexistence of an object" (Kripke 1981, p. 21n).
10. "**Worship**. . . . 1. *trans* . . . to honour or revere as a supernatural being of power, or as a holy thing; to regard or approach with veneration; to adore with appropriate acts, rites, or ceremonies." (*The Compact Edition of the Oxford English Dictionary*, Volume A–0, Oxford University Press, 1971, p. 3826, being p. 320 in the volume for 'W' of *The Oxford English Dictionary* of 1933.) To reverence, veneration, and adoration may be added deference conveyed through 'bending abasement'. That, in the view of many, is an essential element of worship. One can love and adore, but one cannot worship, another as an equal.
11. Richard Swinburne writes, "**both** in virtue of his [nature and his possession of certain] properties . . . **and also** in virtue of his having done of his own free will various actions (e.g., rescued the Jews from Egypt and brought them to

the Promised Land)" (Swinburne 1989, p. 292; bold emphasis added). I am developing only the nature point.

12. Left out because not needed for that perspective, and because I am not up to the task, is elaboration of the "conception of God . . . [of] the Bible and Rabbis [of the Talmud]", which coheres differently with the core attitudinal conception (Wettstein 1997). "The argument from evil [with which we are occupied in Chapters XI and XII] relies upon [the philosopher's conception God as a perfect being]. . . . To put [that conception and with it] the classical problem . . . to the side is not to deny that there are real issues in the neighborhood . . . [to do] with His justice, **indeed with God's being an object of worship**" (Wettstein 1997, p. 427; bold emphasis added).

    Regarding the God of the Bible, of which I can speak only tentatively, there is in the words, "*Be ye therefore perfect, even as your Father which is in heaven is perfect*" (Matthew 5:46.) an unequivocal depiction of Him as morally perfect: As these words can be construed as delivering exactly an injunction to moral perfection, so they can be construed as attributing exactly that perfection. It is said that His knowledge is perfect (Job 37:16). There are words that can suggest that God of the Book knows all that is, at least as regards humankind: "*The Lord looketh from heaven; he beholdeth all the sons of men. From the place of his habitation he looketh upon all the inhabitants of the earth. He fashioneth their hearts alike; he considereth all their works"* (Psalm 33:13–14). There are words that say that God knows all that will be, at least as regards humankind: "*My frame was not hidden from you, when I was being made in secret, intricately woven in the depths of the earth. Your eyes beheld my unformed substance. In your book were written all the days that we formed for me, when none of them as yet existed. How weighty to me are your thoughts, O God! How vast is the sum of them! I try to count them – they are more than sand.*" (Psalms 139:15–18, New Revised Standard Version). And there are words that may say that He knows all that will be (because it will be according to His plan, which He will execute): "*I am God, and there is none like me. Declaring the end from the beginning, and from ancient times the things that are not yet done, saying, My counsel shall stand, and I will do all my pleasure*" (Isaiah 46:9–10) [*"My purpose shall stand and I will fulfill my intention."* New Revised Standard Version]. But there are other words that unequivocally depict Him as knowing neither all that is nor all that will be, even regarding humankind: "*If I find in Sedom fifty innocent within the city, I will bear with the whole place for their sake*" (Genesis 18:26; Fox 1995, p. 79). It is written: "*Now YHWH saw that great was humankind's evildoing. . . . Then YHWH was sorry that he had made humankind on earth*" (Genesis 6:5–6; Fox 1995, p. 33). The implication is that *He did not know when he made humankind, the evil that it was going to do*. The 'message' of The Bible re the knowledge of God is, no surprise given the circumstances of its composition, mixed. Thanks again to Arnold Silverberg.

13. Uses of possible worlds in this book do not tax the idea that they are comprehensive ways things are or might have been. Even so, I say more about my metaphysics, and the logic, of them in Appendix A to Chapter III.

14. To get away from that fiction, we could let the first box represent the actual world, the second box represent some other possible worlds, and the third box represent all remaining possible worlds.

15. A weaker logic, namely, a 'B-logic' ('Brower logic'), which is for 'modalities' defined in terms of an accessibility relation that is only symmetric and reflexive, comes in for brief comment in Chapter IV. While modalities intended in discussions in the philosophy of religion are almost always of the 'truth at all worlds' and 'truth at some world' S5-type, logical principles actually in play in these discussions are rarely specific to S5-modalities. They are almost always valid for all '*alethic*,' or about-or-concerned-with-truth-and-what-is-the-case modalities: these are the modalities that can be defined only in terms of reflexive accessibility relations – these are the modalities for which, that something is necessary entails that it is so. One contrast is with *deontic* modalities: If someone says, "I must not tell a lie – it is *necessary* that no matter what I not lie," and then up and lies, it does not follow that he was mistaken in what he said, for the necessity of which he spoke was 'deontic'; presumably he intended moral necessity. What is morally necessary need not be true. Similarly for '*doxastic* necessity': Perhaps it is true that Henry must have been there, in the sense that we are sure of that, but even so he may not have been.
16. "The Maimonidean way of expressing [this], by saying that it is *idolatrous* to take the things we say about God – that He is 'personal', or that he is 'Good', or that He is 'all-powerful', or that He is 'all-knowing' – as true either literally or 'analogically' – is . . . exceptionally radical. . . ." (Putnam 1997, p. 410). True, but Maimonides' line in passages quoted by Hilary Putnam is not nearly as radical as he supposes. For Maimonides writes there against *not* every predication of an attribute to God, but against only predications of "essential attributes" by which he meant 'intrinsic attributes' – he is against these on the ground that "God is truly one, in the sense that there exists no multiplicity whatever ***in*** Him" (Maimonides, *The Guide to the Perplexed*, quoted in Putnam 1997, p. 407; bold emphasis added).

    Putnam realizes that Maimonides countenances talk of God as merciful, and so on with the explanation that "the meaning is here not that He possesses [intrinsic] moral qualities, but that he performs actions that in us proceed from moral qualities" (Maimonides quoted again, p. 408) and wonders, rhetorically, whether Maimonides could have thought 'He performs actions' is univocal as applied to God and as applied to human beings? Putnam argues: "Logically speaking, if *no* attribute that we can think of can be literally or even analogically predicated of God, then performing actions cannot be predicated of God either" (Putnam 1997, p. 408). True, *but* Maimonides does not say that *no* attribute that we can think of can be literally or even analogically predicated of God. Maimonides does not in passages quoted by Putnam say there is a general problem with anthropomorphic God-talk. He writes in these passages against only what he would cast as 'intrinsic God-talk,' while explicitly welcoming at least some 'extrinsic anthropomorphic God-talk', which he considered to be *univocal* as applied to God and us. It seems that Maimonides' way of rehabilitating would-be attributions of intrinsic moral qualities could be adapted to the rehabilitation of would-be attributions of intrinsic nonmoral qualities such as omnipotence and omniscience, and that his 'negative theology' was not so radical after all. 'Job one' in the project would be to work out a serviceable metaphysics of qualities, intrinsic and extrinsic, and of subjects in which there is and is not multiplicity.

17. David Johnson may think so. Though he defines an 'Anselmian god' to be a necessary being that essentially has the perfections of Swinburne's 'contingent god' 'perfect being,' he uses the idea of an Anselmian god, in a demonstration not of the existence of an Anselmian god, but rather of Swinburne's 'contingent god' (Johnson 1999, p. 98).
18. Mackie's idea is that a simple subject predicate affirmative utterance that would ascribe a *bona fide* property to an object *presupposes*, before it can issue in a statement true or false, not only that its subject-term stands for an object, but that its predicate-term stands for a *bona fide* property. Here a '*bona fide* property' is a *possibly instantiated* property. (I come back to this idea in note 33 to Appendix C of Chapter IV.)

Chapter II

1. So says Hume in his posthumously published *Dialogues* (1777). It is of possible interest that he had spoken differently in his *Treatise* (1739): "Whatever we conceive, we conceive to be existent" (Hume 1888, p. 67). Hume *seems* to imply that, whatever we conceive as existent, we *cannot* also conceive as nonexistent! He had already written: "*That whatever the mind clearly conceives includes the idea of possible existence*" (p. 32). He *seems*, on p. 67, to say that whatever we conceive includes the idea of *actual* existence. His view, reiterated in the Appendix to the *Treatise*, is that we have no "idea of existence . . . separable from the idea of particular objects" (p. 623). That is, I take him to say, the idea of existence we have is an inseparable part of the idea of particular objects.
2. What Leibniz did say is that it "is the privilege of divinity alone" that *it is necessary if it is possible* (Leibnitz 1949, p. 504). That privilege can, however, be seen to come to the same thing for possible natures (i.e., possibly instantiated nature): For any possible nature, its possibility (i.e., its possible instantiation) entails its existence (i.e., its actual instantiation) if and only if its existence is necessary (i.e., it is necessarily instantiated). Leibniz thus implies that it is the privilege of divinity alone *amongst possible natures* that its existence is necessary. Cf., Adams 1987, pp. 203, 210. In symbols, $\Diamond(\exists x)Nx \supset [\Box[\Diamond(\exists x)Nx \supset (\exists x)Nx] \equiv \Box(\exists x)Nx]$. Left as an exercise is derivation of this principle in the system of Section B1 of Appendix B to the next chapter. For this exercise one may let sentence letter 'P' be short for '$(\exists x)Nx$'.
3. Hume may have had in mind Anselm's argument when he composed that speech for Cleanthes, even though 'on the table' was not an ontological but a 'cosmological' argument that would contend nondemonstratively for a necessary being. But this raises a problem with Cleanthes' speech. It opposes demonstrations or 'proofs *a priori*' of existence, though the argument on the table (it is quoted in its entirety in Section 1.2.1 of Chapter VI) is not such a proof. It uses a contingent knowable only *a posteriori* premise, namely, that some things that need not have existed ('particular objects that begin to exist in time') exist. Hume nowhere discusses ontological arguments as such. It is a good question why not. A possible answer is floated in 'a speculation' at the end of the next chapter.
4. Cf.: "We can form no idea of a mountain without a valley, and therefore regard it as impossible." (Hume 1888, p. 33) There is evidence that Hume

sometimes, when writing the *Treatise*, had in mind ontological arguments, sometimes Anselm's, sometimes Descartes's. As said, however, (previous note), Hume does not comment specifically on them. His relation to Spinoza's argument is in fact supportive, inadvertently one supposes (Section 3.4.2 below).

5. A second way to there being at most one perfect being uses the idea that a perfection (some) perfections consist in having *more* of something than anything else has. Cf.: "According to Ockham . . . [i]f 'God' is construed in the . . . sense [of 'something more noble . . . than anything else besides him'] there can only be one God. . . ." (Wainwright 1986, p. 305.) Another argument would use, (i), the premise that a perfect being would be a necessary being, (ii), an extension of the principle of the identity of indiscernibles from necessary things such as numbers, to necessary beings, and, (iii), the idea that a perfect being would have as attributes *only* perfections. Against this argument is that even essentially perfect beings could have 'accidental properties' that are not perfections, and differ in these (pp. 295–6).
6. Probably Descartes noticed this. In his Reply to Pierre Gassendi's objection, to contrast perfect bodies with perfect beings he writes, "let us now take a thing – whatever this thing turns out to be – which possesses all perfections **which can exist together**" (Descartes 1986, p. 101, quoted with context in Section 2.5.1).
7. I do not know that Leibniz did not have a better reason for believing it, for "[i]n January 1678 Leibniz wrote down an elaborate and interesting proof of the conditional [if God's existence is so much as possible, then God actually (and indeed necessarily) exists] (Leibniz 1923–, II, i, 390–1)" (Adams 1995, p. 389), and I have not examined that proof.
8. Similarly for Spinoza's argument discussed in Section 11. Something like the possibility problem is important for Anselm's argument discussed in Part Two, as well as for the argument of Plantinga and Hartshorne (next chapter) and Gödel's proof (the chapter after that).
9. Gaunilon supposed, however, that one could argue similarly for the existence, not of an island than which none greater can be conceived, but of an island "superior everywhere in abundance of riches to all those other lands that men inhabit . . . more excellent [greater] than all other lands" (Reply 6). This slip on Gaunilon's part allowed Anselm to stress in response the difference between being greater than all others that exist and of being than which none greater can be conceived. It is, however, curious in the extreme that Anselm did not acknowledge the possibility of Gaunilon's challenging with the idea of an island than which no greater island can be conceived. Anselm wrote: "Now, I truly promise that if anyone should discover for me existing either in reality or the mind alone – except 'that-than-which-a-greater-cannot-be-thought' – to which the logic of my argument would apply, then I shall find [it] and give it . . . to that person" (Defense 3). To which we can imagine Gaunilon, after having adjusted his objection as indicated, thinking, "Happy days!" Incidentally, though known as *Gaunilo's Objection* – see, for example, Plantinga (1974, p. 89) and Oppy (1995, p. 17) – the problem of the missing blessed isle was not his primary or deepest objection to Anselm's argument, for which objection see Section 3.6.
10. Problems with Penelhum's argument, other than the main one to be developed, are that both [i] and [ii] are false. Suppose I define a 'major number' to be a number that is an even power of a prime number. Examining the concept I

can, contrary to [i], see that it covers several numbers that actually exist, for example, 3^2, though existence is not contained in it, which is contrary to [ii]. But are numbers things? Yes, as is everything. And numbers that exist do actually exist. When speaking of the work of a mathematician, who is often occupied in quests of nonexistent numbers , it may be useful to say that he is, if he is, at last on the track of a number that actually exists. "Good luck to him!"

11. It is plausible that Hume would have said that there *can* be no harm in including existence in the idea of any object, because it *is* included in the idea of every object (see note 2), and certainly not everything we can think of exists. With God serving as an example, Hume writes, to make a general point about the nature of belief, that "'tis certain there is a great difference betwixt the simple conception of the existence of an object, and the belief of it" (Hume 1888, pp. 94–5).
12. But, Ian Hacking 'says,' if the strictures of type theory are well-grounded, it remains a mistake: "according to the [ramified] theory of types . . . the appropriate predicate of existence [for a kind] . . . [cannot be] analytically contained in [the] concept of [this kind]" (Hacking 1978, pp. 629–30). To say something of a kind exists is to predicate existence of this kind, not of things of this kind. The thought is that the existence predicate appropriate to a kind is of a higher-order than the predicates that define this kind. The thought is that to say that there is something of a kind is to predicate existence of this kind, of an order that cannot be predicated of things of this kind. I am not sure what to make of this. But it does seem that things of kinds can exist. And if this is so, then there should be no ban on including the kind of existence appropriate to them in the definition of existent things of this kind. Furthermore, is existence predicated of a kind when it is said that there exists something of this kind? William R. Stirton argues, No. He argues for "the incoherence of regarding 'exists' as anything other than a first-order" (Stirton, 1995, p. 37).
13. I noticed this in 1963 when preparing a lecture at U.C.L.A. The ambiguity of 'a' and its significance for ontological arguments is mentioned in Sobel (1977–8, 1983a) and referenced in Sobel (1987a, p. 259n2). Peter van Inwagen makes the point, and puts it to use in his deflation of Descartes's argument (van Inwagen 1994, pp. 80–2): "The ambiguity is rooted in two different functions performed by the indefinite article" (p. 80). Jerome Shaffer observes that if A's are defined as existing, then, 'A's exist' can express not only that there are A's, but also another proposition that does not entail that there are A's (Shaffer 1962, pp. 319ff.).
14. This amphiboly of 'a'-sentences is in some cases 'idle.' Is there a context in which 'a man is in the next room' can be taken to express a universal generalization? Are there contexts in which 'a man's got to do what a man's got to do' or 'a rat is a rodent' can be taken to express existential generalizations? The first and third of these sentences come from King (1988, pp. 417, 438n1). Also, this amphiboly does not exhaust possibilities for 'a'-sentences. In some contexts 'a man I met in Victoria Station said he knows you' would be taken to express neither kind of generalization. This sentence is adapted from Goldstein (2002, p. 32).
15. Letting the range of quantifiers be wider than the set of 'existents' as naturally understood allows it to be false that "[t]here are no non-existent objects" (Stirton 1999, p. 38). This is a premise of an argument opposed by Stirton that

would prove that "'exists' is [not] a first-level predicate" (Ibid.). He says of that premise that, while "almost everyone believes" it (p. 38), giving it up is "the more straightforward alternative" (p. 47), once one sees "that 'exists' . . . in [for example] 'Santa Claus exists' . . . is a first-order predicate" (p. 46).

16. A universal generalization $(x)(Fx \supset Gx)$ is true if and only if each of its instances is true. An instance '(Fa – Ga)' of this generalization is false, by the truth-table definition of $\supset$, exactly in case 'Fa' is true and 'Ga' false. Otherwise it is true. That means that if '$\sim(\exists x)Fx$' is true, so is '$(x)(Fx \supset Gx)$'.
17. Lines (v*) and (vi*) entail $\sim(\exists x)Sx$, which therefore follows necessarily from (ii*), $\sim(\exists x)(Sx \,\&\, Ex)$, given that (iii*) and (iv*) are necessary. But that is not to say that its necessity, $\Box\sim(\exists x)Sx$, follows from (ii*) given the necessities of (iii*) and (iv*). Suppose that it did. Then, since what follows from something given necessities, follows from it alone, it would be necessary that $\sim(\exists x)(Sx \,\&\, Ex) \supset \Box\sim(\exists x)Sx$. From that, given that $(\exists x)(Sx \,\&\, Ex)$ is redundantly equivalent to $(\exists x)Sx$, would follow the *Leibnizian Result* that a supremely perfect being exists, if its existence is possible, $\Diamond(\exists x)Sx \supset (\exists x)Sx$. Left as an exercise is a derivation in the system of Section B1 of Appendix B to the next chapter for the argument $[(\exists x)(Sx \,\&\, Ex) \equiv (\exists x)Sx]$. $[\sim(\exists x)Sx \,\&\, Ex) \supset \Box\sim(\exists x)\,Sx] \therefore [\Diamond(\exists x)Sx \supset (\exists x)Sx]$. For this exercise '$(\exists x)(Sx \,\&\, Ex)$' and '$(\exists x)Sx$' may be abbreviated by 'P' and 'Q', respectively.
18. I do not say that Descartes has no reasons for the necessity of P_2. Ideas that *may* be implicit in his Fifth Meditation, and in his replies to objections made by Gassendi, ideas that could move his argument 'to another place,' are considered in the next chapter. He 'distinguishes' between necessary existence that would be a property only of God and the existence of, say, a particular physical triangle that would be only actual existence (Descartes 1986, p. 97; cf., pp. 100–102).
19. "Jacques Sans-culotte bowed again. 'Once more I congratulate milord. He is the only Englishman I have ever met who is capable of appreciating our beautiful language. I will pay great attention in future to the article in question'" ("The Entertaining Episode of the Article in Question," in Dorothy Sayers, *Lord Peter Views the Body*, London: Gollancz, 1928, p. 38).
20. His other proofs for Proposition 11 are 'cosmological.' The first uses a universal principle of causes or reasons, as does Demea's in Hume's *Dialogues*, Part 9. The second does not use such a principle and is in this respect like Samuel Clarke's argument for his Proposition 2. We get to the arguments of Demea and Clarke in Chapter V.
21. R. H. M. Elwes also translated Propositions 11 and 7 without articles on 'substance'. Spinoza (1955) contains Elwes's translation that was first published in 1901.
22. "*There are only three possible ways of proving the existence of God by means of speculative reason*. All the paths leading to this goal begin either . . . ; or . . . ; or finally they abstract from all experience, and argue completely *a priori*, from mere concepts, to the existence of a supreme cause . . . , the third [proof is] the *ontological*" (CrPR A590/B618–A591/B619, pp. 500–1).
23. Cf.: "The orthodox translation . . . is 'or.' Spinoza nearly always uses ['*sive*'] to indicate ***an alternative expression***. . . . But the English 'or' is frequently disjunctive. . . . So the unvarying translation . . . by 'or' can be quite misleading. I have therefore usually translated it by 'that is'. . . ." (Shirley 1982, p. 24; bold

emphasis added.) *Sive*: "As a simple disjunctive . . . [i]ntroducing [not an expression for an alternative but] an alternative [expression from something, often one] which is preferred, *or rather, or more accurately, or as I should say*" (*An Elementary Latin Dictionary 1890*.) My friend Wlodek tells me he was taught to translate '*sive*' with 'i.e.'

24. Shifting to words appropriate to Descartes's Fifth Meditation (in translations from his Latin text), the phrases 'a god, that is, a supremely perfect being' and "'a God . . . , that is, . . . a being supremely perfect'" (Descartes 1951, p. 136) are, like the phrase 'God, that is, the supremely perfect being,' grammatical. So are the phrases "'God, that is, . . . a supremely perfect being'" (Descartes 1979, p. 42) and "'God, that is to say, . . . a supremely perfect being'" (Descartes 1969, Volume I, p. 180) are grammatical. *However*, 'that is' and 'that is to say' do not in these phrases have the sense of 'or in other words,' and so there is a question whether they can in these phrases translate '*sive*.'
25. There is on the one hand Spinoza's *argument*. It consists of premises and a conclusion that is supposed to follow from them. And then there is the *reasoning* by which he purports to reach that conclusion from those premises. From now on I will distinguish these in these terms.
26. The demonstration of Proposition 7 cites the Corollary of Proposition 6, the demonstration of which cites Proposition 6, **Axiom 1**, and **Definitions of 3 and 5**. Spinoza, in contrast with Descartes, *toiled* to make existence part of the nature that would be God's. The demonstration of Proposition 6 cites Propositions 2, 3, and 5. The demonstration of Proposition 2 cites Definition 3. The demonstration of Proposition 3 cites **Axioms 4 and 5**. The demonstration of Proposition 5 cites Propositions 1 and 4, Definition 3, and **Axiom 6**. The demonstration of Proposition 1 cites Definitions 3 and 5. The demonstration of Proposition 4 cites Axiom 1 and **Definition 4** in addition to Definitions 3 and 5. In the *Ethics*, Proposition 11 is derived from Axioms 1, 4, 5, and 6; Definitions 3, 4, and 5, along with **Axiom 7**, and, though it is not cited, **Definition 6** of God. My present subject is precisely the first demonstration of Proposition 11 from Axiom 7 and Proposition 7, especially the 'logic' of it, which is somewhat involved and interesting. In this it is like the demonstration of Proposition 5 and unlike the demonstrations of Propositions 2, 3, 6, 6C, and 7, the 'logics' of which are relatively straightforward and uninteresting.
27. Cf.: "Existence is contained in the idea or concept of everything, because we can conceive nothing except as existent" (Descartes 1969, Volume II, p. 57). This line is from Axiom X of René Descartes's arguments in a geometric fashion for the existence of God that are appended to his replies to the Second Objections to his *Meditations*. That axiom adds "that possible or contingent existence is contained in the concept of a limited thing . . . necessary and perfect existence in the concept of a supremely perfect being."
28. Descartes's 'demonstration of the existence of God in a geometric fashion' (Descartes 1969, Volume II, p. 57), in contrast with his ontological reasoning in the Fifth Meditation, resembles Spinoza's ontological reasoning in that it too features necessary existence. Unlike Spinoza's reasoning, however, Descartes's, while it explicitly concludes that God necessarily exists, finally also concludes *explicitly* that He exists: "Hence it is true to affirm that necessary existence

exists in Him, **or that God Himself exists**" (p. 57; bold emphasis added). "Hence it may with truth be said that necessary existence is in God, *or that God exists*" (Descartes 1951, p. 264; bold emphasis added).

There is, to resume the subject of several notes, a problem with these translations, since what follows 'or,' which translates Descartes's '*sive*,' are not other words for the sense of the words that precede it. Nor can '*sive*' express here a disjunction of things it is true to say. Why not? Because Descartes clearly means to conclude, for one definite thing, that necessary existence is in God. One solution to the problem of '*sive*' here is (i) to 'give' Descartes an English word or two, for the conclusion *that it is true to affirm that necessary existence exists in God or*, *equivalently*, *that God* necessarily *exists*, and (ii) to make explicit the further clearly intended 'come-down' conclusion *that God exists*. Another solution is to let '*sive*' be translated here by 'that is,' not in the sense of 'in other words,' but in the sense of '*which is to say*' or '*which implies*.' Query: Is this a sense that '*sive*' sometimes properly has, and so might properly have here?

29. It contrasts with the not unusual exercise of assembling the best argument one can from materials found in an ontological arguer's text and then commenting on this best-one-can-make-of-it argument. Jan Berg (1961) and David Lewis (1983) offer to do that for Anselm. They do not claim to come up with his argument as he saw it, nor do they go into his reasoning for his argument.
30. Spinoza died in 1677. The *Ethics*, published posthumously, does not provide evidence that Leibniz had persuaded him that the ontological reasoning requires a positive demonstration of the possibility of a perfect being. Spinoza would have said that God is not possible only if what would be His nature involves a contradiction. In place of argument, however, to show that there is no contradiction in the nature of a Being absolutely infinite and supremely perfect, he merely says, in his second proof of Proposition XI, that it is absurd to affirm otherwise.
31. In the simplest and soundest argument that David Lewis can find in *Proslogion* II, "there [is] . . . no defining of anything" (Lewis 1983, p. 11). Of the classical arguments, I think that only Descartes's can be fairly charged with pretending to define a being, specifically, a supremely perfect being, into existence, though I suppose that loosely speaking the charge can be (for it is) broadcast.
32. Brackets can be added thus: '{¬x(Ix & Sx)}K[¬x(Ix & Sx)]'. This is often done below for similarly placed symbolic definite descriptions.
33. For

DfGod 'God' shall be short for 'the infinite substance',

we could have

DfGod 'G' = df '¬x(Ix & Sx),

meaning that the name letter G and the Fregean descriptive name ¬x(Ix & Sx) are interchangeable during derivations in the system used for derivations in Appendix B. This, however, would impose on this name letter the anomalies of this descriptive name (for which see the *Description Calculus* of Kalish, Montague, and Mar 1980, Chapter VI). A license to interchange italicized '*G*' and the Russellian description '¬x(Ix & Sx)' could work as well in these

derivations, while staying away from those anomalies. But this interchange could take place only in a formal language in which '*G*' is, one might say, a 'pseudoname letter' fit to abbreviate that 'pseudoname,' a formal language in which, like this description, 'G' was an 'incomplete symbol' in the sense of *Principia Mathematica* (pseudoname letters would not have 'meanings in isolation,' they would not have denotations in models) and in the sense that '*G*' would enter sentences in company with the scope indicator '{*G*}'. This Russellian treatment of the ordinary name 'God' would have occurrences of '{G}' in symbolizations of the conclusion, (6), of Spinoza's reasoning.

There is something to be said for such a Russellian treatment in a formal theory for some uses of ordinary proper names. For "there are, in fact, ways of using proper names to indicate that they should take narrow scope" (Sosa 2001, p. 23; emphasis omitted). Consider a use of 'if there is nothing that is a proper object of obeisance and worship by humans, then God is not *God*.' Stressing the second use of 'God' could indicate that were the description 'the proper object of obeisance and worship' used instead it would take narrow scope. This example argues that 'God' is not always *simply* a designator. Sosa's example argues that names are not always used as *rigid* designators: "One might say, 'you know, if it hadn't been for Plato, Aristotle would never have become *Aristotle* [the last great philosopher of antiquity]" (Ibid.).

34. The scope of the Russelian description '$\neg$x(Ix & Sx)' is narrow to meet the declared intention of the previous section that (4) express a conditional. The wide-scope sentence '{$\neg$x(Ix & Sx)}((∃x)(Ix & Sx) ⊃ [G = $\neg$x(Ix & Sx)])' does not do that. It is, indeed, equivalent to the categorical sentence '{$\neg$x(Ix & Sx)} [G = $\neg$x(Ix & Sx)]' that (4) is designed to get away from.
35. The exact syntax of (6) is important. The similar sentence, 'The infinite substance God necessarily exists', wants to be symbolized not by a conjunction, but by '{$\neg$x[(Ix & Sx) **Λ** x = G]}X($\neg$x[(Ix & Sx) & x = G])' which is equivalent to I(G) & S(G) & X(G)', and does not entail $(\exists_y)(x)$ [(Ix & Sx) ↔ x = y].' Compare 'the infinite substance God' with 'the Canadian philosopher Ian Hacking' (of whom we are very proud!!).
36. Lewis maintains that the simplest and soundest argument to be found in Anselm's *Proslogion* II (Lewis 1983, p. 11) is not modal (*passim*). It is in his view first-order monadic predicate logic without identity.
37. The case against the symbolic argument that comes by replacing (4′) by DfGod 'G' = df '$\neg$x(Ix & Sx)' is simpler. For DfGod is merely a license to perform certain interchanges of symbols in the course of derivations. It is not a premise of the argument that results when it 'replaces' premise (4′).
38. The possibility used to embarrass my economical symbolizations of Spinoza's argument can be used in a case for the invalidity of the 'bells and whistles' symbolization indicated in the previous section. One such case uses, along with the possibility described, the facts that (i) it is false that ⊗∼ *E!*0, zero *cannot* be conceived not to exist, that is, it is true that ∼⊗∼ ***E!*0**, and that (ii) it is true that □ *E!*0, zero necessarily exists, and so equivalently that ∼◊∼ ***E!*0**. Any adequate semantics for a theory for both conceivability and logical modalities would need to accommodate objects whose nonexistence is neither conceivable nor possible.

39. Sentences such as (3) can be 'disamphibolized' by judicious emphasis. 'Disamphibolizing' the sentence '*the* frog in my pond is *not* green' contrasts with 'the frog in my pond is not green'. It is not part of the *sense* of the former that there is a green frog in my pond, but only something deniable that its use implies unless denied (that is, this is only something that is 'Gricean implied'). "For some reason unknown to me, many philosophers have denied the existence of this distinction between wide and narrow negation" (Stirton 1995, p. 47). This is strange given the very difficult to explain away evidence for it in English. I have sampled this evidence and am assembling more. My account of the ease with which one can, in an argumentative context, slide from one scope for negation and correspondingly for a definite description to the other, says that this distinction can be elusive, which entails that it does exist.
40. Putting in place of (4′) the mere license DfGod 'G' = df '¬x(Ix & Sx)' to interchange symbolic names simplifies the case, for then the present point is that (3a) together with (5) do not entail (6).
41. Points (4**) and (4**′) are not equivalent in free logic; only (4**′) makes with (1′), (2′), and (5′) premises that entail (6′).
42. It is not unlikely that, on the occasion of his visit in 1676, Leibniz was familiar with Spinoza's primary demonstration of Proposition 11 and considered it valid.
43. Oppy has it but chooses not to use it in his book: "I do not think that anything would have been added to the discussion that I give by the formalization of the arguments [I discuss]" (Oppy 1995, p. 3). Perhaps not. Though formalization might have discovered that a way he recasts an argument that "Sobel 1983: 197 claims . . . is marred by an equivocation in its use of indefinite descriptions" (p. 290), does not get entirely away from that equivocation. He writes that "Kordig's argument . . . can be recast as follows: '[1] A (unique) perfect being would be deontically perfect. (Hence) [2] It ought to be the case that there is a (unique) perfect being. Hence [3] It is logically possible that there is a (unique) most perfect being. [4] It is the case that there is a (unique) most perfect being'" (p. 291).

    Oppy's sentence for [1] may be ambiguous between a universal and an existential interpretation, though I think it is not and that it has only a subjunctively universal interpretation the 'predicate' of which is '*is* deontically perfect'. But even if the sentence for [1] is in that way ambiguous, it must, for the argument intended by Oppy, be taken to be a universal generalization. Taken as an existential generalization, [1] would 'incorporate' [4] more or less as a conjunct: it would say *inter alia* that there is a (unique) perfect being. (I assume that Oppy's 'most perfect' is long for 'perfect'.) For a similar reason [1] must be taken to be without 'possible existential' import, it must not presuppose the possible instantiation of its 'subject', on pain otherwise of *sub voce* incorporating *inter alia* [3].

    The inference from [1] to [2], which latter sentence presumably affirms the deontic necessity of an existential generalization, is, therefore, to understate, not immediate. It could, however, seem to be valid. One gathers that it has seemed to Oppy to be valid. For there are the following other words for [2],

    (S2′) 'It ought to be the case that a (unique) perfect being exists.'

'in the vicinity' of which are the words,

(S2″) '*A* (unique) perfect being ought to exist.'

'a' is ambiguous in (S2″) between 'any' and 'at least one': (S2″) can be used to express either [2U] – Any (unique) perfect being ought to exist – where this is a universal generalization 'without possible existential import', or [2E] – At least one thing is such that it is a (unique) perfect being, and it ought to exist. [2U] does follow from [1] given the plausible stipulation that any deontically perfect being ought to exist. But then [2U] does not entail [3]. On the other hand, though [2E] entails [3] 'twice over', there are no *good* reasons in sight for thinking that it is entailed by [1]. Incidentally, while the words of (S2′) *are* other words for,

[2] It ought to be the case that there is a (unique) perfect being.,

the words of (S2″) seem *not* to be. There is little, if any, reason to think of [2] and [2U] that either entails the other. Similarly, when one thinks about them, for [2] and [2E]. These comments on Oppy's text might be helped by some symbols. I have followed his lead and eschewed symbols.

44. Using the abbreviation, G: nothing greater than a can be conceived, Berg distinguishes between ∼(G¬xGx), which is in the classical notation, ∼ {¬xGx}G¬xFx, and (∼G)¬xGx, which is in the classical notation, {¬xGx}∼G¬xGx.
45. For example, ∼ *E!* ¬αϕ comes from both ∼ {¬αϕ} *E!* ¬αϕ and {¬αϕ} ∼ *E!* ¬αϕ by deletion of {¬αϕ}. It is only by the narrowest-scope convention of *Principia Mathematica* for deleting scope-indicators that in that work it would be short for ('incomplete' for) specifically the first of these formulas.
46. I think that for Anselm things that 'exist in the mind' comprised *absolutely everything* and would nowadays be said to be things that exist in one possible world or another, that is, all possible things. It is likely that Anselm held that everything that 'exists in the mind' *actually* exists in *a* mind, namely, God's. It is likely that he subscribed to a 'theo-modal realism' according to which possibilities are actual in the mind of God. But this is separate from the position that there is nothing that is not 'in *the* mind,' which is simply a mentalistic way of saying that 'quantifiers are to range over all possible things.' Anselm's likely extra thesis of theo-modal realism might be compared with the modal realism of David Lewis, according to which every possible thing is actually in a possible world that is 'as real as,' since it is the same sort of thing as, this world of ours in which we live and breathe. My modal realism, explained in Appendix A to the next chapter, is different from that. Anselm's likely extra thesis of theo-modal realism might also be considered in relation to George Berkeley's 'master argument' to show that nothing such as a tree can exist "by itself, independent of, and unperceived by, any mind whatsoever" (Berkeley 1965, p. 164), discussed in Sobel (1991b).
47. His argument does not commit Anselm to the view that *whatever* exists in the mind can be thought to exist also in reality. He needs to maintain only that this is so for the something-than-which-nothing-greater-can-be-thought of which he is speaking. He could, without compromising his argument, say that something-than-which-nothing-in-reality-that-is-lesser-can-be-thought can exist only in the mind, since there is not a 'least possible real thing.' He could say

the same of (to coin a word) a *pictoral* where that is, by definition, an realized picture.

48. Gellman stresses that in the third stage Anselm *argues* that this being is God. This is offered as evidence that Anselm did not always consider 'God' and 'that than which none greater can be conceived' to be 'semantically equivalent.' Gellman thinks, however, that there is evidence that Anselm did sometimes consider them equivalent. Gellman cites two passages. First, Anselm promises in his preface a proof of God's existence. Gellman thinks this can only be the proof in *Proslogion* II of the existence of a being than which none greater can be conceived. Second, according to Gellman, Anselm says even the Fool must agree that that than which nothing greater can be conceived exists in his understanding, because this Fool has said there is no God. These passages are I think consistent with Anselm's sometimes view that 'God' and 'that than which nothing greater can be conceived' are not semantically equivalent. First, the promised proof of God's existence can be composed of arguments in *Proslogion* II and III. Its object, expressed metalinguistically, could be said to be to show that the ancient referential equivalents of the name 'God' did not, in what were to be initial acts of naming go astray, that referential chains reaching back to what are supposed to have been initial acts or acts of *bona fide* naming do not "lead back to nowhere" (Gellman 1995, p. 543n2). Second, Anselm does not say that the Fool must agree that he has that being in his understanding, because he has said there is no God. Anselm says that "surely, when this same Fool hears what I am speaking about . . . he understands what he hears, and what he understands is in his mind" (*Proslogion* II). Anselm is saying that this Fool surely understands 'something-than-which-nothing-greater-can-be- thought'.
49. One might prefer a subsidiary argument for (6) from (4) and (5) that 'separates cases' under the disjunction, either j exists in reality or j does not exist in reality. The subsidiary argument I have given is closer to the text of *Proslogion* II.
50. Having proved that (x)[Gx ⊃ (Mx & Rx)] for Anselm in the previous section, there was thus, using that result, a short way from the premise that (∃x)(Gx & Mx) to the conclusion (∃x)[Gx & (Mx & Rx)]. The reader may wish to construct a derivation in the system of Section B3 of Appendix B of the next chapter for the argument, □(x)[Gx ⊃ (Mx & Rx)] ∴ □((∃x)(Gx & Mx) ≡ (∃x)[Gx & (Mx & Rx)]).
51. It makes no difference to the critique of Part One whether existence or necessary existence is included in the concept of a perfect being. In Part One I consider the simpler case of bare existence. Van Inwagen, while saying that it makes no difference (van Inwagen 1994, p. 81), in deference to those who object to the stipulation that existence is a perfection, critiques the case of necessary existence.
52. Construction of a derivation in the system of Section B.3 of Appendix B of the next chapter is left as an exercise.
53. I have implied that 'it is not the case that the frog is green' is amphibolous in the manner of 'the frog is not green' and now say the same of 'it is not the case that the frog is such that it is green'. I think that, in contrast, 'the frog is such that it is not green' is an unamphibolous translation of '{¬xFx}∼G¬xfx'.
54. The system of proof is that of (Kalish et al.). Associated with that elegant text is my *Words and Symbols, Proofs and Invalidations*. This material for students is linked to my home page – http://www.scar.utoronto.ca/∼sobel .

55. Generalizations (1), (2), and (5) have been symbolized 'without existential import.' The invalidity of the argument's symbolization does not depend on that. Let existential import be added thus: (1*) (∃x)Kx & (x)(Kx ⊃ ~ Vx); (2*) (∃x)Sx & (x)(Sx ⊃ Vx); and (5*) (∃x)~Kx & (x)[~Kx ⊃ X(x)]. The argument – (1*), (2*), (4), (5*) ∴ (6′) – is invalidated by the model,

U: {0,1}		
K: {0}	V: {1}	X: {1}
S: {1}	I: {}	G: 0

56. The logic is an extension for Russellian descriptions of the Fregean Descriptions Calculus of Kalish et al., Chapter 6. It is developed in Chapter VIII, "Russell's Theory of Descriptions," of my *Words and Symbols*, which is linked to my home page – http://www.scar.utoronto.ca/~sobel/.

Chapter III

1. Such an argument is outlined in the first chapter of Anselm's reply to Gaunilon's objections. "If 'that-than-which-a-greater-cannot-be-thought' is neither understood nor thought of, and is neither in the mind nor in thought, then it is evident that *either* God is not that-than-which-a-greater-cannot-be-thought *or* is not understood nor thought of, and is not in the mind nor in thought. Now my strongest argument that this is false is to appeal to your faith and to your conscience. Therefore 'that-than-which-a-greater-cannot-be-thought' is truly understood and thought and is in the mind and in thought" (Anselm 1965, p. 169). Anselm says that "God [is] this being" (op. cit., p. 119, *Proslogion* III), on the ground that it is absurd that a creature should be able to think of something greater than its creator. (In fact there is no absurdity in that.)
2. The sentence '□(x)(Px ⊃ □E!x)', which could recommend itself as a symbolization of 'A being . . . [is perfect] in a . . . possible world . . . only if [it] *exists in every possible world*' is not strong enough for this entailment. Derivation of '(∃x)(□E!x & ◇Px)' from '◇(∃x)Px' and '□(x)□(Px ⊃ □E!x)' in the system of Section B3 of Appendix B is left as a nontrivial exercise.
3. Hartshorne, following John Findlay, 'gives' this to Anselm: It is not an idea that can be found full-blown and explicit in the *Proslogion*. Closest is the first sentence of *Proslogion* III, "And certainly this being 'that-than-which-a-greater-cannot-be-thought so truly exists that it cannot be even thought not to exist." This sentence says in other words that this being *exists necessarily*, perhaps 'meaning' that this being exists necessarily 'as described' and is *necessarily perfect*.
4. Confirmation by derivations is left for readers who have constructed the derivation left as an exercise in note 2.
5. The simpler sentence '□(x)(Px ⊃ E!x)' also has the free translation 'perfection is existence entailing'. Hardly anything I say here of '□(x)□(Px ⊃ E!x)' holds of this weaker sentence '□(x)(Px ⊃ E!x)'. Cf., note 2.
6. A slogan of the first Clinton presidential campaign added 'stupid!' to 'It's the economy'.
7. Van Inwagen says that "we cannot consistently adopt" the juridical principle. "'A person is to be presumed innocent of a charge till proved guilty'" (1994, p. 92).

In fact, however, this principle is the law, and its implementation does not lead to inconsistencies. The rule requires that each of Alice and Bertram be found not guilty if neither can be proved to have done the crime, even in circumstances in which it is known that one or the other did it, but there is no inconsistency in that. For findings by courts, *juridical acts* – for example, findings of not guilty in law for Alice and Bertram – are not, do not entail, and cannot be inconsistent with *acts of assertion* such as the judgment that either Alice or Bertram did the crime.

8. Left as exercises are derivations in the system of Section B3 of Appendix B that ***PERFnexEx&necPerf*** and ***PrfExEntlng*** entail equivalences of the strong necessary existence assertion $\Box(\exists x)\Box(E!x \,\&\, Px)$ with, respectively, possibility postulates ***IP*** and ***StrgIP***.
9. Which very point? I count three of possible relevance: (1) that the concept of a being a greater than which cannot be conceived is not self-contradictory; (2) that the words 'a being a greater than which cannot be conceived' are meaningful; and (3) that a being a greater than which cannot be conceived exists in the mind or is possible. Gaunilon was not prepared to concede (3). He conceded (2). He did not question (1).
10. This seems to have been Plantinga's view in 1974. He considers whether the possibility of a greatest possible being is challenged in the way he thinks the possibility of a greatest possible island is by "the idea of a greatest possible island . . . [being *prima facie*] inconsistent" (Plantinga 1974a, p. 91). He defends *the possibility* of a greatest possible being, by defending *the consistency of the idea* of a greatest possible being. The suggestion is that establishing this consistency (which would involve solving the problems of maxima for love and manifestations of love), would establish that possibility.
11. "But is not the concept of a *magican* strange in the way it makes the existence of a world of a **magican** depend on conditions in other worlds?" Perhaps, but not stranger than the concept of an *essentially perfect necessary existent*, the existence of which at a world depends *much more* on conditions in other worlds.
12. Though this equivalence is necessary and true in every world, it is I think *expressible* and *knowable* only in our world. In the explanation of our idea of a *dragoon* there is a definite reference to @, this world of ours in which we live and breathe. Denizens of other worlds could in them refer similarly to them, but not to ours. They could not have the idea of a *dragoon* that we now have, but only the idea of a kind that was similarly related to their world. Cf., Cartwright (1998, p. 77).
13. Those who prefer not to include existence in ideas of kinds can make Rowe's point by starting with a kind K such that (i) K is not instantiated and (ii) such that (i) is not demonstrable *a priori*. Almost everyone supposes that dragons, unicorns, and ghosts are such kinds. Though there are presumably no such things, no contradictions flow merely in virtue of ideas and meanings from the supposition that there not only are such things. The next thing is to form the idea of kind *K*, of a necessarily existent K: A thing is a *K* at a possible world if and only if it exists in every possible world, and is a K in @, this world of ours. And there you have it, the idea of a kind *K* that is, by (ii), *a priori* self-consistent, though, by (i), things of this kind *K* are *impossible*.

For the general point that '*a priori* possibility' of a proposition does not entail logical possibility, let ϕ express a contingent proposition p that is false in @, and let '[A]' abbreviate 'it is true at @ that'. Now consider the proposition p′ expressed by [A]ϕ . This proposition is '*a priori* possible': It is no more knowable *a priori* that p′ is false than it is so knowable that contingent proposition p is false. However, since p is false in @, p′ is false at every world and is not logically possible. Connectedly, and this time without dependence on the assumption that p is false at the actual world, the following express in all worlds valid arguments: ◊[A]ϕ ∴ □[A]ϕ, and ◊[A]ϕ ∴ [A]ϕ (Cf., Schellenberg 1993, p. 9n11). That what is '*a priori* possible' need not be possible (!) is why I have used scare quotes. The words '*a priori* possible' are unfortunate words for what 'not *a priori* impossible' are better words. The words '*a priori* necessary', since what is *a priori* necessary must be necessary, are good words.

14. Theorems of the system of Section B1 of Appendix B are □(Q) ⊃ □(P ⊃ Q) and [□(P ⊃ Q) & □(P ⊃ ∼Q)] ≡ □∼P.
15. Yablo's 'conceiving,' this conjuring of an appearance of possibility, is not the same as and I suspect does not include appreciating that something is 'coherently conceivable' (Yablo 1999, p. 457). That sounds like appreciating that there is no *a priori* inconsistency in ideas of the thing. James van Cleve contrasts 'strong conceiving' with 'weak conceiving.' As one can 'just see' that some things are true, for example, "that nothing is both round and square," so one can 'just see' that some are possible (van Cleve 1983, p. 37, quoted in Tidman 1994). Van Cleve's example is that there are creatures with eyes but not ears. This 'just seeing possible' would be at one end of a continuum of 'Yablo-conceiving,' at the other end of which was the barest semblance of possibility.
16. Thanks to Margaret Cameron for putting me on to this valuable essay.
17. Tidman would say that Yablo-conceiving 'triggers dispositions to believe in possibilities' and that it is this, not the intrinsic quality of the conceiving/picturing, that "gives us reason to think something possible" (Tidman 1994, p. 308).
18. "The central question . . . is why . . . *any connection at all* between what we can or cannot conceive . . . and what is possible. These seem to be two entirely different subject matters, one having to do with . . . minds, the other with . . . how things could be" (Tidman 1994, p. 306). How could "we . . . understand [this] . . . must detecting faculty" (Blackburn 1986, p. 119)? There are two gaps to be closed. First, a theory of this faculty would explain why 'appearances of possibilities' that we experience figure to be *a priori* consistent. Second, and more difficult, it would explain how things and activities of which we have *a priori* consistent conceptions and imaginings figure to be possible.
19. The rationality of theism is defended against "the *evidentialist* objection . . . that none of the theistic arguments . . . is successful" (Plantinga 1991, p. 1). It is found that the theist "has an easy time explaining the notion of our cognitive equipment's functioning properly" in a manner that gives the result that it can be "rational to believe in God without evidential support of other propositions" (p. 8). Plantinga does not mention in this popular Internet article that one of the theistic arguments actually establishes that it is rational to believe in God, or that this is what he used to think.
20. My modal realism may have a certain advantage over Lewis's. Hud Hudson argues that it is a consequence of Lewis's theory that it is itself necessarily

false, since there can be no Lewis-world at which it is true that there is not only that Lewis-world, but another Lewis-world. My theory does not have an analogous consequence: As the number 3 exists in every possible world, so does every one of my possible worlds. Hudson implies that Lewis has agreed in private correspondence that his theory has the awkward consequence of 'saying' that though it is true, it is necessarily false!! (Hudson 1997, p. 79n3). However, Hudson also mentions an easy way out for Lewis (p. 80n4). Let 'there are' in 'there are many possible worlds' be an unrestricted quantifier with the range what I term in the next appendix 'the universal domain', and not what I term this or that 'world domain.'

Hudson's interest in the possibility that Lewis's theory is according to itself necessarily false is that it implies that there can be a *true* proposition that is neither necessary (true at *all* worlds) nor contingent (true at *some but not all* worlds). The theory, he argues, may say that it is itself is such a proposition. And if there can be such propositions, then an argument to the effect that 'the principle of sufficient reason' implies that there are no contingent propositions does not go through (Section 7.2 of Chapter VI 'dresses' this argument of Hudson's). Having indicated alternative elaborations of Lewis's theory, one of which does not have that outrageous consequence, he continues: "Which is the right [elaboration that results in a true extreme modal realism]? The overwhelmingly popular answer seems to be 'Neither!' – There is no true, reductive analysis of modality [such as extreme modal realism would be]" (Hudson 1999, p. 91). My modal realism unelaborated does not have that outrageous consequence and, without pronouncing on the reality of other worlds such as this one of ours, is neither 'extreme' nor 'reductive'.

21. N is not right for 'deontic' necessities, for example, the 'must' of 'you must not do that', which have to do with what ought to be true.
22. Robert Merrihew Adams 'finds' Hartshorne's ***AP*** in the first chapter of Anselm's reply to Gaunilon. But for this 'finding' of Adams he assumes that "[necessarily] if such a being exists . . . its existence is logically necessary" (Adams 1987, p. 231) is fairly symbolized by ***AP***. In fact (Plantinga's point, Section 3) one wants for ***AP*** something like 'necessarily, if such a being exists, the existence of *such* a being is logically necessary'.
23. Neither of the derivations to come 'track' Hartshorne deduction (Hartshorne 1962, p. 51). The reader may wish to constrtuct an unabbreviated derivation in the present system that does that. This unabbreviated derivation will feature for each of the lines in Hartshorne's derivation, other than the lines for ***AP*** and ***IP***, a subsidiary derivation. Here are Hartshorne's lines, in our notation, with 'Q' abbreviating '(∃x)Px': '(1) $\Box(Q \supset \Box Q)$, ***AP***; (2) $\Box Q \vee \sim \Box Q$, excluded middle; (3) $\Box(\sim\Box Q \supset \Box\sim\Box Q)$, modal status is always necessary; (4) $\Box Q \vee \Box\sim\Box Q$, from 2 and 3; (5) $\Box(\Box\sim\Box Q \supset \Box\sim Q)$, from 1; (6) $\Box Q \vee \Box\sim Q$, from 4 and 5; (7) $\Diamond Q$, ***IP***; (8) $\Box Q$, from 6 and 7; (9) $\Box(\Box Q \supset Q)$, modal axiom; and (10) Q, from 8 and 9.
24. Appendix C of the previous chapter contains both standard rules and procedures, and free ones for nonmodal quantified logic with identity.
25. An equivalent procedure allows, for free universal proof, the assumption $E!\alpha$ and makes ϕ sufficient for the proof's completion. This is the procedure in Salmon (1994).

Chapter IV

* Revised and expanded from Sobel (1987a).

1. The three pages that are in Scott's hand, and two in Gödel's, are held in Gödel's 'nachlass' by the Institute for Advanced Study, Princeton, New Jersey. Gödel discussed his proof with Scott in February 1970. Scott presented his own notes to his seminar on entailment sometime during the following academic year. Coming references, unless otherwise indicated, are to Scott's notes.
2. "In 1972 Gödel told me that his study of Leibniz had had no influence on his own work except in the case of the ontological proof, of which Dana Scott had a copy" (Wang 1996, p. 113).
3. John Dawson retails Morgenstern's implied line (Dawson 1997, p. 237, cf., p. 165) but reverses it in his last comment on the subject: "The force of Darwin's observations and deductions caused him to renounce his religious faith. . . . Gödel, however, remained steadfast. . . . He was not only a theist, but . . . attempted to give a formal justification of the ontological argument" (p. 266).
4. Robert Merrihew Adams takes the words 'simpler proof' to refer, not to 'simpler' grounds for axioms in the note *Ontologischer Beweis*, but to a *different* proof for the possibility of a being that has every 'positive' property than the proof given in this note. "Gödel adds that 'this interpretation' supports a 'simpler proof', but he does not give the proof" (Adams 1995, p. 398). Adams suggests that Gödel was alluding to a proof that proceeded from somewhat different axioms. That strikes me as unlikely. If it were so, I think that Gödel would have recorded, perhaps in a note, some indication of those axioms and the simplification they would enable.
5. David Johnson glosses 'positive' with "morally or aesthetically wonderful, with no morally or aesthetically negative aspect" (Johnson 1999, p. 99; 2002, p. 181) and with confidence that I cannot match finds that "the property of being an Anselmian god is, in a moral or aesthetic sense, positive. (Indeed, what property could be more morally or aesthetically wonderful, and without negative aspect, than the property of *being a god in every possible state of affairs*" (Ibid.), where "a *god* . . . is 'a person without a body (i.e. a spirit), present everywhere, the creator and sustainer of the universe, a free agent, able to do everything (i.e. omnipotent), knowing all things, perfectly good, a source of moral obligation, immutable, eternal . . . [*lacuna* original] holy, and worthy of worship" (1999, pp. 98, 180).
6. As in Scott's notes. Hao Wang elaborates the proof in Gödel's notes. A difference is that in Gödel's but not Scott's there is reliance on the identity of logically equivalent properties: "[I]f a system S of pos. props. were incompatible, it would mean that the sum prop. s (which is positive) would be $x \neq x$" (page 2 of Gödel's notes). Wang implies, perhaps inadvertently, that it is an unstated axiom that "there must be some positive property" (Wang 1996, p. 115), "for the whole enterprise to make sense" (Ibid.). In fact, that $\exists\phi P(\phi)$ is entailed by Gödel's (first) Ax 2, $P(\phi) \vee (P(\sim\phi)$.
7. Cf.: David Johnson's idea of "properties [that] are, in a moral or aesthetic sense, positive" as properties that are "morally or aesthetically wonderful, *with no morally or aesthetically* ***negative aspect***" (Johnson 2002, p. 181; emphasis added). To be without a 'negative aspect' is, however, not the same as being without a 'nonpositive aspect'.

8. Axiom 2 is problematic for 'religious or spiritual positiveness'. Consider that the conjunctive property (omniscience and perfect goodness) would tend to make a thing worthy of worship, while thinking that the property of omniscience alone has no such tendency, since an Evil One could be omniscient without tending thereby to be worthy of worship. Also that the conjunctive property of (*unfailingly doing good & being capable of doing bad*) would *contribute* to worshipfulness, though *being capable of doing bad* would *not contribute* to worshipfulness but be only necessary for it.
9. Instead of this 'biconditional definition' there could be the identity

$$G = \hat{x}[\forall\phi(P(\phi) \rightarrow \phi(x)).$$

   Taken with the license to interchange identical names and the principle *Properties* for the abstraction operator in Section 6.1, this identity would afford a recipe whereby any formula with 'G' could be spelled out without i of that operator.
10. "It is worth noticing that there is here an implicit assumption: if we have defined a predicate, then we can straight-away form a name of the property it expresses. (The technically minded will thus wish to note that it is in effect assumed that anything is counted as a property which can be defined by 'abstraction on a formula')" (Anderson 1990, p. 292). This generous understanding of properties is in ample evidence in the notes under discussion, notwithstanding that it "is not" in its full generality "part of Gödel's argument" (Adams in Gödel 1995, p. 402).
11. See "That a Most Perfect Being Exists: November 1676" (Leibniz 1969, pp. 167–8), the first of two notes for discussion with Spinoza. I do not know that this was Leibniz's best attempt to prove the compossibility of perfections, but since Leroy E. Loemker said so, I believe it: "How [to] establish the possibility of a most perfect being? The closest Leibniz comes to an answer is his demonstration, to Spinoza in 1676 . . . a demonstration which he achieves only by defining perfections from the start as simple notions. . . . A more successful proof is never reached . . . though Leibniz later repeats his criticism of Descartes and restates the argument" (Leibniz 1969, p. 52). Leibniz's proof of the compossibility of perfections 'passes over' the possibility of each as if that followed immediately from each perfection's being without negation or limit.
12. To get away from the difficulty with disjunctive properties, one might add a condition to Axiom 2 for the principle that any property that is entailed by a property that is positive, *and not entailed by any property that is not positive*, is positive, $P(\phi)\ \&\ \Box\forall x[\phi(x) \rightarrow \psi(x)]\ \&\ \neg\exists\pi(\neg P(\pi)\&\Box\forall[\pi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)$. But deductions for Theorem 1 cannot use this weakened version of Axiom 2. With it in place, it would be 'back to the drawing board' for this theorem.
13. A better label than 'essence' would be a '*complete property*,' suggesting as it does Leibniz's complete individual concept. It is observed in (Hazen 1999, p. 365) that nowadays 'essence' can suggest a property that comprehends precisely an individuals 'essential properties' without which it would not 'be itself'.

    Def Ess in Scott's notes corrects the definition in Gödel's by adding the conjunct '$\phi(x)$': "Gödel left this clause out . . . but this appears to have been an oversight – it is included in related manuscripts" (Hazen 1999, p. 365). It would

be complicated to spell out without it formulas that use 'Ess', more complicated than for 'G' (note 8).

14. To continue the thought of the previous note, though it is in principle possible, it would be complicated indeed, for sundry formulas with 'NE', to spell them out without it.
15. Scott notes that in the system things that have essences have *unique* essences: "ϕ Ess x & ψ Ess x $\rightarrow \Box\, \phi = \psi$" (Scott, p. 3). So, if ϕ is *an* essence of x, ϕ is *the* essence of x. 'Unique essences' is probably, in Gödel's view, a consequence of the general principle that *logically equivalent properties are identical*: $\Box\forall x(\phi x \leftrightarrow \psi x) \rightarrow \phi = \psi$. It can be seen that if ϕ Ess x and ψ Ess x, then ϕ and ψ are logically equivalent properties.
16. Would a God-like being have *every* necessarily instantiated property? No. For there are properties such that both they and their negations are necessarily instantiated. For example, *being an even number* is necessarily instantiated: It is true at every world that the number 2 is even. And for similar reasons it seems that *not being an even number* is necessarily instantiated: It seems that it is true at every world that the number 3 is not an even number.
17. Not everyone would agree about blue things and conscious beings. Peter van Inwagen would, I think, not agree, though he might be prepared to claim only that no one can *know* there could have been no blue things, and no conscious beings. He writes "anyone who thinks he knows, or has good reason to believe, that there is *no* necessary concrete being is mistaken" (van Inwagen 1977, p. 386). Against an imagined argument – namely, one *from* "all the (relatively) uncontroversial examples of necessary objects are abstract" and "all the uncontroversial examples of concrete objects are material, or, at least, depend for their existence on material objects" and are thus not necessary, *to* the conclusion that no being is both concrete and necessary – he writes that it is a "remarkably weak" quasi-inductive argument (pp. 383–4). This argument from the characteristics of uncontroversial examples may be the best argument the case permits, given that no clear analysis of concreteness is in hand (p. 380) and that, given its premises, it is a very good argument. But I doubt that it is how anyone thinks he knows that there are no necessary concrete beings. I think I know this because, subject to uncertainty regarding 'concreteness' but assuming that 'for God's sake' that it would including 'being in the world with us', I can with some clarity *Yablo-conceive* the possibility of a world devoid of concrete beings. I can almost *imagine* such a world, just as I *can* imagine a room devoid of elephants. That is a sufficient *reason* for believing in the possibility, absent reasons for thinking the contrary.
18. Left to construct, in the system of Section B3 of Appendix B of the previous chapter, is a derivation for $(x)(Gx \rightarrow [\Diamond \text{Worshipful}(x) \rightarrow \sim \text{Worshipful}(x)])$. $(x)[\text{god}(x) \rightarrow \text{Worshipful}(x)] /\therefore \sim(\exists x)[Gx \,\&\, \text{god}(x)]$. Here, Gx, Worshipful(x), and god(x) abbreviate, respectively, 'x is God-like', 'x is worshipful', and 'x is a god'.
19. Kay Borge Hansen said today (June 7, 1999), 'In the early eighties Stig Känger showed me a copy of the notes Dana Scott had sent him. I was suspicious of the axioms, made a mathematical model to see that they were consistent, and noticed – this made the model easier – that there was a collapse of modalities. I thought it might be an artifact of the model. Some time later Wlodek showed me a copy of your paper and I was pleased with the congruence of our findings.'

20. A less radical solution that is specific to the difficulties of Section 6 would consist in confining the essence of a thing to its 'intrinsic' properties. To implement this solution one might take 'intrinsicness,' construed either as a property of individual properties or as a property-to-individuals relation, as a further primitive and set axioms appropriate to it and the ends of the system. It would need to be a consequence of the axioms that, although a thing's essence would include each of its intrinsic properties, it would not include such properties as 'being in the presence of the truth that Q'. Elaborating such a theory of intrinsicness, and adjusting the system to it, would not be a trivial exercise.
21. Too great a one *may* have been Kant's opinion. At any rate he ruled out some kinds of 'adoration': "[L]ove to God as inclincation (pathological love) is impossible, for He is not an object of the senses" (CPracR 1956, p. 86 – academy 84).
22. The recorded-announcement illustration of absurdity comes from Nagel (1971).
23. Johnson, for an example of a recent 'nice' try, starts with a definition of a god as "a person" who amongst other things is "worthy for worship" and defines "an *Anselmian god*" as a being that is essentially a god and that necessarily exists (Johnson 1999, p. 98). He contends that [1] Anselmian Godliness is "positive (morally or aesthetically wonderful, with no morally or aesthetically negative aspect)," and that Gödel is right; [2] "every *positive* property is . . . *possibly instantiated*" (p. 99): He gets this from Gödel's Axiom 2 and the considerably weaker than Axiom 1 principle corollary that not every property is positive. From these premises he infers that [3] "[P]ossibly there is an Anselmian god" (p. 99). It is then observed that [4] necessarily, if there is an *Anselmian god*, then necessarily there is *a god*. He then proceeds in sentential modal logic S5 from [3] and [4] to the conclusion that "in very truth *there is a god*" (p. 99). (The argument of Johnson 1999 is elaborated in Johnson 2002.) I make a '*modus tollens*' out of Johnson's initial '*modus ponens*', and from the impossibility of a necessary existent that is worthy for worship, and thus of an Anselmian god, conclude that either not every positive property (as understood here) is possibly instantiated, or Anselmian godliness is (because of the necessary existence of it) not positive. Another way to this disjunctive result could begin with an argument for the impossibility of *a god*, as defined by Johnson. For, following Swinburne, he says that a god would be, on the one hand, "the creator and sustainer of the universe, able to do everything . . . , and worthy of worship," and on the other hand, "immutable, [and] eternal" (Johnson 1999, p. 98; 2002, p. 181). There are, to say the least, problems with the idea of interactions between the eternal and the temporal (cf., Fales 1997). There are problems with the idea of interactions of the immutable and anything.

    Johnson's definition of an Anselmian god can be symbolized thus:

$$\text{Def}(\mathcal{AG}) \quad \Box(x)[\mathcal{AG}x \equiv Gx \,\&\, \Box(E!x \supset Gx) \,\&\, \Box E!x]$$

    Here '$\mathcal{AG}$' is 'A' overlapped by 'G'. Left as exercises are derivations in the system of Section B3 of Appendix B of the previous chapter that confirm that this definition entails not only [4], $\Box[(\exists x)\mathcal{AG}x \supset \Box(\exists x)Gx]$, but [4*], $\Box[(\exists x)\mathcal{AG}x \supset \Box(\exists x)\mathcal{AG}x]$. Johnson could have made his argument for the existence of an *Anselmian god*, though not without complicating it in ways that might be felt to detract from its considerable pedagogical virtue.

24. I am indebted to Willa Fowler Freeman Sobel for her comments on many drafts of this study, to members of Zepot Metodologiczny Polskiego Towarystwa Filozoficznego, Oddziat w Krakowie, to whom an early version of this work was presented on 21 March l985, and to Judith Jarvis Thomson and George Boolos. Thanks also for comments by Krister Segerberg and conversations with Sten Lindström.
25. Gödel wrote (*c*. 1954): "The ontological proof must be grounded on the concept of *value* (*p* better than $\sim p$) and on axioms" (Gödel 1995, p. 433). The great man was wrong. As Anderson shows, Gödel's primitive 'positive' can be usefully defined in a classically correct manner in terms of 'imperfection', not '*p* is better than $\sim p$'.
26. The replacement of Axiom 1 by Axiom 1* is sufficient to prevent 'modal collapse'. Axiom 1* leaves out the part of Axiom 1 "involved in the proof [Anderson's proof] of the troublesome Corollary 2" (p. 295), that every property of a God-like being is entailed by its God-likeness, which "is at the heart of Sobel's objection" (p. 293). Other changes are designed to secure, redefined, the result that "God-likeness" is necessarily instantiated.
27. An alternative revision at this point of the system, made in Hazen (1999), uses instead of Gödel's essences what might be termed 'positive individual concepts', properties that comprehend precisely the positive properties of individual. Hazen says that this adjustment in the system has the advantage of putting the burden of the proof where it belongs, on the idea of positivity, and the problem of explaining it so that "the axioms . . . are plausible and the conclusion theologically interesting" (Hazen 1999, pp. 374–5). Anderson *focuses* this burden by deriving some of the axioms from a plausible definition of positiveness.
28. Not only that. Anderson also explains how "at least one reasonable way of formalizing" the axioms is shown by a "model of second-order S5 of the sort explained by Nino Cocchiarella" not to lead to a modal collapse (Anderson 1990, pp. 296–7). In this model though the axioms "come out true . . . there are true but contingent propositions – for example, that there are at least two things" (p. 297). The model features two worlds in one of which there is exactly one thing. In the other world, which is designated to be the actual world, there are exactly two things. That formalization works with a second-order definition of 'ϕ is positive': In this definition ϕ is not "in [an] argument place" (p. 297). Modeling without aid of such a definition, modeling in a third-order extension of Cocchiarella's second-order system, would require an account of the extension (p. 300n13). For simplicity, the formalization does not work with exactly Anderson's definition of positive, but that is only for simplicity (cf., p. 301n15).
29. It is observed in Section C4.2 of Appendix C that the possibility and the positiveness* of *perfection* are equivalent in Anderson's system. That is a consequence of his definition of positiveness*.
30. Graham Oppy explains ways of mocking Gödel's proof, as emended by Anderson, to prove the existence of all manner of non-God-like beings (Oppy 1996). Michael Gettings argues that efforts along lines such as Oppy's against Anderson's Gödelian proof fail if they accept as partly definitive of positiveness* the principle that if the property ϕ is positive*, then the modal higher order property that is had by precisely those things that are *essentially* ϕ is positive* (Gettings 1999 – Axiom 7, "if P is positive, then being necessarily P is

positive," p. 310). However, *that principle plays no role in Gödel's proof as emended by Anderson*, as Getting knows (p. 310n3). So, despite Getting's resistance to this assessment (p. 312), his criticisms of Oppy's objections to Gödel's proof as emended by Anderson are not good.

Anselm's parry of Gaunilon's blessed-isle objection was similarly deficient. That objection, generously interpreted, was that Anselm's argument of *Proslogion* II can be adapted to prove the existence of an isle that is not merely more excellent than every other isle that exists, but that is more excellent than every other isle that can be conceived (Anselm 1962, pp. 150–1: Chapter 6 of Gaunilon's comments). Anselm's main response to this objection was that, while a *being* than which nothing greater can be conceived *cannot be conceived not to exist*, every *isle* (and indeed everything that is not a highest conceivable object) *can* be conceived not to exist (p. 160: Chapter 4 of Anselm's reply to Gaunilon). The trouble with this response is that the idea that that than which nothing greater can be conceived cannot be conceived not to exist *does not play a role in the argument of Proslogion* II, in which argument the existence of a being is purportedly proved. That idea makes its first appearance in the opening sentence of *Proslogion* III – "And it assuredly exists so truly, that it cannot be conceived not to exist" (p. 8) – the assertive use of which sentence *presupposes* that '*it*' exists.

*Further to Getting's principle, suppose, Jeff Pelletier has suggested, truthfulness is positive. Suppose it is something to the positive credit, to the 'greatness,' of its bearers. That seems a possibility, but it is doubtful that being *essentially* truthful could be to the positive credit of its bearers. For they would have no choice but to be truthful. Cf., Richard Price's objection to the essentialist dimension of perfect being theology: Chapter I, Section 7.

31. These are distinct properties of an interpretation that features two worlds whose domains of existents are distinct. Were properties in an interpretation functions from worlds to subsets of their domains, nonexistence and nonselfidentity would be the same property in an interpretation. Each would be the function that assigns to each world the empty set. That is one reason for identifying properties in an interpretation with functions from worlds to subsets of the universal domain.
32. Michael Bergmann writes only of objects having properties and exemplifying properties *in* worlds when he explores consequences of Actualism, the thesis "that necessarily, everything that there is exists" (Bergmann 1999, p. 118). Corresponding to that thesis is, perhaps, a restriction on interpretations to those in which world-domains are identical with one another and with the universal domain. Relative to such interpretations, all properties-functions are existence-entailing, and there is no distinction between having or exemplifying a property *in* a world and having or exemplifying it *at* a world. Bergmann argues that one consequence of Actualism is Serious Actualism, "the thesis that necessarily, no object has a property ***in*** a world in which it does not exist" (Ibid.). I make this thesis a tautology and distinguish it from the metaphysical thesis that no object has a property *at* a world in which it does not exist, which thesis I reject.

Bona fide properties (mentioned in the last footnote to Chapter I) are possibly instantiated. If Pr is a *bona fide* property identical with the function *Pr* from worlds to subsets of the universal domain, then, for some world w, *Pr*(w) is a nonempty subset of the domain of w. One might say, of a simple subject predicate utterance that would ascribe a *bona fide* property to an object, that:

(i) it is false, if it is *a priori* determinable from its meaning that the predicate does not stand for a *bona fide* property, and (ii) it is neither true nor false, if, though the predicate does not stand for a *bona fide* property, this is not *a priori* determinable from its meaning. That would allow Mackie to say that it is false that *X is a round square* but neither true nor false that *X is objectively good* .

Chapter V

1. This sentence comes from an examination paper written by Delfina Sclauzero in Philosophy A01 at the University of Toronto in 1979–80.
2. The citation is short for 'St. Thomas Aquinas: *Summa Theologica*, Part One, Question 2, Article 1 in *Basic Writings of St. Thomas Aquinas*, Vol. I, ed. by Anton C. Pegis, Random House, 1945, pp. 18–9.' Citations to follow, when in this style, are all to this two-volume collection of translations by Anton Pegis writings of Aquinas that are "[w]ith one exception [selections from *Contra Gentiles* . . . drawn from the *Summa Theologica*" (p. l). "[T]he *Summa Theologica* consists of there parts (to which a Supplement was added). . . . Each part is . . . subdivided into 'questions', and most of the questions contain several 'articles'. . . . In each article Aquinas first cites objections against the doctrine which he wishes to propose. He than exposes his doctrine or theory. . . . Finally he replies to the objections in turn in the light of the doctrine which he has explained" (F. C. Copleston, *Aquinas*, Penguin Books, 1955, p. 13).
3. But suppose I understand "the name *God* . . . to signify something than which nothing greater can be thought" (ST I q2,a1 p. 20). Then, even though the proposition *that God exists* is not self-evident to me, since this knowledge of the name's significance is not knowledge of its subject's essence, am I not even so in a position to demonstrate this proposition, so that it is in a *sense* 'self-evident' to me, the sense being that its *truth* is self-evident to me, though I do not know what this truth comes to? Aquinas says, No, but lamely, for he at least *seems* to concede to Anselm all that his argument requires: Aquinas writes that "as soon as the name *God* [signifying that than which nothing greater can be conceived] is understood it [a thing than which nothing greater can be conceived] **exists mentally**" (ST I q2,a1 p. 19; bold emphasis added). He says that even so it cannot "be argued that it **actually** exists, unless it be admitted that there **actually** exists something than which nothing greater can be thought; and this precisely is not admitted by those who hold that God does not exist" (ST q2,a1 p. 20; emphasis added). Aquinas seems to 'give' Anselm **existence in the mind** of a being than which nothing greater can be, thought – he seems to give that the thing **possibly** exists, as soon as these words for it are understood. I believe that Aquinas *intended* this concession, and failed to see that to concede that it 'mentally exists' or possibly exists is to give Anselm his argument, and to say that only a *confused* fool can resist its conclusion. This notwithstanding that he had Gaunilon's example of exactly where to put the wrench in Anselm's works, specifically, between having in mind and understanding words for an object, and it, an object signified by them, existing mentally, or Aquinas nearly says, possibly existing.

4. Richard Cartwright provides a minimal statement of the doctrine of The Trinity, considers some construals "to convince . . . of the difficulty of the subject," and suggests other construals (ten others) of which "a full treatment would take account" (Cartwright 1987, p. 198).
5. "But is it not clear that the priority of the Second Way cannot be temporal, since according to Aquinas God is eternal?" No, for though Aquinas does say that God is eternal, he does not equate being eternal with being atemporal. "We . . . reach to the knowledge of eternity by means of time which is nothing but *the number of movement according to before and after* . . . Now in a thing lacking movement, and which is always the same, there is no before and after . . . in the . . . uniformity of what is absolutely outside of movement consists the nature of eternity. Further *those things are said to be measured by time which have a beginning and an end in time*" (S.T. I q10,a1 p. 75.) For Aquinas, to be 'eternal' is to be 'without beginning or end' and 'without succession' of states (loc. cit.). The 'eternal' for Aquinas includes, I think, things such as the number 4 and the proposition that 2 plus 2 is 4, as well as things such as the Northern Star could have been, contingently eternal or merely everlasting things. The number 4 and that proposition are *necessarily* 'eternal' for Aquinas, they are by their natures 'eternal,' and they are atemporal. And God? Aquinas may have thought that God is 'a bit of both,' necessarily unchanging and without beginning or termination like the number 4, and everlasting and at every time without beginning or end as the Northern Star could have been.
6. This ban on infinite regresses of efficient causes cannot be symbolized in our system of quantifiers and predicates using only the relational letter 'C' used in previous sentences. The informal deduction coming of (8) from (1) through (7) cannot be confirmed by a formal derivation in that system.
7. 'Activity' because it is at best one of the 'mysteries' of how the First Cause, God, of Aquinas's philosophical theology could be said, without great violence to the term to be engaged in '*causal activity*' now and then, even given a temporal 'footprint' (previous note); for Aquinas's God would be the same at every time. Nothing would *happen* to God, and it *seems* that God would not *do* anything, ever. At least, this is how it seems to *reason*, which *might* lead Aquinas to say, "Yes, but it is otherwise in *faith*, for which speech that does violence to language is licensed." (That position may be incoherent, for while one can license language, one cannot license *sense*, and it there cannot be belief.)
8. I 'hear' Louis Armstrong's self-deprecatory, "Braggin'," to Ella Fitzgerald in their duet of "St. James Infirmary."
9. Essential to this modeling is that generations of chickens should be of some *minimum duration*. Without that it is conceivable that there should be infinitely many past generations of chickens even though the physical universe came into existence exactly 10,000 years ago today. See Appendix B.
10. The popular consideration, of which Aquinas *disposes* nicely in Question 46, is very similar to one that he *endorses* in Question 7. Suppose that 'a man were to be generated by a man to infinity.' Then an infinite series of fathers and sons would lead to this son, and this series would be "an infinite multitude . . . necessary [for this son to] exist" (ST I q7,a4 p. 61). Aquinas seems to agree with Avicenna and Algazel that "this is impossible, **because** it would mean that something is

dependent on an infinity [in time] for its existence; and hence its generation would never be accomplished, because it is impossible to traverse what is infinite" (loc. cit.; emphasis added). Furthermore, if, as I think, Aquinas in Question 7 goes along with Avicenna and company when they imply that for that reason there cannot be an infinite series of generating fathers and sons, he there contradicts what he says in Question 46, which is that "it is **not** impossible for a man to be generated by man to infinity" (ST I q46,a3 p. 455; emphasis added).

11. "Cantor's theory has been called a theory of the actual infinite. . . . That notion of an actual infinity is sometimes contrasted with that of a potential infinity. . . . But it is not clear what this contrast, as it occurs in philosophy, comes to" (Thomson 1968, p. 186a). Some sense can be made of it for 'developing multitudes.' For example, if there was a first winter and there will always be winters, then one might say that winters are actually finite, though potentially infinite, whereas if there have always been winters, one might say that winters are both actually and potentially infinite. It is, however, doubtful that sense can be given to the contrast for 'static multitudes'. Probably it is best that this traditional terminology be abandoned in these post-Cantor days, but I will abandon only the 'potential' half, and to maintain contact with traditional texts and discussions make liberal use of 'actual' and 'actually' in connection with 'infinity' though I do not intend by them a qualification or distinction. In this practice I follow the great Georg Cantor: "[I]n truth the potentially infinite has only a borrowed reality, insofar as a potentially infinite concept always points towards a logically prior actually infinite conception whose existence it depends" (Cantor 1832, p. 3).

    Rudy Rucker reports that a definition of 'potential infinity' has been given by John Horton Conway. "Conway's [surreal] numbers are introduced as 'gaps' between . . . sets" of numbers (Rucker 1984, p. 83 [85]). "He even gets a definition of the traditional ∞ for potential infinity [which] is defined as the gap between the finitely large and infinitely large surreal numbers, and Conway derives the weird equation $\infty = \Omega/\omega$ [!!], which almost magically ties together [surreal] potential infinity ∞, the simplest [surreal] actual infinity ω, and the [surreal] Absolute Infinite Ω" (p. 83). The use of this peculiar surreal as an explication of the traditional idea of the potential infinite has, I believe, not been tested.

12. "For Carnap . . . questions . . . about which linguistic framework should be adopted . . . are . . . purely practical questions" (Friedman 1997, p. 15). "Carnap characterizes the answers we might reasonably attempt to give . . . as both conventional and pragmatic" (p. 18).
13. If a multitude does not include a distinct thing for every finite number, then, for some finite number n, it contains distinct things only for each number no larger than n. In that case the multitude is of finite size n.
14. "But eventually we do reach a limit to this wonderful hotel's powers of absorption: *alef-one*" (Rucker 1984, p. 75), the number of the 'real numbers,' the numbers whose numerals would be the 'infinite decimals.'
15. Rucker's hotel would be environmentally friendly. "To fix ideas, I have drawn a picture of Hilbert's Hotel. . . . [T]o fit it on a page, I have assumed that each floor is equipped with a science-fictional space condenser, a device that makes each succeeding story two-thirds as high as the one before. The shrinking field also affects the guests" (Rucker 1984, p. 75).

16. Cf.: "The interpretation of Aquinas's view that I propose is the following. [But see *: Rowe is offering not an interpretation, but a 'deep analysis.'] Aquinas is *assuming* that there is or must be an explanation for the fact that causal activity of a certain sort is going on. . . . Consider . . . Aristotle's example of an essentially order series of causes . . . the hand moving the stick that moves the stone. . . . [W]hat we want to explain is the fact that stone-moving activity is going on. Can we hope to explain this by reference to the stick? Clearly not. For . . . the stick is being caused to exhibit that activity by something else [it is pushing because it is being pushed]. . . . [S]o long as our [storied] regress of causes contains only members that are intermediate causes like the stick, there will be no explanation of the fact that this causal activity is now going on. [But] if the series progresses to infinity, each member will be like the stick, an intermediate cause [and so far there will be no answer to the question, From whence all this pushing?" (Rowe 1975, pp. 32–5). [* "It cannot reasonably be maintained that this argument is explicitly or implicitly contained in the argument Aquinas actually presents against the infinite regress of causes. . . . I offer the above . . . as a suggestion of what may underlie Aquinas's rejection of the infinite regress of causes" (pp. 36–7)]. Thanks to Bernie Katz for making me think about pushers. For a column of 'hangers' see note 4 in the next chapter.
17. 'The' counterparts of (8a) and (8b) – (8a'). For every sensible thing that has an efficient cause there is exactly one thing that is not a sensible thing, does not have an efficient cause, and is *the* first cause of that thing. (8b') There is exactly one thing that is not a sensible thing, does not have an efficient cause, and is the first cause *of every* sensible thing that has an efficient cause. These are symbolized in Russell's theory of descriptions by (y)[(Sy & Hy) ⊃ (∃x)(z)([∼Sz & ∼Hz & {⌝*wF(wy)*}⌝*wF(wy)* = *z*] ≡ z = x)] and (∃x)(z)[(∼Sz & ∼Hz & (y)[(Sy & Hy) ⊃ ⌝*wF(wy)* ⌝ *wF(wy)* = *z*)]) ≡ z = x]. (See Appendix A to Chapter II.)
18. There are affinities of Aquinas's Second Way for the First Cause to an argument of Aristotle's for the Highest Good. "Now, if there exists an end in the realm of action which we desire for its own sake, an end which determines all our other desires; if in other words, we do not make all our choices for the sake of something else – for in this way the process will go on infinitely so that our desire would be futile and pointless – then obviously this end will be the good, that is, the highest good" (Aristotle 1962: *Nicomachean Ethics* I, 1, 1094a17–22.) Regarding this passage, Elizabeth Anscombe remarks, "there appears to be an illicit transition . . . from 'all chains must stop somewhere' to 'there is somewhere where all chains must stop'" (Anscombe 1957, p. 34).
19. There is another picky problem with sustaining-cause takes on the argument. They have some trouble with (9), which says that the unique sustaining cause delivered by (8b) for what is now that which everyone correctly gives the name 'God'. The trouble is that there are no obvious reasons why the unique sustaining cause at one time for sensible things existing at that time should be the unique sustaining cause at any other time for sensible things existing at that time. The argument, read as for a single first presently existing sustaining cause, would leave open that this was *a being of just this moment*, and so not worthy of the name 'God', since worshipping, devotion, prayer, and so on, all take time. Not to solve, but to 'bridge' this problem , which is specific to would-be sustaining first causes, (8 ½) could for them be enhanced.

20. Aquinas endorses several Aristotelian arguments against infinite regresses of movers in *Summa Contra Gentiles* (Bk. I, Ch. 13, secs. 12–15). The argument in his First Way is that of Section 14, whose disappointing question-begging themes are merely varied in Section 15. Section 12 harbors two arguments. One rests on the rejectable assertion that members of an infinite regress of movers could not all be moved in a finite time. The second rests on the interesting assumption that infinitely many bodies each of which was moved by another would constitute a body that was itself moved. (But would it be moving?)
21. The Athenian in Plato's *Laws* traces motion back to self-movers (894e–895b) that are souls (896a). These are not said to be in perpetual motion. The hypothesis of an infinite regress of moving movers, none of which moves itself, or moves always, is not considered in the *Laws*.
22. Aquinas endorses several Aristotelian arguments against self-moving beings in general, and perpetually self-moving beings in particular, in *Summa Contra Gentiles* (Bk. I, Ch. 13). My comments are not relevant to those arguments.
23. An argument in Sobel (1998b, Ch. 3, Sec. 8) to show that Universal Causation is consistent with Free Will exercised in 'mini-bangs' uses this construction.
24. "*Kalām* may be simply defined as 'natural theology' or philosophical theism. . . . Taken literally, *Kalām* is simply the Arabic word for 'speech'. . . . [It] became the name of the whole movement within Arabic thought that might best be called Arabic scholasticism. . . . Richard Walzer described [practitioners] as 'dialectical or speculative theologians' and noted that they . . . 'take the truth of Islam as their starting point'" (Craig 1979, p. 4).
25. Craig (1979, p. 63.) reviews the arguments of Ishāq al-Kindi (c. 801–c.873), "[u]niversally recognised as the first true philosopher of the Islamic world" (p. 19); Saadia ben Joseph (882–942), "the 'first important Jewish philosopher' [according to Isaac Husik]" (p. 38); and the thinker "generally known . . . as al-Ghāzāli" (1058–1111). Craig's basic argument is essentially al-Kindi's (see pp. 34–5). Regarding Husiik's elevation of Saadia ben Joseph, Bernie Katz wonders, What about Philo (fl. 20 B.C.–40 A.D.)?

Chapter VI

1. Cf.: "But nothing can never be a cause." (Hume 1888, p. 81). Hume attributes words to this effect to *Locke* for use in a bad argument that purports to demonstrate the general principle that whatever begins to exist must have a cause for its existence. On 'nothing' see "Nothing" by P. L. Heath in *The Encyclopedia of Philosophy*, edited by Paul Edwards, New York: Macmillan, 1967, Volume 5, pp. 524–5. It is an appropriately brief edifying entertainment.
2. **N.B.** 'Contingency,' *as used today in philosophy and logic*, *does not mean* DEPENDENCY. Contingent beings contrast with necessary beings in contemporary terms in that, while a necessary being exists in *every* possible world, a contingent being exists in *some but not every* world. A contingent *existent* exists in the *actual* world, but not in every world. It is not settled *by definition* that every contingent existent depends on some other existent that is its cause or reason. This contemporary usage in philosophy and logic is reflected in the *second* sense mentioned in the entry, "**contingent** . . . dependent on something else: **liable but not certain to happen**: accidental" (*Chambers Twentieth Century: New Edition*).

It is a fair guess that when Leibniz described his 'cosmological argument' as 'a proof *a congentia mundi*' the word '*contingentia*' did mean *dependency,* and similarly for his uses of 'contingent' and 'contingents' in texts of 1714 set out below.

3. "*There are only three possible ways of proving the existence of God by means of speculative reason*. All the paths leading to this goal begin either from determinate experience and the specific constitution of the world of sense as thereby known, and ascend from it, in accordance with laws of causality, to the supreme cause outside the world; or they start from experience which is purely indeterminate, that is, from experience of existence in general; or finally they abstract from all experience, and argue completely *a priori*, from mere concepts, to the existence of a supreme cause. The first proof is the *physico-theological*, the second the *cosmological*, the third the *ontological*. There are, and there can be, no others" (CrPR A590/B618 – A592/B619, pp. 500–1). Immanuel Kant 'established' the adjectives 'cosmological' and 'ontological' for kinds of proofs of God, even if he did not 'coin' them, but got them from Christian Wolff, which, for all I know, he did. Leibniz's way in "On the Ultimate Origination of Things" was 'cosmological' in Kant's sense but evidently not termed 'cosmological' by Leibniz. Cf., "[t]his proof [was] termed by Leibniz the proof *a contingentia mundi*" (CrPR A604/B632, p. 508).
4. Leibniz (1965) has 'complete examination' here. Other translations use 'perfect reason' and 'complete reason'. 'Examination' may be a misprint. In any case, 'explanation' better conveys Leibniz's idea.
5. Marginal note * is not in the text of 1705. Its gist first occurs in a P.S. to Clarke's answer of 10 November 1713 to "The First Letter" from "a Gentleman in Gloucesterhsire" by the name of Joseph, to be Bishop of Bristol, Butler (Clarke 1738, p. 466). Clarke writes that "[m]any readers . . . have misunderstood my Second General Proposition; as if the words [*some one* . . .] meant [*one only*. . . .]" rather than, we might say on Clarke's behalf, 'at least one.' By 'one only' he means 'one just' or better 'one exactly.' It is too bad that Clarke did not own up to Butler that he had himself been confused. That he had been can be gathered from his argument for Proposition II, which argument is unchanged from its first publication in 1705. The third sentence of this argument sets up, as the possibility to be proved, that "there has always existed some one unchangeable and *independent* being, from which ***all other*** beings that are or ever were in the universe, have received their original" (p. 11; bold emphasis added). There could not be two such unchangeable and independent beings. For that would be two beings neither of which, since independent, received its original from any other, though from both of which all other beings received their originals. That Clarke, in 1704, was thinking not '*some at least one*,' but '*some certain exactly one*,' is evident throughout his argument for Proposition II, which ends in the words: "Which, *what* **it** is, remains in the next place to be inquired" (p. 15; bold emphasis added). That he was thinking 'some certain exactly one' is evident also in his Proposition III, whose statement begins thus: "**That** unchangeable and independent being. . . ." (p. 15; emphasis added).
6. There is at this point a footnote in which is quoted "a late able Writer" – William Wollaston, though Clarke does not identify him by name – of "*Religion of Nature – delineated*," which was published in 1724. The quotation is in

part: "Suppose a *Chain* hung down out of the Heavens, from an *Unknown* height. . . . Every link of it gravitated toward the Earth. . . . : And . . . a question should arise, *What* . . . kept up this Chain? Would it be sufficient to Answer . . . that the . . . Lowest [link] hung upon . . . That next above it . . . and so on *in infinitum*? For, *What* holds up the *Whole*? A chain of *ten links*, would fall down; unless something able to bear it, hindered. One of *Twenty*; if not staied, by something of yet Greater Strength. . . . And therefore One of *infinite links*, certainly; if not sustained by Something *infinitely* strong. . . . And Thus it is in a *Chain of Causes and Effects*. . . . [I]f they should be *infinite* . . . they would be but an *infinite Effect* without an *Efficient*. And to assert there is any such Thing, would be as great an *Absurdity* as to say, that a *finite* or *little Weight* wants something to sustain it, but an *Infinite* one . . . does not" (Clarke 1738, p. 13).

As a consequence of Newton's Law, according to which the attraction between the earth and a link of a chain extending infinitely into the heavens would be inversely proportional to the square of the distance between this link's and the earth's 'centers of gravity,' the weight of the infinite chain would presumably be not finite, but the finite limit of the sums of weights of its bottom n links as n increases without limit. But this is a miserable quibble. The argument is, I think, a *good* argument against the possibility of what would be an infinite beginningless series of concurrent *per se* causes.

The problem with this footnote of Clarke's is not with the argument in it, but to understand what Clarke is making of this argument. For he seems *not* be to be conceding to the argument that there cannot be this infinite regress, but to be saying that there *can* be this infinite regress, although only if there is a cause or reason for it involving an *independent* being. Doing what? Holding up the whole chain?! I suspect that Clarke missed the peculiar force of arguments against the possibility of infinite regresses of concurrent *per se* causes.

7. Mark T. Nelson, in an argument he says was Clarke's, uses the premise C1. "Every being (that exists or ever did exist) is either a dependent being or a self-existent being" (Nelson 1998, p. 88b). He says that this is "a corollary" of "PSR 1. For every being that exists or ever existed, there is an explanation of the existence of that being" (p. 88a). In fact, given Clarke's senses of 'dependent' and 'self-existent' as explained by Nelson (p. 87b), C1 and PSR 1 are logically equivalent.
8. Hume attributes to Clarke a bad argument for a principle of causes: "Every thing, 'tis said, must have a cause; for if any thing wanted a cause, *it* wou'd produce *itself*; that is, exist before it existed; which is impossible" (Hume 1888, p. 80). This argument is not in Clarke's argument for Proposition II. I cannot say where, in Clarke's writings, Hume 'found' it.
9. Cf.: "To write is no longer to put pen to yellow paper. It is to enter things into a computer. ('Things?' Well, I don't know what else to say.)" (Cartwright 1987, p. xxii.)
10. These explanations agree with those of note 2. For an entity whose existences *is* entailed by necessary truths necessarily exists, that is, *exists in every possible world*. And *possible* entity the existence of which is *not* entailed by necessary truths (i) since possible exists in some possible world and (ii) does not exist in every world. Why (ii)? Because what exists at every world, exists necessarily and is entailed by every proposition. It is a theorem of the quantified modal

logic of Appendix B of Chapter III that □(x)[□E!x ⊃ □(P ⊃ E!x)]. P is here any proposition.

11. Leibniz writes that "we must pass . . . to something endowed with absolute or metaphysical necessity, *for which no reason can be given,*" *meaning* 'for which no *further* reason can be given, or is *needed*' since what "exist[s] of metaphysical necessity . . . is a being whose essence implies existence" (1697) and 'bears the reason for its existence in itself' (1714).
12. This premise is missing from Demea's argument, which needs it for the transitional conditional, "If there be no necessarily existent being, any supposition, any supposition [regarding the world of contingents] which can be formed is equally possible" (Hume 1991, Part IX, p. 138).
13. Findlay's god would be "a Form, something basically universal" (Findlay 1970, last paragraph). He believed that there is such a 'god' and that we have it, not some excellent entity, to thank for the world, and to worship (absurdly, in my view – Section 7.1 of Chapter IV): "One cannot rationally worship this or that excellent thing or person . . . only Goodness Itself, Beauty Itself, Truth Itself, and so on are rationally venerable, and to bow one's knew to an instance is to commit idolatry" (p. 267). Plotinus 'set above' intelligence in which resided the forms and out of which emanated the world, One, a thing absolutely simple, out of which 'radiated' intelligence. While intelligence sounds like a superentity, one sounds like 'more form than a form', not an entity. Plotinus cast it as 'above being'.
14. As Plato, though it seems not Socrates, wanted to think, Leslie 'says'. He quotes Socrates: "One day I heard someone reading. . . . from a book of Anaxagoras, and saying that it is Mind that directs and is the cause of everything. . . . I thought that if this were so, the directing Mind would direct everything and arrange each thing in the way that was best. . . . [But in fact the man did] not believe that the truly good and 'binding' binds and holds together" (*Phaedo* 97c–99c). Socrates does not say that the truly good binds and holds together *and creates*, and he does not say that it binds and holds together *directly* without mediation of Mind. Leslie reports with approval that Plato takes these extra steps. "The *Republic* makes clear that goodness, rather than Mind's love for it, can be immediately effective" (Leslie 1979, p. 209) in these lines: "[T]he sun not provides visible things with the power to be seen but also with coming to be, growth, and nourishment, although it is not itself coming to be. . . . [Y]ou should also say that not only do the objects of knowledge owe their being known to the good, but their being is also due to it, although the good is not being, but superior to it in rank and power" (*Republic* 509b).
15. Leslie wishes not to offend the faithful, but, in my view, his conciliating options are not 'live'. The second would apply the name 'God' to something that I have argued is certainly not worshipful (Section 7.1 of Chapter IV), and the first, of a 'divine fifth wheel' for which we would have nothing to thank, should, I think, be similarly unsatisfactory to a religious mind. Calling a spade what I consider a spade, I will say that Leslie's cosmology is godless.
16. Cf.: "VII. *The Self-Existent Being, must of necessity be but One*. This evidently follows from his being Necessarily-Existent. . . . Again: To suppose *Two* . . . implies this plain *Contradiction*; that, *each* of them being *independent* from the other, they may *either of them* be supposed to exist *alone* . . . and consequently *neither of them* will be Necessarily Existing. Whatsoever therefore Exists necessarily, is the

One Simple Essence of the Self-Existent Being. . . . " (Clark 1738, p. 47). Joseph Butler (1692–1752) wrote in "The First Letter" of November 4, 1713 (Clarke 1738, pp. 459–463) that to say of things that they are independent is to say that either can exist "without any relation to, or dependence on the other: But *Where* is the . . . Idea . . . *that it will be no Contradiction to imagine the other not to Exist?*" (p. 461). Butler elaborated this telling point on pages 461–3. Clarke's response of November 10, 1713 was that "[w]hat exists *necessarily*, not only *must* so exist *Alone*, as to be *independent* of any thing else; but (being *Self-sufficient*,) *may* also so exist *Alone*, as that every thing else may *possibly* (or *without any Contradiction* in the *Nature* of Things) be supposed *not to exist at all*: And consequently . . . is not *necessarily* existent" (p. 465). Clarke condescended that "[t]hese sorts of things are indeed very difficult to *express*, and not easy to be *conceived* but by very *Attentive Minds*" (p. 467). Butler wrote in "The Second Letter" (undated), "I am sorry I must tell you, your Answers to my objections are not satisfactory" (p. 467). He did not, however, revisit the business of Uniqueness, but addressed only Clarke's response to the objection in "The First Letter" to his arguments under Proposition VI for the Omnipresence or, in Butler's term, Ubiquity, of a self-existent being.

17. There is the question, What *kind* of causes are at issue here? I assume that Cleanthes has in mind '*generating efficient causes*' for the *existence* of these particles. Generating efficient causes are themselves entities. Demea's argument can be seen to proceed on the suppressed premise, here in Leibniz's words, that "the reason for any existent can be only another existent" (Leibniz 1965, p. 86). Cleanthes's resistance to Demea's argument does not challenge, and indeed assumes, this premise. *Perhaps* Philo, without saying so, should be read as challenging this premise in his 'cannot forbear' addition to 'Cleanthes' reasonings,' though he *explicitly* suggests only that possibly the intimate nature of bodies, if penetrated, would reveal why natural beings are *ordered* and *disposed* as they are, *not* why they *exist*.
18. Thinking that he had in this way covered the infinite case, Cleanthes would not have been bothered by the suggestion that there *could not* be generating efficient causes for infinite, beginningless, successions of causes, since generative causes must be temporally prior to their effects. Nor need he have been, since all causes in such successions of cause can be subsequent to some time, as could be the causes of such successions (cf., Appendix B of the previous chapter).
19. Cf., Demea: "whatever exists must have a cause" (Part 9, p. 148). Hume, though he considered it "neither intuitively nor demonstratively certain" (Hume 1888, p. 79), was of the firm "**opinion** of the necessity of a cause to every new production" (p. 82; bold emphasis added). 'We' *are* of this opinion. The only question is how we have come to it.
20. Pruss (1998) argues well to conclusions of this section and more. An innovation of its arguments is use of the following principle for "causal explanation of collections (or, perhaps, preferably, aggregates)" **by** collections: "[A] collection B (causally) explains a collection A, if for every element of A, there is an element *b* of B such that *b* (causally) explains *a*" (p. 154). This is described as a "reasonable sufficient condition for causal explanations of collections" (p. 154). Cleanthes would be pressed not to agree. He all but says to Demea that if for every element of A there is an element of B that causes it, then *the elements of B cause A*. And it

is not clear how he could say that, though the elements of B cause A, *the aggregate B of these elements* does not cause A. Now come problems that this principle would make for Cleanthes's objection and reasons why Cleanthes would want somehow to reject it. Assume the principle, and suppose we show Cleanthes' a cause of each temporal state of the material universe in a state preceding it. Then, by the new principle, we have shown him that the aggregate of temporal states is *a cause of itself*, and he would need to explain how that is consistent with this aggregate's not being a necessarily existent Being, with its not being "as impossible for [it] not to exist as for twice two not to be four" (Hume 1991, p. 149). Entangled with that problem would be the problem of in what *way* it would be 'cause of itself'. Even if it has a beginning in time in the sense of a time at and before which it has not temporal states, but after which it does, it is not temporally prior to itself. So it cannot be its own *generating* efficient cause.

21. Again, if an argument with conclusion *Con* and necessary premises conjoined in *Prs* is valid, then, (1) since the argument is valid, it is not possible for *Con* to fail to be the case, though *Prs* is the case, $\sim \Diamond(\sim Con \ \& \ Prs)$, or equivalently, (1′), $\Box(Prs \supset Con)$; and, (2), since the premises of the argument are necessary, so is their conjunction *Prs*, (2′), $\Box(Prs)$. It follows from (1′) and (2′) that it is necessary that Con, $\Box(Con)$: For according to (1′) and (2′), each of $(Prs \supset Con)$ and *Prs* is necessary, and *whatever follows from things that are necessary is itself necessary.*
22. For this banishment there are just two cases to consider, since every proposition is either true or false. *Suppose a proposition P is true*. Then PrSuffRsns says there is a demonstration for P. *So P is necessarily true, □P. Suppose P is false*. Then its negation, ∼P, is true, and PrSuffRsns says there is a demonstration of this negation, so it is necessarily true., □ ∼P, and *P is necessarily false, or, impossible,*$\sim \Diamond P$.
23. This is a case of the principle that, if there is a truth of a kind, then there is a truth of this kind that entails every truth of this kind, which principle may be compared with "if there are facts of type ϕ, then there is an aggregate or sum of all the ϕ facts" (Koons 1997, p. 194a). Robert Koons say that this principle of his system "corresponds to the premodern denial of infinite regress" (p. 204). This edification is one of the 'good bits in between' referred to in Appendix C. Another is the contrast of "the Paleyian argument" analogical to a designer, in which "we start with only three terms, human artifacts, human creators, and the cosmos," with "the Thomistic argument" analogical to the intelligence of the First Cause, in which "we start with four . . . terms," human artifacts and creators, and The First Cause and The Cosmos (pp. 201–2).
24. Richard Gale and Alexander R. Pruss imply that certain theists can reject PrDedExpl as spelled out by (1) and (2) and bolstered by (3) and (4). The theists in question are Libertarians, who say that there is a contingent truth CT to the effect that there is a necessary, very powerful, intelligent, and good supernatural being who freely brings about the Big Conjunctive Contingent Fact, BCCF. There cannot be a 'deductive explanation' of CT. For *suppose* that DX is a deductive explanation of CT. Then there is a single-premise deductive explanation DX′ of CT. So *DX′ is a deductive explanation of CT*. Let P be the premise of DX′. P entails CT, from which, since CT is contingent, it follows that P is contingent. But then CT entails P, for CT entails BCCF, which evidently entails every contingent truth (Gale and Pruss say that it is "comprised of all contingent

propositions that are true" – 1999, p. 470). So by (2) *DX′ is not a deductive explanation of CT*. Gale and Pruss say that these Libertarian theists can accept that there is no explanation for CT, since according to them "that a being *freely* performs an action. . . . stands in need of no further explanation [beyond] the Simple Libertarian explanation 'She did it of her own free will'" (Ibid.). However, these Libertarian theists are *saddled* with a reason or explanation *of sorts* why their supernatural being freely chose to bring about BCCF, for *that* This One freely chose this, since itself a contingent truth, is included in it. They must concede that This One freely chose freely to choose it, BCCF, and since this too is a contingency, that This One freely chose freely to choose that it freely chose freely to choose it, *and so on* – shades of Leibniz (see A2 of Appendix A) – *ad infinitum*. They are saddled with '*many* Simple Libertarian reasons too many'.

25. Van Inwagen maintains similarly that "the collapse of all modal distinctions" is a consequence of "the famous Principle of Sufficient Reason" (1983, p. 202). I formulated more or less the arguments of Sections 6 and 7.1 in 1963 when preparing for a class at U.C.L.A.
26. Let P be a necessity and Q a truth. Either P entails Q or Q is *logically independent* of P: $(\Box P \,\&\, Q) \supset (\Box(P \supset Q) \vee [\sim\Box (P \supset Q) \,\&\, \sim\Box(P \supset \sim Q)])$ is a theorem of SMC (Appendix B of Chapter III). And Q is *probabistically independent* of P, that is, P is of no *evidential relevance* to Q: $(\Box P \,\&\, Q) \supset [Pr(Q/P) = Pr(Q)]$ is a theorem of 'Kolmogorov probability theory' for which see Appendix A of Chapter VIII.
27. The putative explanations of Hawking and Leslie challenge a premise of a 'new cosmological argument': "There is no proposition consistent with the claim that there are only contingent beings which, if true, would explain why there are contingent beings" (Katz and Kremer 1997, p. 64). I believe they intended 'logical consistency,' for it can go without saying that no proposition that is in *every way* compatible with a proposition's falsehood can explain its truth.
28. I contend similarly in Sobel (1998) that necessities "cannot be just in themselves proper subjects for dismay or relief or any human attitude" (p. 43), which coheres with the conclusion of Section 7.2 of Chapter IV, that it *was* an ill day when St. Anselm hit upon the idea that God must be a *necessary* being, since "[n]ecessary existence . . . far from contributing to proper greatness and *worshipfulness*, is *prima facie*, and I think not only *prima facie*, at odds with that."
29. Leibniz's God freely chooses amongst possibilities in the sense of possible finite or contingent entities that are to be actual. He does not, as it seems that Descartes's God would, *choose* what is to be possible. Perhaps, however, in Leibniz's view, the existence of possibilities depends on God's existence. Walker, striking a diminished Cartesian chord for Leibniz, says that since, according to Leibniz, "possibilities have their being in God"s *intellect* . . . a world without God would be a world without modality" (Walker 1997, p. 117).
30. "Si quis enim a me quaeret cur decrevit creare Adamum, dico: quia descrevit facere perfectissimum, seu cur vult perfectissimum. Si jam quaeris a me cur decrevit facere perfectissimum, seu cur vult perfectissimum (quid enim est aliud velle quam decernere facere?) respondeo id voluisse liber, seu quia voluit. Itaque voluit quia voluit velle, et ita in infinitum." ("Reflexions Sur Bellarmin 1680–1682?," in G. W. Leibniz, *Textes Inédits* publiés et annotés par Gaston Grua, Paris: Presses Universitaires de France 1948, p. 302.)

31. That would it seems be thanks in only minuscular part to 'earthly essences,' for "says Fabian . . . we are a middle-sized planet going around a relatively small star in a not-very-big galaxy that lies in a not very large group of galaxies. We have two or three big galaxies in our local group with 20 members altogether, but there are clusters in the sky with as many as 1,000 big galaxies. That pretty much puts us in our place" (*Cambridge Alumni Magazine*, Easter Term 2000, p. 23). Further to this 'reality-check,' "Our own galaxy, the Milky Way, contains [upward of] a . . . billion stars" (p. 20); probes of the Hubbles Space Telescope lead "astronomers to estimate that our universe holds around one trillion galaxies" (p. 25).
32. "The Platonic suggestion is that a thing could exist *because* its existence was [ethically] needful or required . . . we could use the idea of *synthetic necessities* here" (Leslie 2001, p. 172). The 'because', Leslie is saying, is that of synthetically necessary entailment.
33. Leslie now stresses that this is not to say "that *our universe* is the best possible" (Leslie 2001, p. 135). He identifies 'the world' with 'the divine mind and its thoughts' (or the divine minds and their thoughts) and writes: "Covering everything worth knowing, the divine thoughts would presumably extend to the structures of vastly many possible universes. The structures would all of them be known in all their details so that . . . those universes would one and all be more than merely possible. Like our universe, they would actually exist inside the divine mind. It would be odd if ours were the best of all" (p. 136). But the *world* – this mind (or these minds) with thoughts that extend in excruciating detail to this universe of ours, and to many other universes – is the best of all possible *worlds*. It is better, for example, than a mind that comprehended only the best possible *universe*, and better than any mind that did not comprehend this miserable 'island universe' in which we live our little lives. It is only "reality as a whole [that is in Leslie's current cosmology] guaranteed to be infinitely good" (p. 135), it is only the infinite mind or minds. Whatever.
34. As in Sergeant Friday's signature-line in the television series and movie *Dragnet*: "The facts, ma'am, just the facts." With appreciation and respect.
35. Regarding propositions, one cannot do better first than to read "Propositions" in Cartwright (1987), which regarding a particular example explains why (i) "what A asserted, namely, that *p* [the proposition that Botvinnick uses it, the French Defence]," is not; (ii) "A's asserting on that occasion that *p*"; (iii) "asserting that *p*" (p. 36); (iv) "what A predicated of Botvinnick"; (v) "what A uttered, namely the words 'Botvinnik uses it'; (vi) "A's uttering those words on that occasion"; (vi) "uttering those words" (p. 37); (vii) "the token of 'Botvinnick uses it' produced by A"; or (viii) "the meaning of the words 'Botvinnick uses it'" (p. 51); *and* that (i), that proposition, is what B referred to when he said "That"s true" (p. 36). Having indicated several things that propositions are not, questions remaining are said to include what these things are that we often say are true, how they are to be identified and discriminated one from another, and how they are related to things (perhaps other things) that we sometimes say are true such as 'persons's beliefs' (what they believe, and their believings of them – cf., pp. 72–3), 'thoughts' (again, thinkables, and thinkings of them), and so on.

36. This is so far verbatim for (1) in "A Neglected Theory of Truth" (Cartwright 1987, p. 74), but for the deletion of 'that' which in (1) follows 'the fact.' This deletion solves the problem of (1), which is that "the final occurrence of 'p' [in (1)] does not occupy a variable-accessible position" (Ibid.). The first conjunct of my formula has, for example, the instance, 'if *that today is Sunday* is true, then *that today is Sunday* is identical with the fact *that today is Sunday*.' In this conditional sentence, '*that today is Sunday*' is a referring expression that stands for a proposition. It stands for a particular proposition that, given that the antecedent of the conditional proposition expressed is true, is a particular fact. This makes 'the fact *that today is Sunday*' grammatically like 'the dowager Clementine' in which referring expression the referring expression 'Clementine' is preceded by what in a suitable context can be a complete referring expression in its own right, as in 'the dowager was not amused.' Similarly for 'the fact' as in 'the fact, I confess, does me no credit'.
37. He indicates that, even if it is true that it is not raining, it is not a fact that it is not raining – "¬ p is typically not verified by any fact, but instead by what I shall call a 'negative condition'" (p. 194b). And he is against at least some disjunctive facts: If it is a fact that p, and a fact that q, there is not "a third disjunctive fact" that p v q (Ibid.). I have resisted a liberal use of warning quotes, since it seems unnecessary in that way repeatedly to remind that these things that Koons says of the objects of which he would speak, his 'facts,' are clearly false of facts properly (ordinarily) so-termed.
38. There are ready-to-hand things that agree with some of Koons's views regarding 'facts'; there are the things picked out by gerundives such as 'today's being a weekday,' '2's being greater than 1,' 'there exist some dependent things,' and so on. If asked what makes it true that today is a weekday, an answer could be, "Today's being a weekday!" That would not be an informative answer, or an answer to the question intended. But it would be an answer that could be offered perversely, *if*, as I *think* could be made out, when today is a weekday, *today's being a weekday* – that *state* of things – is not *identical* with the proposition *that today is a weekday*, or, equivalently, with the fact. Similarly, perhaps, for every true proposition. Cf.: "[W]e can hold facts to be whatever it is that makes true sentences (or utterances) true; in the simplest case, the fact that makes 'S is P' true is merely *S's being P*" (Nelson 1998, p. 90). Mark Nelson, however, usually makes do with 'that' – referentials for his 'facts,' and so at least then courts confusion of his 'Russellian facts' with *bona fide* facts.
39. Similarly for Mark Nelson's "Bertrand Russell's Defence of The Cosmological Argument" (Nelson 1998), which, under the name 'fact', makes much of not facts, but 'Russellian facts.' It is still, on page 1 of Koons (2000) by facts that "the truth or falsity of . . . propositions is determined," facts, however, that are soon to be the "relata for causal relations, whether we call these objects possible 'facts', 'situations', or 'states of affairs'" (pp. 4–5). It is not important to Koons whether or not his 'facts' that make propositions true and that (some of them) stand in causal relations are facts. The latter 'facts' "are identified with pairs consisting of an actual situation-token and a type that it supports" (p. 57; cf., p. 16). 'Fact' in excerpts from Koons (1997) is replaced throughout by 'situation' in Koons (2000, pp. 110–2, 113–9 – Section 8.5 on p.113 is new material). The same switch is made without comment in Koons (2001).

Chapter VII

1. Richard Swinburne finds intimations of the design argument in the Bible. "The prophet Jeremiah lived in an age in which the existence of a creator-god of some sort was taken for granted. What was at stake was the extent of his goodness, knowledge, and power. Jeremiah argued from the order of the world that he was a powerful and reliable god, that god was God" (Swinburne 1994a, p. 48b).
2. Immanuel Kant considered decisive a radical objection of the kind Philo tries and Cleanthes embarrasses. "[T]he concept of a being which is the original basis of nature, *viz.*, a being such as cannot at all be given us in experience . . . is . . . inadequate for dogmatic determinations" (*Critique of Judgment* 397, quoted in Pereboom 1996). That the concept of such a being cannot be given *at all* in experience, that we have experience of no beings at all like it, is false. It is, I think, less of a stretch from things directly experienced than are many entities that figure in widely accepted theories of matter. And whether this concept can figure in a 'dogmatic determinations' is beside the point of a discussion of evidence bearing on the probability of a hypothesis in which this concept figures. Kant devotes several pages of his *Prolegomena to Any Future Metaphysics* (350ff) of 1783 explicitly to Hume's *Dialogues*, but there is at most one where one would expect to find *many*, in his *Critique of Judgment* of 1793: A reference on 420 may be to the *Dialogues*, Part 4.
3. Cannot an event "cry out for some explanation other than coincidence" (Johnson 1999, p. 44) whether or not an explanation other than coincidence can be envisioned? I think not, and I suspect that David Johnson agrees. For his Big Break that 'spells out' π to the first 1,000 million digits on a really big 'checkerboarded' billiard table, I am sure he has in mind an explanation. He does not say, but I think intends, that in the case, since there is available an explanation other than 'coincidence', the best such other explanation is to be believed. I do not agree with that. Before deciding what, if anything, to believe regarding what was going on, I would want to think about the intrinsic plausibilities of explanations. They *could* be as inferior to that of 'coincidence', as would have been the likelihood of the break supposing it, is inferior to what would have been its likelihoods supposing them. William Dembski offers a recipe for 'eliminating chance' *whether or not there* is available some other explanation, and so of course without regard to the intrinsic plausibilities of alternative explanations (Dembski 1998). 'Nice work if we could get it if we tried!' Cf.: Sobel (forthcoming).
4. Similarly for 'the surprising' and 'the extraordinary'. "It is unsurprising that a monkey types 'nie348n sio 9q;c', but when she types 'I want a banana!' we are astonished [for we assume that the monkey is typing randomly on an unrigged typewriter]. . . . The crucial feature of surprising events seems to be that they challenge our assumptions about the circumstances in which they have occurred. . . . The difference [between the monkey-cases] is that in the second case there [are] alternative but not wildly improbable hypothes[es] concerning the conditions in which the event took place, upon which it is much more probable [than it is on the random-unrigged circumstances we assumed]. . . . The event is surprising in that it forces us to question whether [it] really was an accident [by random, unrigged chance]" (White 2000, p. 270).

5. r is of relevance to q independent of the relevance of p to q *if and only if* $P(q/r) \neq P(q)$ and $P[q/(p \& r)] \neq P(q/p)$.
6. This is the account of conditional probabilities elaborated in Sobel (1997a). In that paper, however, conditional probabilities are 'freestanding' and not, as is more usual, identified with ratios, so that they exist only when these ratios exist. The 'freestanding' conditional probability P(q/p) is, however, identified with the ratio P(p & q)/P(p) when P(p) is greater than zero. In the 'model' of these 'freestanding,' learning something e can lead 'by conditionalization' to a change in the probability for h even when initially this probability was zero. I do not, as Bradley Monton does, consider it a defect of this 'model' that in it "the only way to change from a zero to a non-zero probability assignment [to e] is by updating on a proposition [h] that was assigned zero prior probability" (Monton 2002, p. 51). Richard Jeffrey advocated 'freestanding' conditional probabilities in "Methodology Probabilized," Lecture 2 of his Hägerström Lectures in Uppsala (19 May 1991).
7. P_n is the subject's probability function *at* the time he learns p; P_0 is his probability function through an interval *to* but not including the time he learns p. The principle is for ideal learners subjects for whom learning is *instantaneous*.
8. Conditionalizing on the strongest proposition learned for sure is not always reasonable (Cf., Jeffrey 1992, pp. 122–4). For a case otherwise, suppose probabilities as you approach a curve beyond which lies a traffic light include $P(\text{red}) = P(\text{green}) = 1/3$, and that upon rounding the curve and as a consequence of seeing the light they are $P_n(\text{red}) = 1/3$ and $P_n(\text{green}) = 2/3$. You are now sure that the light is functioning and that it is not yellow, but being red/green color-sight deficient, you are only somewhat more inclined to think it is green than that it is red. You have learned something for sure, namely, that it is either red or green. Let this be the strongest proposition you have learned. Assume your old probability for a safe drive through conditional on that disjunction, $P_o(\text{safe/red} \vee \text{green})$, is $\frac{1}{2}$. You will not, if rational, conditionalize for $P_n(\text{safe}) = \mathbf{1/2}$ if your new conditional probabilities include $P_n(\text{safe/red}) = 0$ and $P_n(\text{safe/green}) = 1$. Given these new conditional probabilities, your new probability for a safe drive through should be **2/3**, for your new probabilities include $P_n(\text{red}) = 1/3$, $P_n(\text{green}) = 2/3$, $P_n(\text{red} \vee \text{green}) = 1$, and (we are taking for granted) $P_n(\text{red \& green}) = 0$. In (Earman 1992) the rule is that "if it is learned for sure that E and . . . E is the strongest such proposition, then . . . $Pr_{new}(.) = Pr_{old}(./E)$" (p. 316). But in Earman (2000) it is rather that "when an agent has a learning experience **and the content of the experience is fully captured** by a proposition E, the agent's degree of belief function Pr_{new} after the learning experience is related to her degree of belief function . . . before the learning experience by the rule . . . $Pr_{new}(.) = Pr_{old}(./E$" (p. 26; emphasis added). Cf.: "Conditionalization . . . is applicable. . . . [when there is a] proposition E . . . of which it can correctly be said that **what the agent learned . . . is that E is true**" (Jeffrey 1990, p. 165; emphasis added). In the case of this note, it cannot be correctly said that 'what you have learned' is that (red $\vee$ green), for this disjunction does not 'fully capture' the content of your learning experience. In such a case, "probability kinematics . . . a generalization of conditioning [is] apt" (Jeffrey 1992, p. 124).
9. The idea graphically illustrated, expressed now algebraically, is that – *given the Bayesian postulate that conditional probabilities measure evidential relevance* – if Pr(q/p) is my probability for q conditional on p before becomingcertain of

p, Pr′(q/p) is my probability for q conditional on p upon becoming certain of p, and *Pr(q/p) = Pr′(q/p)*, then, assuming my probabilities are 'coherent' and that they conform to elementary principles of probability, my probability for q upon becoming certain of p, Pr′;(q), equals Pr(q/p). Why? Because we have, **(i) Pr(q/p) = Pr′(q/p) and (ii), Pr′(p) = 1**. And so, by the 'definition of conditional probability,' (iii) Pr′(q/p) = Pr′(q & p)/Pr′(p), and, from (ii), by an elementary principle of probability, (iv) Pr′(q & p) = Pr′(q). It follows from (ii), (iii), and (iv) that, (v) Pr′(q/p) = Pr′(q). Therefore, from (i) and (v), **Pr′(q) = P(q/p)**.

10. Additivity is, for ideally consistent and well thought out degrees of confidence, only slightly stronger than the principle that, for any propositions p and q, if $\sim\Diamond$(p & q), then P(p v q) = P(p) + P(q). Let this latter principle be *Additivity**. This slightly weaker condition would have sufficed present purposes, since Additivity (our condition) is entailed by Additivity* and Equivalence. For a conditional proof of this entailment, we may assume the antecedent of Additivity,

(1)	P(p & q) = 0, and deduce its consequent, P(p v q) = P(p) + P(q)thus:.	
(2)	P(p v q) = P([(p & ∼q) v (∼p & q)] v (p & q))	Equivalence
(3)	P(p v q) = P([(p & ∼q) v (∼p & q)] + P(p & q)	2, Additivity*
(4)	P(p v q) = P([(p & ∼q)] v (∼p & q)]	1, 3
(5)	P(p v q) = P(p & ∼q) + P(∼p & q)	4, Additivity
(6)	P(p) = P[(p & q) v (p & ∼q)]	Equivalence
(7)	P(p) = P(p & q) + P(p & ∼q)	6, Additivity*
(8)	P(p) = P(p & ∼q)	1, 7
(9)	P(q) = P[(p & q) v (∼p & q)]	Equivalence
(10)	P(q) = P(∼p & q)	9, Additivity*, 1
(11)	P(p v q) = P(p) +P(q)	5, 8, 10: QED

11. This covers Earman's explicit qualification: "satisfying (L01) and (L02)" (pp. 134–5). "(L01) . . . all logical truths . . . are transparent to the . . . agent" and have probability in a probability function representing his credences. "(L02) . . . the agent is aware of every [possible] theory [and has a credence for it" (pp. 121–2).
12. Did I do that right?! There is the step when I *combined* with evidence Y with evidence on R, to calculate your probability for A conditional on this combined evidence. For this calculation I used your initial probabilities that you had before receiving evidence Y, and presumably *updating* your probabilities on it. Should not I have proceeded similarly in my calculations, first 'updating' as you had on Y the probabilities needed for an application of Bayes's Theorem to update on R your previously updated on Y probability for A? Might not this more complicated and time-consuming calculation, that followed the course of your actual mental operations, have produced a different mathematical result? Fortunately, no. "The result of successive conditionalization on two statements is [necessarily] the same as that of conditionalization once on the conjunction of those statements. . . ." (Jeffrey 1981, p. 83), which is "straightforward to verify" (Ibid.).* To illustrate in our case, updating the initial function P on Y leads to P′(A) = ½, P′(B) = 1/6, and P′(C) = 1/3 and can (with some effort) be seen to lead to P′(R/A) = 1/9, P′(R/B) = 2/6, P′(R/C) = 4/4 and so to

$P'(A/R) = [(\frac{1}{2})(1/9)]/[(\frac{1}{2})(1/9) + (1/6)(2/6) + (1/3)(4/4)] = 2/16$. Let the result of updating $P'(A)$ on R be $P''(A)$. We see that $P''(A) = 2/16 = 1/8$. That is, as Richard Jeffrey says it must be, the same as the result we reached by updating $P(A)$ on (Y & R): $P[A/(Y \& R)$, calculated above, is 1/8. [*A corollary of this result is that when 'learning by conditionalization' it does not matter in what *order* things are learned. It is observed in (Jeffrey 1990, pp. 182–3) that this is not true for 'learning by Jeffrey-conditionalization, or probability kinematics', a 'generalization' of learning by conditionalization mentioned in note 8.]

13. It is taken for granted by Cleanthes and Philo that a designer would be a *temporal* being, there being no prospect of *inferring* from what would be the work of a designing or creating of nature being and *eternal* being. Even allowing the *possibility*, Philo might say, there is no hope of a *proof from experience* of such a being. (Cf., Fales 1997.)
14. Philo makes a related observation when suggesting that there is no explanatory advantage in referring order in the material world to an intelligent working from a plan, which is an orderly arrangement of ideas. "We have . . . experience of ideas, which fall into order, of themselves, and without any *known* cause: But, I am sure, we have a much larger experience of matter, which does the same; as in all instances of generation and vegetation, where the accurate analysis of the cause exceeds all human comprehension" (Part 4, p. 127). Optimistic investigators of 'principles of order inherent in the world' (Part 6, p. 137) believe that much in their way is not beyond human comprehension.
15. I make a questionable assumption regarding Philo's 'Infinite Time', namely, that *intervals* of it, and *the events that take place in these*, are *separable*, so that a given interval could have been occupied by entirely different events, and in particular, an interval during which the world is very orderly could have been an interval during which it was chaotic.
16. "[D]espite N. Kemp Smith's editorial note, it is not entirely clear when one looks at the manuscript whether Hume intends this [Part 12, p. 177n18] as a note written in his own person" (J. C. Gaskin in Hume 1998, p. 209n121).
17. This option in which belief is suspended has a strong and weak form. Suppose total evidence is E, and available hypotheses are $H_1, \ldots, H_n$. The weak form says distributively of each of the hypothesis, h, that it is not believable: $P(h/E) < \frac{1}{2}$. Let N be the null or 'none of the above' hypothesis: N is equivalent to $\sim(H_1 \vee \cdots \vee H_n)$. The strong form says that N is to be believed, that is, that is to be believed of the available hypotheses collectively, than none is true: $P(N/e) > \frac{1}{2}$.
18. "Inferences to the best explanation are extremely common in science and everyday life" (Katz and Kremer 1997, p. 68). True. "The general form of such reasoning is this: Of the available and competing possible explanations of the fact that F, E is the best. . . . [It is reasonable to infer E]" (Ibid.). It is false for the following reasons. [So is a premise of a new 'cosmological argument' that is derived from this general form: "Given that (i) there is a possible explanation of the fact that F and (ii) any possible explanation of the fact that F entails P, it is reasonable to believe that P" (p. 65). Another premise of that argument is at least troubled: see note 30 of the previous chapter.]

    When all of the available and competing possible explanations of a fact are very bad, it can be *unreasonable* to believe any of them. If a prosecutor has

produced one fantastic theory of the crime after another, all of which say Jones did it, a judge may direct an acquittal without hearing from the defense to learn whether it has a better theory that says that Jones did not do it. "If that is the best you can do," he might say, "there is no case to go to the jury. Not only have you not ruled out reasonable doubt. You have not given grounds for reasonable belief." Also, when the only explanations in hand *or expected* for a fact are very poor, it can for a person be reasonable to conclude, not that the fact has an explanation, but that it quite lacks an explanation: A case to this point for some minds can be the fact that there are contingent entities when the only explanations expected run in terms of the creative wills or necessary beings. (Contrary to Katz and Kremer, p. 69.)

19. Salmon does not agree. Grounds for his opinion that Hume was an atheist, grounds that he finds in the *Dialogues*, run together, however, the question of whether the book's arguments are strong against the existence of a god who would be good in human terms and whether they are strong against the existence of an intelligent designer, even one devoid of a moral character.
20. Salmon describes Part 8 as "as a rather clear anticipation of a . . . theory of biological evolution" (Salmon 1978, pp. 159–60). But he disarmingly confesses that in this he may be "indulging in wishful thinking" (p. 160).
21. Cleanthes cites interesting empirical evidence. He has the good taste to pass over without mention James Ussher's biblical research, notwithstanding that: "Nearly all Hutton's [James Hutton, 1726–97] Edinburgh contemporaries believed that the earth had been created in 4004 BC, the date which Archbishop Ussher [1581–1656] had calculated by correlating Middle Eastern and Mediterranean chronologies with Holy Writ. Ussher's chronology was printed in the margins of bibles from 1701 onwards. . . ." (Jones 1986, p. 124). Of possible interest are calculations conducted in another theological tradition. "The *Sefer ha-Temunah* speaks of Sabbatical cycles. . . . This is based on the Talmudic teaching that 'the world will exist for six thousand years, and in the seven-thousandth year, it will be destroyed. The *Sefer ha-Temunah* state this . . . is merely one Sabbatical cycle . . . [of] seven. . . . According to [one opinion we are in the seventh cycle and] the universe [was] forty-two thousand years old when Adam was created. . . . Rabbi Isaac of Akko [1250–1350] writes that since the Sabbatical cycles existed before Adam, their chronology must be measured, not in human years, but in divine years. . . . This has startling consequences, for according to many Midrashic sources, a divine day is 1,000 earthly years long, and a divine year . . . is equal to 365,250 earthly years. Thus, according to Rabbi Isaac of Akko, the universe [is] 15,340,500,000 years [old]" (Kaplan 1993, pp. 6, 9). According to some recent 'popular science,' the physical universe is about 15 billion years old.
22. There are certain ideas relevant to the present subject that could have come to Hume's attention that are not in evidence in the *Dialogues*. The *catastrophic theory* of natural history floated by Philo contrasts with *uniformitarian theories* that see certain processes working continually, and at more or less even rates, throughout history and even now. Such was the view of James Hutton's of Edinburgh, and was probably 'in the air' in Edinburgh, several years before Hume's death. Hutton's ideas, first published in 1785 but largely formed by 1767, culminate in a view of an open past and open future, and of an ancient earth. He writes: "The result, therefore, of our present enquiry is, that

we find no vestige of a beginning, – no prospect of an end" (James Hutton, *Theory of the Earth*, volume I, 1795, p. 304; from a page reproduced by Jean Jones, op. cit., p. 121). He was, furthermore, the first articulate proponent of geological uniformitarianism: "Hutton declared that the earth processes which are at work today, such as erosion, deposition, uplift, folding and volcanic activity, operated in the same manner in the past and will continue to do so in the future. . . . Although Hutton declared that the surface of the earth had been recycled many times . . . he denied that these events had been brought about by sudden and major catastrophes. . . . 'We are not to suppose, that there is any violent exertion of power, such as is required in order to produce a great event in a little time; in nature we find no deficiency in respect of time, nor any limitation in regard to power' [*Theory of the Earth*, volume I, p. 182]" (Jones 1986, pp. 123, 124). This view was directly opposed to that of many of his contemporaries who favored 'catastrophist' explanations" (Jones 1986, p. 124). Hutton's uniformitarian geology, developed by Sir Charles Lyell in *Principles of Geology* (1830–3) is far more congenial to evolutionary theories of living organisms that feature gradual accumulations of chance adaptive modifications than are catastrophic models that propose periodic local and even global reductions to rubble, with annihilations of all local and even global life-forms.

There is no evidence in the *Dialogues* that Hume was conversant with Hutton's uniformitarian speculations. Philo retails the more popular catastrophic views. There is no evidence of Hume's contact with James Hutton: "Hutton's early years in Edinburgh [following his return in 1767, after an absence of thirteen years] coincided with the heyday of the Scottish Enlightenment and he counted the most brilliant men in the city among his friends, notably William Robertson; Adam Ferguson; Lord [James Burnett] Monboddo . . . ; Adam Smith; and Joseph Black. Only Hume is missing from the list and no-one has yet discovered how well Hume and Hutton knew each other" (Jones 1986, p. 120). James Hutton does not appear in the indices to either Mossner (1980) or Mossner (1943).

23. Developments, without 'informed interventions', *to* these precocious molecules, '**prebiotic**' evolution, are matters of speculation on which I gather that no clear consensus has emerged. Samples of this speculation can be found in (Dawkins 1976), (Dawkins 1986), and (Kauffman 1993). Lines of (Kauffman 1993) are glossed in (Dembski 1998, p. 61), with references to work from 1984 to 1995 by four like-minded speculators. It is a research program in which numerous experimentalists and theoreticians are engaged. Dean H. Kenyon "date[s] the inception of this field of research to Stanley Miller's pioneering work in the early 1950s" (Thaxton, Bradley, and Olsen 1984, p. v). Skepticism concerning prospects of this research program is painstakingly argued in (Thaxton, Bradley, and Olsen 1984). They contend that a feature of the "prebiotic simulation experiments" to which these speculations *appeal* – namely, that these simulations "owe their success to [a] crucial but *illegitimate* [for purposes of that appeal] role of the investigator" (p. 184) – has these experiments actually telling against the program's object, and for "what Michael Polanyi has called a 'profoundly informative intervention'" in the development of 'prebiotic' earth (p. 184). They argue that, by the investigators's invariably establishing experimental conditions that deviate from those probable for early earth, it is as if they were themselves

exercising "intelligent influence . . . over 'prebiotic' earth" (p. 184). Notwithstanding this criticism of 1984, the research program to make out natural, without 'informed intervention,' processes of prebiotic evolution continues apace. Probably this is because of the profound intrinsic implausibility to researchers' minds, of possible informed-intervention hypotheses. There are, for such hypotheses to confront, the Humean questions, how, and by what manner of Intelligence, this intervention would be done. Pending *some* elaboration please, Polanyi's 'profoundly informative interventions' are *words only*.

24. Hume *wondered* whether greater success along the lines of his new theory *could* be reasonably expected, which was reasonable. Kant was *certain* that greater success *could not* be reasonably expected, which was not reasonable. "It is . . . quite certain that we can never get a sufficient knowledge of organized beings and their inner possibility, much less get an explanation of them, by looking merely to mechanical principles of nature. Indeed, so certain is it, that we may confidently assert that it is absurd for men even to entertain any thought of so doing or to hope that maybe another Newton may some day arise, to make intelligible to us even the genesis of but a blade of grass from natural laws that no design has ordered. Such insight we must absolutely deny to mankind" (*Critique of Judgment* 400, translated by James Creed Meredith). "[A] source of . . . organized beings . . . without . . . [operation of] a design, *could* . . . be buried among the secrets . . . of nature" (Ibid.). But, Kant was *certain* that, if it is, finding it out is absolutely beyond the capacity of human inquirers.
25. While [E & ∼(E + D)] entails the denial of that involvement, it does not entail the denial of every involvement of a designer or guiding intelligence. It simply does not entail anything of that sort.
26. Cf. Jon Dorling's Bayesian analysis of the realist/anti-realist debate in philosophy of science. If T is a realist theory of some subject, O is "observational consequence set" and R is its "central realist ontological claim" (concerning, for example, protons, electrons, and such), then of course P(O) "will always and for everyone be greater than or equal to P(T)" (Dorling 1992, p. 363). But, even if P(O) is much greater than P(T), that is not a reason for "accepting positivism" (p. 362) as far as T, and believing instead of it (*sic*) only O, let alone a reason for not only not believing R, but believing ∼R. For (I now elaborate in a manner of which I believe Dorling would approve) perhaps T = (O + R) is on the evidence much more probable for you that Pos = [O& ∼(O + R)]. Let me remind you that + here is *combining*, not merely *conjoining*.
27. "McMullin (1993, p. 378: *Studies in the History and Philosophy of Science* 24) summarizes some of the relevant facts as follows: If the strong nuclear force were to have been as little as 2% stronger (relative to the other forces), all hydrogen wold have been converted to helium. If it were 5% weaker, no helium at all would have formed and there would be nothing but hydrogen. If the weak nuclear force were a little stronger, supernovas could not occur, and heavy elements could not have formed. If it were slightly weaker, only helium might have formed. If the electromagnetic forces were stronger, all stars would be red dwarfs, and there would be no planets. If it were a little weaker, all stars would be very hot and short-lived. If the electron charge were ever so slightly different, there would be no chemistry as we know it. Carbon (^{12}C) only just managed to form in the primal nucleosynthesis. And so on" (Sober 2002).

28. Roger White argues that "while E′ [some universe is life-permitting,] is more probable given [MU] than it is given ∼[MU], [MU] has no effect on the probability of E [which is that α, our universe] is life-permitting" (White 2000, p. 262). His argument for each conjunct depends on the mistake of identifying the negation ∼MU with (∼M & R). His trouble starts with the defective stipulation, "let . . . M = m [the number of universes is] large (the Multiple Universe hypothesis" (p. 262). The defect is that the hypothesis that the number of universes is large, for which I use M as White does, is not identical with his Multiple Universe hypothesis for which he also uses M, and I use MU. In terms of my abbreviations, M is only a conjunct of MU. As implied above, ∼(MU) is ∼(M & R), which is equivalent to (∼M ∨ ∼R), and to [(M & ∼R) ∨ (∼M & R) ∨ (∼M & ∼R)]. Lee Smolin's Evolutionary Many Cosmoi theory (Section 8.5) is an (M&∼R)-theory. It makes life-permitting cosmoi especially likely. For any 'typical' cosmos, including this cosmos of ours, if as his theory assumes it is 'typical', it makes very likely that it is life-permitting. ['Typical' in what respect? I believe the answer goes to the chance-process of a cosmos by which its parameters are determined. In the 'typical' cosmos the chance of this process for life-permitting parameters is high. If I am right about Smolin's use of 'typical', then he did not need to include in his theory that our cosmos is typical, since the rest of his theory implies that it is overwhelmingly probable that our cosmos is typical. I come back to this matter in a note below to text of Smolin's.]

    Here is White's argument somewhat simplified: Let M say that the number k of universes is large. Let n be the number of possible parameter-assignments. Then $P(E'/MU) = 1 - (1 - 1/n)^k$. Therefore, P(E′/MU) > P(E′/∼MU). For the inference we have that if k is very large, P(E′/MU) is nearly 1: $(1 - 1/n)^k = [(n - 1)/n]^k$; the limit of $[(n - 1)/n]^k$ as k goes to ∞ is 0. But what about *P(E′/∼MU)*? We have that $P[E'/ (\sim M \& R)] = 1 - (1 - 1/n)^j$,, where j < k.: ∼M says that the number j of universes is not large. This conditional probability is smaller than P(E′/MU). What we *want*, however – since MU is (M & R) – is that P[E′/∼(M & R)] is smaller: White has no argument for this. I think there is none.

29. Van Inwagen adds that "[t]here are various conceivable mechanisms that are more realistic. . . . [For example] that the [cosmoi arose as fluctuations] in some pre-cosmic analogue of the quantum field. . . . If there were a large number of 'possible [pre-cosmic analogues of the quantum field]' and only a very few of them had the right properties to be random cosmos generators, then we should have made no progress in appealing to [a random generating Field] in our attempt to deal with the teleological argument" for a pro-life Tuner (van Inwagen 1994, p. 143). Does van Inwagen think there is a difference here between this field theory and pro-life Tuner theories *vis-à-vis* the evidence of fine-tuning for life? There seems not to be. A field-theorist could say back that if are many possible Tuners, and only a few are pro-life Tuners, no progress is made by an appeal to a pro-life Tuner in an attempt to deal with his cosmic argument: as "[o]ne might raise the question . . . why [this pre-cosmic field" (Ibid.), so one can ask why this Tuner. I believe that in thinking about the contest between many cosmoi theories of various sorts and pro-life Tuner theories, we can get back to Bayes's Theorem for a Hypothesis in a Partition, and that van Inwagen has not found a Bayesian-relevant difference between theories of fine-tuning. These,

recall, will none of them be extraordinary theories with which Leibniz would say 'we can stop'.

30. The intelligent fine-tuning of which I speak, though with little comprehension, would take place *in time*. To my mind no other fine-tuning activity makes any sense *at all*. I view similarly many cosmoi theories that locate random processes for determinations of a cosmos's parameter *in the time of this cosmos*, and those that gesture towards supercosmic *processes* for the parameters of cosmoi that *take place out of the times of their cosmoi.*

31. White reasons that "the more universes there are, the more likely it is that [E′] *some* universe [thanks to its 'tuning'] supports life. . . . So . . . **[i]f** the extent of our knowledge was just [that *some* universe supports life], then this would count as evidence for M [the Multiple Universe Hypothesis]" (White 2000, p. 264; bold emphasis added). [The inference 'so' depends, for White, on something that he mistakenly thinks he has proved, namely, that P(E′/the Multiple Universe Hypotheses) > P(E′/ ∼the Multiple Universe Hypotheses). See note 28.] By this argument, however, for *any* particular 'tuning,' if the extent of our knowledge were that there is some universe with that tuning, this would count as evidence for M, and *equally good* evidence for M, for White assumes that there is exactly one tuning that supports life: He writes, "Let T_1 be the configuration which is necessary to permit life to evolve" (p. 262). By an analogous argument, if the extent of my knowledge of the history of bridge were that (i) every hand dealt in bridge has been fairly dealt and (ii) that a hand of a particular constitution has been dealt, 'this would count as evidence that many bridge hands have been dealt.' It would be evidence, and just as good evidence, regardless of the character of that hand, and whether it was made up of thirteen spades, or of the cards, 4♡, K♤, 7♢, 8♢, A♤, 10♤, K♤, 10♡, Q♢ , J♡, 2♤, 5♤,Q♤. This, I think, 'cannot' be right, though it is not entirely clear to me why not.

To think about the evidence of knowledge, we should think about probabilities 'without this knowledge'. Here comes, for a special case concerning tunings, the beginning of an argument that I have not fully assembled. The case is one in which I am sure that I will acquire the knowledge that some universe has 'this' or 'that' tuning without learning anything other than this that is relevant to the Multiple Universe hypothesis, and that I will conditionalize on this knowledge. Let Ej′ say that *there is a universe with 'tuning' T_j*. Suppose, for the case, that: (a) I am sure that there are n possible 'tunings'; (b) for every i, $1 \leq i \leq n$, I am not sure that not Ei′; **(c) I am sure that, for some i, $1 \leq i \leq n$, I will, at a time when I know nothing of relevance to MU, learn that Ei′ without learning anything other than Ei′ that is relevant to MU, and that I will 'update' my probability for MU on this learning by conditionalizing;** and (d) there is a probability p such that, for every i, $1 \geq i \leq n$, my probability P(MU/Ei′) = p. Then, assuming that I 'can put two and two together'

(1) I am sure that *my probability for MU **will be*** p. from (c) and (d)

From this, if I am an ideal intellect, it follows by Bas van Fraassen's Principle of Reflection* (van Fraassen 1984) that,

(2) *my probability for MU **is** p*.

(3) for every i, $1 \geq i \leq n$, $P(MU/Ei') = P(MU)$. from (d) and (2)
(4) learning Ei′, and nothing else of independent relevance to MU, *would* ***NOT*** *(tend to) confirm MU.* from (3)

[*The principle says that the probability of proposition q conditional on the proposition that the probability of proposition q *will be* x *is* x. It is maintained in (Sobel 1987c) that Reflection is valid for ideal intellects.] A similar argument begins with the sentence, 'Let 'Ej' say that *this universe of ours has 'tuning' Tj*', and ends with the line, '(4) learning Ei, and nothing else relevant to MU, would not (tend to) confirm MU.' "But what if, contrary to (c), I am not sure that I will have such a learning experience? What if I am indeed sure that I will not have such a learning experience? It seems that that should not matter, and that even given only (a), (b), and (d), my present probability for MU should be p. What remains is to prove that this is so, as I think, or that it is not so.

32. You draw a straw from a bundle of 1,048,576, knowing that you will be 'killed before you know it' unless you draw the shortest straw. For all you know there have been millions of millions of such draws. You survive. What can you say? "Only this: that you didn't know whether you had an unobserved benefactor or whether you were 'surrounded' by millions of millions of such drawings. . . ." (p. 145). If you considered both of these hypotheses to be unlikely, you could, and presumably would, say more, for example, "or neither." [Van Inwagen's setup includes in addition that "as far as your knowledge went, the . . . two hypotheses [benefactor, millions of draws] were about equally probable" (p. 145). However, if the idea is that their 'prior probabilities were for you equal', you should say that in all probability you have an unobserved benefactor, since only that hypothesis predicts your survival. And if the idea is that their 'posterior probabilities were for you equal,' then *of course* you can say only that you didn't know whether it was one or the other.
33. A review by Graham Oppy of an anthology (Oppy 2001) sent me to Smith's paper. He cites it when commenting on Shimony's discussion of the possibility that some laws of nature are results of evolution (Shimony 1999). I hoped that I would find such a theory in Smith's paper. I think I see in it an *opening* for such a theory.
34. On White's Multiple Universe theories, "[t]he events which [determine parameters of] universes are **not** causally related in such a way that the outcome of one renders the outcome of another more or less probable. . . . Wheeler universes [for example], like dice, 'have no memories'. . . . Previous big bangs in the sequence have no effect on the outcome of any other big bang" (White 2000, p. 263).
35. I am not sure of the exact form of this 'addition'. Let S be the settings of parameters in a cosmos C and S′ the settings in a cosmos C′ that 'bounces out of C'. Probably the idea is that S′ is selected by a random chance process that assigns equal chances summing to 1 to each setting of parameters in a range that includes all settings 'close to' S. A possibly more appealing idea is that S′ is selected by a chance process that is not random, a chance process that assigns chances summing to 1 to all possible settings, with settings closer to S receiving greater chances: for example, closest settings could together have a chance of $\frac{1}{2}$, next closest settings could together have a chance of 1/4, and so on.
36. This assumption (3), if I understand the sense of 'typical' in it, does not work in Smolin's theory. In the theory, cosmoi can have different probabilities for the

random processes by which their parameters are selected, resulting in values that are conducive to black holes. A cosmos that comes from one with parameter-values that *were not* highly conducive to black holes has a lower probability for parameter-values that are conducive to black holes than does a cosmos that comes from one with parameter-values that are conducive to black holes. *However*, in the theory, not only do most cosmoi have parameters conducive to black holes, but also most cosmoi came from cosmoi with parameter-values that were conducive to black holes. So in the theory, *for most cosmoi, the random-chance processes by which their parameters are determined are 'heavily biassed' in one way or another (see previous note) toward the determination of parameters conducive to black holes.* In the overwhelming majority of cosmoi, not only is there a chance for parameters conducive to black holes, but a great chance. Therefore, in the theory, "that our universe is with overwhelming probability 'typical'" (Shimony, quoted above) in the sense that its chance-process for determining its parameters was heavily biassed toward the selection of parameters conducive to black holes, did not have to be *assumed*: It is a consequence of the theory without assumption (3), and if that is what Smolin means in assumption (3), he did not need to assume it.

According to his theory, the case regarding cosmoi in which parameters conducive to black holes have a chance, and cosmoi in which this chance is great is different from what may be the case regarding planets suitable for life in the sense of life's having a chance of evolving on them and the chances for life on these planets. "Although α is the probability that a randomly selected suitable planet will have life evolve [on it], different suitable planets still might have different probabilities [for life evolving on them]; since α is the value for the ***average*** [suitable] planet, some [suitable] planets may have values that are greater than α while others may have values that are lower" (Sober 2003). Not only might different suitable planets have different probabilities for life, but the story of suitable planets could be such as to predict that different suitable planets have different probabilities for life 'flat across the spectrum of probabilities'. In that case, there would still be (in ordinary parlance) an average suitable planet in terms of the chance for life on suitable planets, but there would not be (in ordinary parlance) a typical suitable planet in terms of that chance. It would be as it is for sizes of families if there were equal numbers of families of sizes two to ten. The average family size would be six, but there would not be a typical size.

37. "How long has this been going on?"*. If the theory says there have been universes with black holes *forever*,** then that a randomly selected universe would have parameters near a peak of the production of black holes presumably has, in the theory, a probability of 1. Indeed, this presumably is so whether or not the theory says that this has been going on forever, if it says that it will go on *forever*. [*George and Ira Gershwin. "Ella and Oscar," Los Angeles, May 19, 1975; Pablo Super 2310 759. Please listen. **'There have been universes with black holes *forever*', what does it mean? It means there is a universe that has infinitely many ancestral universes, a universe that lies in a beginningless line of universes each member of which comes from the preceding member. Similarly for 'there will be universes with black holes forever'.]
38. The theory predicts with some probability that any randomly selected member of the collection of universes strewn in 'supertime'* has the ingredients for life

as we know it, for carbon-based life. The theory as so far stated leaves somewhat open the measure of *this* probability. It says that "**one way** for a universe to make a lot of black holes requires there be carbon and other organic elements," and then that "ingredients for life . . . are **typical** of universes" (p. 204). [*Only universes in the same branch are related in 'supertime', and no universes are contemporaries in 'supertime'.]

39. In particular, the evidence that *a* world is as if fine-tuned for life, E′, is entailed by the evidence that *this* world of ours is as if fine-tuned for life, E, and though the former is predicted by many cosmoi, P(E′/many cosmoi) > P(E′), the latter is not, P(E/many cosmoi) = P(E).
40. Smith thinks Hawking's quantum cosmology is confirmed over "the standard hot big bang model [that says] the universe began . . . from a physical singularity" (Smith 1994a, p. 236 [237n2] – *Analysis*), though he was of another opinion in (Smith 1994b), where he wrote "it is possible that there are no observations that discriminate between Hawking's theory and the theory that the universe began with a lawless singularity" (Smith 1994b, p. 319 – *Dialogue*). "The Dialogue paper was written much earlier than the Analysis paper. The main observational evidence is that Hawking's wave function predicts the initial conditions for the inflationary era, whereas the standard big bang theory does not. So I think Hawking's wave function is better confirmed by the evidence" (Smith, December 9, 1995, correspondence). Let SBG, WSBG, and HAWK be, respectively, a big bang model that features an initial real lawless singularity, a big bang model in which such a singularity is only an unreal limit approached by early states (see the first note to Section 8.4), and Hawking's theory. Let IC be the initial conditions for the inflationary era. Then the likelihood of IC on HAWK is certainly greater than the likelihoods of IC on SBG and WSBG. And the prior probabilities of HAWK, with its wave of real chances when there was nothing, and SBG with its singularity of infinite density and zero size, are perhaps to most minds equal. But the prior probability of WSBG, which is burdened neither by that singularity nor that wave, will to most minds be greater. And so, though HAWK may be better confirmed by the evidence than is SBG, it is unclear that it is to most minds better confirmed than is WSBG.
41. Smith exaggerates in claiming inconsistency. He should have maintained only that the intrinsic plausibility of (AcausTh + Hawk) is much greater than (ClassCausTh + Hawk) because the latter would require beliefs in divine reasons "'of which we have no idea'" (Craig and Smith 1993, p. 243; Smith is paraphrasing a suggestion made by Craig).
42. "Unless the AcausTh cobbled with Hawk is itself thoroughly necessary," an Anselmian might interject. Cf.: Regarding "difficulties attending [Smith's] analysis of the probability of the universe's existing on the HH [Hartle-Hawking] wave function compared to the probability of its existence on the HH wave function plus theism," William Craig notes that, "In a recent colloquium, Plantinga pointed out to Smith that since according to classical theism God exists in all possible worlds, the probability of the universe on the wave-function cannot differ from its probability on the wave function plus theism" (Craig 1997, pp. 292–3). This objection to my use of Smith's argument would fail, however, for lack of relevance. It would implicitly begin a change of subject from possible late-science permutations on the argument from design in which contingent theisms would figure in explanations of the kind familiar in ordinary life, courts of law, and

science, eventually to an extraordinary would-be (see Section 5.2 of Chapter VI) explanation of the kind of cosmological metaphysicians that would explain in entirely necessary terms.

43. Craig favors big bang cosmologies that do not postulate real initial singularities, extraordinary lawless states of infinite density, zero size, and no temporal duration. (Cf., Craig 1991, pp. 496ff; Craig and Smith 1993, pp. 258ff.) A theory can avoid such states while still affirming a beginning to history in the sense of a time when there was nothing physical and nothing happened, at all times after which there was something and at many if not all times after which something happened. For a Big Bang one can add that after that time there was always *a lot*, and that at times closer and closer to that boundary things occupied less and less without limit space, and change increased without limit in rapidity, so that history 'began' in a 'fast-starting' beginningless series of exceedingly small and then larger and larger states. (Cf. Sobel 1989, pp. 88–9: It is observed that a certain causal determinism is consistent with such a 'beginning'.) In this scheme every state is connected to an earlier state by natural laws, though the earlier the state the more indeterministic the applicable natural laws, and though *very* early (before 10^{-43} seconds into time, it is sometimes written) changes took place before the forces, gravitational, strong, weak, and electromagnetic, had separated and their strengths had been determined, at which times the chances according to applicable natural laws of these strengths being as if fine-tuned for life were vanishingly small. The *prima facie* inconsistency of without-real-singularities Big Bangs and God's rationality is hardly less than the *prima facie* inconsistency of with-real-singularities Big Bangs and God's rationality.
44. Cf.: "If it is within any person's power to do do A or (exclusive disjunction) B, and A certainly or probably advances the person's goals and B does not, then (all other things being equal) the person is rational with respect to A and B if and only if the person does A rather than B" (Craig and Smith 1993, p. 243: Smith's words). In place of 'A certainly or probably advances the person's goals and B does not' might be 'the person's expected value for A, given his goals and probabilities, exceeds that of B'; in place of 'does A rather than B' one might put 'does not do B'.
45. I am grateful to Paul Gooch, William Seager, and Paul Thompson and most particularly to Willa Fowler Freeman Sobel for comments and criticism on precursors of parts of this chapter.
46. This equivalence-claim assumes that conditional probabilities it addresses 'exist', that is, that their conditions are positively probable. Similarly for all claims concerning conditional probabilities in this appendix.
47. To see this equivalence, let $P(G \,\&\, E) = a$, $P(G \,\&\, \sim E) = b$, and $P(\sim G \,\&\, E) = c$ and assume $a + b > 0$ and $a + c > 0$. It can then be established that *P(E/G) > P(E/∼G)* if and only if $a/(a + b) > c/[1 - (a + b)]$. This is algebraically equivalent to $a/(a + c) > a + b$. Working from this back through the identities one can reach *P(G/E) > P(G)*.
48. Recalling that Swinburne's E is for some 'uncertain evidence', we have that someone persuaded by Swinburne that E tends to confirm G might take more interest in the equivalent condition that ∼E tends to disconfirm G. It is a valid principle that $[P(e) > 0 \,\&\, P(\sim e) > 0] \supset ([P(h/e) > P(h)] \equiv P(h/\sim e) < P(h)]$. Demonstration is left as a nontrivial exercise.

49. Suppose, for a false instance of cumulative confirmation, that a fair die has been cast, and that I have no idea which number came up. Assume the abbreviations, A: either 1 or 3 came up; B: either 1 or 2 came up; C: either 2 or 3, and observe that the conjunction (B & C) entails ∼A. It can be seen that

$$\begin{array}{ccccccc} [P(A/B) > P(A)] & \& & [P(A/C) > P(A)] & \supset & (P[A/(B \,\&\, C)] > P(A)) \\ 1/2 \quad 1/3 & & 1/2 \quad 1/3 & & 0 \quad 1/3 \\ \quad T & & \quad T & & \quad F \\ \multicolumn{3}{c}{T} & & \multicolumn{3}{c}{F} \end{array}$$

Confirming evidence B and C for A, when combined, disconfirms: P[A/(B & C)] < P(A). Whichever I were to learn first, B or C, would raise my expectation of A, which expectation would, however, be dashed as soon as I learned the other of these two individually favorable for A conditions. Cumulative confirmation is Jeffrey's Conjunction Condition II (if e confirms h, and f confirms h, then e-and-f confirms h), against which he brings a similar counterinstance (Jeffrey 1985, p. 108). And the reverse is possible: Disconfirming evidence when combined can confirm. "To see this consider the following example due to Carnap [Carnap 1950, pp. 382, 395]. Suppose ten players participate in a chess tournament. Some of the players are local players, some are from out of town, some are junior players, some are senior players, some are men (M), and some are women (W). Their distribution is given as follows:

	Local players	Strangers
Juniors	M,W,W	M,M
Seniors	M,M	W,W,W

. . . . Suppose . . . that hypothesis W is that a woman wins, evidence L is that a local player wins and that evidence J is that a junior wins. The probability of W on the background information is 1/2. [Five of the ten players are women.] The probability of W given L is 2/5, because two of the local players are women. Thus evidence L Disconfirms hypothesis W. The probability of W given J is 2/5, because two of the five junior players are women; thus evidence J also disconfirms W. But the probability of W given L&J [is 2/3, because two of the three local junior players are women. So evidence L&J confirms W], even though L disconfirms W and J disconfirms W" (Otte 2000, p. 7; I have changed Carnap's letters.)

50. Assume the probability distribution, $P(e \,\&\, f \,\&\, h) = 1/2$, $P(e \,\&\, f \,\&\, \sim h) = 0$, $P(e \,\&\, \sim f \,\&\, h) = 0$, $P(e \,\&\, f \,\&\, \sim h) = 0$, $P(e \,\&\, \sim f \,\&\, \sim h) = 0$, $P(\sim e \,\&\, f \,\&\, h) = 0$, $P(e \,\&\, \sim f \,\&\, \sim h) = 0$, $P(\sim e \,\&\, \sim f \,\&\, h) = 0$, $P(\sim e \,\&\, \sim f \,\&\, \sim h) = 1/2$. Let $e' = \sim e$. Then

$$([P(h/e) - P(h)] + [P(h/e') - P(h)])/2 = ([1 - 1/4] + [0 - 1/4])/2 = \mathbf{1/4}$$

and

$$\begin{aligned} &([P[h/(e \vee f)] - P(h)] + [P[h/(e \ \vee \sim f)] - P(h)] + [P(h/e') - P(h)])/3 \\ &\quad = ([1 - 1/4] + [1 - 14] + [0 - 1/4])/3 = \mathbf{5/12}. \end{aligned}$$

51. "But surely if evidence is more likely on a hypothesis than on its negation, P(e/h) > P(e/∼h), then the hypothesis is more likely on the evidence than is its negation, P(h/e) > P(∼h/e), so that P(h/e) > ½." No. Applications of Bayes's Theorem for a Hypothesis and Its Negation, and simple algebra, yields that P(h/e)/P(∼h/e) = [P(h) · P(e/h)]/[P(∼h)· P(e/∼h)]. From that it follows by more algebra that P(h/e) > P(∼h/e) *iff* **[P(h)/P(∼h)]** > [P(e/∼h)/P(e/h)]. It is not sufficient for P(h/e) > P(∼h/e) that P(e/h) > P(e/∼h), since P(e/h) > P(e/∼h) is consistent with [P(h)/P(∼h)] ≯ [P(e/∼h)/P(e/h)]. Both 'prior probabilities' and 'likelihoods' matter to 'posterior probabilities.' Now comes an application of this simple point to a common argument in the philosophy of perception. Let **Experience** be the fact that we have experiences that are *as if* due to interactions between bodies we have with continuously existing external objects, and let **Realism** be the hypothesis that we *have* bodies and have experiences that *are* due to interactions between our bodies and continuously existing external objects. Consider the following argument for Realism. "It is conceivable . . . [∼Realism], but that hypothesis *does not predict* what we experience [it does not predict Experience]. . . . A far better hypothesis, is . . . [Realism]. . . . *That* hypothesis *does predict* the inner experiential lives we live" (Sobel 1996, p. 127). The *premise* of this argument is that the 'likelihood' of Experience on Realism is *greater* than that on ∼Realism: *P(Experience/Realism) > P(Experience/∼Realism)*. The *conclusion* of this argument is that Realism is a far better hypothesis than ∼Realism, by which was meant that it is 'far better *on the evidence of Experience*': leaving out 'far', this is that *P(Realism/Experience) > P(∼Realism/Experience)*. The argument is a *non sequitur*: Its conclusion does not follow from its premises.

Necessary and sufficient for a valid argument is the single premise ***[P(∼Realism)/ P(Realism)] ≯ [P(Experience/Realism)/ P(Experience/∼Realism)]***, as well as the set of premises {*P(Experience/Realism)* > P(*Experience/∼Realism*), ***P(∼R) ≤ P(R)***}. Are the boldly emphasized premises 'available'? The probabilities in them are to be what would have been our probabilities *before experience*. At that time, or from that perspective, we would have no *reason* to think that Realism is true. Any probability we had for Realism 'then' would have to be 'natural', and an opinion we were 'so made' to have. Therefore, these premises may be, I think they are, 'available' in the sense that they are *true*. But they are *not* available for an 'argument for Realism', an argument that provided *reasons* for the truth of Realism, an argument that *justified* belief in Realism.

52. In practice the partition is often not exclusively of 'hypotheses' naturally so-termed and includes implicitly, to make a *partition*, a 'none of the above' hypothesis. In the simplest case, the partition includes a 'hypothesis' naturally so-termed, and its negation, which is not a 'hypothesis' naturally so-termed.
53. Here, paraphrased from Meierding (1998) with implicit material bracketed, are premises for the implication in his case: That evidence tends to support, or undermine, an hypothesis [and by how much] can be determined by comparing the likelihoods for it of this hypothesis and its negation (p. 278). Combining the support provided by pieces of evidence can provide sufficient support for rational belief in an hypothesis [or rational disbelief], it can 'tip the scales' for [or against] this hypthesis (p. 273).

54. A relevance that, strangely, he conceals, by putting in place of $P(\sim G) \cdot P(E/\sim G)$ in his application of Bayes, $P(E \ \& \ \sim G)$, which equals that product, given that $P(\sim G) > 0$.
55. It is surprising that Swinburne should say this without qualification, given that he holds that the likelihood on his theism of the evidence of his teleological argument is 1.

Chapter VIII

* This chapter integrates and expands revisions of Sobel (1987b, 1991). Not relevant but wonderful is Dorothy L. Sayers's, *Clouds of Witness* (Victor Gollancz 1926), in which she writes once of "a cloud of witnesses."

1. Hume (1902, p. 125). Citations, unless otherwise indicated, are to this work, in all cases but one, to "Of Miracles" in it.
2. The difference, which we get to in Section 2, is not that miraculous happenings are much less probable, for the marvellous and extraordinary is already very improbable. Similarly, the difference between the marvellous, the extraordinary, and the 'unusual' and the multitude of *humdrum* facts is not that the former are much less probable, which often they are not. It is not, however, to the purposes of "Of Miracles" to go into this difference.
3. "[T]he probability of . . . the falsehood of the witness becomes as much greater as the fact attested is more extraordinary. Some authors have advanced the contrary. . . . Simple common sense rejects [their] strange assertion . . . " (Laplace 1917, p. 114; also see p. 17).
4. Why are these things certain? First, "Of Miracles" was published in 1748, whereas Thomas Bayes's essay did not appear in print until 1763. Second, there is little evidence that Hume ever read Bayes's essay, for though it is cited by Richard Price in a footnote to his dissertation on historical evidence and miracles, a copy of which Hume acknowledged receiving, we have no evidence that Hume followed up this somewhat obscure reference. (Richard Price, *Four Dissertations*, Fifth Edition, 1811, p. 290, n. O to p. 229 of the text. First published in 1767.) Third and most important, Bayes's Theorems as we know them, complete with places for possibly unequal prior probabilities, are not in Bayes's essay. This makes somewhat problematic David Raynor's saying that "Price explicitly invokes Bayes's theorem" in his dissertation; and misleading at best is Raynor's saying that "it is regrettable that Hume's knowledge of Bayes's theorem has gone unnoticed for so long" (Raynor 1980, p. 107).

   When Hume wrote "Of Miracles" he had no knowledge of Bayes's not yet published essay. Hume, though he learned of its existence and relevance to his view concerning the extraordinary improbabilities of miracles prior to testimony for them, may never have seen this essay. And he certainly never had knowledge of what we know as Bayes's theorems. It is possible that *Bayes* had no knowledge of these theorems. See Section 7 for some elaboration.
5. Agents capable of violating the laws of nature by particular volitions would, in these volitions, not be subject to the laws of nature and so would be at least to that extent 'above nature's laws' and 'supernatural'. Hume equates the miraculous with the supernatural. He evidently counts miracles, along with 'prophecies', as "supernatural events" (p. 118). He writes of "a strong presumption against all

supernatural and miraculous relations" (p. 119). *Perhaps* Hume does not write 'some *other* invisible agent' to leave open the question whether or not God would be invisible. Jesus, whom some believe was God, was not invisible. More likely, I think, is that Hume intended 'other' to be *understood*. That is what I shall assume.

6. However, when Hume explicitly defines the word, he does not (contrary to Earman 2000, p. 8) write that 'a miracle can be *more* accurately defined' but simply that "[a] miracle can be accurately defined"(p. 115n).
7. This despoiling of water is said to have been done so that the Pharaoh should have known with whom he was dealing (*Exodus* 7: 16–7). It is said that it did not work, since the Pharaoh's magicians, who had cast their staffs before him to become serpents, did also this striking of water into blood before him (*Exodus* 7: 22). David Johnson, of lines including these of the water of the Nile, "Such of these reported physical occurrences as would have been observable by the witnesses I believe in very truth to have occurred, and I believe this on the testimony of those witnesses" (Johnson 1999, p. 41). He does not say whether he also believes, on the testimony of the same witnesses, in the marvels of the Pharaoh's magicians. It was, on the evidence of these passages of *Exodus*, a time of not only divine interventions into the natural order, but also of court magicians matching the Lord trick for trick. I wonder.
8. Jaqueline Mariña writes that "the position of the thoroughgoing naturalist . . . [makes] only two options . . . open . . . when confronted with a report of a violation of nature: . . . revise [one's] previous understanding of the laws of nature . . . [or] conclude that the report was simply due to a mistake" (Mariña 1998, p. 310). To this, of 'thoroughgoing naturalists' I say, "Perhaps, if by this are meant causal determinists." She continues, "**It is . . . the firm acceptance of this . . . that allowed Hume to argue the way that he did concerning the negative probability that miracle reports are true**" (Ibid.; emphasis added). To this of Hume I say, with restraint, "I don't think so."
9. Evidence that Hume himself took for granted (i) that miracles as accurately defined are *possible*, (ii) that there have been and will be few if any miracles, and (iii) that a 'very perfect Being' would, without our knowing, make many more miracles for a happier world is that these are Philo's opinions in Part 11 of the *Dialogues* (Hume 1991, p. 164). There is nearby evidence that Hume was *inclined* to think that "every thing in the universe [is] **conducted** by general laws" (op. cit., p. 165; emphasis added). That opinion implies that no miracles ever have or will happen.
10. Clarke has proposed the following revision of Hume's definition of a miracle: "A miracle is an intended outcome of an intervention in the natural world by a supernatural agent" (Clarke 1999, p. 54). Let a 'supernatural agent' be an invisible agent capable of making miracles in Hume's sense of violations of the laws of nature by particular volitions. Let the natural world comprise all things that are spatially and temporally related to things that are governed by the laws of nature. It seems a possibility, that is, we seem to have 'ideas' (suspect 'ideas' Hume would remind) of the possibility that not all events in the natural world are governed by laws (cf., p 51) and of the possibility that some events in the natural world are governed only by indeterministic laws that detail '*objective chance* connections' between natural events. And so we seem to have 'ideas' of *intervening*-miracles

that would not be *violation*-miracles. Hume would, I think, not be interested in working out details of 'ideas' of these 'miracles' or of the nature of evidence for them, because, in so far as these 'ideas' are new, they depart from the ordinary 'ideas' of believers in miracles with whom he wishes to converse. Clarke, when arguing that it can be rational to believe in miracles, 'although he does not insist that miracles necessarily violate laws of nature' (Clarke 1997, p. 95), works with a Humean definition that he knows has this consequence (p. 96). It is easier to envision evidence for 'violation-miracles' than for merely 'intervening-miracles'.

11. Peter van Inwagen, in his *Essay on Free Will*, labors not to resolve, but to insist upon, 'the problem of liberty and necessity' and so works with the 'idea' of necessarily quite exceptionless laws of nature. An agent, invisible or not, who "*violated the laws of nature*, that is . . . worked miracles" would thereby *falsify* them (van Inwagen 1983, p. 14); he would, assuming the necessity of Determinism, *change* them. There is no possible world *in which* van Inwagen's 'laws of nature' are violated, as there are no possible worlds in which husbands are unmarried.
12. Hume's arguments are not addressed to sophisticated modern Humeans who do not believe in objective necessary connections and hold 'Humean views' of laws as kinds of exceptionless generalizations. Hume's arguments are not addressed to such believers in 'miracles', or at least in the possibility of them. David Johnson may be such a believer in 'miracles'. Earman may be such a believer in the possibility of 'miracles'. David Lewis may be such a believer in the possibility of 'small miracles' that would be wrought by us. (Cf., Lewis 1973; Sobel 1998, pp. 159ff.)
13. Let 'M_D' say that a particular event M occurred and was a transgression of a law of nature by the Deity, and let 'M_a' say that it occurred and was a miracle by another invisible agent. Suppose that E is a conjunction of testimonies and other statements of evidence for the occurrence of this particular event. Then, as said, it can be that E is evidence *for* the occurrence M and its having been a miracle according to Hume's 'accurate definition', $P[(M_D \vee M_a)/E] > P(M_D \vee M_a)$, though it is *against* its having been *a miracle by the Deity*, $P(M_D/E) < P(M_D)$. Suppose, to demonstrate this possibility, the following probability distribution to conjunctions in which M_D, M_a, and E and their negations are mixed:

$$\begin{array}{ll} P(M_D \,\&\, M_a \,\&\, E) = .001 & P(\sim M_D \,\&\, M_a \,\&\, E) = .1 \\ P(M_D \,\&\, M_a \,\&\, \sim E) = 0 & P(\sim M_D \,\&\, M_a \,\&\, \sim E) = 0 \\ P(M_D \,\&\, \sim M_a \,\&\, E) = 0 & P(\sim M_D \,\&\, \sim M_a \,\&\, E) = 0 \\ P(M_D \,\&\, \sim M_a \,\&\, \sim E) = .1 & P(\sim M_D \,\&\, \sim M_a \,\&\, \sim E) = .799. \end{array}$$

Then, by Equivalence, Additivity*, and the Definition of Conditional Probability of Section 4.1 of the previous chapter:

$$\begin{aligned} P(E) = {} & P(M_D \,\&\, M_a \,\&\, E) + P(M_D \,\&\, \sim M_a \,\&\, E) + P(\sim M_D \,\&\, M_a \,\&\, E) \\ & + P(\sim M_D \,\&\, \sim M_a \,\&\, E) = .001 + 0 + .1 + 0 = .101 \end{aligned}$$

$$\begin{aligned} \boldsymbol{P[(M_D \vee M_a)/E]} = {} & P[E \,\&\, (M_D \vee M_a)]/P(E) = [P(M_D \,\&\, M_a \,\&\, E) \\ & + P(M_D \,\&\, \sim M_a \,\&\, E) + P(\sim M_D \,\&\, M_a \,\&\, E)]/P(E) \\ & = (.001 + 0 + .1)/.101 = \mathbf{1!} \end{aligned}$$

$$\begin{aligned}\boldsymbol{P(M_D \vee M_a)} &= \mathrm{P(M_D \,\&\, M_a \,\&\, E) + P(M_D \,\&\, {\sim}M_a \,\&\, E) + P(M_D \,\&\, {\sim}M_a \,\&\, E)} \\ &\quad + \mathrm{P({\sim}M_D \,\&\, M_{\boldsymbol{a}} \,\&\, E) + P({\sim}M_D \,\&\, M_a \,\&\, E)} \\ &\quad + \mathrm{P({\sim}\,M_D \,\&\, M_a \,\&\, {\sim}E) = .001 + 0 + 0 + 0 + .1 + .1 = \mathbf{201/1000}}\end{aligned}$$

$$\begin{aligned}\boldsymbol{P(M_D/E)} &= \mathrm{P(E \,\&\, M_D)/P(E) = [P(M_D \,\&\, M_a \,\&\, E) + P(M_D \,\&\, M_a \,\&\, {\sim}E)} \\ &\quad + \mathrm{P(M_D \,\&\, {\sim}M_a \,\&\, E) + P({\sim}M_D \,\&\, {\sim}M_a \,\&\, {\sim}E)]/P(E)} \\ &\quad = (.001 + 0 + 0 + 0)/.101 = \mathbf{1/101}\end{aligned}$$

$$\begin{aligned}\boldsymbol{P(M_D)} &= \mathrm{P(M_D \,\&\, M_a \,\&\, E) + P(M_D \,\&\, M_a \,\&\, {\sim}E) + P(M_D \,\&\, {\sim}M_a \&E)} \\ &\quad + \mathrm{P(M_D \,\&\, {\sim}M_a \,\&\, {\sim}E) = .001 + 0 + 0 + .1 = \mathbf{101/1000}.}\end{aligned}$$

14. The best hypothesis for reconciling sporadic and arbitrary miracle-making with God's being just and loving posits (i) that *freely* turning of creatures to God in love and not for the good it can do them is of great value, (ii) that a necessary condition for that is 'religious ambiguity,' which suggests no links between belief and rewards and includes that there is compelling evidence of God existence or activity, and (iii) that a just and loving God's disposition to gift creatures through miracles is tempered by (ii). This hypothesis either makes God's sporadic arbitrary miracle-making not unfair or makes its unfairness a necessary feature of God's unimprovable-on-balance work. (Cf., Keller 1995, pp. 70ff.) However, while reconciling sporadic and arbitrary miracle-making with God's being just and loving, the availability of this hypothesis does not significantly reduce the potency of the evidence against the existence of a God that findings of sporadic and arbitrary miracles would afford.
15. Johnson reports that evidence for some miracles is to his mind sufficient evidence for God. He finds testimonial evidence in the Bible for miraculous occurrences – divisions of the Red Sea, water of the Nile into blood, water-walks, The Resurrection – sufficient to establish beliefs in these occurrences (pp. 40–1), and he holds "that the best explanation of these extraordinary occurrences . . . postulates the existence of 'the God of Abraham, and of Isaac, and of Jacob" (p. 69). There are arguments from miracles in the sense of *natural occurrences that are contrary to the laws of nature* (or to the apparent or presumed laws of nature) to God variously identified. The 'logic' of these arguments from 'miracles' is the same as that of arguments from design, from evil, and from common consent. They want to be folded into an omnibus argument from their evidence combined. (See the last paragraph of Section A4.4 of the appendix to the previous chapter.)
16. Hume does not say this 'in so many words'. His only use of 'strong presumption' is, I think, in "[i]t forms a strong presumption against all supernatural and miraculous relations [affirmations], that they are observed chiefly to abound among ignorant and barbarous nations" (Hume 1902, p. 119).
17. That there are not causes from which an event can be derived does not entail that it violates a causal law of nature. It is logically possible for an event to be 'not lawful' without being 'unlawful'. However, if it is part of the 'idea' of causal laws of nature that they are in all possible applications consistent, so that a law cannot require in a situation what another law prohibits, then that there *are* causes from which an event can be derived entails that it is *not* a violation of a law of nature, and conversely that an event violates a law of nature entails

that there are not causes from which it can be derived, and that it is not only 'unlawful' but 'not lawful'.

18. Earman would, I expect, consider this a 'plus' for those reasons. He says that "an adequate epistemology should deliver the conclusion that in most (all?) actual cases, when all the evidence is weighed up, little credibility should be given to [UFO landings, water-walks, and resurrections]" (Earman 2000, p. 4). Earman does not claim that his own epistemology delivers this conclusion and implies that it does not (op. cit., pp. 59–61).
19. The idea of the theorem developed in correspondence with Donald A. Gillies, who proposed something similar and proved it in a letter of 6 June 1989. Axioms of the theory are Kolmogorov's plus the ratio-definition of conditional probability, for which see Appendix A.
20. Cf.: "Hume . . . famously argued that it could never be rational to accept that an event is, or was, miraculous" (Keller 1997, p. 95). Hume did not argue for that; he did not believe it .
21. Hume did not suppose that he had imagined in his eight-days-of-darkness case testimonial evidence sufficient to establish a miracle as accurately defined. While Hume says that such testimony is *possible*, he does not *illustrate* this possibility in "Of Miracles." Probably, however, he would say that testimony sufficiently "extensive and uniform" (Ibid.) to establish the fact of Cleanthes' "articulate voice . . . in the clouds" (Hume 1991, 117) is imaginable and that testimony sufficient to establish that fact would be at the same time sufficient to establish it as a miracle not merely in the sense of a departure from, or exception to, the laws of nature, but in the sense of a *violation of a law of nature by a particular volition of an invisible agent*.
22. Was the establishment of order in nature 'the miracle of miracles' in Hume's view?! Did he think that probably the cause of order, for example, of the solar system, and of the adaptation of living means to ends, was Intelligence through directly efficacious Will? I think he did, at least when he was not deep in his skeptical funk regarding causes.
23. Hume implies, I think, that at least for the wise and learned who give credit to testimony for miracles of their religion, testimony for some miracles that were only of other religions would be found to have the same force. This is not obvious (cf., Johnson 1999, pp. 82–7).
24. While Johnson suspects that Hume may depend on the false principle that "evidence *for* a theory – evidence which magnifies the theory's probability of being true – must at the same time be evidence *against* any *incompatible* theory" (Johnson 1999, p. 80), he concedes (p. 81) that since Hume writes of miracles that would *establish*, and not merely *tend* to establish, particular religions, he can be interpreted as relying on a true principle: While it is not a valid principle that $\sim\Diamond(t \,\&\, t') \supset ([P(t/e) > P(t)] \supset [P(t'/e) < P(t')])$, it is a valid principle that $\sim\Diamond(t \,\&\, t') \supset ([P(t/e) > \frac{1}{2}] \supset [P(t'/e) < \frac{1}{2}])$. Johnson thinks that probably, "for reasons we will not go into" (p. 82), Hume had in mind not merely probable but *certain* establishment. One reason would be that when evidence that would alone probably establish a theory is combined with evidence that would alone to the same probability establish another theory, the evidence does not 'destroy itself': It can be sufficient to establish, even to 'prove', one of these theories. To see this suppose that a marble is to be drawn at random from an urn, of which

there are two containing. Each contains marbles that are rough or smooth, and white and green: Urn A (1 rough and white,1 smooth and white, 1 smooth and green), Urn B (2 rough and green, 1 smooth and white). Though $P(A/W) = P(B/R) = 2/3$, $P(A/R \& W) = 1$.

25. Which it may do, even if it would not according to arguments in Sobel (1987a, 1997), were I a 'perfect intellect.'
26. The problem broached here is for a Bayesian theory of certain learning to draw in a principled way a line between what is *learned in* experiences and what is *gathered from* that but not learned. I have assumed in the text the 'free-standing' conditional probabilities of note 7 of the previous chapter that are defined for all logically possible conditions. In that theory 'certainties of background information' can be 'dislodged' be new maximally surprising information: In that theory it is possible for $Pr(h/e) \neq 1$ though $P(h) = 1$, provided that $P(e) = 0$.
27. Hume *may* be exercising some license in this use of the word 'miracle'. He may not mean that in that case the false testimony would be a miracle as 'accurately defined,' that is, a natural impossibility brought about by God or some (other) invisible agent. He *may* here mean by 'miracle' a natural impossibility and contrary to the laws of nature, a *violation* of the laws of nature, but not a violation by a particular volition of God, or the interposition of some other invisible agent. "But if not a violation by invisible agents, by whom then?" An answer could be, "By *visible* agents, in particular, in the case of miraculous testimony, by the persons testifying."

    There is no evidence that Hume took a *dimmer* view of the possibility of transgressions of laws of nature by volitions of visible agents than by volitions of invisible agents: *That is*, there is no evidence that he 'found' such a dimmer view, when engaged with, and in the language and thought of, the sophisticated vulgar, as I have said (Section 2.5) he is in "Of Miracles" and is not in "Of Liberty and Necessity." "But he equates the miraculous with the supernatural, and visible agents – you and I, say – are *natural* agents." Hume could say, "Yes, on the theory of Free Will, free human actions, though transgressions of laws of nature by volitions agents, or at least not pursuant to laws of nature, would not be miracles as accurately defined. But there is nothing deep in the restriction of the accurate use of this word to the unlawful work of invisible agents." Cf., Mill (1874, pp. 226–7) on the analogy of products of exercises of human will, that is, free actions, with those of exercises of powerful wills by what would be invisible agents, that is, miracles, and (op. cit., p. 228ff) on *differences* that make "the antecedent presumption against [the latter] extremely strong" (p. 228). Regarding the former, Mill considered the 'antecedent presumption against them' either weak or nonexistent. He leaves "the case of human volition an open question" (p. 233) and may have considered it to be a nest of open questions. He would not have been the first, or the last, of that consideration.
28. Cf.: "Hume's . . . view that a wise man does not believe that Christ rose from his grave, is based on a general rule of (testimonial) evidence. When A testifies that p to B, B should estimate (1) the prior probability that not p, and (2) the credibility of A as a witness that p. . . . If (1) > (2), then B should not believe that p based on A's testimony that p" (Root 2001, p. 19). Assuming that 'the credibility of A as a witness that p' is measured by $P[p/t(p)]$, the view here attributed to

Hume for the wise assessment of testimony in hand is $(P[t(p)] = 1) \supset [((P(\sim p) > P[p/t(p)]) \supset (P[\sim p/t(p)] > \frac{1}{2})]$, which is equivalent to

$$(\mathbf{P[}t\mathbf{(}p\mathbf{)]} = \mathbf{1}) \supset (P[((p/t(p)] \geq \tfrac{1}{2}) \supset (P(p) \geq P[t(p)\&\sim p],$$

the 'generalization' of Hume's Theorem,

$$[(\mathbf{P[}t\mathbf{(}p\mathbf{)]} > \mathbf{0}) \,\&\, (P[p/t(p)] \geq \tfrac{1}{2})] \supset (P(p) > P[t(p)\& \sim p])].$$

The main difference is that the view attributed in the quotation to Hume is *specifically* for 'testimony in hand,' whereas Hume's theorem is for all positively probable testimony.

29. The 'only-if' half of Testimony for Miracles, which is equivalent to,

$$[(P[t(M)] > 0) \,\&\, (P[M/t(M)] > \tfrac{1}{2})] \supset (P[t(M) \,\&\, M] > P[t(M) \,\&\, \sim M]),$$

entails Hume's theorem. Even so, the proof in Appendix A of Hume's Theorem can be diverted after line (9) for a proof of the above stronger conditional. The 'if' half of Testimony for Miracles, which is equivalent to

$$(P[t(M) \,\&\, M] > P[t(M) \,\&\, M]) \supset [(P[t(M)]) > 0) \,\&\, (P[M/t(M)] > \tfrac{1}{2})],$$

holds, since its antecedent (i) rather obviously entails the first conjunct of its consequent, and (ii) entails the second conjunct by way of $P[M/t(M)] = P[t(M) \,\&\, M]/P[t(M)]$, and thus

$$(P[t(M) \,\&\, M]/P[t(M) \,\&\, M] + P[t(M) \,\&\, \sim M].$$

30. After finding a reading of Hume's maxim, Earman denigrates it as "just the unhelpful tautology that no testimony is sufficient to establish . . . a miracle unless it is sufficient to make the occurrence more probable than not" (Ibid.). That, I suppose, tells against the 'goodness' of his reading of Hume's maxim. For Hume's maxim seems helpful, as it can be even if, as it is on my reading, a trivial theorem of probability. Consider the praise Earman pays to Bayes's Theorem as "a trivial consequence [of the definition of conditional probability], but one with profound implications" (p. 27).
31. In general, for declarative sentences ϕ and ψ, the sentence ϕ *unless* ψ is equivocal in the manner of the sentence ϕ *or* ψ between weak and strong truth-functional interpretations (not to mention other interpretations including in particular subjunctive ones). *Weak*: $(\sim\phi \supset \psi)$ which is equivalent to $(\phi \vee \psi)$. *Strong* $[(\sim\phi \supset \psi) \,\&\, (\psi \supset \sim\phi)]$, which is equivalent to $[(\phi \vee \psi) \,\&\, \sim(\phi \,\&\, \psi)]$ and also to $(\sim\phi \equiv \psi)$.
32. It follows in Bayes's problem "that if all of the first n trials have yielded Bs, then the probability that the next trial will also yield a B is $(n + 1)/(n + 2)$. So as $n \to \infty$, the probability that the next instance will be B approaches 1" (Earman 2000, p. 28). "[I]t follows [also] from Bayes's assignment of priors that the probability that *all* future trials will yield Bs remains flatly 0 no matter how large n becomes" (p. 29). Hume, if he saw these consequences, could have recast his discussion in Part I to be founded on the first result while he remained in reasonable doubt concerning the relevance of the second to the 'logic' of learning from experience. Earman stresses that the second result is a creature of Bayes's priors.

33. Pearson (1978, p. 378).
34. Price does distinguish between "the *capacity* of testimony to report truth" and "the *credit* of testimony" and allows that prior improbability of a reported fact can affect the latter. But I think that he did not understand exactly how it could affect the latter. He was only close to bringing under formal control the distinction that is necessary and largely sufficient to an appreciation and acceptance of the common sense of Hume's informal position.
35. 'The veracity of the witness' is defined in a case if and only if in this case, for any S and S′, such that P(S), P(S′), P[*t(S)*], and P[*t(S′)*] > 0, P[*t(S)*/S] = P[*t(S′)*/S′]. In the case in the text, the reporter's veracity is taken to be .9. *The reporter's 'veracity'* relative to 79 is in this case the same as *the credibility of his report*. But it is not the same as *his 'reliability'* in the sense given to this term in Appendix B. The reporter's 'reliability' relative to 79 is (.9/[.9 + (.001(.1)/.999))]), which, according to my pocket calculator, is .99988879.... The reporter's 'reliability' relative to 79 reflects not only his 'veracity' relative to 79 but also his lack of bias toward 79, and the great improbability of his misreporting 79 if some other number is drawn: r – for its formula see the Rule at the end of Section B1.1 of Appendix B – depends not only on P[*t(79)*/79] but also on **P[*t(79)*/∼*79*]**, which latter conditional probability is, in the case, .0001001....
36. This idea can be found in Jeffrey (1985). Also see van Fraassen (1984, pp. 251–252).
37. See Cohen (1981, pp. 365–366) and Ellsberg (1961).
38. Qualities of relevant 'singular probabilities' – qualities of 'priors' and of 'likelihoods' that enter into a reporter's reliability measure (in the sense made exact in Appendix B) – can be equal. Suppose, for example, that I know that a well-mixed urn contains 50 white balls and 50 black ones, and that Alice and Betty have observed a random draw from this urn. Suppose that in my view Alice and Betty are in this kind of case equally reliable reporters. Let Alice tell me that the ball drawn was white. And then let Betty contradict her and tell me that it was black. Shall I after this second report ignore my then prior probability for the ball's being black – ignore, that is, Alice's testimony – and believe to the order of Betty's reliability that the ball was black? Shall I believe (with due reservations) Betty rather than Alice just because Betty spoke last?! Surely not. At this point *qualities* on the one hand of 'priors', and on the other hand of 'likelihoods' that enter into my measure of Betty's reliability, can be (and so far would seem to be) *equal*, and consequent to Betty's testimony in which she contradicts Alice's testimony, I should be back to square one as far as my opinions concerning the drawn ball's color. I might also reconsider to downgrade my measures of the reliabilities of Alice and Betty as reporters in this kind of case.
39. I believe that the second part of this idea was proposed (without endorsement or rejection of the first part) by L. Jonathan Cohen in a discussion following the presentation of this paper on August 29, 1986 during the Hume Conference held in Edinburgh under the joint auspices of the Institute for Advanced Studies in the Humanities and the Hume Society.
40. In so far as there is legitimate controversy regarding applications of Bayesian principles to testimony, it concerns not the relevance of prior probabilities, but the relevance of probabilities – the relevance of 'priors' *and* 'likelihoods' – of various qualities. Discussion has concentrated on 'priors' because in many cases they are of different qualities from, and have different bases than do,

'likelihoods' that enter into reliability measures, and because not infrequently these differences seem to tell against 'priors', their qualities seeming inferior and their bases such as to render them less relevant.

Subjects in the second experiment, who were told that 85 to 15 was the ratio of green to blue taxicabs *involved in accidents* in the town, had, one assumes, higher quality 'priors' than did subjects in the first experiment, who were told only that that was the ratio of green to blue taxicabs *in the town*. Subjects would, one assumes, have had yet higher quality 'priors' regarding the color of the taxicab involved in an accident on a certain night in a particular part of town, if they had been told that 85 to 15 was the ratio of green taxicabs to blue ones involved in accidents *at night, in that part of the town, within a month of that night*. One supposes that they would have had much lower quality 'priors' if they had been told instead only that every taxicab in the town was either green or blue and that the ratio of green taxicabs to blue ones *in the world* (and not necessarily also in the town) was 85 to 15. Persons who had nothing to go on other than one or another of these ratios would presumably agree in the quantity of their 'singular probabilities' for the proposition that the taxicab involved in an accident in the town on that recent night was blue, but persons possessed of information at the extremes of relevance indicated would presumably, and reasonably one feels, *differ* in their readiness to base bets on their 'singular probabilities.' While differing little if at all in the quantities of their 'singular probabilities', they would, one assumes, differ markedly in their confidence in these probabilities, which would be of very different qualities.

41. Consider Venn (1888, pp. 412n, 421n) and Niiniluoto (1981, p. 349).
42. It is a matter of his reliability *when testifying to S*, which can be very different from his reliability when testifying to other propositions. The importance to our subject of the likelihood of erring by testifying to S (say that 79 was drawn) when S is false (say because 78 was drawn), rather than testifying erroneously to something else (say that 93 was drawn), is the main point of Section 9.2.
43. Please see the first paragraph of Section 2.2 for my understanding, continued here, of 'proofs', 'infallible experience', and 'firm and unalterable experience' as used by Hume for evidence for laws of nature and against miracles.
44. Earman identifies *the proofs* that Hume says lie against miracles given that they would be violations of laws of nature with something Earman describes as Hume's '*proof* against a miracle' in Part I. Explaining the words 'this argument' in a letter of 1762 that Hume wrote to George Campbell, Earman writes: "[T]he most reasonable supposition is that he was referring to the 'proof against a miracle' he gives in Part I of his essay . . . , a proof he touted to be 'as entire as any argument from experience can possibly be imagined'" (Earman 2000, p. 6; cf., pp. 23, 43). In fact, 'the argument' of Part I is not 'an argument from experience' of any kind, but a philosophic argument that culminates "a general maxim worthy of our attention" (Hume p. 115), the maxim discussed in Section 4. He considered *this* argument to be deductively valid and of only true premises, but he does not tout it as 'as *entire* as any argument of *its* kind can possibly be imagined.'
45. There is this problem for every Bayesian interpretation of Hume that would assign probabilities of 1 to what are perceived as laws of nature and of 0 to what would be perceived as miracles: They do not allow for the possibilities, in which

Hume believed, of 'degrees of last assurance' or 'degrees of miraculousness.' There is not for *every* such Bayesian interpretation of Hume the additional problem that it says that *absolutely no evidence* E can 'tend to establish a miracle' M. Bayesian interpretations in which "the conditional probability Pr(Y/X) is defined by Pr(Y&X)/Pr(X) when Pr(X) $\neq$ 0" (Earman 2000, p. 26) have this problem (p. 31). For in these interpretations, if P(Y) = 0 and P(Y/X) is defined, P(Y/X) = 0. A Bayesian interpretation in which P(Y/X), while constrained by the quotient condition, is defined for all logically possible X does not have this additional problem. In the theory of (Sobel 1997a), P(Y/X) can be > 0 even when P(Y) = 0, though only if also P(X) = 0: It is a theorem of this theory that, *for possible X,* $\sim$*([P(Y/X) > P(Y)] & [P(Y) = 0])* $\vee$ *[P(X) = 0]*, words for which theorem can have a familiar ring: The words I have in mind are that *no evidence can tend to establish what is absolutely improbable, unless it is of such a kind as to be itself absolutely improbable*.

46. See Henle and Kleinberg (1979, chapters 1–4) for an introduction to the theory of hyperreal numbers pioneered in the early 1960s by Abraham Robinson. Also see "Hyperreals and Decision Theory" in the appendix of Chapter XIII.
47. Responding to Johnson (1999, p. 58), I do not mean that the person has assigned probability i to M, but that his credences for propositions are represented by a probability function that includes this assignment.
48. When Hume contrasts proofs with probabilities and writes that proofs are "such arguments from experience as leave no room for doubt or opposition" ("Of Probability" in Hume 1902, p. 73), I take him to be contrasting proofs with ordinary probabilities and to have in mind opposition by ordinary nonproof arguments from experience. I am grateful to Andreas Weber for useful questions on these matters.
49. See Butler (1961, pp. 147 – 8). I owe this reference to Dorothy P. Coleman.
50. Quoted with apparent approval. Consistently with that Earman himself would not believe in alien abductions, even if "a worldwide . . . poll found hundreds of millions of [self-described] witnesses to alien abductions," *unless* persuaded (as he can only *imagine* happening) that the witnesses were reliable and independent (Earman 2000, pp. 60–1).

Chapter IX

1. c. 342–292 B.C. "Translated by F. C. Allinson (Loeb Classical Library).... Unidentified fragment.... Also attributed to Aristophanes by Lucian...." (John Bartlett, *Familiar Quotations*, sixteenth edition, general editor J. Kaplan, Boston: Little, Brown, and Company, 1992, p. 81.)
2. Cf., Penelhum (1971, p. 15), where the problem of perfect justice and mercy is seen as difficult and not pursued.
3. Single quotes are used because usually we write of 'probabilistic evidence' and say that e is evidence for p if and only if P(p/e) > P(p). Nothing can be evidence in this sense for possibilities or necessities, since, for every e, p, and probability function P, each of P($\Diamond$p) and P($\Box$p) is either 1 or 0; for every e, p, and probability function P, P($\Diamond$p/e) = P($\Diamond$p) and P($\Box$p/e) = P($\Box$p).
4. Margaret Cameron, with whom I have had useful conversations about the mysteries of essential properties.

5. Similarly, only evidence for her being unilingual can be evidence for her being essentially unilingual, since it follows from the definition of 'Ess(x,ϕ)' that (x)(ϕ)([Ess(x,ϕ) & E!x] ⊃ ϕx).
6. Bill Seager and I reached these conclusions in conversations, refreshing as always, for which I am thankful. The formal point can be confirmed by a short derivation in the system for quantified modal logic in Appendix B of Chapter III as elaborated for second-order quantification in Appendix C of Chapter IV: Of use in a derivation can be the S5-equivalence of □(E!x ⊃ ϕx) and ◊□(E! ⊃ ϕx). We have that, given that Margaret exists, It is possible that she is essentially unilingual, only if she is unilingual. This result is 'in the neighborhood' of the result for *dragoons* in Section 8.2 of Chapter III: It was found that they are possible, only if dragons are actual. Funny things can happen when we 'modalize'.
7. Cf., Savage (1967), which is in part a response to Mavrodes (1963).
8. Cf.: "This leads us to what I call the Paradox of Omnipotence: can an omnipotent being make things he cannot subsequently control? . . . It is clear this is a paradox: the [question] cannot be answered satisfactorily either in the affirmative or in the negative. If we answer 'Yes', it follows that if God actually makes things which he cannot control . . . he is not omnipotent once he has made them: there are *then* things which he cannot do. But if we answer 'No', we are immediately asserting that there are things which he cannot do, that is to say he is already not omnipotent" (Mackie 1973, pp. 214–15).
9. William E. Mann starts the last in his series of paradoxes with the line, "Can God create a stone too heavy for God to move?" (Mann 1995, p. 559.) All 'Stones' I know of work with something like self-referential tasks.
10. Mackie's question of the previous note can be answered in the affirmative. There is no need to "distinguish between first order omnipotence . . . to act, and second order omnipotence . . . to determine what powers to act things shall have" (Mackie 1982, p. 216). An omnipotent can and would have all powers to act, and to determine powers of things including itself.
11. Line (2′) corresponds to (3) in argument A in Savage (1967), whereas (1′) – see below – corresponds to (2) in that argument. I approve of (1′) and point the finger at (2′). Savage oppositely approves of his (3) and points the finger at his (2). He does not look into the modal structures of propositions in his argument.
12. On the chance that necessary existents would be not everlasting, but eternal, when it matters I making the temporal condition explicit.
13. Swinburne disagrees: "[T]he theist surely does not require that the object of his worship be a logically necessary being.*" (Swinburne 1993, p. 301). I agree with that. (*"That God would need to exist of logical necessity in order to be worthy of worship was a claim made by Findlay" (footnote).) "For if the existence of God is the tremendously exciting thing which theists believe it to be it is not to be expected that anything can show why God exists" (Ibid.). I am not sure about that. "Yet why something exists of logical necessity can be shown in terms of the incoherence of supposing that it does not exist; men can be brought to see why it exists, how it could not but exist" (Ibid.). On this I disagree. Logical impossibilities need not be *a priori* impossible.
14. Many attributes can be had nonessentially. Omnipotence can be had nonessentially. Some attributes are had only essentially. Included here is the 'attribute' of self-identity. The test recall is that an attribute is essential to a thing if it is

necessary that this thing has it in every world in which it exists. Thus, for self-identity, we have that it is provable that $(x)[x = x \supset \Box(E!x \supset x = x)]$. Which kind of attribute is existence, and necessary existence?

15. The conjunctive conditions (i) and (ii) for being an ONSLP is I think equivalent to Wierenga's condition (C) for being omnipotent: "An omnipotent being need not be able to bring about a state of affairs which it is impossible that ***that*** being bring about" (Wierenga 1989, p. 16; bold emphasis added).
16. Wierenga writes of McEar as "a character introduced by Plantinga" (Wierenga 1989, p. 28n34), realizing that Plantinga left open whether McEar is *essentially* "capable of only scratching his ear" (Plantinga 1967, p. 170).
17. Geach's objection to the idea that God can do so and so if and only if it is logically possible that *God* does so and so is that it may be too late to bring it about that Miss X never loses her virginity (Geach 1973a, p. 15). This is not an objection to God's being an ONSLIP, which is to be understood along lines drawn in Section 4.3 as a 'career-condition.'
18. As I do not stipulate that 'being an ONSLIP' and 'being omnipotent' mean the same, so I do not stipulate that 'God's being *sans* ONSLIP' and 'God's being omnipotent' mean the same. Contrast: "[W]e may say that God's omnipotence means that *God can perform any action the performance of which is logically consistent, and consistent with God's own nature*" (Peterson et al. 1991, p. 56). We may, as Humpty Dumpty taught, say whatever we please.

Chapter X

* With grateful acknowledgement to Joseph Heller, *Gods Knows*, New York: Alfred A. Knopf, 1984 (on the dust jacket of my copy of which God says in a cartoon, "Go figure").

1. These thinkers would be well advised to say that God's knowledge would not be broken up into proposition-sized bits *without remainder*. They should not say that God does not know true propositions, but that This One's knowledge *goes beyond propositions*, as Aristotle may have been the first to say that our knowledge can go beyond propositions expressible by us in certain ways: "[T]he equitable is just, but not the legally just but a correction of legal justice . . . all law is universal but about some things it is not possible to make a universal statement which shall be correct" (*Nicomachean Ethics* V, 10, 1137b11–5, translation by W. D. Ross). Aristotle *may* have had in mind only a 'practical impossibility.' W. D. Ross makes something similar an absolute impossibility: "But no act is ever, in virtue of falling under some general description, necessarily actually right; its rightness depends on its whole nature and not any element in it" (Ross 1930, p. 33).
2. Patrick Grim interjects *"de re*, let us say" in his book (Grim 1991. p. *95)* in a report of a version of my argument prepared in 1976 for an introduction to philosophy class and sent to him on the occasion of his "Some Neglected Problems of Omniscience," *American Philosophical Quarterly* 20, 1983. Consider, for example, me and the set of propositions {the proposition that 2 exceeds 1, the proposition that I know that 2 exceeds 1}. Perhaps Grim's interjection signals that he would distinguish (i) the proposition that, of me and the propositions that 2 exceeds 1, and that I know that 2 exceeds 1, says that I know each of them

from (ii) the proposition that, of me and the set of propositions {the proposition that 2 exceeds 1, the proposition that I know that 2 exceeds 1}, says that I know each of its members. I do not, since I do not distinguish propositions that would, though distinct, be logically equivalent. Late amendment to this note: "I think I just wanted to avoid problems of *de dicto* enunciability" (Grim in private communication in 2002).

3. Simmons's paper of 1993, concerned as it is exclusively with Grim (1984, 1988), relates curiously to Grim's work. In particular it takes no notice of Grim (1991). Simmons writes that "Grim does not consider the matter . . . what kind of entities are truths?" (p. 23; sentences reordered), whereas Grim considers this question in Chapter 1 of his book wherein he entertains both sentential and propositional options. Also, Simmons, citing Georg Cantor's idea of multiplicities that cannot be conceived as unities, writes as if Grim does not consider this possibility, though he in fact does (see Grim 1991, p. 124 on Cantor's idea of a set), in the vicinity of these lines: "There is, we might admit, no set, no class, no collection, and no totality of all truths. . . . But it might appear that we could nevertheless speak of *all* truths" (p. 114).
4. Cf., 'Talk about *good*!' Also consider, 'For what else is there to talk about?'
5. For example, I am not in a position to judge the philosophical adequacy, as collectors of all truths, of certain 'proper classes' of which I have heard mention. A reader of a draft of this chapter has said, with the suggestion that one of these might provide a satisfactory container for all true propositions, that "there are theories of proper classes that do allow them to be members, even to be self-membered, without being too set-like. (The trick is to allow truth-value gaps, as in Kripke's theory of truth.)"
6. What follows relates in several ways to the section in Grim's book titled "The Appeal to Quantification" (Grim 1991, pp. 113–22). That section, along with the section titled "A Self-Reflective Problem for the Central Thesis" (pp. 122–4), may have benefited from correspondence between Grim and Plantinga (see pp. 148n45–116, l48n48–118). This correspondence is condensed in Plantinga and Grim (1994), which includes reflections on it (Plantinga and Grim 1994, pp. 297–305*)* that I assume postdate the composition of those sections in the book. Perhaps the correspondence from Alvin Plantinga to Grim on pages 291–7 also postdates composition of those sections in Grim's book. This chapter of mine essentially reached its present form before I learned through Plantinga and Grim (1994) of that correspondence. [Late amendment to this note: "[T]he book was off before much of the Grim-Plantinga correspondence. . . . I had started to ponder the 'self-reflective problem' before Plantinga pressed the point. . . . I don't remember whether that correspondence fed directly in to the final draft of that section" (Grim, private correspondence, 2002.)]
7. "*The Incomplete Universe* suggests that the conclusion should be drawn by more indirect means. . . . But I think it may also be . . . that there is no explicit conclusion that can be drawn. It may be that the argument is . . . a garden path . . . that [for those who believe that omniscience is possible, that one can speak meaningful of all propositions, or the like] . . . will lead to contradiction and consternation. I do a bit more regarding this line . . . in the last section of 'The Being That Knew Too Much,' *International Journal of Philosophy of Religion* 47 (2000), 141–54" (Grim, private communication, 2002).

8. "But is it for you to *decide* what propositions are about?" No, though I do get to guess! It is a question of what the best theory of propositions says they are about. This theory will, in so far as possible, count as propositions what, before theory, we are convinced are things that can be true and false, things of the sort that are sometimes expressed by sentences. Several things that propositions are not are detailed in "Propositions" (Cartwright 1987). The best theory must *not* say that there is a proposition answering to some description such that the assumption that there is a proposition answering to this description leads to a contradiction. I am guessing that the best theory will restrict the aboutness of propositions in the manner indicated. Theories of grammar, theories in particular of sentences and their subjects, make a somewhat different case. The assumption that some string of words is a sentence, while it can be false, cannot lead to a contradiction.
9. It is not, given my strictures on what propositions are about, an easy corollary of this conclusion that, contrary to (Cartwright 1994), it is not possible to speak of everything, though it would be surprising, were speaking of everything possible, though speaking of all true propositions was not.
10. In order that it should have any plausibility at all, it would need to be a principle that would secure 'diagonal propositions' only for *some* propositions about some propositions. For 'diagonal propositions' would be by design impossible, and no one, except as part of an argument against propositions in general, would say that there are no propositions about any propositions.
11. "You say that there are true propositions about all true propositions. I gather, cutting through the complexity of your argument, that you do not deny that there are true propositions that are not amongst the things they are about." This is true. "And you will say that an omniscient would know all about these true propositions'." Yes. "But you deny, to protect the existence of true propositions about all true propositions, that there is a valid principle of Separation for Propositions by which one can infer, from there being true propositions about all true propositions, that there is a true proposition about precisely *these* true propositions, that is, let us say, about precisely all '*irrelexive truths*'." Just so. "And yet you say God would have *knowledge* of precisely all 'irreflexive truths', and I suspect you think that you have knowledge of precisely these truths, knowledge that is 'beyond propositions', indeed, 'beyond truth'." You guess right. "Incredible!" Not to me. I find it hard to deny that there are propositions about all true propositions. And it is demonstrable that there is not a true proposition that is about precisely all 'irreflexive truths'.* As for my knowledge of precisely all 'irreflexive truths' – which I conclude cannot be knowledge of propositions or truths – I confess that though I cannot doubt that I have it, I do not know what to make of it. [*Assume the 'abbreviations' – T^1: *a* is a true proposition; R^2: *a* is amongst the things that *b* is about. 'There is not a true proposition that is about precisely all 'irreflexive truths'' is, as I understand it, symbolized by' $\sim(\exists x)[Tx \ \& \ (y)(R(yx) \equiv [Ty \ \& \sim R(yy)])]$'. Left as an exercise is to derive this theorem of first-order logic.] The challenges of this note are paraphrased and elaborated from Grim (private correspondence, 2002).
12. Arguments against the possibility of propositions about all true propositions can, we are told, stay clear of "the vagaries of 'about'" (Grim in Plantinga and Grim 1994, p. 301). But all depend on the reality of less likely propositions. And all threaten to 'self-deflate' (Plantinga in same, pp. 296–7, 302–5).

13. Grim presents "a standard proof of Cantor's theorem in [his] note 28" (Grim 1991, p. 147) that is similar and in another place takes "a moment to exhibit the full Cantorian argument phrased specifically in terms of sets and truths" (p. 93).
14. The principle relied upon here is from ($\sim p \supset p$) to infer p. A principle soon relied upon is from ($p \supset \sim p$) to infer $\sim p$. For these principles consider that ($\sim p \supset p$) is equivalent to ($\sim\sim p \vee p$) and thus to p, and that ($p \supset \sim p$) is equivalent to ($\sim p \vee \sim p$) and thus to $\sim p$.
15. Hunter's reference is to Abraham Adolf Fraenkel, *Abstract Set Theory*, Amsterdam: North-Holland, 1961.
16. I am indebted to Willa Fowler Freeman Sobel for comments and discussion, to Jeffrey Pelletier for a valuable challenge, and of course to Patrick Grim for many good ideas and lately valuable comments on this chapter, including a vigorous argument against the main conclusions drawn in Section 12.

Chapter XI

1. Jeremy Collier, *A Short View of the Immortality and Profaneness of the English Stage*, 1697, quoted in *The Compact Edition of the Oxford English Dictionary*, Oxford: Oxford University Press, 1971, p. 2644.
2. There is a presumption to overcome against the existence of a 'god of love', a god that would have us love it and would to that end make itself known to us. Prior to the establishment of reasons for the existence of such a god, there is, I think, a presumption against that existence. (Cf., Schellenberg 1993.)
3. A *god* would presumably satisfy *several logically independent* conditions that are individually necessary and jointly sufficient for *godliness*. If that is right, then an arguing atheist can make his case by showing that conditions in some proper subset *G'* of the set *G* of these necessary and sufficient conditions for godliness are not all satisfied by any one being. An arguing theist cannot make his case in a similar manner. He must show that the conditions in *G* itself (and thus those in each of its nonempty proper subsets) are all satisfied by some one being.
4. Is Alvin Plantinga such a theist? He thinks that "the argument from evil . . . really does give the believer (some believers, anyway) something to worry about" (Plantinga 1998, p. 543). He is himself such a worrying theist, who cannot be satisfied with merely the logical compatibility of the evil that abounds and the existence of his god.
5. This section revisits matters that received attention in Chapter I and in Section 7 of Chapter IV.
6. Cf. "The religious frame of mind. . . . desires the Divine Existence both to have that inescapable character which can, on modern views [of necessity], only be found where truth reflects arbitrary convention, and also the character of 'making a real difference' which is only possible where truth doesn't have this merely linguistic basis" (Findlay 1955, p. 55).
7. This argument *against* theism, from **objectively obligatory attitudes** being '**too queer**' to be believed, is opposed by a Kantian argument for theism in Mavrodes (1986), according to which: (i) **We are objectively obligated to do things** the doing of which costs us dearly, if there is no God who balances accounts; (ii) the world would be a **crazy place**, if we are objectively obligated to do things the doing of which costs us dearly; (iii) the **world** is not a crazy place; therefore, there is a

God to balance accounts. There is, Mavrodes realizes, available a *modus tollens* that, rejecting his argument's conclusion, holds on to its second premise and concludes that either we are not objectively obligated to do things that would cost us dearly (perhaps because objective obligations would be to queer to be believed) or the world is a crazy place.

8. Other inductive atheistic arguments, arguments that do not bring the evidence of evil against theism, can be gathered from late sections of Chapter VII, which can also serve atheistic demonstrations *quia*. Evidence for some Big Bang theories (Section 8.9 of that chapter) can be seen, on reflection, to be evidence against theisms that feature rational creators, whether or not moral. Big Bang theories that insist upon the lawlessness of the initial state that banged pose problems of motivation. A world with such a beginning is not what one would expect beforehand from a rational creator. Somewhat similarly, evidence for Stephen Hawking's wave function quantum cosmology (Section 8.8) can be evidence against theisms. The focus of the present section, however, is inductive arguments not from the recondite evidence that fuels modern physical cosmologies (e.g., "COBE satellite observations of the density fluctuations in the background radiation" – Smith 1995, p. 236), but from facts of apparently unnecessary evil that all can see and understand.
9. Hume narrows his sights to 'ills that *molest* sensible creatures'. Left out of his case (pp. 163–69) are *atrocities* and *wickedness* and *moral evil*, even though he allows that "in the opinion of many [it] is much more predominant above moral good than natural evil above natural good" (p. 169). Regarding moral evil, regarding evil *proper*, that sensible creatures not merely *suffer* but *perpetrate*, Hume is brief and challenging: "What I have said concerning natural evil will apply to moral, with little or no variation" (p. 169). ('Little or no'?) Theistic responses to evil are sometimes lopsided in the other way. Alvin Plantinga, after his 'modal number' on the 'logical challenge' of *moral evil* (Plantinga 1974a, pp. 24–57), dispatches that of natural evil with the thought that some may be required for greater moral goods, and the rest could be really moral evil, because the work of Satan and his cohorts, and so already 'solved' (pp. 57–9).
10. As in "Of Miracles," so in the *Dialogues*, Hume speaks with the sophisticated vulgar and engages in what, according to his deep philosophy, is nonsense.
11. Cf.: Cleanthes. "[I]f we abandon all human analogy, as seems your intention, Demea, I am afraid we abandon all religion, and retain no conception of our adoration" (p. 161). Cf.: Swinburne. "[I]t seems to me deeply central to the whole tradition of Christian (and other Western) religion that God is loving . . . and morally good. . . . There is no doubt more to loving someone than not kicking them in the teeth. But it does (barring special considerations) seem to involve at least not kicking them in the teeth" (Swinburne 1998, p. 7).
12. The second conjunct of the conclusion reached by Philo here is left out of what I gather, from the marvellous periodic rhetorical question with which Hume has Philo begin his last speech, is the stated conclusion of the *Dialogues*, namely, "*that the cause or causes of order in the universe probably bear some remote analogy to human intelligence*"and that "the analogy, imperfect as it is, can be carried no farther . . . and cannot be transferred with any appearance of probability, to the other qualities of the mind" (p. 185). Philo's not mentioning the great *improbability* of a transfer of the analogy to human *goodness* is, I think, generous

discretion on his part. He is about to open the door to "contempt of human reason" and flight "to revealed truth" of "the nature, attributes, and operations of the divine object of our faith" (Ibid.), Christ, the God of goodness and love of the Christian Bible. Hume allows his readers 'to contempt' human reason "that it can give no solution more satisfactory with regard to so extraordinary and magnificent a question" (Ibid.), and to go to faith to *complete* reason, rather than to *contradict* it, as Hume believed they would do. He believed that persons can of course *unreasonably* do that 'all by themselves'. He wrote only of the "reasonable person" as being, if "moved by *Faith* to assent to [the Christian religion] . . . conscious of a continued miracle in his own person, which subverts all the principles of his understanding, and gives him a determination to believe what is most contrary to custom and experience" (Hume 1902, p. 131), which words of Hume's were perhaps *not* meant in jest (peace to Mackie 1982, p. 29), or at least not *entirely* in jest.

13. Though bothered by 'Humean qualms', I believe in 'objective chances' on conditions.
14. Cf.: CORNEA, 'the condition for reasonable epistemic access': "On the basis of cognized situation s, human H is entitled to claim 'It appears that p' only if it is reasonable for H to believe that, given her cognitive faculties and the use she has made of them, if p were not the case, s would likely be different than it is in some way discernible by her" (Wykstra 1984, p. 85). The 'appearing' of CORNEA is said to involve (i) having an inclination to believe that p is caused by s and (ii) 'taking' s to harbor some evidence for us that p. My claim that 'it appears that p' is understood here, in CORNEA, to entail the report that I have an inclination to believe that p, the claim that I have this inclination, because (causal notion) of s and the claim that s (to which an exercise of my cognitive faculties has given rise) harbors evidence for p.
15. Stephen Wykstra writes of 'cognized situations' to which uses of 'cognitive faculties' give rise: See the previous note. I write instead of 'appearances' to which they give rise. I follow him in his use of 'appear'.
16. Something like condition RB is assumed in the sceptical challenge of Section 4.3.
17. Does ModerateNCECA make trouble, for what it would cost, for a claim on my part, based on the same cognitive situation, that it appears that I am not a brain in a vat? No. Would I even so not be entitled to that claim on that basis? I think not, being in part persuaded by (i) that I want to say that I am entitled on that basis to claim that it appears that I have feet and (ii) by the plausibility of the following principle (suggested by Wykstra in conversation) that says that entitled-appears-thats are 'closed under entailment': For any cognitive situation s, and propositions p and q, such that p entails q, if I am entitled to claim on the basis of s that it appears that p, then I am entitled to claim on the basis of s that it appears that q.
18. Nor does ModerateNCECA trouble my confidence that I am not a brain in a vat and say I am not entitled to it. That condition is for entitled-claims to how things appear, which would be based on appearances generated by exercise of my cognitive faculties. And while confident that I am not a brain in a vat, I may hesitate to claim that it *appears* that I am not a brain in a vat, that it appears *to me* that *I am* not a brain in a vat. Also, my confidence that I am not a brain in a vat does not pretend to be based on *appearances* generated by exercises of my cognitive faculties.

19. Not every perfect-being theist is a skeptic, witness Swinburne, and not every sceptic is a perfect-being theist, witness Hume. So far symmetry. But probably nowadays a greater percentage of perfect-being theists are sceptics than are sceptics who are perfect-being theists for 'proportional asymmetry'.
20. This argument, which in 1986 Rowe titled 'The Empirical Argument from Evil,' is, he tells us, the argument he previously referred to as 'the evidential argument from evil'. He goes back to that title: It is indeed part of the title to Rowe (1996). Premises of the argument of Rowe (1979) are similar; its conclusion is the same. But there is no talk of probability. Rowe's argument contrasts with Hume's in that it is specifically against a *perfect* being and cannot be extended plausibly to oppose nearly perfect beings. The plausibility of the exceptionless generalization B depends on its attendance to nothing less than a perfect being. Another contrast is that the form of Rowe's probabilistic argument is not implicitly 'Bayesian'.
21. Because premises (i) and (ii) entail that $P(\sim G) \geq P(E)$. For this inference, we have that for any p and q: **(a)**, $P(p \supset \sim q) = P(q \supset \sim p)$, since $\Box[(p \supset \sim q) \equiv (q \supset \sim p)]$; and, **(b)**, $[P(p \supset q) = 1] \supset [P(q) \geq P(p)]$, since, for any p and q, $P(p \supset q) = P(p \,\&\, q) + P(\sim p \,\&\, q) + P(\sim p \,\&\, \sim q)]$, $P(p) = P(p \,\&\, q) + P(p \,\&\, \sim q)$, and $P(q) = P(p \,\&\, q) + P(\sim p \,\&\, q)$. So, when $P(p \supset q) = 1$, we have that $P(p \,\&\, \sim q) = 0$, and thus that $P(p) = P(p \,\&\, q)$ and $P(q) = P(p \,\&\, q) + P(\sim p \,\&\, q)$. Regarding the argument, (i), (ii), therefore (iii): From (ii) it follows that (iv) $P(E \supset \sim G) = 1$ by (a). Premises (i) and (iv) entail conclusion (iii) by (b).
22. So is the argument with (*) $P(\sim E/G) = 1$ in place of (ii). Indeed, (*) is logically equivalent to (ii). For the entailment of (ii) by (*), assume (*) $\mathbf{P(\sim E/G) = 1}$. It follows that: $P(G \,\&\, \sim E) = P(G \,\&\, \sim E) + P(G \,\&\, \sim\sim E)$, and so that (**) $P(G \,\&\, \sim\sim E) = 0$. $P(G \supset \sim E) = P(G \,\&\, \sim E) + P(\sim G \,\&\, \sim E) + P(\sim G \,\&\, \sim\sim E)$. From (**) it follows that $P(G \,\&\, \sim E) + P(\sim G \,\&\, \sim E) + P(\sim G \,\&\, \sim\sim E) = P(G \,\&\, \sim E) + P(G + \sim\sim E) + P(\sim G \,\&\, \sim E) + P(\sim G \,\&\, \sim\sim E)$. $P(G \,\&\, \sim E) + P(G + \sim\sim E) + P(\sim G \,\&\, \sim E) + P(\sim G \,\&\, \sim\sim E) = 1$. So $\mathbf{P(G \supset \sim E) = 1}$, which is of course (ii). A somewhat similar series of inferences confirms the reverse entailment. The first step is to see that $P(G \supset \sim E) = 1$ entails $P(G \,\&\, \sim\sim E) = 0$.
23. The nearby argument (i') $P(E) > \frac{1}{2}$, (ii') $P(G \supset \sim E) = (1 - i) \approx 1$, and therefore (iii) $P(\sim G) > \frac{1}{2}$ is not valid. Let probabilities of 'basic conjunctions' of G and E have the values,

$$P(G \,\&\, E) = i,\ P(G \,\&\, \sim E) = (\tfrac{1}{2} - i),\ P(\sim G \,\&\, E) = .5,\ \text{and}\ P(\sim G \,\&\, \sim E) = 0$$

Then $P(E) = (\frac{1}{2} + i) > \frac{1}{2}$, $P(G \supset \sim E) = (1 - i)$, and $P(\sim G) = \frac{1}{2} \not> \frac{1}{2}$. The argument that puts (ii″) $P(\sim E/G) \approx 1$ in place of (ii″) is also invalid. The displayed values for 'basic conjunctions' entail that $P(\sim E/G) = P(G \,\&\, \sim E)/P(G) = (\frac{1}{2} - i)/\frac{1}{2} = (1 - 2i) \approx 1$, for 2i is an infinitesimal, given that i is an infinitesimal.
24. As always I write of probabilities as measures of persons' levels of confidence in propositions, in which persons' can without fault or irrationality differ. Rowe is not explicit about the concept of probability he intends in Rowe (1986), but it is a fair guess that he intends a measure of objectively reasonable levels of confidence in propositions that is singular and not person-relative. It can be gathered from the gloss on my conception of probability that I do not believe in objectively reasonable levels of confidence. It is not a difference that matters at the moment :$\sim$G and P(G) = 1 are logically consistent for all epistemic conceptions of probability.

25. She was "brutally beaten, raped, and strangled in Flint, Michigan on New Year's eve" (Rowe 1998a, p. 545). I am sorry to have to write of this case, and to come even that close to entertaining the possibility of its having been all-in-all worthwhile, and tolerated by an omnipotent, omniscient, and perfectly good by-standing witness. My God!!
26. 'We' covers both theists and atheists. The theists probably make for Alston an easy case, if they are justified in their theisms. For then they are justified in judging that these evils are *not* gratuitous. The substantial thesis may be that an *atheist* cannot be justified judging that these evils are gratuitous or, for that matter, that they are not gratuitous, since all they, as anyone, has to go on is that as far as anyone can see these evils are gratuitous.
27. Why only 'may be'? Because, for all we know, we may not be ignorant of *any* of what would be reasons for such a being. Do we know that such a being would not be a simple Benthamite utilitarian?
28. I write 'at least then' because, while I think that an empty search of the dimensions of Alston's would be sufficient for an atheist's confidence regarding the two cases, it is not necessary. I doubt that Rowe agonized over whether he was being fair to perfect beings and thought long and hard about ways the awfuls of his cases could have been for the best, before he was confident that they had *not* been for the best in them, or anyone. I think he was *justified* even in his *first* confidence and that, as Hume (1991, p. 160) and Swinburne would say, 'it was already then the theist's turn to tug the heavy oar' and shake that confidence with positive reasons for thinking that those awfuls might well have been for the best It is *not* enough for them to say that, *for all we can ever know*, these awfuls were for the best, that is, that we do not know that there are not positive reasons unknown to us for thinking that they were for the best. Compare this to the lame argument against The Resurrection, that we do not know that there are not positive reasons quite beyond our ken – for example, properties of light, forever lost to us in space, that reflected from the site of the tomb – for thinking that Jesus did not die on the cross.
29. Similarly for believing with justification that the total consequences of an action, reaching into 'the infinite future,' would be at least as good as those of any alternative to it. We can be justified in believing that about an action, when, as far as we can see, it would have best consequences, and we have no reason to think that further investigation might well cast this action in a different light. *Pace*: (Moore 1993, p. 199, Section 90): "[W]e never have any reason to suppose an action is our duty [and]... will produce the greatest value possible"; and (Bergmann 2001, pp. 292–3).
30. Rowe has omitted reference to the argument for P that, while not proving it, makes it probable. He should have said that his 1988 argument went something like this: It is probable that P: $Pr(P) > \frac{1}{2}$. P makes Q probable: $Pr(Q/P) > \frac{1}{2}$. Therefore it is probable that Q: $Pr(Q) > \frac{1}{2}$. Q entails ∼G. Therefore it is probable that ∼G: $Pr(\sim G) > \frac{1}{2}$. The second inference is valid, but the first inference is not. See discussion under the third problem in Section 6.1.2.
31. This objective is not well served by including k in the condition of every conditional probability, since that does not 'say' it is in the probabilities 'background information'. What recommends itself is a convention for subscripts that indi-

cate the total background information of probability functions: The one that Rowe is interested in would be the probability function Pr_k. That would take k out of the conditions of the probabilities with which he is concerned and bring in unconditional probabilities such as $Pr_k(P)$, along with conditional probabilities such as $Pr_k(\sim G/P)$. We could, to identify Pr_k, say that it is the probability function Pr in which $Pr(p) = 1$ if and only if $\Box(k \supset p)$.

32. Rowe does not explain his use of 'information'. I assumed that something is 'information' for a person if and only if he is certain of it (whether or not it is true) and that 'background information k' is closed under entailment.
33. There is the possibility that some shared information for theism will need to be removed, so that $Pr(G/k) = \frac{1}{2}$. And – a point to which I return in 6.1.3.5 – for an argument *specifically* from *the particular evils E1 and E2*, there is likely other shared information that 'tells' against G that will need to be removed, for example, that our experience is of only *embodied* intelligence and of wills that are not 'immediately efficacious'. And there is the likely shared information of *other* particular evils and of "the . . . **multitude** of horrendous evils in the world" (Rowe 1996, p. 265; emphasis added) that should not be part of 'background information' for an argument from the particular horrors E1 and E2.
34. Cf.: "[P]roblem, Given: $P(C|A) = p, P(C|B) = q$, find: $P(C|AB)$. ['Solution'] . . . **if no additional information, probabilistic or logical, is given** the probability of the conclusion on the conjoint evidence can be any value in the unit interval" (Hailperin 1996, p. 235; emphasis added). "But Plantinga 'gives' additional logical information. He says that $P = (\sim G \vee \sim J)$, $P^* = (G \vee \sim J)$, see pp. 541 and 542. So it 'looks' as if he is saying that $Pr(G/(\sim G \vee \sim J)\ \&\ k) = 1/3$ and $Pr(G/(G \vee \sim J\ \&\ k) = 2/3$ entail that $\mathbf{Pr(G/\sim J\ \&\ k) = \frac{1}{2}}$." True, but *this* entailment does not hold. An urn could contain three balls that are not jet black ($\sim J$), two of which are graphite (G) and the other not ($\sim G$). In this urn two-thirds of the balls that are $(G \vee \sim J)$ are G, and one-third of the balls that are $(\sim G \vee \sim J)$ are $\sim G$, and two-thirds of the balls that are$\sim J$ are G, **not 1/2**.
35. I think that the way is, for propositions p and q, to let $Pr(p/k) = Pr_k(p)$ and $Pr[q/(p\ \&\ k)] = Pr_k(q/p)$, where Pr_k would be your probability function "so long as [you] had only k to go on" (Rowe 1996, p. 272) and hope you are 'every man'.
36. Cf.: "P: God does not exist or (God exists and no known good justifies him in permitting E1 and E2" (Rowe 1998, p. 545).
37. $\sim J^*$ is similar to "P*: No good we know of *would* justify God (*if he exists*) in permitting E1 and E2" (Rowe 1996, p. 283n8). Rowe has against P* in place of P for his argument only that it is not entailed by $\sim G$. $\sim J^*$ is close to Plantinga's $\sim J$, understood in the first of the two ways that Rowe discovers in (Plantinga 1998, p. 541): See (Rowe 1998, p. 549). $\sim J^*$ is closer to Rowe's X, which he says entails P (p. 550), as he says $\sim G$ does. So $(\sim G \vee X)$ entails P. Rowe does not say, but I assume he would agree, that P *should* entail $(\sim G \vee X)$, which would be to say that P should be revised to entail $(\sim G \vee X)$.
38. We should want excluded from the background information of all of these arguments from evidence of evil against G, evidence against G that has nothing to do with evil. Otherwise we might reach the conclusion, for some proposition p to do with *evil*, that $Pr[G(p\ \&\ k)] < 1/10$, *because*, for example, of the improbability on k of 'directly efficacious volitions.'

39. Cf.: Given the restriction of k "to information that is shared by most theists and nontheists" and "not to beg any questions. . . . We will say that k by itself makes neither God's existence nor his nonexistence more likely than not . . . [and that] aspects of k – if they . . . impact positively or negatively . . . in some way balance out . . . " (Rowe 1996, p. 265).
40. Framework Assumption II can remind one of the assumption that Rowe makes that "aspects of k – if they impact positively and negatively on the likelihood of God's existence – in some way balance out so that the totality of k leaves the probability of the existence of God at 0.5" (Rowe 1996, p. 266). He wants k trimmed if necessary for this result. The argument I am assembling would have established to the satisfaction of all in its first stage that, subject to the exclusion of considerations of evil from their background information, considerations for and against 'balance out'.
41. Bergmann would agree that we have *some* ground for believing that *Q* but not enough to make *Q* more probable than not. "[C]ontrast . . . the belief that, *because you have tried your best to think of what possible goods there are, you have a fairly good idea of what possible goods there are* . . . [with the belief] that the earth is more than 100 years old or that there is an external world?" "Is that belief **sensibly** held **with anything like the degree of confidence** with which [these beliefs are]. Certainly not" (p. 290; emphasis added). Implied is that it *is* sensibly held with *some*, but *not much*, confidence. We have 'been here before.' The belief that you have a fairly good idea of what possible goods there are that might justify E1 and E2, is, at this late date, sensibly held, as long as you do not have a reason for thinking that there remain vast arrays of goods possible up to that task of which you have no idea. "But you have such a reason, if you have reason to believe in God!" True. However, this move does not interrupt the *present* argument for a high probability for Q*, proceeding as it does under Framework Assumption II that says in part that we agree that we have no reasons for believing G.
42. The argument is valid without this premise: If an argument with p as a premise is valid, and $\Box(p \supset \Box p)$ is true, then the argument without this premise is valid. Relevant to the present case is the principle that for any p, $\Box[\Box p \supset \Box\Box p)$.
43. Neither K nor K' is the weakest certainty that entails each proposition *other than* Æ of which our subject is certain, if his certainties are 'closed under entailment'. If his certainties are 'closed under entailment', that weakest certainty is still TI. (He is certain of Æ and, so, also of $(Æ \vee P)$ and $(Æ \vee \sim P)$ for every proposition P. For any contingent proposition P these disjunctions are not logically equivalent to Æ, so they are *other than* Æ. Every proposition that entails both of these disjunctions entails Æ. So every proposition that entails every certainty other than Æ also entails Æ and thus every certainty. TI is *the* certainty that entails every certainty.) Describing K as TI *minus* Æ thus calls for 'scare quotes', as, for a related reason, does describing TI as K plus Æ in view of the equivalence of TI and (K & Æ). (K is equivalent to $(TI \vee \sim Æ)$, and $[(TI \vee \sim Æ) \& Æ]$ entails T. So *(K & Æ) entails TI*. TI is the logically strongest proposition of which our subject is certain, so it entails every proposition of which he is certain. He is certain of Æ . So TI entails Æ, and thus $[(TI \vee \sim Æ) \& Æ]$. Since again K is equivalent to $(TI \vee Æ)$, it follows that *TI entails (K & Æ)*.)

44. There are solutions for other cases in Section 4.3.2 of Chapter VII and Section 5.5 of Chapter VIII. My present management of the problem is meant to be substantially as explained for Hume's evidential argument from evil.
45. I use 'faith' here in a manner that entails strong religious conviction in the sense of a high degree of personal probability. For a very interesting account of faith that lacks this entailment, see Koons (1993, pp.145–60).
46. I do not consider being represented by a single probability function or set of functions to be a condition for reasonable credences, since I think that this reasonable is possible in the presence of limited knowledge of necessities. For example, not too long ago it was perfectly reasonable to have a credence less than certainty for Fermat's Last Theorem, as I assume many top mathematicians did.
47. I am relating van Inwagen's sceptical strategy to my Bayesian form for an evidential argument from evil. He actually addresses this strategy to a two-stage argument that consists first of a *prima facie* case against theism from a consideration of likelihoods of evil on theism and alternative hypotheses, and second of a defence of these likelihoods and resistance to attempts to provide overriding reasons for theism. The exact logic of the argument he opposes is to me obscure.

Chapter XII

1. The logical problem of evil is a problem for the *nonmodal core* of perfect-being theism that says that there exists a perfect being, omniscient, omnipotent, and perfectly good. It is not part of this core that there exists *necessarily* an *essentially* perfect being. Rowe implies otherwise (Rowe 1999, p. 101a), the other bits only developing his 'second problem' for theists, which is that "there is a [certain] serious conflict, if not incoherence, **within** the theistic picture, of God" (p. 103b; emphasis added.). Swinburne, to deal with the problem of evil, assembles a theodicy for his essentialist perfect-being theism. He thinks that an essentially omniscient everlasting being would not know what free choices his creatures are going to make, or what free choices his free creatures would be going to make, if he had created a somewhat different world. He thinks that such a being would not know these true propositions, because such a being *could not* know them (p. 131). I doubt that he establishes that (cf., footnote 48). In any case, his theodicy does not 'solve' the problem of evil, since that is a problem for *omniscient* beings, and his theodicy depends on an 'essentially omniscient' being's not being omniscient. An essentially omniscient being would not have the foreknowledge of free choices that a 'merely omniscient' being would have!! It depends on an essentially omniscient being's lacking foreknowledge that a 'merely omniscient being' would have, and so in not knowing everything that is knowable.
2. Grover Maxwell said in a conference paper in 1953–4 (I was present) that ordinary language is notoriously ambiguous and vague! O. K. Bowsma, in his role as commentator, explained, 'Herbert told May, May told Grover, he told Herbert, and that is the way it got around.'
3. Nor does Richard Gale, unless he has recently changed his mind. "Almost everyone now believes that adequate defenses have been devised to neutralize [the logical problem of evil]. . . . In particular, it is claimed that the free-will defense,

in at least one of its many versions, succeeds in reconciling God's existence with moral evil. In my book . . . I argued that no version of this defense works, and thereby the logical problem posed by moral evil is still with us" (Gale 1996, p. 206).

4. The second conjunct '$\sim\Diamond$(Evl & PrfBng)' says that it is impossible for both evil and a perfect being to exist, which could be, not because they 'fight' or are *incompatible*, but because one or the other is impossible *full stop*. The first conjunct excludes this possibility.
5. A proponent of the argument who maintains that a perfect being is possible must think that (1) is not necessary. For suppose $\Box$Evl and $\Diamond$PrfBng. Then $\Diamond$(Evl & PrfBng), which contradicts part of (2).
6. Looking into this concession, it is one thing to say that moral evil and suffering are a necessary means to the greatest good of the world and *another* thing to say that therefore evil is compatible with the existence of a perfect being. For a perfect being would be *morally* perfect, and, a Kantian might say, it is always wrong to treat humanity in a person as a means, and a perfect being would not *do* it 'though the Cosmos fell'. Cf. Pereboom (1996, p. 512) on Kant's "merciless . . . rejection" as meriting "no refutation" of the theodicy that would justify moral evil and suffering as being "in relation . . . to the highest wisdom precisely the most fitting means to . . . the greatest good of the world" (Kant 1996, 27). But see note 10.
7. Cf.: "Virtually everyone who endorses an argument from gratuitous evil accepts" that "if His purposes included anything like the greater goods involved in our most plausible theodicies, He would be justified in permitting some intense suffering, perhaps even a great deal" (Howard-Snyder 1999, p. 127). This conditional that gratuitous evil arguers are said to 'accept' implies that it is plausible that God's purposes would justify His permitting even a great deal of intense suffering. No doubt some who run arguments from gratuitous evil accept *that* 'for purposes of moving on' and say that there remains the ample evidence of *gratuitous* evil against His existence. But I doubt that any who runs arguments from gratuitous evil *believes* that God's purposes would justify His permitting a great deal of intense suffering.
8. Or equivalently, $(\exists x)$[Bst(x) & $\sim$Bst@]. Bertrand Russell would count 'a best of all possible worlds' as "'denoting phrase'" (Russell 1905, p. 479), "an indefinite description" (Russell 1919, p. 167). In a formal treatment other than the familiar one, he could handle indefinite descriptions in a manner that discriminated between possible scopes when they are embedded in compound sentences (see Neale 1998). In terms of such notation premise, (1′) accords its indefinite description 'widest scope'. Plainly stated, it is intended that (1′) entails that there *is* a best possible world.
9. Let 'Perf(x)' say that x is a perfect being. The propositions that $(\exists x)$Perf(x) and that P@ are 'materially equivalent' but they are not logically equivalent, unless either $\Box(\exists x)$Perf(x) or $\sim\Diamond(\exists x)$Perf(x). If *not* this disjunction, then either P@ is true at every world and $(\exists x)$Perf(x) is false at some world, or P@ is false at every world and $(\exists x)$Perf(x) is true at some world.
10. Would Plantinga question (b)? It can *seem* that Kant would: See note 6. *However*, by factoring in the value of a *divine* 'good will', Kant *could* have said that God

necessarily created a best world he can create, though there can be worlds he could have created instead that would have been '*mundanely* better' on *partial* accounts that left out values accruing to them from the good will of the divine person Himself. Cf.: "It is hard to see how the additional good to be gained by God's violating a moral duty could ever outweigh the good that would thereby be lost" (Langtry 1996, p. 320).

11. A consequence of this truth condition for 'counterfactuals of freedom' is that, if (p $\Box\!\!\rightarrow$ q) is 1, then **(p & q) entails (p $\Box\!\!\rightarrow$ q)**. I will ignore this entailment *and* (except for this note) the fact that, by *relying* on it, and a generalization of the coming assumption *[MA2]*, one can prove that, *for each possible person, it is possible that this person* suffers from 'transworld depravity relative to this world of ours,' as defined below (which is *not* to say that 'universal transworld depravity' is a possibility, that is, that *it is possible that, for each possible person, that this person* suffers from transworld depravity relative to this world of ours). Michael Bergmann explains such a proof (Bergmann 1999, pp. 338–9). I ignore these things for two reasons. First, because Bergmann reports that Plantinga now "insists that A & B does not imply *If A were the case, then B would be the case*" (p. 34), so that Plantinga would not himself be satisfied by that proof, and second, because I believe that the correct logic for subjunctive conditionals in general is 'weakly centred' (Lewis 1973, p. 29; cf., Sobel 1986, Section 3.2), as does Plantinga now: "[T]he counterfactual is true only if there is no *sufficiently close* possible would in which p is true but q is not" (Plantinga 1996, p. 329). The suspect highlighted entailment is not valid for 'weakly centred' conditionals. The simplest retrenchment for Plantinga that left as much as possible in place would be I think that truth conditions for its 'counterfactuals of freedom,' in addition to those he states, include that their antecedents are false, so that, if '(if it were that p, then it would be that q)' is a 'counterfactual of freedom' – or, not to confuse, let us say a 'Plantinga counterfactual of freedom' – then it is *true* if and only if it true that **[$\sim$p & (p $\Box\!\!\sim$ q)]**, wherein the counterfactual (p $\Box\!\!\rightarrow$ q) has precisely the strongly centered 'Stalnaker' truth conditions stated in (Plantinga 1973a,b). Then this 'Plantinga counterfactual of freedom' is *possible* if and only if there is a possible world at which is it true that [$\sim$p & (p $\Box\!\!\sim$ q)]. That there is a possible world at which it is true that (p & q) does not entail that there is one like that. Assuming this retrenchment, one 'loses' Bergmann's demonstration of something that would be near enough (consider note 16) to 'the possibility of universal transworld depravity' for Plantinga's 1973 purposes.
12. Plantinga would I think add that this must also be 'supernaturally possible'. He would I think say that S is *not* free in the intended sense if, though it is *causally and naturally possible* both that *U* hold and S take that action, and that *U* hold and S refrain from it, this is not *really possible* since were S about to take the action God would intervene and get S to refrain. In the sense intended, freedom should be incompatible not only with most forms of Causal Determinism for the natural world, but also incompatible with settled policies of divine intervention. There are here two kinds of 'incompatibilist freedom,' a weak kind that is incompatible only with kinds of Causal Determinism and a strong kind that is incompatible as well with settled policies of divine intervention.

13. The assumption is that sometimes it is not true only that an agent who has a choice between doing something and not doing it *might* freely choose to do it, and *might* freely choose not to do it. Plantinga's assumption is that sometimes it is true that he *would* do it, or that he *would* not do it. Adams rejects this assumption, and that only 'might'-counterfactuals of freedom are true. For example, according to Adams, in a circumstance in which Paul will be exercising his freedom to accept or not, it is true that Paul *might* accept, and *might not*: [(Offer $\Diamond\!\!\rightarrow$ Accept) & (Offer $\Diamond\!\!\rightarrow \sim$Accept)], but *not* true that he *would* accept, (Offer $\Box\!\!\rightarrow$ Accept), or *would not*, (Offer $\Box\!\!\rightarrow \sim$Accept) (Adams 1987, p. 91n4).
14. P is a *possible person* if and only if there is a possible world W such that were W the way things are, then P would be an actual person.
15. That is, "always does what is right" (p. 52), or "does only what is right" [p. 186], where these are understood as expressing equivalent generalizations that have 'existential import.'
16. Reflecting on the argument for $\Diamond$Beyond, including this argument for $\Box$(UTD $\supset$ Beyond), one may see that Plantinga could have used, to reach $\Diamond$Beyond, something considerably weaker than $\Diamond$UTD. Perhaps the weakest replacement along the same lines is it is possible that, for every morally perfect world W in which there is some moral good and not moral evil, there is in W a person P who does of his own free will a right action X, and it is true in the actual world that ($\mathrm{MWS_W(X)}$ $\Box\!\!\rightarrow \sim$X).
17. Since [$MWST_{\mathrm{PAW}}(\mathrm{X_1})$ $\Box\!\!\rightarrow$ $\mathrm{freelyX_1}$] is true at PAW, there is a world in which, though it is true that $MWST_{\mathrm{PAW}}(\mathrm{X_1})$, it is not true that ($\mathrm{freelyX_1}$), but it is true that (freely $\mathrm{X_1'}$), where $\mathrm{X_1'}$ is something Adam could in PAW have freely done instead of $\mathrm{X_1}$. That means that ($MWST_{\mathrm{PAW}}(\mathrm{X_1})$ & $\mathrm{freelyX_1'}$) is possible. But that does not entail that [$MWST_{\mathrm{PAW}}(\mathrm{X_1})$ $\Box\!\!\rightarrow$ $\mathrm{freelyX_1'}$] is possible, *if* this is taken to be a 'Plantinga-counterfactual conditional' in the sense explained in note 11, and, as said in that note, an argument for the possibility of transworld depravity that relied on that entailment would not now satisfy Plantinga.
18. Mackie missed the opportunity provided by his confusion of Plantinga's idea of 'transworld depravity' with that of 'essential conditional depravity,' to observe that there is authority against any actual person's being *thus* handicapped. It is written (*Matthew* 5:48) that He enjoined, *Be ye therefore perfect, even as your Father which is in Heaven is perfect*, thereby implying that He considered this to be possible for everyone.
19. "Plantinga proposes the hypothesis that all possible free creatures (or their essences) have *trans-world depravity*. . . . Plantinga does not claim that the hypothesis is true, or even that it is plausible. He argues only that it is logically possible, because he is using it to defend the view that it is logically possible that both God and evil exist. I do not doubt that the latter is logically possible; but religious thought must seek an account . . . that is credible, as well as logically possible" (Adams 1987, p. 89). I read Adams as 'saying' that he does not doubt that *the former* is also logically possible. He continues: "It is worth asking, therefore, whether the hypothesis of universal transworld depravity is plausible, **on the assumptions about truth of conditionals that Plantinga shares with. . . . Molina and Suarez**" (Ibid.; emphasis added). Adams explains reasons they would have against the fact of universal transworld depravity that are

similar to my reasons against its possibility. Adams must say that, those assumptions aside, universal transworld depravity is *impossible*. His position on Plantinga's 'would-counterfactuals of freedom,' which is that they are all false, entails that no one can suffer transworld depravity: The condition, explicit in its definition, for a person P being transworld depraved in a world W is that enormously many, 'would-counterfactuals of freedom' are true of P in W.

20. There is evidence that Alston accepts Premise (2″). He argues "that we are [not] rationally justified in accepting" that (1) "there exist instances of intense suffering which an omnipotent, omniscient being could have prevented without thereby losing some greater good or permitting some evil equally bad or worse" (Alston 1996, p. 98). There is evidence that he believes that if we were justified in believing in one such instance, we would be justified in believing that there is no such a being. "No doubt the theist is committed to regarding (1) as false" (p. 121n2). He believes that there is such a being, from which I gather that he believes that there is no such instances of intense suffering and 'leap' to the conclusion that he believes that this mess of a world is as good as a perfect being could make it. That is not 'Pangloss', but it is a son of it.
21. Counterfactuals of these forms are counterfactuals *of freedom* when q says that someone, of his own free will, will do (has done, is doing) something.
22. The argument says that God could not make sure that this is a best possible world: Let that be the proposition that *there is no divine guarantee of a best world* , NDG. An argument from the possibility of what Bergmann terms 'universal transworld untrustworthiness', ◊TU, says that *it is possible that* NDG and explains why this is sufficient: "[I]f *every* essence is transworld untrustworthy, then there is no way God could create free creatures and *ensure* that no evil occurs. . . . [or] *guarantee* that no free creature will do what is wrong. It follows from ◊TU (and the rest of [Plantinga's] . . . original FWD), therefore, that God's existence is compatible with the existence of evil" (Bergmann 1999, p. 342).
23. Albert Einstein said it, though one should not, Anthony Flew explains, gather from this often-cited dictum that Einstein was a believer. "Einstein was once asked . . . whether he believed in God. He replied that he believed in Spinoza's God. Since for Spinoza the words 'God' and 'Nature' were synonymous, Einstein was, in the eyes of Judaism, Christianity, and Islam, unequivocally an atheist. It was in this Spinozistic understanding of the word 'God' that Einstein protested against quantum theory: 'The Lord God does not play dice'" (Flew 1996, p. 57).
24. The sequence can be of length 1. A perfect being may make one choice 'in the beginning'. Consistent with that would be this being's activity at all times in the execution of this choice.
25. Instead of assuming *divine knowledge of objective chances* for worlds on a creative choice, one can assume and work with *divine probabilities* for worlds conditional on this choice. Divine conditional probabilities, since they would be ideally well informed (!), would be equal to objective conditional chances, if such chances exist and would, I assume, in any case 'guide' a Divine's choices. It can be seen that either way, not only can a best divine bet world not be a best divinely creatable world, but a best divinely creatable world need not be a best divine bet world. If of two slot machines each of which takes one loonie

(that's a one canadian dollar coin), and one pays $1,000 one time in 20,000, while the other pays $10 one time in 100, the second one is the machine to play, if I want to maximize my expected monetary return.

26. Though an omniscient's '*middle knowledge*' of what free agents other than those there are would do may be limited, because there is nothing they *would*, but only chances for what they might do; an omniscient's *foreknowledge* of what actual free agents *will* do is *complete*.

27. *A note on the Infinite Goodness of God and the Problem of Evil*. Let me sketch a way to reject a theo-mathematical argument for there *being* a best possible *world*, in contrast with **worlds**. It goes like this:

    God is *the* infinitely good possible thing. There is a world in which God exists and no *infinitely* bad thing exists. Let W be such a world. W is infinitely good and a best world. For *suppose* there is a world that is better than W. Let W' be such a world. W' is only finitely better (for it is given that God is *the* infinite good): That is, *Value(W′) > Value(W)*, and, for some finite n, Value(W′) = Value(W) + n. However, since Value(W) = ∞, Value(W) + n = Value(W, and *Value (W′)≯ Value(W)*. Contradiction. Therefore there is *not* a world that is better than W.

    This argument raises some issues of value theory, but they are not well raised by it, since to reject the argument it is sufficient to say that values of worlds are best measured not by standard real numbers and Cantorian infinities, but by nonstandard, hyperreal numbers that run down into *infinitesimals* greater than zero and smaller than every 'hyperreal real' and up into *infinimals* greater than all 'hyperreal reals'. The relevant point about infinimals, or infinite hyperreals, is that for any infinimal *I* and 'hyperreal real' n, *I* + n is a *greater* and *I* − n is a *lesser* infinimal. (Hyperreals are discussed in notes and the appendix to the next chapter. Using them for values, if there is not a best *world*, then there is not a best *world*.)

28. Using , for illustration, Premise (2′), (◇[(∃x)Bst(x) & ∼Bst@] & ◇PrfBng) & ∼◇([(∃x)Bst(x) & ∼Bst@] & PrfBng), *assume* that it is necessary that there is not a best world: □∼(∃x)Bst(x). Then ∼◇(∃x)Bst(x), which entails, ∼◇[(∃x)Bst(x) & ∼Bst@]. That entails ∼◇[(∃x)Bst(x) & ∼Bst@] & ◇PrfBng), which is the negation of the first conjunct of Premise (2′).

29. How are premises (*) and (') related? We have that the world is a *bettered-world* if and only if it is *not a best world*, (Bttrd@ ≡ ∼Bst@), and let us assume[#] that it is necessary that there is a best world if and only if it is possible, □(∃x)Bst(x) ≡ ◇(∃x)Bst(x). Now come relations between the premises in the partition of cases: (a) □(∃x)Bst(x); (b) □∼(∃x)Bst(x). If (a), (1*) and (2*) are logically equivalent to (1′) and (2′) respectively. If (b), then: (1*) is logically necessary and (1′) logically false; (2′) is logically necessary; (2*) entails (2′); and (2*) is not entailed by (2′), unless (2*) is true, in which case it too is logically necessary. ([#]The assumption is that values of worlds are not 'world-relative', so that if there is a world w that is at some world a best world, then w is a best world at every world. This assumption is a consequence of a view of value that is taken for granted in all discussions of God and evil with which I am familiar. According to that view, the values of 'things' depend in the sense of being entailed by their 'natures'.)

30. Cf.: "It might be objected, 'If V is better than W, and God can actualise V without violating any moral duties, then it follows that God has better reasons for actualising V than for actualising W, and so that God would be acting irrationally if he actualised W rather than V.' Either these things do not follow, or else an omnipotent, omniscient and perfectly good being is logically impossible, or else there are prime worlds [worlds God could actualize such that he could not actualise better worlds]" (Langtry 1996, p. 320). I think 'these things do follow'.
31. Similarly regarding: "[I]f there is no maximum degree of perfection among possible worlds, it would be unreasonable to blame God, or think less highly of his **goodness**, because he created a world less excellent than he could have created" (Adams 1987, p. 52; emphasis added). This is true even if his preferences for worlds go by their 'degrees of perfection'. From the assumption of no maximum perfection and this rule for His preferences, only something disjunctive follows like, either He did not know there were better worlds to choose or He was not up to choosing and making the better worlds, or He was dazzled by the infinite richness of His options, so he chose unreasonably. If you say none of that and more would be consistent with His *nature*, then I say He did not create the world, and if you say *that* is inconsistent with His nature, then. . . .
32. For the existence of such strategies it is not enough *that there is not a most valuable possible world*, since this could be, though the value of every world is in the interval 0 to 1. There are, however, strategies of infinite objective expected value, if for every positive natural number there is a world of this value. Suppose that, and assume the limit-rule for denumerably infinite sums. Let worlds with natural number values be numbered by them. Consider the mixed strategy that, for every positive natural number n, assigns the chance $1/2^n$ to world 2^n. This strategy is well-defined, since, by the limit-rule, $\Sigma 1/2^n = 1$. And by the limit rule the objective expected value of this strategy is infinite. This mixed strategy is not what would ordinarily be meant by a '*random* strategy', which is a mixed strategy that assigns *equal* chances to outcomes.
33. X obtains 'just before' a time t if and only if there is a prior time t' such that, for every time t'' subsequent to t' and prior to t, X takes place at t''.
34. His knowledge according to this assumption is confined to the objective chances for outcomes on this mixed strategy. Suppose the mixed strategy is M, one of its outcomes is O, and the chance of O in M is c. Then what he knows about O could be symbolized $\Diamond_c$O. For contrast, if M were a strategy other than the one he knows he will commit himself to, then his knowledge of O would be of its chance conditional on that strategy and could be symbolized (M $\Diamond_c\!\!\rightarrow$ O).
35. Similarly for a person's infinite expected value for a St. Petersburg game for money.* Contrary to Richard Jeffrey, the lesson of these games is *not* that only a fool would have an infinite expected value for one, but that everyone else would realize "there is not that much money in the world" (Jeffrey 1990, p. 155). The lesson of these games is that *their infinite, their greater than any finite measure, expected values* are *prima facie* not relevant to what an agent should be prepared to pay to play them, since he knows that they are bound to return a finite sum. [*"The player tosses a coin repeatedly until the tail turns up on, say, the *n*th toss, at which point the game ends, and the player is paid 2^n dollars" (Jeffrey 1990, p. 150).]
36. "Is this a well-defined mixed strategy? Chances in a mixed strategy must sum to 1. A device that 'randomly selected' a number would establish the same chance

for every number n. The sum of an infinite series of some number n, no matter how small, is infinite." The last is true of standard, but not of nonstandard, real numbers. "Though the chance for one of an infinite number of equally likely possibilities, 'cannot be a positive real number . . . [w]ith the development by A. Robinson of nonstandard analysis . . . another alternative presents itself: [that it] should be a positive infinitesimal. . . . [A] nonstandard measure can be constructed on the unit interval such that **the measure of any real number in the interval is a fixed infinitesimal**' (Bernstein and Wattenberg 1969, p. 171; emphasis added)" (Sobel 1994b, pp. 73–4). Similarly, I assume, for the 'spectrum' of positive natural numbers. The strategy is well defined, and I *assume* that its objective expected value can be defined by an extension of the limit-rule for denumerable sums, and that so defined it is infinite.

37. The second strategy *dominates* the first under the infinite partition of possible numbers delivered, and there is no infinite partition under which the first dominates the second. In such a case, though his expected values for these strategies, since both are 'infinite', i.e., greater than finite, are the same, the dominating strategy is preferable. Cf., Sobel (1994, p. 84.)
38. Is foresight that is not 'fore-figuring' *possible*? It is if *hindsight*, which is not *memory*, that is, cognition caused by the fact remembered, is possible. While without evident incoherence in their concepts, they may, for all I know, not be possible cognitions. If foresight is impossible, this second case may not be possible, and there may be only the first case against the 'high-tech' solution to the problem for perfect-being Creators, if there is not a best possible world.
39. The possibility of omniscience alone is defended in Chapter X. Its compatibility with 'freedom of choice' is the subject of the present appendix. Now come brief remarks relevant to how well it goes with other characteristics that might be considered divine. I have in mind for them these remarks, the truth that the sun is shining *here and now*. Suppose that a being is 'omnipresent throughout space-time? and so no more *here-now* than *there-then*. Could this being know that it is not raining *here* and *now*? Suppose a 'polar opposite' being who is aspatial and atemporal and *never anywhere*. Could *this* being know that it is not raining here and now? I see no time-and-tense reason why not in either case, as long as here and now are in universal features distinct, though such beings would not express such propositional knowledge in sentences such as 'it is raining here and now.' Suppose that a being is *immutable* in itself but spatial and temporal just like us. Could *it* have use for such sentences in the expression of its knowledge? Eddy Zemach and David Widerker have sketched a way in which an essentially omniscient being can be in itself the same in different possible worlds: See Section A5.2.1. This sketch implies a way in which an immutable essential omniscient that was spatial-temporal like us could on occasion say knowingly to itself, "[T]he sun is shining [here] *now*" (Alston 1986, p. 305): As it 'successively occupied different spatial-temporal perspectives' its fixed inner-state-words 'here' and 'now' could take on different referents. Alston says that "an immutable *or* timeless being cannot . . . successively occupy different temporal perspectives" (p. 305). While a timeless being cannot do that, and so cannot 'know in temporal indexicals', an immutable being can occupy different temporal perspectives and so – in the way sketched by Zemach and Widerker – may be able to 'know in temporal indexicals'.

40. There is evidence, in passages quoted by Hunt, that Augustine wanted more: "Surely this is the question that troubles and perplexes . . . how can the following two propositions, that God has foreknowledge of all future events, and that we do not sin by necessity but by free will, be made consistent with each other? 'If man must sin, his sin is not a result of the will's **choice**, but is instead a fixed and inevitable necessity. . . . '" (Hunt 1999, p. 4; *On Free Choice of the Will*, III.3; emphasis added). "Evodius's [surviving] doubts . . . 'Of course I do not deny any of these points. Yet I still cannot see how God's foreknowledge of our sins can be reconciled with our **free choice** in sinning. God must, we admit, be just and have foreknowledge. But I would like to know by what justice God punishes sins which must be. . . . '" (p. 6; III.3; bold emphasis added).
41. Chapter I of Sobel (1998b) discusses *che sarà sarà* naive fatalism, which it finds similarly flawed. God has nothing to do with the present bad argument for the incompatibility of His foreknowledge and freedom. So foreknowledge has nothing to do that fatalism, the argument for which would be complete in the 'insight' that *necessarily what will be, will be*.
42. Cf., (6) in Pike (1989, p. 63), and the inference from (6) to (7) in Plantinga (1989, p. 185).
43. McCord Adams implies that someone C is like merely omniscient G in all respects except that C at t′ mistakenly believes A will do Y at t, might 'take G's place' and be everlastingly omniscient, if A were to do Y at t (McCord Adams 1989, p. 304n6). But consider that, for every proposition p that is false and that is logically independent of the proposition that A will not do Y at t: Were A to do Y at t, C as described would (as G does and would) still believe the proposition that (either A will not do Y at t, or p), though this proposition (which is true) would be false. C, as specified by McCord Adams, though very similar to G belief by belief, would be *very different* from G, in that C's beliefs would be massively inconsistent.
44. For any thing X and property P, if X is essentially P, then X is essentially essentially P (and so on!).
45. Swinburne does not reject this way of reconciling an essentially omniscient's incorrigible foreknowledge, with the freedom of an action foreknown. He does not consider it. (See Swinburne 1998, pp. 131–4.) He cannot block it on the ground that God would necessarily exist, for he does not believe that about God.
46. This is according to standard translations, though not the best translations (see note 1 in Chapter I).

Chapter XIII

* Revised from Sobel (1996).

1. "Unless," one must add, "beliefs in the case at hand can be 'properly basic', for then, by definition of 'proper', it is reasonable to believe despite the absence, by definition of 'basic', of intellectual reasons." I bracket and do not go into definitions of' 'properly basic' beliefs and "whether belief in God can be properly basic for intellectually sophisticated modern adults" (Rowe 1993, p. 170), who are aware of the variety of religious beliefs and the to and fro arguments concerning the existence of gods variously conceived.

2. Pascal (1962, p. 136). Ian Hacking provides the following bibliographic information: "Blaise Pascal, *Pensées sur la religion et sur quelque autres sujets*, the first edition, *édition de Port-Royal*, was in 1670. The standard text is that of Louis Lafuma (Paris, 1951); or *Oeurvres Complètes*, ed. by Lafuma (Paris, 1963). Pascal's wager is the fragment headed *Infinirien*, which is no. 418 of Lafuma. Many older editions and translations use Leon Brunschvicg's numeration, where *Infinirien* is 233." (Hacking, p. 186). The passage is 343 in Martin Turnell's translation. It is 223 in H. F. Stewart's translation.
3. This just *might* make 'me' a fictional character that differs from every real person living or dead. Chances can get *very* small, and there are for most people imaginable possible goods of enormous value. I wonder whether *anyone* would in fact, on reflection, accept for nearly sure every imaginable sacrifice of things they value, not just for himself, but for the world, for a nearly no chance for eternal bliss, for himself. That is, I think, the wonder relevant to whether personal eternal bliss has ever been for anyone of *infinite* value. Has there ever been anyone that selfish? It is more plausible that personal eternal bliss has been for a person 'of infinite value' in relation to every imaginable purely worldly personal good, but still I think not very plausible. No matter how bad things get in a life, they could always get worse. "But eternal bliss *is* of infinite value, so that a chance for it would be worth the certainty of every pain." The question in the present context is not is it a purported 'objective value' for a person, but persons's 'subjective values' for it, as reflected in their responses to possible gambles. Those who have been prepared to suffer enormously for their chances for eternal bliss have, I think, all believed their chances for it to be not so bad.
4. See *La logique, ou l'art de penser*, Paris, 1662 (the work of Antoine Arnauld, Pierre Nicole, and possibly others such as Pascal associated with Port Royal des Champs), Partie IV, Chapitre XV, p. 384. Chapter XV of the first edition is chapter XVI in the fourth edition of 1674 and in subsequent editions. In every edition it is the last chapter of the book. The book "was constantly modified, augmented, and rewritten by its authors; by 1685 six editions . . . had appeared" (Steven Nadler in *The Cambridge Dictionary of Philosophy*, 1995, p. 632).
5. A *probability-partition* is a set of propositions so related that it is certain that exactly one of its members is true. There is, for present purposes, a further condition on the partition of circumstances C. It needs to be a *practical partition for option a*: "[T]he agent [must] be sure that there is exactly one c in C such that [were the agent to do a, c would obtain]" (Sobel 1989, p. 76). Practical partitions will not be mentioned outside of this footnote. It can be seen that a set of propositions can be a probability-partition without being a practical partition for an option, and vice vera. Being a practical partition is enough for the theorem coming up. Other partition-theorems call for probability-partitions. If circumstances in a probability-partition are certainly independent of an option, this partition is a practical partition for this option. Practical partitions will not be mentioned again in this chapter.
6. A partition C is 'sufficiently exclusive' if "for any distinct c′ and c″ in C, the agent is sure that not both (a & c′) and (a & c″) are open" (Sobel 1989, p. 77). In what follows, 'sufficient' when applied to partitions or divisions of circumstances is always short for 'sufficiently exclusive'.

7. This (subject to footnote 5) is Theorem 1 of Sobel (1989, p. 78). My general *definition*, for purposes of this chapter, of the expected value of a proposition (for example, a conjunction of an option and circumstance) makes it a probability-weighted average of values of worlds at which this proposition is true. For any proposition p the probability of the entertainability of which for subjunctive speculation is positive, the expected value of p is,

$$\sum_{\text{w a p-world}} \mathrm{P}(\mathrm{p}\Box\!\rightarrow \mathrm{w}) \cdot \mathrm{V}(\mathrm{w}).$$

This is a definition "for the special case of agents who do not believe in objective chances" (Sobel 1989, p. 71), that is, for agents for whom, for any proposition entertainable for purposes of subjunctive speculation p, and proposition q, either if it were the case that p, it would be the case that q, or if it were the case that p, it would be the case that ∼q. For agents who give some credence in some cases to 'mights' and to 'objective chances' between zero and one, I favor a definition that has in place of $\mathrm{P}(\mathrm{p}\,\Box\!\rightarrow \mathrm{w})$, a term for the probability-weighted objective chance for w on p. That the chance for w on p is x can be symbolized by either $(\mathrm{p}\,\Diamond_{\mathrm{x}} \rightarrow \mathrm{w})$ or $\mathrm{Ch}(\mathrm{w/p}) = \mathrm{x})$. Using the latter symbolization, the probability-weighted objective chance for w on p is,

$$\sum_{0\leq \mathrm{x}\leq 1} \mathrm{P}[\mathrm{Ch}(\mathrm{w/p}) = \mathrm{x}].$$

This more general, and complicated, approach to expected value is assayed in Sobel (1986).

8. * Hyperreal numbers can be thought of as including all standard real numbers along with many more numbers. Some additional numbers are at once positive and greater than zero and yet smaller than every standard positive real number. These are the *infinitesimals* of hyperreal number theory. Some other additional numbers are larger than every positive real number, and still others are smaller than every negative real number. These are the *infinimals*, the positive and negative infinities of hyperreal number theory. Relations of note between infinitesimals and infinimals include that if i is an infinitesimal, then 1/i is an infinimal, and if *I* is a positive infinimal, then 1/*I* is an infinitesimal. It is a most important fact about hyperreal numbers, including the infinitesimals and infinimals amongst them, that *they are governed by the same mathematical principles that govern standard real numbers*.

For completeness I observe that mathematical usage features what can seem to be a third kind of infinity: '∞' and '−∞' are sometimes used for infinite sums, when there is not a finite limit for the series of subsums: for example,

$$\sum_{\textit{natural number } n\,>\,o} n = \infty.$$

It is doubtful that '∞' in this use stands for some one transfinite Cantorian cardinal. If it did it would presumably stand for $\aleph_0$. But there are very similar infinite sums none of whose subsums are whole cardinal numbers, and it would be odd to identify their sums with a transfinite cardinal. It is also doubtful that ∞ in this use stands for some one infinimal. I think that in this limit-use '∞' does not name a determinate number of any kind, and that, for example, $\sum_{\textit{natural number } n\,>\,o} \mathrm{n} = \infty$. says not that the indicated sum has some particular

numerical value, but rather only that it exceeds every, and thus does not have any, finite value. Koons (1993) contains a valuable application of hyperreal values and probabilities to ideas of Kierkegaard.

9. * There is not a smallest Robinsonian infinimal. Indeed, Elias Zakon shows that for every positive infinimal integer I (Zakon writes of 'natural infinimals'), there are at least c (the cardinality of the set of standard real numbers) smaller infinimal integers (Zakon 1969, p. 201). Could the greatness of the value 'I' am supposed to attach to eternal bliss be measured in hyperreal decision theory by a positive infinimal? This depends. The answer is no, if 'I' am supposed to prefer absolutely *every* chance of eternal bliss, even to a certainty of every worldly loss. If the worldly loss is measured by finite hyperreal f and eternal bliss measured by infinimal I, then 'I' must be indifferent between the *infinitesimal* chance f/I for eternal bliss and the certainty of this loss: As for standard reals, so for hyperreals finite and infinite, x = (x/y)y. The answer is yes, if 'I' am supposed to prefer only every *noninfinitesimal* chance of eternal bliss to even a certainty of every worldly loss. If the worldly loss is measured by finite hyperreal f and eternal bliss measured by infinimal I, then, for every finite hyperreal integer h no matter how great, and regardless of the greatness of f, 'I' must prefer the chance $1/h$ for eternal bliss to the certainty of this loss: It is true of any such hyperreals that $(1/h)I > f$, since $(1/h)$I, though less than I, is still an infinimal.

   While limited incommensurabilities – for example, incommensurabilities relative to all finite chances – of some possible gains to some possible losses are representable in hyperreal decision theory, extreme incommensurabilities – incommensurabilities relative to all chances including infinitesimal ones – are not. This is not a regrettable limitation of hyperreal decision theory if, as I think, such extreme incommensurabilities of objects of preference make no more sense than do intransitive preferences. Decision theories, as theories of rational choice, aspire to representations *only of systems of preferences that make a certain kind sense* for which transitivity of preference and at least limited commensurabilities of objects of preference are, I think, proper conditions. [These remarks concerning transitivity pertain to preferences properly so-called: what I term 'preferences *tout court*' and contrast with 'pairwise preferences' in Sobel (1997b).]

10. This is the probability of a conditional, a subjunctive conditional, not a conditional probability. It is P[Bel(G) $\square\!\!\rightarrow$ G], not P[G/Bel(G)].

11. * The case is in each of these connections different for hyperreal infinities. For any positive 'natural hyperreal' n and positive infinimal I, $I + n$ is a greater infinimal. And for any positive 'real hyperreal' r less than hyperreal 1, and positive infinimal I, $r \cdot I < I$. While "[t]he finite is annihilated in the presence of the [Cantorian] infinite" (Pascal 1962, p. 132), finite additions and subtractions, and multiplications and divisions, though they always yield infinimals, yield different infinimals. Such finite operations make exactly the differences to infinimals that one might naively expect them to make to any numbers, but which they do not make to Cantorian infinities.

12. I have for several lines been denying what Hacking asserts, namely: "If one act [weakly] dominates the rest, then it will [no matter what the probabilities] be recommended by" an expectation argument (Hacking 1972, p. 187).

13. * What chance is that? *In Cantorian terms* it is either 'no chance' or zero. If the infinity is $\aleph_0$ (the number of natural numbers), then either $1/\aleph_0$ is *not defined* or it is defined as the limit of 1/n as finite n increases without limit – $\lim_{n \to \infty} 1/n$ – which is 0. *In Robinsonian terms*, taking some liberty with James's words, 'there is one in an infinite number of chances' can be understood as a nice way of saying that there is an "infinitesimal chance" (cf., McClennen 1994, p. 120), or equivalently that there is the $1/I$ chance, where I is a Robinsonian infinimal. Cf.: "*Infinitesimal* came to mean unity divided by infinity, $(1/\infty)$" (*The Compact Edition of the Oxford English Dictionary*, 1971, volume I, p. 1429). Divisions of positive numbers by infinimals are defined, and are never zero, but always infinitesimals (hyperreals greater than zero and less than all hyperreal reals). For any infinimal I, $I \cdot (1/I) = 1$, and $1/I$ can, furthermore, be said to be precisely the '*Ith* part of 1'. Regarding the *liberty* mentioned, the view that "there [are] an infinity of chances" (James) cannot be that there are *an infinimal of chances*, since infinimals are not 'set-sizing' numbers. They are numbers for 'magnitudes' with which to answer 'how much?', but not for 'quantities' with which to answer 'how many?'.

    If someone is sure that there is a chance, an 'objective chance', no matter how small for God's existence, then, by David Lewis's Principal Principle that relates subjective probabilities to views of objective chances, this person, if consistent, has that much confidence in God's existence. According to this principle, the probability of p conditional on its objective chance being x is x: $Prob[p/(ch(p) = x)] = x$. The Principal Principle argues that if 'there were an infinity of chances, and only one for God' means that the speaker's probability for God is positive and smaller than every finite n, that is, that it is infinitesimal.
14. Values in these matrices are *expected values* of option–circumstances conjunctions. In the case, conjunctions that include either C1 or C3 as conjuncts have zero probability. This would be a problem if I defined expected values in terms of standard conditional probabilities, but I define them in terms of probabilities of conditionals and require not that bearers of expected values are positively probable, but only that there 'entertainabilities' are positively probable. (See footnote 7.)
15. T. H. Huxley says "that it is wrong for a man to say that he is certain of the objective truth of any proposition unless he can produce evidence which logically justifies that certainty" (Huxley 1904, p. 310). That is wrong, I would say, because it is misleading, presumably deliberately so, since when a person says that he is certain he implies that he can produce 'justifying evidence'. Nothing similar attaches to his merely *being* certain, or to his saying that he is certain with the disclaimer that he cannot produce 'justifying evidence'.
16. * Cf.: "[A] gamble is advantageous on the basis of [the] expected-value standard whenever:

$$\frac{\text{chance of winning}}{\text{chance of losing}} > \frac{\text{cost of stake}}{\text{size of prize}} = \frac{\text{potential loss}}{\text{potential gain}}.$$

    And if the potential gain is infinite, this standard favors the gamble as long as the chance of winning is nonzero." (Rescher, pp. 16–17; bold emphasis added).

Not necessarily, I say, if we countenance infinitesimal positive chances and positive infinimal gains, even when, as Rescher in the context is assuming, potential losses are finite. Suppose the potential loss is 1; the potential gain is, for some infinitesimal i, the positive infinimal 1/i; and the chance of winning is infinitesimal i/2). Then, contrary to Rescher's stated principle, it is *not* the case that

$$\frac{\text{i}/2}{1-\text{i}/2} \quad > \quad \frac{1}{1/\text{i}}.$$

For, since everything true of standard reals is true of the hyperreals, this inequality entails $\frac{1}{2} > (1 - \text{i}/2)$, which is patently false: i/2 is an infinitesimal, so that 1 - i/2 is *very* close to 1.

17. * Case 3 is not so simple in nonstandard hyperreal arithmetic. Infinitesimal chances for infinimal rewards need not be worth every finite price. If, for example, for infinimals I and I', Prob(G) = $1/I$, and I' is the great value that Bel(G) in conjunction with G has for 'me', then

$$ExVal[\text{Bel}(\text{G})] = (1/I) \cdot I' + (1 - 1/I)(-1)$$

is greater than

$$ExVal[\text{~~Bel~~}(G)] = 0,$$

if and only if

$$I' > I - 1.$$

That is, an infinitesimal $1/I$ chance for the infinimal reward I' is not worth the price –1, if $I' < I - 1$.

18. Possible reward policies in a related case could be bliss for believers who believe without, or against, intellectual reasons, and none for anyone else. Whether or not attaining and sustaining belief in the requisite circumstances in this case would (in Pascal's words) make you stupid, would depend on which of the two circumstances worked for a person.

19. *A hyperreal analysis in which ∞ was a positive infinimal would make relevant the relative magnitudes of *Prob*(G & ReBel) and *Prob*(G & ReRat). Then *ExVal*[Bel(G)] would equal *ExVal*[~~Bel~~(G)] if and only if *Prob*(G & ReBel) equalled $[Prob(\text{G} \,\&\, \text{ReRat}) \cdot \infty + 1]/(\infty + 1)$.

The calculation of this result assumes that still, in a hyperreal analysis, *ExVal*[Bel(G) & (G & ReBel)] would equal *ExVal*[~~*Bel*~~(G) & (G & ReRat)]. The general idea for the calculation that results (last sentence previous paragraph) can, of course, be related to more plausible numbers for these *ExVals*, for example $(\infty - 1)$ and ∞, respectively. Calculations for more plausible numbers would (no surprise this!) yield different results. (Thanks to Hermann Weidemann for comments that pointed to the substance of this paragraph.)

20. *George Schlesinger assumes standard Cantorian infinities and mounts this argument for his tie-breaking principle. Roy Sorensen, commenting on a formulation of that principle published by Schlesinger in 1988 without supporting argument, says that "even if . . . true, it needs to be shown true" (Sorensen 1994, p. 142). Relevant to Sorensen's demand is the present availability not only of Schlesinger's supporting argument, but of a better argument. It is a

theorem, a trivial theorem, of hyperreal decision theories that if each of two options makes noninfinitesimally probable to different degrees the same infinimal gain, and all other possible gains and losses are finite, then the option that makes more probable that infinimal gain is of *greater infinimal expected value* than that other option. Even if one does decision theory with standard reals and Cantorian infinities, the fact of this theorem of hyperreal decision theory should be reason enough for Schlesinger's tie-breaking principle.

21. Using Cantorian infinities to represent more and less attractive afterlives promised by different gods makes the relative probabilities of these gods irrelevant as long as they are all greater than zero. In contrast, using Robinsonian infinimals for values and hyperreals (including infinitesimals) would give point not to the qualification "other things being equal" (Lycan and Schlesinger, p. 85), but at least to the less demanding qualification that other things be not extremely unequal. Robinsonian infinimals are all commensurable: For any infinimals I and I', if $I > I'$, there is the hyperreal $h = I'/I$ such that $h \cdot I = I'$ and $h < I'$. Not so for Cantorian infinities: For Cantorian infinities ∞ and ∞', $\infty > \infty'$, if $x \cdot \infty = \infty'$, then $x = \infty'$.
22. These policies would be in a way less demanding, and His punishment policy would be in one dimension less draconian than that of the god of *Exodus*, wherein Moses quotes God as saying: "Thou shalt not bow down thyself to them [other gods, and graven images of things in the heaven, earth, or water]: for I the Lord thy God am a jealous God, visiting the iniquity of the fathers upon the children unto the third and fourth generation of them that hate me; and shewing mercy unto thousands of them that love me, and keep my commandments" (*Exodus* 20:4–6).
23. * In contrast, sums involving positive and negative infinimals are defined and well-behaved. For example, for any infinimal I, $I + (-I) + -Prob(\sim G) = -Prob(\sim G)$, and infinimals have negative inverses. For any infinimal I, $-I$ is the infinimal such that $I + -I = 0$. William Martin (1990) claims to refute Pascal's Wager with the observation that for every hypothesis that there is a God who rewards believers there is a hypothesis that there is a God who punishes believers and rewards nonbelievers. This hypothesis, since logically possible, should, he maintains, be assigned some probability (cf., p. 234). The result, Martin thinks, is that "infinite expected values cancel each other out" (p. 232). This would-be refutation of Pascal's Wager involves two moves. First, Martin assumes that for any probability p and infinity ∞, $p \cdot \infty = \infty$. Cantorian infinities, but not Robinsonian infinimals, annihilate finitudes in this manner. Second, Martin assumes that $\infty + -\infty = 0$. Robinsonian infinimals, but not Cantorian infinities, behave like that. Martin also assumes (p. 233) that $-\infty + -\infty + \infty = 0$. Neither Cantorian nor Robinsonian large numbers behave like that; there are no prospects for intelligible infinities that do.
24. *Mark* 8:36 and *Matthew* 16:26 as in the Cambridge edition of The Holy Bible translated and revised for King James 1611. The Revised Version of 1881 has 'life' in place of 'soul', as does the Revised Standard Version of 1952 and the New Revised Standard Version of 1989.
25. James quotes from Clifford, with an indicated omission, lines that contain this "degradation" statement. It is this strong statement itself that James leaves out (James 1956, p. 8).

26. 'I' am led by reflection on the present case to think that exclusive maximization and strong ratifiability is *not* sufficient for pragmatic reasonableness. It seems that strong ratifiability may be merely one kind of decision stability, and that only exclusive maximization and strong decision-stability of *every* kind is sufficient for pragmatic reasonableness.
27. *Duff maintains that, for almost everyone, the infinite desirability of belief in a Christian God could not provide a pragmatic justification for Christian praxis as a way to belief. His argument is that Christian praxis is not the only life, the only course of action, which, if you are reasonable in your views, makes the establishment of that belief somewhat probable. Indeed, if you are at all reasonable in your views and probabilities, every way of life leaves the establishment of that belief at least somewhat probable: "No course of action can make it absolutely certain that I will not come to believe in God: therefore, every course of action has an infinite expected value [for me if I believe that God would reward believers with goods of infinite value]. . . . I have, therefore [supposing I believe that] no reason to try to increase the probability that I will come to believe. . . . [H]owever I live, whatever I do, the expected value of my actions is infinite! [Supposing I place no credence in possibilities of infinitely bad consequences]" (Duff 1986, p. 108). It is obvious that Duff's entertainment depends on the infinity of eternal bliss *not* being a Robinsonian infinity and so needs to be taken seriously only if there are good reasons for eschewing such infinities in decision theory, which, I think, there are not.
28. *A Wager of Another Kind*. Suppose a Being in whose power to predict your choices you have enormous confidence. Suppose that you are not merely almost certain, but absolutely certain, that this Being's prediction about your choice in the situation to be discussed are correct. There are two lives you can live. This Being – your Creator – has made you a free agent. He has given you the power to choose whether to live piously and correctly according to His wishes or to indulge your fancies, yield to temptation and, prudently as regards this-world consequences, have much more fun. *And* He has decided what will be your prospects after death. Eternal bliss has been arranged for you if he predicted that you will exercise your freedom according to his wishes – otherwise nothing: *infini – rien*. All that is settled. There is nothing you can do to affect your eternal prospects. He not only will not alter the arrangements he has put in place for you after your death. No matter what – no matter how you were to live – he would not alter these arrangements. However, while there is nothing you can do about your objective prospects, you *can* affect your subjective expectations for them. Your life, though you are sure that it cannot be a cause of the Being's grace, is for you, given your theology, a possible sign of it. What do you do? What life do you choose? Does it matter to this question how much longer you expect to live in hope or despair?

    This 'wager' may be compared with Newcomb's Problem. "Suppose a being in whose power to predict your choices you have enormous confidence. . . . You know that this being has often correctly predicted your choices in the past (and has never, so far as you know, made an in correct prediction about your choice), and furthermore you know that this being has often correctly predicted the choices of other people, many of whom are similar to you, in the particular

situation to be described below. One might tell a longer story, but all this leads you to believe that almost certainly this being's prediction about your choice in the situation to be discussed will be correct.

"There are two boxes, (B1) and (B2). (B1) contains $1000. (B2) contains either $1000000 ($M) or nothing. What the content of (B2) depends upon will be described in a moment. . . . You have a choice between two actions: (1) taking what is in both boxes; (2) taking only what is in the second box. Furthermore, and you know this, the being knows that you know this, and so on: (I) If the being predicts you will take what is in both boxes, he does not put the $M in the second box. [And] (II) If the being predicts you will take only what is in the second box, he does put the $M in the second box. The situation is as follows. First the being makes its prediction. Then it puts the $M in the second box, or does not, depending upon what it has predicted. Then you make your choice. What do you do?" (Nozick 1969, pp. 114–15. There are discussions of Newcomb's Problem in Sobel (1994, 1998b). There is discussion of infallible predictors in Sobel (1988), which is revised in Sobel (1994).

29. I thank Willa Fowler Freeman Sobel for many discussions of the ideas of James and Pascal, and much more. I have also benefited from comments of anonymous readers, and from an undergraduate senior seminar session on infinities that featured a report by Sammy Jakubowicz and useful participation by William Seager.
30. Rudy Rucker writes that one reason why "Conway's surreal numbers . . . have not attained any wide usage among more practically minded mathematicians . . . could be that it is hard to define the higher-order operations (such as exponentiation and tetration) . . . on [them]. Instead of using the surreal numbers, those mathematicians who need infinitesimals use a smaller, and somewhat different, extension of the reals, the so-called *hyperreal numbers* . . . introduced by Abraham Robinson. . . . Robinson's hyperreal numbers are best thought of as sequences of reals. . . . This is convenient, since all of the operations on the ordinary reals can be carried over in a 'pointwise' fashion to . . . hyperreal numbers [so thought of or constructed]" (Rucker 1984, pp. 84–5).

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